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How Efficient are Decentralized Auction Platforms?

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We model a decentralized, dynamic auction market platform in which a continuum of buyers and sellers participate in simultaneous, single-unit auctions each period. Our model accounts for the endogenous entry of agents and the impact of intertemporal optimization on bids. We estimate the structural primitives of our model using Kindle sales on eBay. We find that just over one-third of Kindle auctions on eBay result in an inefficient allocation with deadweight loss amounting to 14% of total possible market surplus. We also find that partial centralization—for example, running half as many 2-unit, uniform-price auctions each day—would eliminate a large fraction of the inefficiency, but yield lower seller revenues. Our results also highlight the importance of understanding platform composition effects—selection of agents into the market—in assessing the implications of market redesign. We also prove that the equilibrium of our model with a continuum of buyers and sellers is an approximate equilibrium of the analogous model with a finite number of agents.

Key words: Dynamic auctions, Approximate equilibrium, Internet markets

JEL Codes: C73, D4, L1

1. INTRODUCTION

Online market platforms are increasingly important in today's economy. For example, in 2014 eBay reported USD$82.95 billion in sales volume and 8.5% annual growth after nearly two decades in business.[1] Other platforms with hundreds of millions or billions of dollars of transactions include StubHub (event tickets) and Upwork (contract workers). Since many

1. Information downloaded from https:investors.ebayinc.comsecfiling.cfm?filingID=1065088-15-54&CIK=1065088 on 11/17/2015.

The editor in charge of this paper was Aureo de Paula.

participants are exchanging a broad array of products on these platforms, each platform has sophisticated search tools to help users find partners to transact with.

Given the power of modern search algorithms and the thickness of the markets, one might conjecture that these platforms would do an excellent job of matching buyers and sellers, eliminating market frictions, and generating efficient trade. This conjecture is particularly compelling in cases where the products are homogenous and buyer and seller reputation are not significant barriers to trade. Our goal is to test this conjecture by estimating a novel model of the eBay auction platform using data on sales of new Amazon Kindle Fire tablets.

On the eBay platform a large number of participants compete in a large number of auctions each day, and buyers and sellers can participate across successive days. In this article, we provide a rich model of such an auction platform in which a continuum of buyers is matched to a continuum of seller auctions each period. After matching has occurred, each single-unit auction is executed independently, auction winners (and the associated sellers) exit the market, losing bidders move on to the next period, and new bidders enter at the end of each period. We include a costly per-period entry decision to capture the time and effort costs of participation. We use an extensive dataset on new Amazon Kindle Fire tablets to estimate the structural model primitives such as the matching process that allocates potential buyers to auction listings, the monetized cost of participation, and the steady-state distributions of buyer valuations and seller reserve prices. While the participation cost we find is low, on the order of \$0.10, it turns out to be an important regulator of the number and types of buyers in the market.

On the empirical front, we make several contributions to the literature on identifying auction models. A key feature of our estimator is that it requires only observables that are readily available on many platform websites. In particular, we are able to identify the average number of buyers per auction without assuming that we observe all of the bidders in each auction. If bid submission times are randomly ordered, then some auction participants with an intent to bid may be prematurely priced out of the spot-market before they get a chance to submit their bid. Therefore, the total number of unique bidders within a given eBay auction constitutes a lower bound on the actual number of competitors. Our non-parametric identification argument for the dynamic structural model requires only that we observe this random lower bound on the number of competitors, the seller reserve price, and the highest losing bid within each auction.

Our identification strategy lets us separately identify bid shading (*i.e.* bidding strictly below one's private valuation) due to the use of a non-truthful pricing rule (*e.g.* a first-price auction) and bid shading due to intertemporal incentives. From a buyer's perspective, failure to win an auction today is no tragedy since he can return tomorrow and bid again, which implies there is an opportunity cost to winning today. We show that when the spot-market pricing rule is non-second price—so that the winner's bid may directly affect the sale price—then the additional, static demand shading incentive is layered on top of the dynamic demand shading incentive in an intuitive way that allows for straightforward econometric identification. This represents what we refer to as a "plug-and-play" property: identification of the dynamic structural model is obtained under any auction format where identification is known for the static, one-shot game. This is important since many electronic auction pricing rules (including eBay's) deviate from the second-price form in empirically relevant ways. Given a value for the time discount factor, we show that the degree of demand shading is non-parametrically identified.

To make this concrete, Figure 1 plots the equilibrium bidding strategies under first and second-price auction rules in a dynamic auction market given the economic primitives we estimate.[2] The 45-degree line can be interpreted as the equilibrium of a static second-price auction that omits

2. We chose to plot bidding strategies under first- and second-price spot-markets because they represent the polar extremes of static demand shading incentives among common pricing mechanisms.

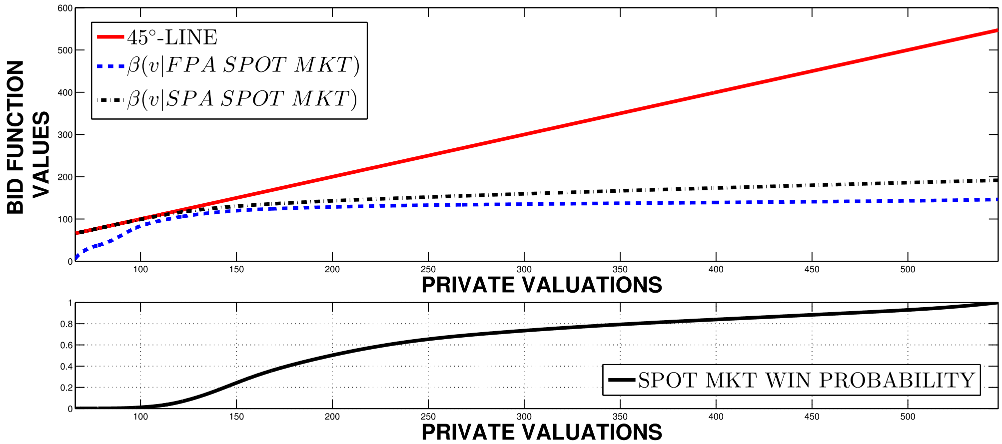

FIGURE 1

Static versus dynamic demand shading incentives

the dynamic opportunity cost. The bid shading caused by the intertemporal opportunity cost is represented by the difference between the bidding strategy under the second-price auction (SPA) mechanism in a dynamic setting and the 45-degree line. The additional demand shading caused by switching to a non-truthful mechanism such as a first-price auction rule is represented by the gap between the first and second-price bidding strategies in our dynamic setting. We also include the probability of winning in the bottom pane for reference. The conclusion we draw from the plot is that dynamic incentives tied to opportunity costs play a clearly dominant role in shaping behaviour: for all bidder types with non-trivial win probabilities, the demand shading caused by intertemporal opportunity costs is an order of magnitude larger than the static demand shading. In other words, it is more important for bidders to understand intertemporal opportunity costs than how to strategically bid under a non-truthful pricing rule.

Having estimated the structural model, we move on to investigate the efficiency of the market. Platform markets like eBay exist for the purpose of reducing frictions that impede trade and converting some of the resulting efficiency gains into profits for the platform's owners. With this in mind, it is natural to assess how closely eBay approaches the ideal of fully efficient trade. To fix ideas, suppose there are exactly two listings and four bidders, two bidders with high valuations and two bidders with low valuations. The social planner's preferred outcome is one where each auction attracts one high value bidder as this would guarantee that the high value bidders win in any monotone bidding equilibrium. However, when there is randomness in the bidder-listing match process, there will be a positive probability that one auction listing will not have a high-value participant, meaning a low value bidder wins at a low price and a high-value bidder loses. Another way of putting it is that the matching frictions mean some auctions have too much competition and others too little relative to an efficient allocation.

We begin our counterfactual analysis by using two separate methods to measure inefficiency under the current market conditions. Our first method relies on the raw data. Using our estimated buyer–seller ratio we can count the number of times in our data that a bidder with an inefficiently low value (*i.e.* low bid) won an auction and prevented a high value bidder from receiving that item. This method gives a lower bound on the prevalence of inefficient allocations because, for example, it cannot detect scenarios where multiple high-value losers attended the same auction. We find that within the listings for new Kindles, at least 27.6% of all auctions allocate goods to buyers with inefficiently low valuations.

In our second method, we use the structural estimates to get a precise value for the fraction of auction listings that award an object to a buyer whose private value is inefficiently low. This method also allows us to quantify the deadweight loss, which is defined as the average difference in value between high-value losers and low-value winners. We calculate that 36% of new Kindle listings result in inefficient allocations and that the total deadweight loss amounts to roughly 14% of potential market surplus.

Next, we explore the implications of alternative spot-market mechanisms eBay could use to improve efficiency. Specifically, we consider the welfare cost of eBay's choice to use single-unit auctions, which we refer to as *decentralization*. We use our estimates to analyse outcomes of alternative markets where, instead of single-unit auctions, eBay runs u-unit, uniform-price auctions, and we use u as a measure of the market's *centralization*. The most extreme version of this counterfactual would be a single multi-unit, uniform-price auction each day. We find that aggregating auctions together so that eBay runs half as many auction listings for 2-units each day recovers 35% of the welfare loss by improving the efficiency of the allocation, while running a quarter as many 4-unit auctions recovers over 57% of the welfare loss. However, centralization reduces the expected sale price, which in turn reduces eBay's revenues. In addition to being a vehicle for analysing the welfare losses, we believe that centralizing auctions is a practical design strategy in settings wherein the goods are homogenous (*e.g.* new Kindles).

While we conduct our estimates within the particular eBay context, we believe that the degree of welfare losses we find should temper optimistic expectations that online platform markets can eliminate all market frictions. In addition, the market centralization solution we propose is far more broadly applicable than a single, isolated eBay market, and we believe it could be worthwhile considering similar centralization-oriented designs in other platform market contexts. For example, centralization may be possible for standardized back-office tasks that are bought and sold on Upwork.

