Over the past decades there has been huge interest in the study of supersymmetric quantum field theories and in particular the computation of exact results. This study is primarily carried out with the hope that a understanding supersymmetric theories will ultimately lead us towards a better understanding of non-supersymmetric theories, such as the standard model of particle physics. This is also of great interest from a mathematical point of view because such study may also lead us towards a mathematically rigorous definition of quantum field theories, similar to that achieved for conformal field theories in two dimensions.

Much of the recent study has been dedicated to theories with $\mathcal{N} = 2$ supersymmetry. In this thesis we have initiated a program towards the study and computation of exact results for certain $\mathcal{N} = 1$ theories, namely those said to be of class $\mathcal{S}_k$. Within this class lies theories which are of the type of $\mathcal{N} = 1$ SQCD; namely $\mathcal{N} = 1$ vector multiplets coupled to chiral matter in fundamental representations. $\mathcal{N} = 1$ SQCD displays many of the same interesting phenomena as non-supersymmetric QCD, such as confinement. However, class $\mathcal{S}_k$ presents a framework for exploiting new dualities, due to the string and M-theory constructions.

In Chapter **??** we have demonstrated that $\mathcal{N} = 1$ analogues of Higgs and Coulomb branches may be defined for theories of class $\mathcal{S}_k$. We have computed the Hilbert series for them and described many of their properties. We have also defined and discussed many interesting limits of the superconformal index.

In Chapter **??** we have computed the partition function of instantons for a certian subset of class $\mathcal{S}_k$ theories. Our work relied on the correspondence between instantons and D(-1)-branes we describe the ADHM construction for these theories as a matrix model. This partition function is then identified with conformal blocks of the $\mathcal{W}_{kN}$ algebra [**?**], pointing towards the possible existence of further 2d/4d relations for class $\mathcal{S}_k$ theories.

In Chapter **??** we change our attention to four dimensional $\mathcal{N} = 3$ theories. This is motivated by the recent discovery [**?**] of genuine $\mathcal{N} = 3$ supersymmetric theories engineered using a special quotient geometry within F-theory dubbed S-fold. The existence of yet more new $\mathcal{N} = 3$ theories was conjectured in [**?**], by means of gauging a discrete symmetry of $\mathcal{N} = 4$ that emerges at strong coupling. These constructions bypasses the no-go theorems stating that every CPT-complete $\mathcal{N} = 3$ theory automatically enhances to $\mathcal{N} = 4$ supersymmetry; by means of the theories having no Lagrangian description. We focus out attention on the theories constructed as in [**?**] and their higher rank generalisations. We review the details regarding both the construction of these theories as well as their various universal properties. We then focus on the computation of two type of exact results for these theories; being the Coulomb limit of the supersymmetric index and the Hilbert series for the Higgs branch. These quantities encode several important features of the moduli space of supersymmetric vacua for these theories. In certain special cases we are also able to compute the fully refined supersymmetric index.

Chapter **??** is dedicated to $\frac{1}{2}$-BPS defects in six dimensional $\mathcal{N} = (1, 0)$ SCFTs. The SCFTs that we focus on, when compactified on a punctured Riemann surface

$\mathcal{C}$, are precisely those which give rise to theories of class $\mathcal{S}_k$, namely the $(1,0)_{A_{k-1}}$ theories of type $\mathfrak{g} = A_{N-1}$. The $\frac{1}{2}$-BPS defects are amongst those which may be inserted at the punctures to engineer various $\mathcal{S}_k$ theories. In this chapter we focus on the self-dual (tensionless) strings of the 6d theories in the presence of the defect. In the class $\mathcal{S}_k$ picture wrapping these strings on $\mathcal{C}$ gives rise to point-like BPS states in the 4d theory. The strings admit a dual effective 2d-gauge theory description living on their worldvolume. We describe this gauge theory and compute its Elliptic genus. These strings provide the main contribution to the $T^2 \times \mathbb{R}^4$ BPS-partition function for the 6d theory and it can be written in a expansion over Elliptic genera. We also perform a similar computation for the supersymmetric index of 5d $\mathcal{N} = 1$ theories in the presence of defects.