

```
ClearAll;
```

```
Action = (((r * Sin[θ[r]]) ^ 2) * ((1 + (r ^ 2)) ^ (-1) + ((r * (θ' [r])) ^ 2))) ^ (1 / 2)
```

$$\sqrt{r^2 \sin[\theta[r]]^2 \left(\frac{1}{1 + r^2} + r^2 \theta'[r]^2 \right)}$$

```
RHSr = FullSimplify[D[Action, θ[r]]]; (*Compute Euler Lagrange Equations*)
```

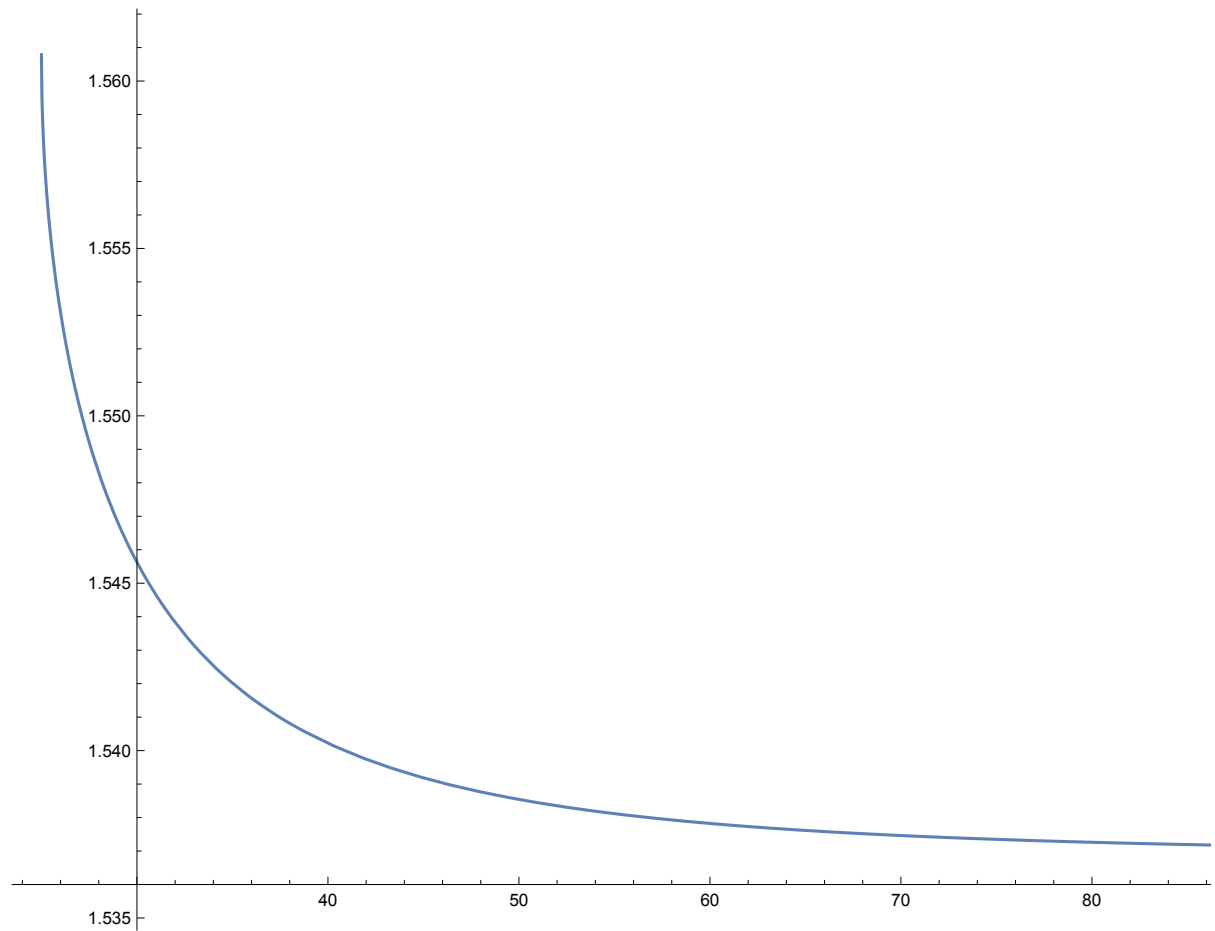
```
LHSr = FullSimplify[Dt[D[Action, θ' [r]], r]];
```

```
r0j = 25;
```

```
Jointed =
```

```
NDSolve[{RHSr == LHSr, θ[r0j] == Pi / 2 - 0.01, θ' [r0j] == -80}, θ, {r, r0j, 100}];
```