Our last counterfactual explores the importance of controlling for platform composition when computing counterfactuals or redesigning platform markets. We repeat our welfare and revenue exercises, but we hold the distribution and number of buyers fixed. Under this misspecified model, one would predict that welfare gains from redesign are larger and that centralizing the market increases seller revenue. These results are both primarily driven by the effect that the composition of the pool of buyers has on bidder behaviour.

Finally, our article also makes a contribution to the theory underlying the large market models we use. The notion of a large market approximation, sometimes referred to as an oblivious equilibrium, is not novel to this article. However, proving a formal relationship between a model with a continuum of players and the finite markets that exist in reality is difficult when the market mechanism admits discontinuities, and an auction setting provides several points where such discontinuities can arise. In Supplementary Appendix C, we prove that despite these issues, one can view an equilibrium of the model with a continuum of buyers and sellers as an ε-equilibrium of the model with a finite number of buyers and sellers. We view this result as a justification for our use of the continuum model in our estimation and counterfactual exercises.

The remainder of this article has the following structure. In Section 2, we develop a theory of bidding within a dynamic platform based on a model with a continuum of buyers and sellers. In Section 3, we use this model to specify a parsimonious structural model of eBay, which we show is identified from observables. We also propose a semi-nonparametric estimator based on B-splines. In Section 4, we present our model estimates, and Section 5 contains our counterfactual analyses. Proofs for the technical claims in the main text are relegated to Supplementary Appendix A. Supplementary Appendix B provides an algorithm for computing counterfactuals and testing for uniqueness. In Supplementary Appendix C, we prove that our model with a continuum of agents approximates an analogous model with a large, but finite, number of participants. Supplementary

Appendix D provides details on our estimation techniques and algorithms. Supplementary Appendix E empirically investigates our random matching assumption. Supplementary Appendix F provides a variety of additional counterfactuals on revenue, participation costs, and seller incentives.

1.1. *Related literature*

The most closely related predecessor to our article is Backus and Lewis (2016), which studies a model of eBay where bidders participate in a sequence of single-unit, second-price auctions. Backus and Lewis (2016) focus on identifying a buyer demand with flexible substitution patterns across different goods and the possibility that individual bidder demand evolves over time. Their model could be used to compute welfare counterfactuals like ours in a setting with homogenous goods, but this (obviously) excludes substitution between products. However, since Backus and Lewis (2016) does not include participation costs or a procedure for estimating the measure and type distribution of buyers entering the market, the model remains silent on how market structure influences entry and exit through the interaction of changing auction format and participation costs. Our structure also decomposes static and dynamic demand shading incentives, which allows us to compare the relative strengths of these forces. This decomposition also links existing results on estimating static auction models with the literature on dynamic auction markets. Finally, we prove that our large market model approximates a more realistic model with a finite number of agents. Due to the different foci and contributions of each work, we view our papers as complementary.

Another related paper is Hickman (2010), which shows that the pricing rule on eBay is actually a hybrid of the first-price and a second-price auction formats. This is because minimum bid increments imply there is a positive probability that the winner will pay her own bid. Hickman et al. (2017) explore empirical implications of the non-standard pricing rule on eBay within a static auction model and show that estimates may become biased in an economically significant way if it is ignored. We build on these two papers in the following ways. First, our model incorporates both dynamic demand shading incentives and static demand shading incentives. Second, we extend the estimator of Hickman et al. (2017) to allow for binding reserve prices, which affects identification of the bidder arrival process and the private value distribution in complicated ways.

Our methodology analyses approximate equilibria played by a large number of agents, which has been a prominent theme in the microeconomics and industrial organization literatures. Due to the broad scope of this literature, we provide a brief survey and a sample of the important papers related to the topic. Early papers focused on conditions under which underlying game-theoretic models could be used as strategic microfoundations for general equilibrium models (*e.g.* Hildenbrand, 1974; Roberts and Postlewaite, 1976; Otani and Sicilian, 1990; Jackson and Manelli, 1997). Other papers focused on conditions under which a generic game played by a finite number of agents approaches some limit game played by a continuum of agents (*e.g.* Green, 1980, 1984; Sabourian, 1990). A recent branch of this literature applies these ideas to simplify the analysis of large markets with an eye to real-world applications (*e.g.* Fudenberg et al., 1998; MacLean and Postlewaite, 2002; Budish, 2008; Weintraub et al., 2008; Krishnamurthy et al., 2014; Azevedo and Leshno, 2016).

Nekipelov (2007) and Hopenhayn and Saeedi (2016) develop models of intra-auction price dynamics with repeated bidding in a single auction. Their goal is to rationalize common empirical patterns concerning the timing of bids. In our model, we abstract away from intra-auction dynamics, and instead we concentrate on inter-auction dynamics and how future periods shape bidding incentives today. Peters and Severinov (2006) develop a model of a multi-unit auction

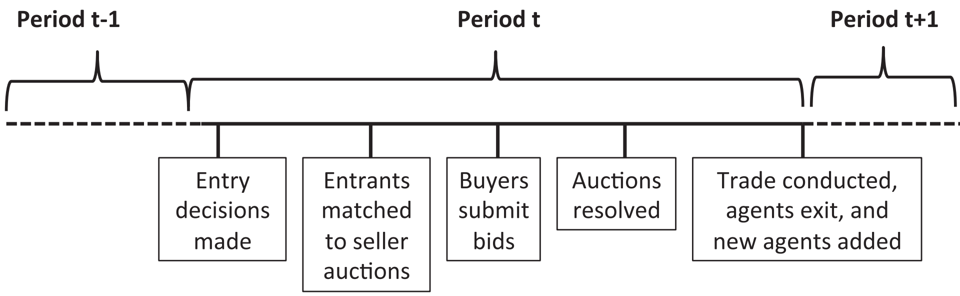

FIGURE 2
Timing within a period

environment similar to eBay with the goal of studying the sorting of buyers into sellers' auctions in a static setting.

The first paper we are aware of that attempts to estimate a model of a sequence of auctions is Jofre-Bonet and Pesendorfer (2003), which applies the pioneering identification and estimation strategy of Guerre et al. (2000) to a dynamic setting. Building on Jofre-Bonet and Pesendorfer (2003), Balat (2013), and Groeger (2014) also focus on capacity constraints and/or learning-by-doing in procurement settings (highway construction contracts). In this line of research, allocative efficiency *within a given spot-market auction* is a concern because with a relatively small number of bidders, past auction outcomes induce bidder asymmetry through capacity constraints and/or learning by doing. In contrast, we study a large market setting with single-unit demands, where consumer types remain constant over time. While a given spot-market auction is guaranteed to allocate efficiently within the set of bidders that show up, the set of auctions that run within a period will be collectively inefficient due to the role of randomness in matching of bidders to auctions. Donald et al. (2006) estimated a model of sequential timber auctions where a small number of firms with multi-unit demand compete over time.

2. A MODEL OF PLATFORM MARKETS

Before entering the eBay market, the buyer considers her own value for the good, makes an assessment of the opportunity cost of winning, and formulates her bid. We assume that the time a bidder chooses to enter the market is driven by factors exogenous to eBay (*e.g.* the schedule of work breaks), which means the buyer only considers bidding on a small and randomly selected fraction of the auctions that close during that day. If a buyer wins the spot-market auction, then she does not participate in future days. Remaining buyers return to eBay the next day to place a bid. We summarize the timing in Figure 2.

2.1. *Model primitives*

The market evolves in discrete time with periods indexed $t \in \{0,1,2,...\}$. We treat sellers as a source of exogenous supply. This modelling choice is driven by the fact that sellers face very weak incentives to set the optimal starting price,[3] which makes estimates based on a model of seller behaviour less credible. In each period, there is a measure 1 of sellers with starting prices

3. In Supplementary Appendix F, we show that the sellers earn less than \$1 in increased revenue by moving from a starting price of \$0 to the revenue maximizing starting price.

described by cumulative density function (CDF) G_R with support $[0,\bar{r}]$.[4] G_R may have a mass point, but only at the lowest possible reserve price, $r=0$, and has a probability density function (PDF) $g_R(R|R>0)$ that is strictly bounded away from zero over the rest of its support.

We refer to the set of buyers present at the start of period t as *potential entrants*; at the beginning of each period they make decisions based on the observed number and type distribution of the other potential entrants and their own types. Each period, the first choice a potential entrant must make is whether or not to enter the market and participate in the platform. We denote the choice to participate as *Enter* and refer to the agents that make this choice as *entrants*. The choice to not participate is denoted *Out*, and any agent choosing Out leaves the game permanently and earns a payoff of 0 in every future period.

Throughout, we assume that the goods for sale are homogenous and that buyers have demand for a single unit. Each buyer's value for the good is her private information, which we denote as v. A buyer that wins a good on the eBay platform and pays a price of p receives a payoff in that period of $v-p-\kappa$, where κ is a per-period participation cost paid by entrants regardless of whether they win. We assume $\kappa>0$; this may reflect the opportunity cost of time spent searching for a listing and participating in the market, or it may reflect an actual monetary participation fee that the platform designer imposes. If an entrant does not engage in trade, her payoff is simply $-\kappa$.

A continuum of potential entrants with measure μ is added to the economy at the end of each period, and each of the newly added agents has a value drawn independently from CDF $T_V(\cdot)$ with PDF $t_V(\cdot)$. These private values are persistent and do not change with time. We assume that t_V is strictly positive over the support $[0,1]$. The total set of potential entrants therefore includes both the newly added agents and those buyers that failed to win in the previous period and are carried over to the current period.[5] A generic, measure 0 buyer is denoted using the subscript i.

After choosing *Enter*, each entrant formulates a strategic bid in a simultaneous-move spot market without knowing either the number or identity of the other agents participating in the particular auction to which she is matched. The form of the random matching process, the distribution of entrant types, and the exogenous distribution of starting prices is known to agents at the point when they choose their bids. If a measure $\mathcal{C}$ of buyers chooses to enter the auction market—including some new potential entrants and holdovers from previous periods—they are randomly assigned to auctions, with each auction receiving a random number of bidders $K\sim\pi(k;\mathcal{C})=Pr\{K=k\}$. We refer to $\mathcal{C}$ as the *market tightness* parameter since it is the buyer–seller ratio.

Assumption 2.1 *We require that* π *satisfy the following conditions:*

(1) $E[K]=\mathcal{C}$.
(2) $\pi(1;\mathcal{C})>0$.
(3) π *is continuous in* $\mathcal{C}$.

Part (1) requires that all of the buyers be matched into auctions. If we allowed $E[K]>\mathcal{C}$, then in aggregate the auctions would be assigned more bidders than entrant buyers that exist in the market. If we allowed $E[K]<\mathcal{C}$, then a positive measure of bidders would not be assigned to any auction. This is why $\mathcal{C}$ appears as an argument of the bidder arrival process π, because the latter

4. The lowest starting price on eBay is \$0.99, but this does not affect our theoretical results. eBay also allows sellers to choose reservation prices that are hidden from buyers, but this is done so infrequently that we ignore it in our modelling.

5. See Supplementary Appendix A for details on topologies used in our analysis.

must be consistent with the former in equilibrium. However, our theory requires no functional form assumptions on π, as there are many such distributions that could be consistent with a given value of $\mathcal{C}$. Part (2) ensures that an entrant with the lowest value amongst all entrants can win an auction with positive probability. Part (3) is a continuity assumption that we use in the proof that an equilibrium exists.

From the perspective of a single bidder, let M denote a random variable representing the number of competitors she faces, and let $\pi_M(M;\mathcal{C})$ denote its probability mass function (PMF). Her beliefs over M are given by (see Myerson, 1998)

$$\pi_M(m;\mathcal{C}) = Pr\left[m \text{ opponents}|\mathcal{C}\right] = \pi(m+1;\mathcal{C})\frac{(m+1)}{\mathrm{E}[K]}. \tag{1}$$

The random matching process results in the stochastic independence of the valuations of the bidders assigned to any particular auction. To complete the description of the game, we need to characterize the matching of bidders to auctions when the measure of entering bidders vanishes.[6] In this "no participation by buyers" limit (*i.e.* $\mathcal{C} \to 0$), each bidder that does choose to enter is the sole bidder in the auction with arbitrarily high probability.

Assumption 2.2 $\pi_M(0;\mathcal{C}) \to 1$ *as* $\mathcal{C} \to 0$.

Examples that satisfy Assumptions 2.1 and 2.2 include a Poisson distribution with parameter $\mathcal{C}$, generalized Poisson, and a geometric distribution with parameter $\mathcal{C}^{-1}$.

Finally, note that our continuum model is "large" in the sense that the actions of individual bidders have no effect on the aggregate distribution of auction outcomes. However, the actions of individual bidders have a large effect on the auction to which they have been assigned.

We now address a few potential objections to our framing of the bidding model. First, we assume that the buyer formulates her bid before entering the market, rather than updating her bidding strategy in real time as the observed price path within the current auction unfolds. As we discuss in the Section 1 (see Figure 1) and in Section 4, intertemporal incentives that are independent of any single auction play a far more important role in determining optimal bids than the competitive environment of today's auction. In equilibrium, the bidder takes into account the expected static bid-shading incentive, but she has relatively weak incentives to update her bid based on the particular details of the auction she is matched into. The weak incentives to update her bid suggest it is not unreasonable to assume she does not tailor her bid to the observed price path.

Second, we assume a bidder does not repeatedly bid within the same auction. As we discuss in Section 4.1, we only consider bids arriving in the last 60 min of the auction. One reason we do this is to avoid the question of how to handle dynamic behaviour within an auction. As it stands, we see few instances in the data of a bidder returning to increase her bid if she has already bid within the narrow window we consider at the close of each auction.

Third, by assuming a buyer participates in an auction closing near her time of entry, we rule out strategic selection into particular auctions. This assumption would be violated if, for example, at the time of entry, bidders could predict which auctions were likely to close with a low price. To investigate this possibility, we refer to the raw data and define the *relative price* (RP) as the median price for auctions ending within 24 h before/after a given auction minus the final sale price for that same auction. When the RP is large and positive, the bidder chose well; otherwise,

6. Such an assumption is necessary to evaluate deviations by a buyer from a candidate equilibrium wherein no buyers enter the market.

she did not. To test whether bidders can predict an auction's RP, we first restricted attention to the set of bidders whose bidding affected the price path during the final 60 min of the auction. We regressed the RP on the current price when those bidders first entered the auction, and found a small R^2 value of 0.109. Because many of those bidders who shape the terminal price path enter prior to the final 60 min of the auction, we also regressed RP on the current price at various points in time. We found that the R^2 of these regressions with 60 min remaining was 0.34, meaning that price predictability was low even for auctions closing in the near future. Moreover, at only 60 s prior to close of bidding, over a third of the variation in relative price cannot be captured by current price. Unless a bidder is willing to return to eBay dozens of times each day, price predictability is minimal. We conclude that bidders have limited ability to select into auctions due to the low predictability of relative prices, and therefore, randomness plays a significant role in determining who is matched to each auction. We present further details of this analysis and other metrics on price predictability in Supplementary Appendix E.

Fourth, one might also worry that buyers can see features of the auction that are unobservable to us that might be proxied for by, for example, the starting price of the auction (Roberts, 2013). If this were a significant issue, one might find correlation between the starting price and the closing price of the auction. We find in our data that the correlation coefficient between the starting and the closing price is -0.015 and statistically indistinguishable from zero. A lack of significance, of course, does not prove that there is no selection at work. We discuss this issue in more depth in Section 4.

2.2. *Equilibrium*

Our analysis focuses on stationary equilibria where the aggregate market states are constant across periods.[7] We let C denote the measure of the set of all potential entrants (newly added and carried over from the prior period), and we denote the distribution of all potential entrant types by F_V.[8] Thus, μ and T_V are primitives, while C and F_V are equilibrium objects. The state variables describing the economy in a given period are a vector (C, F_V, G_R). A symmetric bidding strategy is a function $\beta : [0,1] \to [0,1]$ with a typical bid denoted $\beta(v)$. We do not consider asymmetric bidding strategies.[9] The entry decision is a function $\zeta : [0,1] \to \{0,1\}$, where $\zeta(v) = 1$ indicates the agent enters the market and $\zeta(v) = 0$ that the agent stays out of the market. $\sigma = (\beta, \zeta)$ denotes the full strategy. These strategies explicitly condition on the agent's private information and implicitly condition on the equilibrium values of the (stationary) aggregate state variables.

The spot-market auction mechanism is defined by an allocation rule and a pricing rule that are functions of the reservation price and bids. The allocation rule $x(b, C, F_V, G_R|\sigma)$ is a binary random variable taking on a value of 1 (0) in the event that the buyer wins (loses) an auction with a bid of b, given the aggregate state (C, F_V, G_R) and a common strategy σ. The pricing rule $p(b, C, F_V, G_R|\sigma)$ is a random variable describing the transfer from the buyer to the seller conditional on a bid of b, given (C, F_V, G_R) and σ. We assume throughout that $p(b, C, F_V, G_R|\sigma) \leq b$. Both x and p are defined by the auction mechanism used on the platform. We develop theoretical results in terms that apply for arbitrary spot-market auction formats—*e.g.* first-price, second-price, etc.—so that our model may serve as a general framework for platform markets in a variety of settings.

7. In Supplementary Appendix C, we discuss how to extend our model to the non-stationary case and what is lost by assuming stationarity.

8. The character roman-C here is the measure of the set of potential entrants, whereas calligraphic-$\mathcal{C}$ (used in Section 2.1) is the measure of the set of actual entrants. The difference between the two is that C includes new potential entrants arriving in the current period who will choose not to enter, while $\mathcal{C}$ does not.

9. Proposition 2.3, part 3, proves that only a countable set of buyer types have multiple best responses, which in turn means that asymmetric bidding strategies would have to agree for almost all types.

To simplify notation we also define

$$\chi(b,C,F_V G_R|\sigma) = E[x(b,C,F_V,G_R|\sigma)] \quad \text{and} \quad \rho(b,C,F_V,G_R|\sigma) = E\big[p(b,C,F_V,G_R|\sigma)\big].$$

Hereafter, we suppress the notation for the aggregate state variables C, F_V, and G_R and strategy σ in the functions χ and ρ. Note that ρ represents expected transfers that are not conditional on sale. That is, each entering bidder has an ex ante expectation of paying $\rho(b)$ in the spot-market, although under any winner-pay pricing rule only one bidder will pay a positive amount, ex post.

All agents discount future payoffs using a per-period discount factor $\delta \in (0,1)$. The value function given a (symmetric) strategy vector $\sigma = (\beta,\zeta)$ played by all agents is:[10]

$$\begin{aligned}\mathcal{V}(v,C,F_V,G_R|\sigma) =& \zeta(v)[\chi[\beta(v)]v - \rho[\beta(v)] - \kappa + (1-\chi[\beta(v)])\delta\mathcal{V}(v,C,F_V,G_R|\sigma)] \quad (2)\\ & + (1-\zeta(v))\delta\mathcal{V}(v,C,F_V,G_R|\sigma)\\ =& \zeta(v)[\chi[\beta(v)](v-\delta\mathcal{V}(v,C,F_V,G_R|\sigma)) - \rho[\beta(v)] - \kappa)] + \quad (3)\\ & \delta\mathcal{V}(v,C,F_V,G_R|\sigma).\end{aligned}$$

Where confusion will not result, we suppress the notation for the aggregate state variables C, F_V, and G_R and strategy σ in the value function $\mathcal{V}$. When describing agent behaviour, we refer to "agents best-responding to the aggregate state," which we think better captures the economic intuition than the more common "agents best responding to the actions of the other players." This is analogous to describing an agent in a general equilibrium economy as best responding to prices rather than the actions that generate the prices.