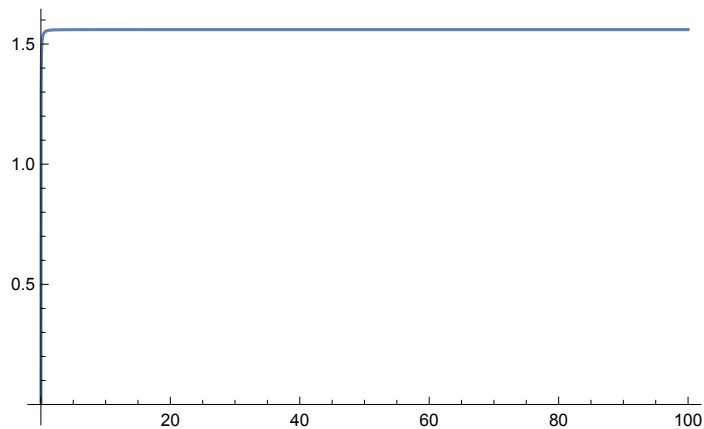
```
Plot[Evaluate[θ[r] /. Jointed], {r, r0j, 100}, PlotRange -> All]
```



```

r0d = 0.01;
DisJointed =
  NDSolve[{RHSr == LHSr,  $\theta$ [r0d] == 0.01,  $\theta'$ [r0d] == 80},  $\theta$ , {r, r0d, 100}];
  (*Solve for the solution which does cross from one side at  $\theta=0$  r =r0 *)
  (*at small r0 expect this to be dominant solution*)
Plot[Evaluate[ $\theta$ [r] /. DisJointed], {r, r0d, 100}, PlotRange -> All]

```



```

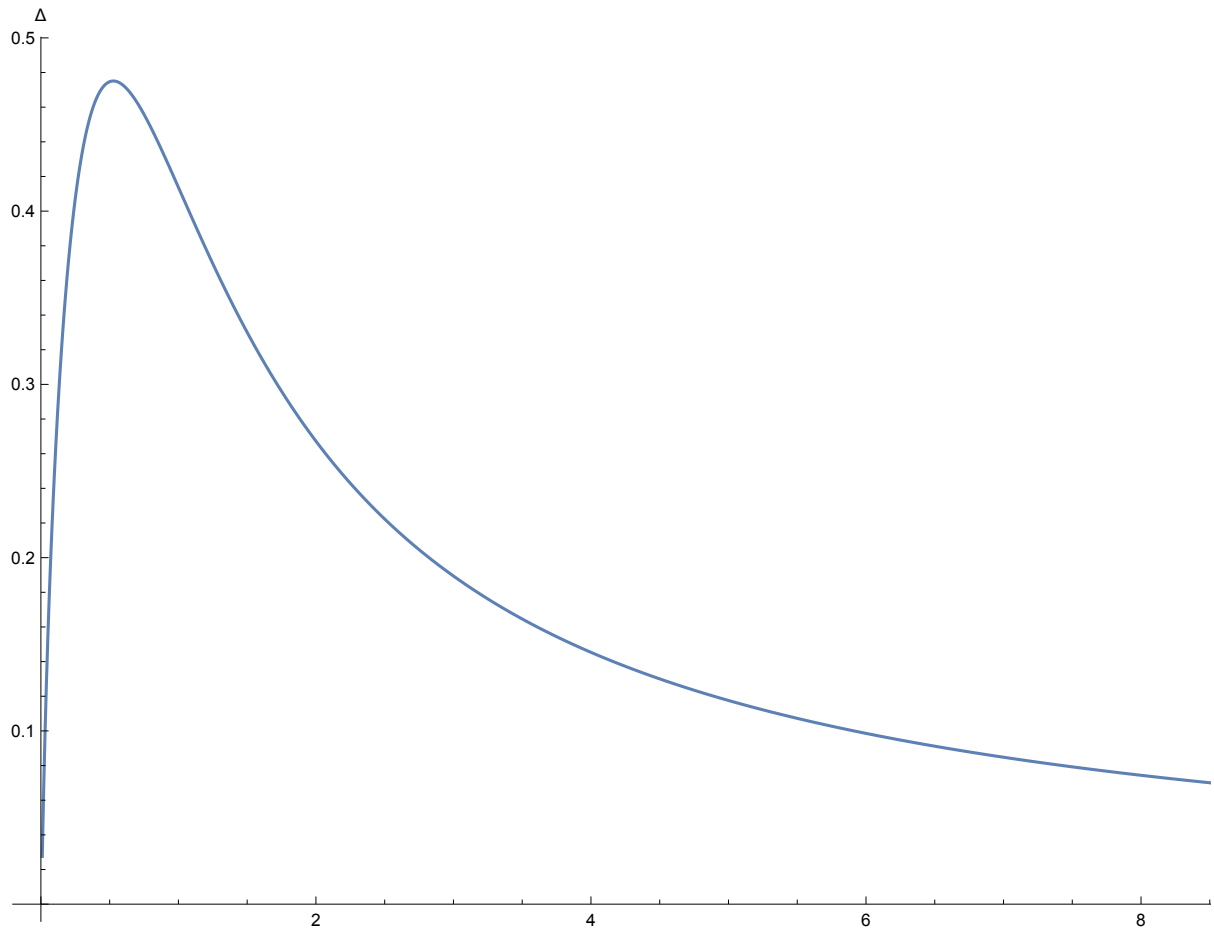
beg = 0.01;
end = 10.0;
step = 0.01;