Suppose that the distribution of types F_V and the strategy σ generate a distribution of bids G_B in the auctions on the platform. We let β^s denote the best-response in the static version (*i.e.* when $\delta = 0$) of the spot-market mechanism given a distribution of bids G_B and starting prices G_R. One useful property of our model is that we can describe the best-response to the aggregate state variables in terms of β^s. To see this, we first define a bidder's private value minus her opportunity cost as her *dynamic value*, denoted $\tilde{v}_v \equiv v - \delta\mathcal{V}(v)$. If a buyer enters the market, her optimal bid is defined by:

$$\underset{b}{argmax}\ \chi[b](v-\delta\mathcal{V}(v)) - \rho[b] = \underset{b}{argmax}\ \chi[b]\tilde{v}_v - \rho[b]. \quad (4)$$

But this is the problem faced by a buyer with value $\tilde{v}_v$ in the static form of the spot-market. Thus, $\beta(v) = \beta^s(v - \delta\mathcal{V}(v))$ is a best-response bidding strategy to the aggregate state (C,F_V,G_R) and a strategy vector (β,ζ) played by the other buyers.

In most static spot-market mechanisms, one's equilibrium bid is chosen to balance out opposing forces: a higher bid will increase the chance of winning, but it may also raise the price one pays as well. Whenever the second force is present, bidders shade their demand; henceforth, we refer to this as *static incentives*. Intertemporal dynamics introduce an additional demand shading incentive: if the spot-market price is sufficiently high today, then a bidder would prefer to wait in expectation of lower prices tomorrow. Therefore, even when the spot-market game follows a second-price rule, rational bidders in equilibrium engage in demand shading; we refer to this as *dynamic incentives*.

Proposition 2.3 summarizes four useful properties of the best-response, $(\widetilde{\beta},\widetilde{\zeta})$, to (C,F_V,G_R).

10. Proposition 2.6 proves an equilibrium exists, and agents must have a weakly positive expected utility in equilibrium since they would otherwise exit. Recall that private values $v \in [0,1]$ are bounded and therefore $p \in [0,1]$ in equilibrium. Since an agent can only win one auction and an infinite discounted sum of the entry fees is finite, expected equilibrium payoffs must be finite. This implies the existence of a value function.

Proposition 2.3 *The best-response strategy $(\widetilde{\beta},\widetilde{\zeta})$ satisfies:*

1. *$\widetilde{\beta}(v)=\beta^s(v-\delta\mathcal{V}(v))$ is a best-response bidding strategy.*
2. *$\widetilde{\beta}(v)$ is increasing in v if $\chi(b)$ is increasing.*
3. *$\widetilde{\beta}(v)$ is strictly increasing in v if $\chi(b)$ is differentiable and strictly increasing in b. In addition, $\widetilde{\beta}(v)$ is uniquely defined for a set of v of Lebesgue measure 1.*
4. *There exists a cutoff $\widetilde{e}$ such that $\widetilde{\zeta}(v)=1$ if and only if $v\geq\widetilde{e}$.*

Part (1) summarizes the relationship between the static and dynamic auctions. Part (2) uses the supermodularity of the buyer's decision problem to show that best-responses by the bidders must be monotonically increasing. Part (3), which holds in our application, proves that if $\chi(b)$ is strictly increasing, then the best-response function is strictly increasing. The strict monotonicity implies that almost all buyer-types have a unique best response, which also implies that the best-response function is symmetric across agents. Part (4) implies that we can describe the best-response strategy as $(\widetilde{\beta},\widetilde{e})$, where $\widetilde{\zeta}$ has been replaced with the cutoff $\widetilde{e}$.

With these results in hand, we can define our notion of stationary equilibrium.

Definition 2.4 *The strategy vector $\sigma=(\beta,e)$ and the states C and F_V are a Stationary Competitive Equilibrium (SCE) if for all bidder values v we have:*

1. *Optimal bids: $\beta(v)=\underset{b}{argmax}\ \chi[b](v-\delta\mathcal{V}(v))-\rho[b]$*
2. *Optimal entry: The entry cutoff is determined by the equation:*

$$\chi[\beta(e)]e-\rho[\beta(e)]=\kappa \tag{5}$$

3. *Stationarity: The measure and distribution of agents entering the market equals those of the exiting agents:*

$$\textit{For all } v\geq e,\ \ \mu t_V(v)=\chi(\beta(v))f_V(v)\mathcal{C} \tag{6}$$

Part (1) of Definition 2.4 implies the bidding strategy of a buyer is optimal given that she enters. Part (2) requires that the lowest valuation buyer that chooses to enter must be indifferent between entering and staying out. Since the equilibrium is stationary, buyers with a value $v<e$ prefer to stay out in every period (and hence buyers with these values exit the game immediately), while buyers with a value $v>e$ strictly prefer to enter in every period. The marginal buyer, $v=e$, is indifferent between entering and staying out and earning a payoff of 0, which means this buyer's continuation value is 0 (and the buyer faces no dynamic incentives). Therefore, the marginal agent's utility in any given period is:

$$\mathcal{V}(e)=\chi[\beta(e)]e-\rho[\beta(e)]-\kappa=0 \tag{7}$$

which yields the equation in part (2) of Definition 2.4. Part (3) of Definition 2.4 requires that the state variables C and F_V be consistent with the laws of motion of the game. For an economy to be stationary, the distribution and measure of buyers that win auctions and exit the game must be replaced by an identical distribution of new entrants. Recall that μ is the measure of buyers entering each period with distribution t_V, so the left-hand side of Equation 6 describes the mass of agents of each type entering the game each period. $\chi(\beta(v))$ describes the probability an agent of type v wins and exits the economy, so $\chi(\beta(v))f_V(v)\mathcal{C}$ is the mass of agents of type v who win an auction and exit the game each period.

To get some insight into the forces driving agent behaviour in our model, consider the SPA spot-market mechanism. Since it is an equilibrium in weakly undominated strategies for a bidder to bid her value in a static SPA, the SCE strategy is $\beta(v)=v-\delta\mathcal{V}(v)$.[11] In the static, one-shot setting, the opportunity cost (*i.e.* $\mathcal{V}(v)$) is 0 as outside options are assumed not to exist. In our dynamic model, the opportunity cost of winning today is the continuation value the bidder receives if she returns to the market to bid again in a future period.

The following assumption restricts the best-responses of the bidders. It is stated in terms of the (static) spot-market best-responses since the properties of the static models have been characterized in the past. For example, Assumption 2.5 is trivially satisfied in the symmetric, weakly undominated equilibrium of the SPA.[12] Note that Assumption 2.5 places restrictions on the best-reply correspondence (not the equilibrium).

Assumption 2.5 *Let $\mathcal{Q}\left[0,\overline{q}\right]$ denote the space of measures over $[0,1]$ that admit pdfs bounded from above by $0<\overline{q}<\infty$. For any $\overline{q}$, the best-response bidding strategy of the spot-market mechanism, $\widetilde{\beta}^s$, is continuous with respect to any $G_B \in \mathcal{Q}\left[0,\overline{q}\right]$; any distribution of starting prices, G_R, that has full-support with atoms only at 0; and the market tightness parameter, $\mathcal{C}$.*

We can choose $\varphi \in (0,1)$ such that for any static best response $\widetilde{\beta}^s$ to $(\boldsymbol{\lambda}, G_B, G_R)$ where $G_B \in \mathcal{Q}\left[0,\overline{q}\right], \overline{q}<\infty$, and any $v>v'$ we have:

$$\widetilde{\beta}^s(v)-\widetilde{\beta}^s(v')\in\left[\varphi(v-v'),\frac{v-v'}{\varphi}\right]. \tag{8}$$

Assumption 2.5 provides continuity properties that are essential for our proof of equilibrium existence. Our proof relies on a fixed point argument that requires the best-responses of the buyers and the market aggregates be continuous with respect to each other, which is the crucial contribution of the first half of Assumption 2.5. The remainder of the assumption mandates bounds on the slope of the bidding function. The lower bound insures that there will not be atoms in the bid distribution, which would destroy the model's continuity. The upper bound insures the set of bidding strategies we need to consider is Lipschitz continuous (and hence equicontinuous and compact). Together continuity and compactness allow us to apply Schauder's fixed point theorem to prove an equilibrium exists. We discuss the issue of uniqueness of the equilibrium in Section 5.

Proposition 2.6 *A stationary competitive equilibrium exists, and a positive mass of buyers choose to enter the market if κ is not too large.*

Finally, one must acknowledge that our model assumes a continuum of agents, whereas the actual eBay market is only used by a finite number of buyers and sellers. The size of the state-space of an analogous finite model grows exponentially in the number of agents, making it impossible to solve the model or generate counterfactuals. In Supplementary Appendix C, we prove that an SCE is an "approximate equilibrium" of a finite model with sufficiently many agents, meaning that any individual agent has at most a small incentive to deviate from the stationary strategy when all agents play the SCE strategy. Intuitively, the stationary state of the continuum model

11. Similar equations appear in Jofre-Bonet and Pesendorfer (2003), Backus and Lewis (2016), and Iyer et al. (2014). Jofre-Bonet and Pesendorfer (2003) study a first-price procurement auction, so the formula includes a static demand-shading incentive. The latter two study SPA mechanisms that admit only dynamic bid-shading incentives.

12. Lizzeri and Persico (2000) established it for the first-price auction and the all-pay auction; and Hickman (2010) proved Assumption 2.5 for the auction mechanism on eBay.

is a good approximation of the aggregate state of a finite model with many agents, so the SCE bidding strategy will be approximately optimal for the state realized in the finite model. If there is little to be gained by costly monitoring of and optimizing with respect to the state realized in the finite-agent model, then bidders would find it optimal to act "as if" the state is fixed at the SCE value.

Proving such a result forces us to state concretely the finite-agent market structures that we believe our model approximates. In our case, we assume that as the market grows, a large number of auctions occur each day and the agents can only participate in a randomly chosen auction each day. In addition, approximation results typically rely on the continuity of the model, and our auction setting includes a large number of potential discontinuities that make it non-obvious that such an approximation result holds.

3. AN EMPIRICAL MODEL OF DYNAMIC PLATFORM MARKET BIDDING

We now shift to developing a structural model based on the SCE concept. Letting L denote the number of sampled auctions, the observables, $\{\tilde{k}_l, r_l, y_l\}_{l=1}^{L}$, include $\tilde{k}_l$, the observed number of bidders within the lth auction; r_l, the reserve price; and y_l, the highest losing bid. We assume only that the highest losing bid is available for reasons discussed in Section 4.1.[13] We assume throughout that $\left(\tilde{k}_l, r_l, y_l\right)$-triples are independent across auctions $l = 1, \ldots, L$ and that the data-generating process (DGP) is a SCE of our model. Our DGP corresponds to only a single realization of the market tightness parameter $\mathcal{C}$, but we are able to non-parametrically identify the bidder matching process $\pi(\cdot;\mathcal{C})$ present in the DGP. For that reason we suppress the second argument during our discussion on identification. However, our counterfactual exercises will induce exit by low-value bidders, thus resulting in values of $\mathcal{C}$ that are not present in the data. To facilitate pinning down counterfactual market structures later on, our estimator will use a finite-dimensional parametric assumption about $\pi(\cdot;\mathcal{C})$.

For simplicity of discussion, consider the decision problem of a bidder who has decided to enter and finds herself competing within a spot-market auction; we will refer to her as bidder 1. As before, the number of opponents she faces is $M \equiv K - 1 \geq 0$, which follows distribution $\pi_M(\cdot)$. Prior to bidding, bidder 1 observes her private valuation v and she views her opponents' private values as independent realizations of a random variable $V \sim F_V$ having strictly positive density f_V on support $[\underline{v}, \overline{v}]$ with $\underline{v} \geq 0$.

The theory from the previous section depicted a set of potential buyers, some of whom choose to enter the bidding market and some of whom don't, with Equation 5 determining the relevant entry cutoff. Since we are unable to collect real-world observations on non-entrants, we adopt the convention that F_V is the steady-state distribution of buyer types who choose *Enter* in the DGP. Let $\underline{v} = e$ denote the type that is just indifferent to entering, leading to the following formula that we refer to as the "zero surplus condition:"

Assumption 3.1 $\mathcal{V}(\underline{v}) = 0.$ [14]

Bidder 1 views the bids of her opponents as a random variable $B = \beta(V)$, which follows distribution $G_B(B) = F_V\left[\beta^{-1}(B)\right]$ with support $[\underline{b}, \overline{b}]$. Let B_M denote the maximal bid among all of 1's opponents, and we adopt the convention that $B_M = 0$ if there are no other opponent bidders. The distribution of B_M is then $G_{B_M}(B_M) = \pi_M(0) + \sum_{m=1}^{\infty} \pi_M(m) G_B(B_M)^m$. R is the starting price

13. Our identification results would be similar if we used the winning bid instead.
14. See discussion of Definition 2.4 for intuition on why this must be true.

of the auction, which is randomly drawn from CDF G_R. In order to win, player 1's bid must exceed the realized value of the random variable $Z \equiv \max\{R, B_M\}$. Note that the distribution of Z is the same as the win probability function:

$$\chi(b) = G_R(b) \sum_{m=0}^{\infty} \pi_M(m) G_B(b)^m = G_Z(b). \tag{9}$$

3.1. *Model identification*

The structural primitives to identify are μ, κ, π, T_V, and G_R.[15] Throughout, we assume that the bids within each auction and across auctions are independent. The model is identified if there is a unique set of structural primitives that rationalizes the joint distribution of observables, $\{\tilde{k}_l, r_l, y_l\}_{l=1}^{L}$. The primary econometric challenge is correcting for two forms of sample selection in order to identify $\pi(\cdot)$ and $G_B(\cdot)$, which are required to identify the structural primitives. We first establish non-parametric identification of these objects in Section 3.1.1. Section 3.1.2 proves that this initial result implies non-parametric identification of the structural primitives when the spot market is a SPA. This result is straightforward, but it lacks the generality needed for a viable empirical framework since many platform markets (including eBay) use non-SPA formats. In 3.1.3, we prove a second, more novel result extending our method to platforms with broad varieties of selling mechanisms, which formalizes the "plug-and-play" property. Since static and dynamic bid-shading incentives can be decomposed in a parsimonious way, identification of the dynamic structural model is obtained under any auction format where identification is known for the static, one-shot game. We relegate all identification proofs to Supplementary Appendix A.3.

3.1.1. Identification of the bid distribution and bidder arrival process. If the econometrician knew exactly how many bidders attended each auction, then the empirical problem here would be simple. For example, if the number of bidders was known to be k, then the CDF of the (observable) highest losing bid, call it $H_k(b)$, is related to its (unobserved) parent distribution through the following bijective mapping:

$$\begin{aligned} H_k(b) &= G_B(b)^k + k G_B(b)^{k-1}[1 - G_B(b)] \\ &= k(k-1)\int_0^{G_B(b)} u^{k-2}(1-u)du \equiv \phi_k^{-1}(G_B(b)). \end{aligned}$$

From the above integral, it is easy to see that ϕ_k is a bijection mapping quantile ranks of $H_k(b)$ into quantile ranks of $G_B(b)$: the upper limit of integration ranges from zero to one (onto), and since the integrand is always positive, it is monotone in G_B (one-to-one). Our setting is more complex though: the number of bidders K is a random variable, following PMF $\pi(K)$, so the observed distribution of the highest losing bid, $H(\cdot)$, takes the form

$$H(b) = \sum_{k=2}^{\infty} \frac{\pi(k)}{1 - \pi(0) - \pi(1)} (H_k(b)) \equiv \phi^{-1}(G_B(b)). \tag{10}$$

$H(b)$ is a weighted average of the distributions of second order statistics from samples of varying k, where the weights are the probability that a given k will occur as the number of bidders matched

15. Our exposition might appear to treat F_V as a primitive, but in reality it is endogenously pinned down by T_V (and other variables) through equation (6).

to a particular listing. Note that since $\phi^{-1}(G_B(b))$ is a convex combination of monotone bijective transformations $\phi_k^{-1}(G_B(b))$, it must also be a bijection.

A more pressing challenge to empirical work is that the observed number of bidders in each auction, $\tilde{K}$, is only a lower bound on the actual number of bidders matched to the auction, K. Due to random ordering of bid submission times across all bidders who watch an item with intent to compete, some may find that their planned bid was surpassed before they had a chance to submit it to the online server. These bidders will never be visible to the econometrician, even though they were matched to the auction and competing to win.

To solve this problem, we incorporate an explicit model of the sample selection process into our identification strategy. We adopt an approach similar to that of Hickman et al. (2017) who proposed a model of a *filter process* executed by Nature that randomly withholds some bidders from the econometrician's view.[16] For a given auction with k total matched bidders, this filter process first randomly assigns each bidder an index $\{1,2,\dots,k\}$, where one's position in the list determines the ordering of bid submission times. Nature then visits each bidder in the order of her index within the list, keeping a running record of the current lead bidder and current price as she goes. As Nature visits each bidder in the list, she only records bids that cause the price to update because they exceed the second highest previous bid. Otherwise, Nature skips bidder i's submission as if it never happened and reports to the econometrician only the record of price path updates, which reveals $\tilde{k} \leq k$ observed bidder identities. This filter process is meant to depict the way information is recorded on real-world platform markets, where some bidders will not appear to have participated even though they had intent to bid. This view of intra-auction dynamics assumes that the ordering of bidders' submission times is random, but we are agnostic as to how the bidders choose the moments in calendar time to place their bids.

Note that the distribution of $\tilde{K}$ conditional on a given k can be characterized without knowing the form of $\pi(\cdot)$ or G_B. Since a bidder's visibility to the econometrician only depends on whether her bid exceeds the second-highest preceding bid, the researcher can easily simulate the filter process based on quantile ranks to compute conditional probabilities $\Pr[\tilde{k}|k]$ for various $(\tilde{k},k)$ pairs. We adopt a special notation for this function, $P_0(\tilde{k},k) \equiv \Pr[\tilde{k}|k]$, and treat it as known since it can be computed without data on bidding. Since $\tilde{k}$ is observable, we can use this information to express its PMF, denoted $\tilde{\pi}(\tilde{k})$, as a function of the primitive distribution: $\tilde{\pi}(\tilde{k}) = \sum_{k=\tilde{k}}^{\infty} P_0(\tilde{k},k)\pi(k)$.

However, this equation will not suffice in our case: unlike Hickman et al., our data exhibit binding reserve prices, which introduce a further random wedge between actual participation k and observed participation, $\tilde{k}$. Not only do some bidders go unobserved because the filter process withholds them from view, but an additional fraction of bidders, who would have otherwise been reported by Nature, go unobserved because their bids fall below the reserve price. This second layer of selection produces substantial complications since G_B and G_R now determine how the second source of selection influences the relation between the distribution of observed $\tilde{K}$ and the underlying distribution of actual K.

We propose an adjusted filter process. First, for each auction Nature randomly draws k from $\pi(k)$ and r from G_R. The k randomly chosen bidders formulate their strategic bids without knowing the realization of k or r. Nature then compiles a reported list of bidders for the econometrician in two steps. First, she dismisses any bidders whose bids do not exceed the reservation price. Second, she randomly assigns a timing index $i \in \{1,\dots,k'\}$ to the remaining $k' \leq k$ bidders and

16. In a similar setting, Platt (2015) explored parametric inference assuming that K is Poisson distributed. However, our empirical estimates with a more flexible model strongly reject the Poisson assumption in favor of a more flexible model with greater variance of bidder participation in each auction.

then executes the standard filter process on that subset. Finally, Nature reports $\tilde{k}$ and r to the econometrician.