```

```

JointedTab = Flatten[
  Table[{r0, (Pi / 2 -  $\theta$ [100])} /. NDSolve[{RHSr == LHSr,  $\theta$ [r0] == Pi / 2 - 0.0001,
     $\theta'$ [r0] == -80},  $\theta$ , {r, r0, 1000}], {r0, beg, end, step}], 1];
ListLinePlot[JointedTab, PlotRange → All, AxesLabel → {r0,  $\Delta$ }]

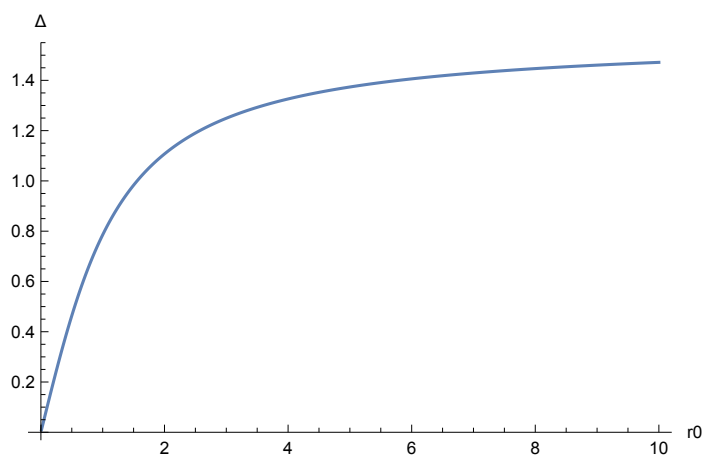
```