In order to characterize the conditional distribution of $\tilde{K}$ given r, first note that if there are $K=k$ total bidders, the probability that j of them are screened out by r is $\binom{k}{j}G_B(r)^j[1-G_B(r)]^{k-j}$. Now suppose there are $\tilde{K}$ observed bidders in an auction with K total bidders. We combine the two levels of selection in the adjusted filter process with the following equation:

$$\Pr[\tilde{K}=\tilde{k}|K=k,r]=\sum_{j=0}^{k-\tilde{k}}\binom{k}{j}G_B(r)^j[1-G_B(r)]^{k-j}P_0(\tilde{k},k-j). \tag{11}$$

The sum is to account for the fact that any number of bidders between 0 and $k-\tilde{k}$ could be screened out by selection on reserve prices. The trailing term involving P_0 is to account for the standard filter process running its course with the surviving set of bidders. Equation (11) characterizes the distribution of $\tilde{K}$, conditional on reserve price r, as

$$\tilde{\pi}(\tilde{k}|r)=\sum_{k=\tilde{k}}^{\infty}\Pr[\tilde{k}|k,r]\pi(k). \tag{12}$$

Lemma 3.2 *Suppose that there exists a set $k_1<k_2<\cdots<k_{\mathcal{I}}$ where $\sum_{i=1}^{\mathcal{I}}\pi(k_i)=1$. Then for any finite $\mathcal{I}$ it follows that there is a unique $(\pi(\cdot),G_B(\cdot))$ pair that is consistent with the joint distribution of the observables $\left(\tilde{K},R,Y\right)$.*

Lemma 3.2 does not require any shape restrictions on G_B, and it does not depend on the specific value of $\mathcal{I}$ in any way (aside from its finiteness). In other words, the identification result holds for models of the bidder arrival process that become arbitrarily flexible as $\mathcal{I}$ grows. In that sense, the preceding result may be considered as non-parametric. However, the proof of the lemma is non-constructive in that it does not immediately suggest a method for estimating the parameters. For a more intuitive understanding, first note that bijectivity of the mapping ϕ implies that fixing π pins down a unique $G_B^{\pi}(b)=\phi(H(b);\pi)$ through equation (10). Therefore, if we condense equations (10)–(12) we can write

$$\tilde{\pi}(\tilde{k}|r)=\sum_{k=\tilde{k}}^{\infty}\pi(k)\sum_{j=0}^{k-\tilde{k}}\binom{k}{j}\phi(H(b);\pi)^j[1-\phi(H(b);\pi)]^{k-j}P_0(\tilde{k},k-j), \tag{13}$$

which depends only on the function $\pi(\cdot)$ and the observables. In principle then, for any finite, $\mathcal{I}$-dimensional, restriction of the bidder arrival process, equation (13) allows the researcher to choose from a continuum of moment conditions, with each possible moment condition depending on a distinct $(\tilde{k},r)$ pair.

3.1.2. Baseline model: second-price, sealed-bid spot-market auctions.

Using Lemma 3.2, we are now ready to develop our main identification results concerning the structural primitives. A SPA spot-market mechanism implies a specific form for the expected payment function,

$$\rho(b)\equiv\mathrm{E}[p_B(b)]=\int_0^b tg_Z(t)dt. \tag{14}$$

In steady state, we can express the Bellman equation and bidding strategy as

$$\mathcal{V}(v) = \max_{b \in \mathbb{R}_+} \left\{ \chi(b)v - \rho(b) - \kappa + [1-\chi(b)]\delta\mathcal{V}(v) \right\} \tag{15}$$

$$\beta(v) = v - \delta\mathcal{V}(v) \tag{16}$$

The demand shading factor given by bidder 1's continuation value, $\delta\mathcal{V}(v)$, is uniquely characterized by four things: the per-period entry cost, κ; the distribution of bids, $G_B(b)$; the distribution of starting prices, G_R; and the bidder arrival process $\pi(\cdot)$. Note that (16) implies $\mathcal{V}(v) = \frac{v-\beta(v)}{\delta}$. By substituting this expression into equation (15) and using the shorthand notation $b^* = \beta(v)$, we can rearrange terms to get

$$v = b^* \frac{1-\delta\left(1-\chi(b^*)\right)}{1-\delta} - \frac{\delta}{1-\delta}\left(\rho(b^*)+\kappa\right) = \beta^{-1}(b^*). \tag{17}$$

Proposition 3.3 *For a given discount factor δ, suppose that there exists a set $k_1 < k_2 < \cdots < k_{\mathcal{I}}$ where $\sum_{i=1}^{\mathcal{I}} \pi(k_i) = 1$. Then when the spot-market mechanism is a sealed-bid, second-price auction, for any finite $\mathcal{I}$ there is a unique configuration of model parameters $\mathbf{\Theta} \equiv (\mu, \kappa, \pi, T_V, G_R)$ that is consistent with the joint distribution of the observables $\left(\tilde{K}, R, Y\right)$.*

3.1.3. Model identification under alternative spot-market mechanisms.

We now extend our identification result to cover platforms that use general spot-market formats, which may cause both static and dynamic demand shading incentives. Recall that a buyer with valuation v has a dynamic value equal to $\widetilde{v}_v = v - \delta\mathcal{V}(v)$. Proposition 2.3 implies that each buyer bids as if they are playing a static auction with their dynamic bid shading incentives collapsed into their dynamic value. We show that if the right set of observables are available to identify the mapping β^s that would arise in a static auction with allocation rule χ and pricing rule ρ, then the value function $\mathcal{V}$ and the private value v from the dynamic auction market are also identified. We refer to this as our "plug-and-play" identification result since it demonstrates how existing identification results for static auction models can be reused in the context of a dynamic platform market. Note that plugging β^s into equation (15) yields:

$$\mathcal{V}(v) = \frac{\chi\left[\beta^s(\tilde{v}_v)\right]v - \rho\left[\beta^s(\tilde{v}_v)\right] - \kappa}{1-\delta\left(1-\chi\left[\beta^s(\tilde{v}_v)\right]\right)}.$$

Using the shorthand $b^* = \beta^s(\tilde{v}_v) = \beta(v)$ and substituting in the definition of $\tilde{v}_v$, we can rearrange terms further to get:

$$v = \tilde{v}_v \left(\frac{1-\delta\left[1-\chi\left(b^*\right)\right]}{1-\delta} \right) - \frac{\delta}{1-\delta}\left(\rho\left(b^*\right)+\kappa\right) = \beta^{-1}(b^*). \tag{18}$$

In the case of a SPA spot-market, $b^* = \beta^s(\tilde{v}_v) = \tilde{v}_v$, so (18) reduces to (17) above.

Proposition 3.4 ***Plug and play identification.*** *For a given discount factor δ, suppose that there exists a set $k_1 < k_2 < \cdots < k_{\mathcal{I}}$ where $\sum_{i=1}^{\mathcal{I}} \pi(k_i) = 1$. Then for any finite $\mathcal{I}$ the model parameters $\mathbf{\Theta} \equiv (\mu, \kappa, \pi, T_V, G_R)$ are uniquely identified under any spot-market mechanism for which the* *optimizer of equation 4, $\beta^s(v)$, could be identified from the available observables $\left(\tilde{K},R,Y\right)$ if they were generated from a sample of static, one-shot auction games.*[17]

Once again, Proposition 3.4 may also be considered as non-parametric in the sense that the logic of its proof need not involve shape restrictions on G_B, F_V, or T_V, and it holds for models of the bidder arrival process that become arbitrarily flexible as $\mathcal{I}$ grows. Proposition 3.4 is also useful because it broadens the applicability of our model and methodology to allow for empirical work for any spot-market mechanism that admits a monotone equilibrium in the static setting, and for which the pricing and allocation rules can be expressed in terms of $\pi(\cdot)$, G_R, and G_B. For example, any platform model where the spot-market is a first-price auction will still be identified from commonly available observables (see Guerre et al., 2000). In our empirical application, the eBay pricing rule is a non-standard hybrid of the first-price and second-price formats, which causes bidders to engage in additional demand shading due to static, strategic incentives.

3.2. *A semi-parametric estimator*

Following our identification argument, we recover the structural primitives using a strategy inspired by the two-stage approach pioneered by Guerre et al. (2000) and Jofre-Bonet and Pesendorfer (2003). In the first stage, we flexibly estimate $\pi(\cdot)$, G_B, and G_R, and in the second stage we construct the remaining objects $\chi(\cdot)$, $\rho(\cdot)$, κ, $(\beta^s)^{-1}(\cdot)$, $\beta^{-1}(\cdot)$, $\mathcal{V}(\cdot)$, $F_V(\cdot)$, μ, and T_V as functions of first-stage parameter estimates. However, Stage I is an estimation step, while Stage II is a purely computational step based on the outputs from Stage I.

Thus far, we have left the bidder arrival process $\pi(k)$ unrestricted in order to demonstrate that the observables and the theoretical model on their own are sufficient to identify the structural primitives. In this section, we develop an estimator to implement our identification strategy, but for the purpose of computing counterfactuals we now assume K follows a generalized Poisson (GP) distribution (Consul and Jain, 1973) with PMF:

$$\pi(k;\boldsymbol{\lambda})=\lambda_1(\lambda_1+k\lambda_2)^{k-1}\frac{e^{-(\lambda_1+k\lambda_2)}}{k!},\ \lambda_1>0,\ |\lambda_2|<1. \tag{19}$$

The GP reduces to a regular Poisson distribution when $\lambda_2=0$, with fatter tails when $\lambda_2>0$ and thinner tails when $\lambda_2<0$. Thus, we refer to λ_1 as the *size parameter* and λ_2 as the *dispersion parameter*. The GP model implies that $\pi_M(m,\boldsymbol{\lambda})=\pi(m+1;\boldsymbol{\lambda})(m+1)\frac{(1-\lambda_2)}{\lambda_1}$.

Following our identification argument, G_B and $\boldsymbol{\lambda}$ must be jointly estimated, which rules out many common methods such as kernel smoothing. We choose to specify G_B as a B-spline, which is a linear combination of globally defined basis functions that mimic the behaviour of piecewise, local splines (the name "B-splines" is short for *basis splines*). By the Stone–Weierstrass theorem, B-splines can be used to approximate any continuous function to arbitrary precision given sufficiently many basis functions. B-splines provide a combination of flexibility and numerical convenience that is ideally suited to our application.

Let $\mathbf{n}_b=\{n_{b1}<n_{b2}<\cdots<n_{b,I_b+1}\}$ be a set of knots on bid domain $[\hat{\underline{b}},\hat{\bar{b}}]=[\min_l\{y_l\},\max_l\{y_l\}]$ that create a partition of I_b subintervals. This need not be a uniform partition, but we do require that $n_{b1}=\hat{\underline{b}}$ and $n_{b,I_b+1}=\hat{\bar{b}}$ so that the partition spans $[\hat{\underline{b}},\hat{\bar{b}}]$. The knot vector, in combination

17. Alternatively, if the optimizer of equation (4) is unique and the allocation rule $\chi(b)$ and pricing rule $\rho(b)$ can be identified from the available observables $\left(\tilde{K},R,Y\right)$, then $\beta^s(v)$ is identified.

with the Cox–de Boor recursion formula, defines a set of I_b+3 cubic B-spline basis functions $\mathcal{F}_{bi}:[\hat{\underline{b}},\hat{\overline{b}}]\to\mathbb{R}$, $i=1,\dots,I_b+3$ that parameterizes the bid distribution:[18]

$$\hat{G}_B(b;\boldsymbol{\alpha}_b)=\sum_{i=1}^{I_b+3}\alpha_{b,i}\mathcal{F}_{b,i}(b).$$

We also use this approach to estimate G_R and F_V. Let $\mathbf{n}_r=\{n_{r1}<\cdots<n_{r,I_r+1}\}$ and $\mathbf{n}_v=\{n_{v1}<\cdots<n_{v,I_v+1}\}$ denote knot vectors that span the reserve prices, $[\underline{r},\hat{\overline{r}}]=[0.99,\max_l\{r_l\}]$, and the space of buyer values, $[\hat{\underline{v}},\hat{\overline{v}}]$ (with the bounds to be estimated).[19] These knot vectors determine our other basis functions $\mathcal{F}_{r,i}:[\underline{r},\hat{\overline{r}}]\to\mathbb{R}$, $i=1,\dots,I_r+3$ and $\mathcal{F}_{v,i}:[\hat{\underline{v}},\hat{\overline{v}}]\to\mathbb{R}$, $i=1,\dots,I_v+3$ which then parameterize $\hat{G}_R(r;\boldsymbol{\alpha}_r)=\sum_{i=1}^{I_r+3}\alpha_{ri}\mathcal{F}_{r,i}(r)$ and $\hat{F}_V(v;\boldsymbol{\alpha}_v)=\sum_{i=1}^{I_v+3}\alpha_{vi}\mathcal{F}_{v,i}(v)$.

3.2.1. Stage I: $\boldsymbol{\lambda}$, G_B, and G_R. Recalling that the matrix of conditional probabilities $P_0(\tilde{k},k)$ is known beforehand, we now define the model-generated conditional PMF of $\tilde{K}$ given r as

$$\tilde{\pi}(\tilde{k}|r;\boldsymbol{\lambda},\boldsymbol{\alpha}_b)=\sum_{k=\tilde{k}}^{\overline{K}}\left\{\sum_{j=0}^{k-\tilde{k}}\binom{k}{j}\hat{G}_B(r;\boldsymbol{\alpha}_b)^j\left[1-\hat{G}_B(r;\boldsymbol{\alpha}_b)\right]^{k-j}P_0(\tilde{k},k-j)\right\}\pi(k;\boldsymbol{\lambda}), \tag{20}$$

where $\overline{K}$ is an upper bound on the auction sizes we consider.[20] We also adopt the following as the empirical analogue of the conditional PMF:

$$\hat{\tilde{\pi}}(\tilde{k}|r)=\sum_{l=1}^{L}\mathbb{1}(\tilde{k}_l=\tilde{k})\frac{\mathcal{K}\left(\frac{r-r_l}{h_R}\right)}{\sum_{t=1}^{L}\mathcal{K}\left(\frac{r-r_t}{h_R}\right)} \tag{21}$$

where $\mathbb{1}(\cdot)$ is an indicator function, $\mathcal{K}$ is a boundary-corrected kernel function, and h_R is an appropriately chosen bandwidth.[21] The model-generated highest loser bid distribution is

$$H(b;\boldsymbol{\lambda},\boldsymbol{\alpha}_b)=\sum_{k=2}^{\infty}\frac{\pi(k;\boldsymbol{\lambda})\left(G_B(b;\boldsymbol{\alpha}_b)^k+kG_B(b;\boldsymbol{\alpha}_b)^{k-1}[1-G_B(b;\boldsymbol{\alpha}_b)]\right)}{1-\pi(0;\boldsymbol{\lambda})-\pi(1;\boldsymbol{\lambda})}, \tag{22}$$

18. A standard text on B-splines is de Boor (2001). See also (Hickman et al., 2017, Online Appendix).

19. For discussion on choice of knot locations used in our empirical implementation, see Supplementary Appendix D.1.

20. The GP distribution has unbounded support, but for implementation we impose a finite bound $\overline{K}$. In practice, $\overline{K}$ is chosen so that there is essentially 0 probability it would be exceeded in the set of 1,732 auctions we analyse.

21. The boundary-corrected kernel we use follows Karunamuni and Zhang (2008). See Hickman and Hubbard (2015) for an in-depth discussion of its advantages and uses in structural auctions models.

and its empirical analogue is $\hat{H}(b)=\sum_{l=1}^{L}\mathbb{1}(y_l\le b)/L$. Using these separate pieces, we can define a method of moments estimator as

$$(\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b)=\underset{(\boldsymbol{\lambda},\boldsymbol{\alpha}_b)\in\mathbb{R}^{I_b+5}}{\operatorname{argmin}}\sum_{l=1}^{L}\left\{\left[\tilde{\pi}(\tilde{k}_l|r_l;\boldsymbol{\lambda},\boldsymbol{\alpha}_b)-\hat{\tilde{\pi}}(\tilde{k}_l|r_l)\right]^2+\left[H(y_l;\boldsymbol{\lambda},\boldsymbol{\alpha}_b)-\hat{H}(y_l)\right]^2\right\}$$
$$\text{subject to} \tag{23}$$
$$\alpha_{b,1}=0,\ \alpha_{b,I_b+3}=1,$$
$$\alpha_{b,i}\le\alpha_{b,i+1},\ i=1,\ldots,I_b+2.$$

The estimate $(\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b)$ is chosen to make the model-generated conditional distribution of $\tilde{K}$ match its empirical analogue as closely possible.[22] The constraints on the empirical objective function enforce boundary conditions and monotonicity of our parameterization for $\hat{G}_B$.

Finally, we separately estimate $\hat{G}_R$ by a simpler method of moments procedure as

$$\hat{\boldsymbol{\alpha}}_r=\underset{\boldsymbol{\alpha}_r\in\mathbb{R}^{I_r+3}}{\operatorname{argmin}}\sum_{l=1}^{L}\left\{\left[\hat{G}_R(r_l;\boldsymbol{\alpha}_r)-\ddot{G}_R(r_l)\right]^2\right\}$$
$$\text{subject to} \tag{24}$$
$$\alpha_{r,1}=\ddot{G}_R(\underline{r}),\ \alpha_{r,I_r+3}=1,$$
$$\alpha_{r,i}\le\alpha_{r,i+1},\ i=1,\ldots,I_r+2,$$

where $\ddot{G}_R(r)=\sum_{l=1}^{L}\mathbb{1}(r_l\le r)/L$ is the empirical CDF of reserve prices.

3.2.2. Stage II:.

Having these estimates in hand, we are able to directly re-construct the remaining structural primitives. Some Stage II objects will depend on the time discount factor, and where this is the case we so note by including δ as a parameter argument for the relevant functional. Another difficulty is that eBay employs a hybrid of the usual first- and second-price auction formats that uses a bid increment denoted $\Delta>0$. Typically, the price is set equal to the second highest bid plus the increment. However, if the top two bids are within Δ of each other, then the second-price rule yields a price above the highest bid. In that case, the price is set equal to the high bidder's own bid as in a first-price auction. Thus, eBay's pricing rule is $p(b)=min\{Z+\Delta,b\}$.

Hickman (2010) proved existence and uniqueness of a monotone Bayes–Nash equilibrium under this pricing rule in a static auction where the number of bidders is known. This equilibrium involves demand shading because there is positive probability that the winner's own bid will determine the price she pays. Hickman et al. (2017) showed, in a static bidding game with stochastic participation and no binding reserve prices, that a bidder's private value is identified from the distribution of bids through the equation:

$$v=b+\frac{G_Z(b)-G_Z[\tau(b)]}{g_Z(b)},\ \tau(b)=\begin{cases}\underline{b} & \text{if } b\le\underline{b}+\Delta\\ b-\Delta & \text{otherwise,}\end{cases} \tag{25}$$

22. A fully non-parametric estimator for π is possible, but with additional complications. The main challenge is that only finitely many $\Pr[K=k]$ can be estimated with finite sample size L. Thus, one could choose an upper bound $\overline{K}_L<\infty$ and restrict $\Pr[K=k]=0$ for each $k>\overline{K}_L$. The estimator must also specify the rate at which $\overline{K}_L$ should grow with the sample size. In a simpler setting than ours—a static bidding model of eBay laptop computer auctions with no binding reserve prices—Hickman et al. (2017) found strong evidence that a GP model produced estimates that could not be improved upon by relaxations of its parametric form in a sample size of roughly 750 auctions.

where $\tau(b)$ is a threshold function determining the point below one's own bid which, if the random variable Z surpasses it, will trigger a first-price outcome. Proposition 3.4 enables us to adapt equation (25) above for the static inverse bid function $(\beta^s)^{-1}$ in our model. Our inverse static bid function is given by:

$$\left(\hat{\beta}^s\right)^{-1}(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)=\hat{\tilde{v}}_b=b+\frac{\hat{G}_Z(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)-\hat{G}_Z\left[\tau(b);\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r\right]}{\hat{g}_Z(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)}. \tag{26}$$

Using Stage I estimates, we can construct the allocation rule:

$$\chi(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)=\hat{G}_R(b;\hat{\boldsymbol{\alpha}}_r)\sum_{m=0}^{\infty}\pi_M(m;\hat{\boldsymbol{\lambda}})\hat{G}_B(b;\hat{\boldsymbol{\alpha}}_b)^m. \tag{27}$$

Taking into account the hybrid pricing rule, we can construct the payment function:

$$\begin{aligned}\rho(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)=&\underline{r}G_Z(\underline{r};\hat{\boldsymbol{\alpha}}_r)+\int_{\underline{r}}^{\tau(b)}(t+\Delta)\hat{g}_Z(t;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)dt\\&+b\left(\hat{G}_Z[b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r]-\hat{G}_Z[\tau(b);\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r]\right).\end{aligned} \tag{28}$$

The first two terms on the right-hand side are for the event where a second-price rule is triggered, and the third is for the event where a first-price rule is triggered. Recall that we allow for the possibility that G_R has a mass point at the lower bound of its support.