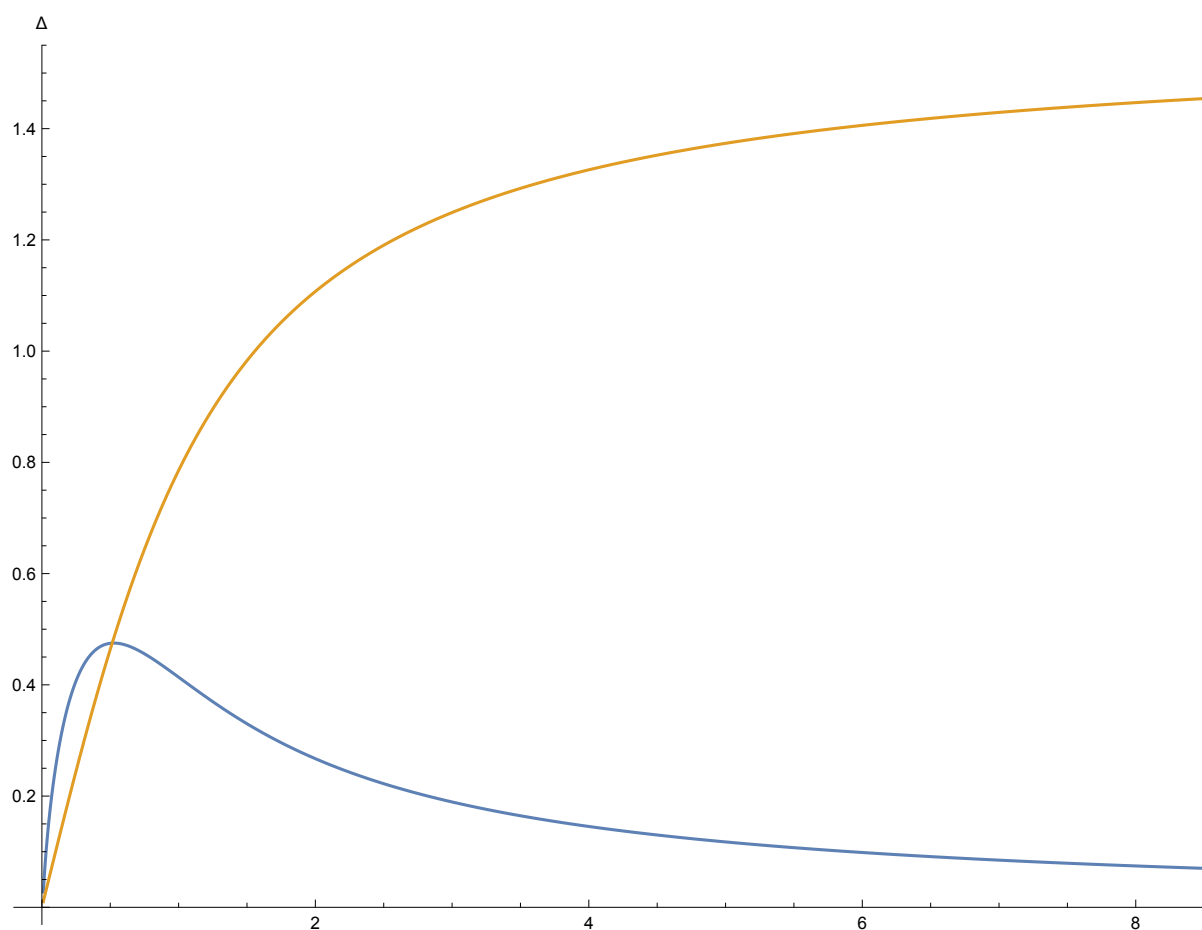
```

DisJointedTab = Flatten[
  Table[{r0, Pi / 2 -  $\theta$ [100]} /. NDSolve[{RHSr == LHSr,  $\theta$ [r0] == 0.001,  $\theta'$ [r0] == 80},
     $\theta$ , {r, r0, 1000}], {r0, beg, end, step}], 1];
ListLinePlot[DisJointedTab, PlotRange → All, AxesLabel → {r0,  $\Delta$ }]

```



```
ListLinePlot[{JointedTab, DisJointedTab}, PlotRange -> All, AxesLabel -> {r0, Δ}]
```

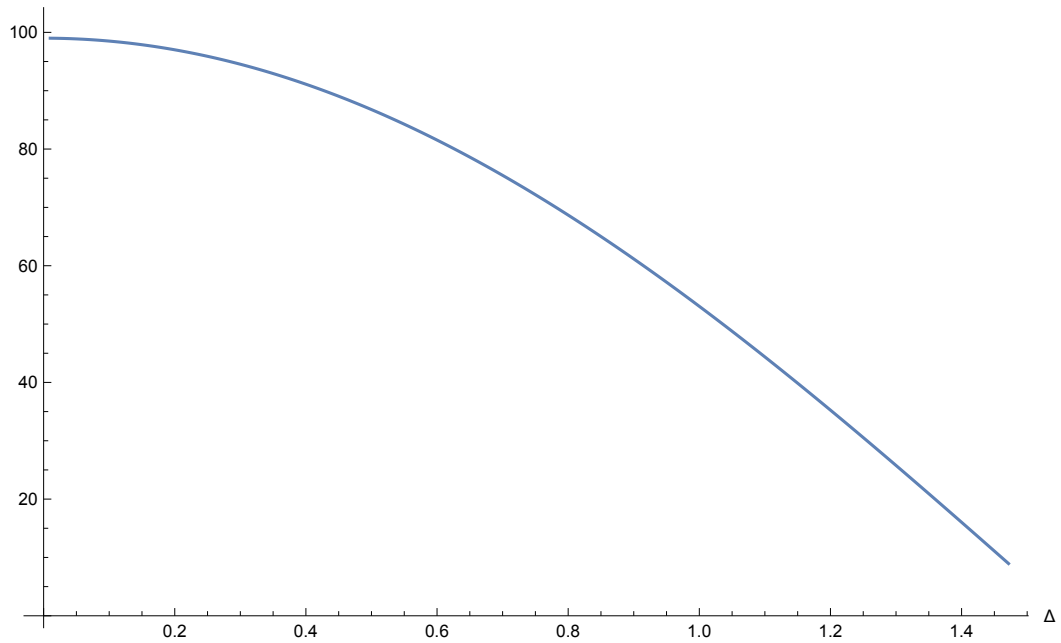


```

tabDeltaDisjoined =
  Partition[Flatten[Table[{(Pi / 2 -  $\theta$ [100]) /. NDSolve[{RHSr == LHSr,
     $\theta$ [r0] == 0.001,  $\theta'$ [r0] == 80},  $\theta$ , {r, r0, 100}], NIntegrate[Action /.
    NDSolve[{RHSr == LHSr,  $\theta$ [r0] == 0.001,  $\theta'$ [r0] == 80},  $\theta$ , {r, r0, 100}],
    {r, r0, 100}]}], {r0, beg, end, step}], 2], 2];
ListLinePlot[tabDeltaDisjoined, PlotRange -> All,
  AxesLabel -> {" $\Delta$ ", "ActionDisJointed"}]

```

ActionDisJointed

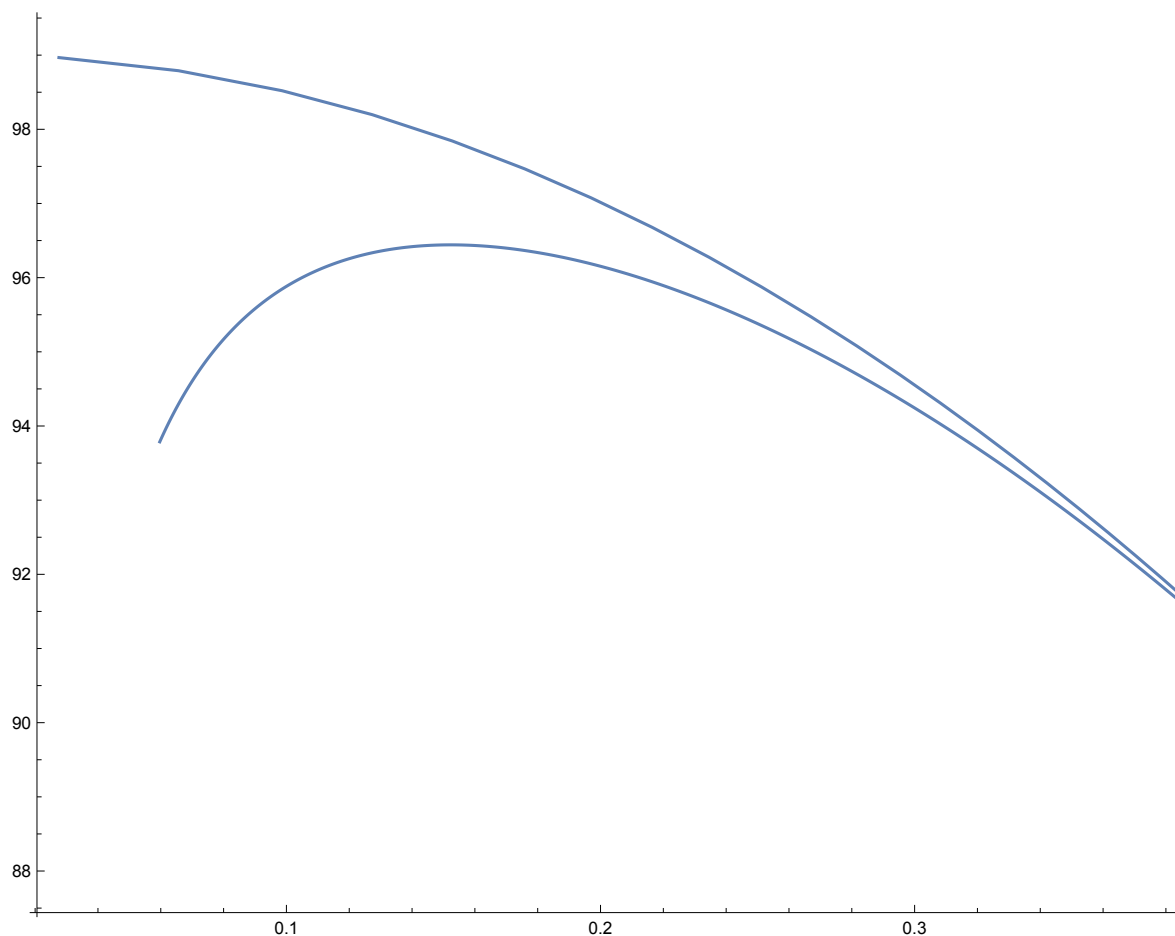


```

tabDeltaactjoined = Partition[
  Flatten[Table[{(Pi / 2 -  $\theta$ [100]) /. NDSolve[{RHSr == LHSr,  $\theta$ [r0] == Pi / 2 - 0.0001,
     $\theta'$ [r0] == -80},  $\theta$ , {r, r0, 100}], NIntegrate[
    Action /. NDSolve[{RHSr == LHSr,  $\theta$ [r0] == Pi / 2 - 0.0001,  $\theta'$ [r0] == -80},
     $\theta$ , {r, r0, 100}], {r, r0, 100}]], {r0, beg, end, step}], 2], 2];
ListLinePlot[tabDeltaactjoined, PlotRange -> All, AxesLabel -> {" $\Delta$ ", "ActionJoined"}]

```

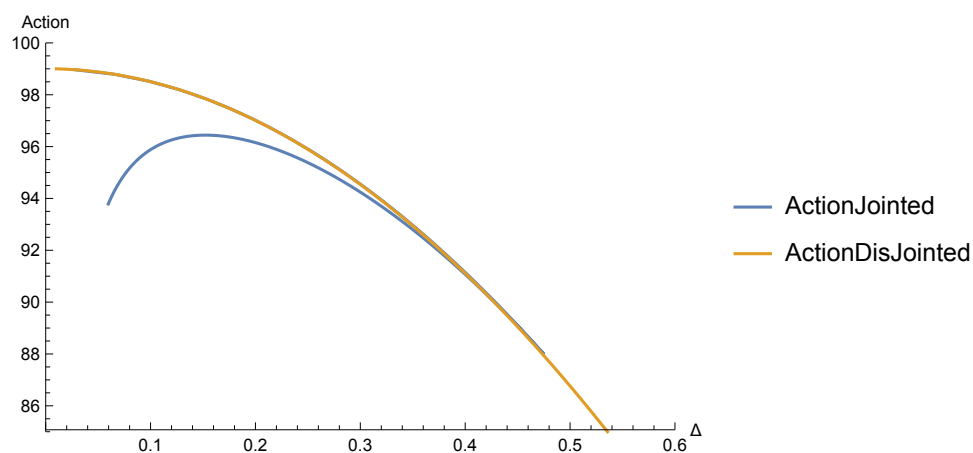
ActionJoined



```

ListLinePlot[{tabDeltaactjoined, tabDeltaactdisjoined},
  PlotLegends -> {"ActionJoined", "ActionDisJoined"},
  AxesLabel -> {" $\Delta$ ", "Action"}, PlotRange -> {{0, 0.6}, {85, 100}}]

```



```
ListLinePlot[{tab $\Delta$ actjoined, tab $\Delta$ actdisjoined},
  PlotLegends  $\rightarrow$  {"ActionJointed", "ActionDisJointed"},
  AxesLabel  $\rightarrow$  {" $\Delta$ ", "Action"}, PlotRange  $\rightarrow$  {{0.35, 0.5}, {87, 93}}]
```