Using the zero surplus condition, we can recover the per-period entry cost as:

$$\hat{\kappa}=\chi(\underline{\hat{v}}_v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)\underline{\hat{v}}_v-\rho(\underline{\hat{v}}_v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r) \tag{29}$$

as well as the dynamic inverse bid function and value function which are:

$$\hat{v}=\hat{\beta}^{-1}\left(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right)=\hat{\tilde{v}}_v\frac{1-\delta\left[1-\chi(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)\right]}{1-\delta}-\frac{\delta\left(\rho(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r)+\hat{\kappa}\right)}{1-\delta} \tag{30}$$

$$\hat{\mathcal{V}}\left(v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right)=\frac{v-\hat{\tilde{v}}_v}{\delta}. \tag{31}$$

We parameterize the private value distribution F_V using B-spline functions to obtain a flexible representation that is convenient for computing counterfactuals. We begin by specifying a grid of $J=I_v+1$ points spanning the bid support, $\mathbf{b}_J=\{b_1,\ldots,b_J\}$, and a knot vector $\mathbf{n}_v$ that spans $\left[\underline{\hat{v}},\hat{\bar{v}}\right]=\left[\hat{\beta}^{-1}\left(\underline{\hat{b}};\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right),\hat{\beta}^{-1}\left(\hat{\bar{b}};\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right)\right]$. This defines our basis functions $\mathcal{F}_{v,i}:\left[\underline{\hat{v}},\hat{\bar{v}}\right]\to\mathbb{R}$, $i=1,\ldots,I_v+3$, from which we can compute $\hat{\boldsymbol{\alpha}}_v$:

$$\begin{aligned}\hat{\boldsymbol{\alpha}}_v=\underset{\boldsymbol{\alpha}_v\in\mathbb{R}^{I_v+3}}{\arg\min}&\sum_{j=1}^{J}\left\{\left[\hat{G}_B\left(b_j;\hat{\boldsymbol{\alpha}}_b\right)-\sum_{i=1}^{I_v+3}\alpha_{vi}\mathcal{F}_{vi}\left[\hat{\beta}^{-1}\left(b_j;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right)\right]\right]^2\right\}\\&\text{subject to}\\&\alpha_{v,1}=0,\ \alpha_{v,I_v+3}=1,\\&\alpha_{v,i}\le\alpha_{v,i+1},\ i=1,\ldots,I_v+2.\end{aligned} \tag{32}$$

Finally, the steady-state measure and distribution of new agents each period are:

$$\hat{\mu}=\left[1-\pi\left(0;\hat{\boldsymbol{\lambda}}\right)\right]G_R\left(\underline{b};\hat{\boldsymbol{\alpha}}_r\right)+\int_{\underline{b}}^{\overline{b}}g_R\left(r;\hat{\boldsymbol{\alpha}}_r\right)\left(\sum_{k=1}^{\infty}\pi\left(k;\hat{\boldsymbol{\lambda}}\right)\left[1-G_B\left(r;\hat{\boldsymbol{\alpha}}_b\right)^k\right]\right)dr \tag{33}$$

$$t_V\left(v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b;\hat{\boldsymbol{\alpha}}_r,\delta\right)=\frac{\chi\left[\beta\left(v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right);\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r\right]f_V\left(v;\hat{\boldsymbol{\alpha}}_v,\delta\right)\frac{\hat{\lambda}_1}{1-\hat{\lambda}_2}}{\hat{\mu}}. \tag{34}$$

3.2.3. Asymptotics and standard errors. In Supplementary Appendix D.1, we give recommendations for B-spline tuning parameters (*i.e.* choice of number and locations of knots) that achieve a reasonable mix of utility and tractability in our empirical application. A fixed choice of tuning parameters can be viewed as defining a flexible, though finite-dimensional, parametric model. In Supplementary Appendix D.2 we show why, under this view, our Stage I parameter estimators $\hat{\boldsymbol{\lambda}}$, $\hat{\boldsymbol{\alpha}}_b$, and $\hat{\boldsymbol{\alpha}}_r$ fall within the class of GMM estimators, meaning that standard asymptotic theory applies to establish sampling distributions for all Stage I and Stage II parameters.[23] In our implementation we use the standard non-parametric bootstrap as a computationally convenient method of accounting for the role of sampling variability. We re-sample from our auction-level observations (with replacement) to construct 1000 bootstrap samples of size L. Then, we execute Stages I and II of our estimator as described above to obtain confidence bounds on the parameters.

4. DATA AND RESULTS

We use a unique dataset on Amazon Kindle Fire tablet devices that we scraped from eBay during March through July 2013. Our scraping algorithm captured all Kindle listings on eBay during that period, and for each one we downloaded and stored various `.html` files including the item listing page and the bid history page. During the sample period we observed a total of 1,732 Kindle Fires listed as "new" (*i.e.* unused in a factory sealed box) or "new other" (*i.e.* unused in an unsealed box) for an average of 11.25 per day.

Each Kindle tablet had eight gigabytes of internal storage and a seven-inch screen with standard-definition resolution of 1024×600. The Kindle Fire tablets came pre-loaded with Amazon's proprietary version of the Android-based operating system that prevents the user from accessing the full Android app market.[24] This makes the Kindle Fire a poor substitute for a standard tablet (*e.g.* Apple iPad) that can serve a dual role as a productivity tool. The Kindle Fire is designed to exclusively access Amazon's electronic media market, which includes e-books, periodicals, audiobooks, music, and movies.[25] All transactions were covered by the eBay Money Back Guarantee to insure consumers against potential unscrupulous sellers.[26]

23. In Supplementary Appendix D.2, we also sketch how one might extend our analysis using semi-parametric sieve methods by (for example) increasing the number of B-spline knots as more data become available to increase the flexibility of the parametric model. However, proving the requisite estimator properties that yield point-wise asymptotic normality of the underlying functionals is non-trivial and beyond the scope of this paper.

24. It requires specialized knowledge to uninstall the proprietary operating system, and doing so is costly since it invalidates all product guarantees issued by Amazon.

25. Amazon also maintains its own limited app market—primarily dedicated to entertainment and online shopping, but in June 2013 it contained less than one tenth the number of apps available in Apple's App Store for iPhones or Google Play for Android devices. See https://en.wikipedia.org/wiki/App_Store_(iOS); https://en.wikipedia.org/wiki/Google_Play; and https://en.wikipedia.org/wiki/Amazon_Appstore; information retrieved on 15 July 2016.

26. As of 15 July 2016, details on eBay's consumer protection program were available at http://pages.ebay.com/ebay-money-back-guarantee/questions.html.

In order to further probe the homogeneity of our Kindle auctions sample, we manually examined a 10% sample of the raw `.html` files. Unlike many other tablet devices, accessories are rarely coupled with Kindle Fires: of these listings, only one mentioned an accessory (a Kindle case) that the seller had bundled into the sale. The majority of the listings with a condition of "new" had been opened. A common explanation was that the seller was checking that all parts (*e.g.* charging cord) were present. We conclude that the "new" listings are best interpreted as items that are like new and essentially unused.

Because listings with a low closing price identify the participation cost, κ, we manually scrutinized all of these items. We identify \$66 as the minimal observed sale price, but we examined all listings with a closing price of less than \$80. Of these, we removed listings that (for example) were selling Kindle accessories (*e.g.* cases) rather than the actual device or were offering a Kindle running a user-modified version of the Android OS. These atypical listings were largely isolated to the lower tail of the price distribution.

One final concern is that there may be residual auction-specific variation which our manual survey missed, and which is not included in our econometric model. Unobserved heterogeneity (UH)—auction characteristic that bidders see but the econometrician does not—is a common problem, and various approaches have been developed to deal with it (e.g., see Krasnokutskaya, 2011; Balat, 2013; Roberts, 2013). Each approach assumes bidder valuations are separable in the UH and the idiosyncratic component, which makes it possible to deconvolve UH from agent-specific variation in bids. Roberts (2013) proposed a method to correct for UH when only one bid is observed per auction. Since we can only be confident that the highest losing bid in each of our auctions is fully reflective of equilibrium strategies (see discussion below), Roberts (2013) is the most relevant approach to the current context. Assuming reserve prices are a separable function of the UH variable, he shows that one can use joint movement in reserve prices and bids to deconvolve the UH and identify private valuations.

In our data we observe non-trivial variation in sellers' reserve prices with roughly one third of them being binding for a positive fraction of the bidder population. Therefore, one might reasonably suspect that if UH is present then higher values of the unobserved characteristic prompt sellers to increase reserve prices.[27] A necessary condition for UH in the Roberts (2013) model is co-movement of bids and reserve prices, which is testable. We find in our data that the correlation between seller reserve price and the highest losing bid is statistically indistinguishable from zero and very small in a practical sense, at -0.015. We conclude that there is not unobserved heterogeneity as captured by Roberts (2013), but we cannot rule out other forms of unobserved heterogeneity as per Krasnokutskaya (2011) or Balat (2013). The combination of market features (*e.g.* our manual survey of `.html` pages, uniform buyer insurance, proprietary operating system and limited app market, and uniform characteristics of the Kindle Fire tablets) and the lack of evidence from our UH test is consistent with our assumption of a homogeneous goods market. These characteristics of the eBay Kindle data allow us to avoid significant complications covered by other work, such as identifying UH or complex substitution patterns (see Backus and Lewis, 2016), and instead focus on questions of bidding behaviour, allocative efficiency, and market design.

4.1. *Intra-auction dynamics*

For each auction listing, we observe the timing and amount of each bid submission as well as an anonymized hash of the bidder identity. As previous empirical work has recognized, one challenge

27. Roberts (2013) assumes only that reserves are a monotone separable function of UH. For example, it does not matter whether reserves are chosen to optimize projected revenues or whether they are chosen to hedge against the risk of selling at an unacceptably low price, since both scenarios would satisfy monotonicity.

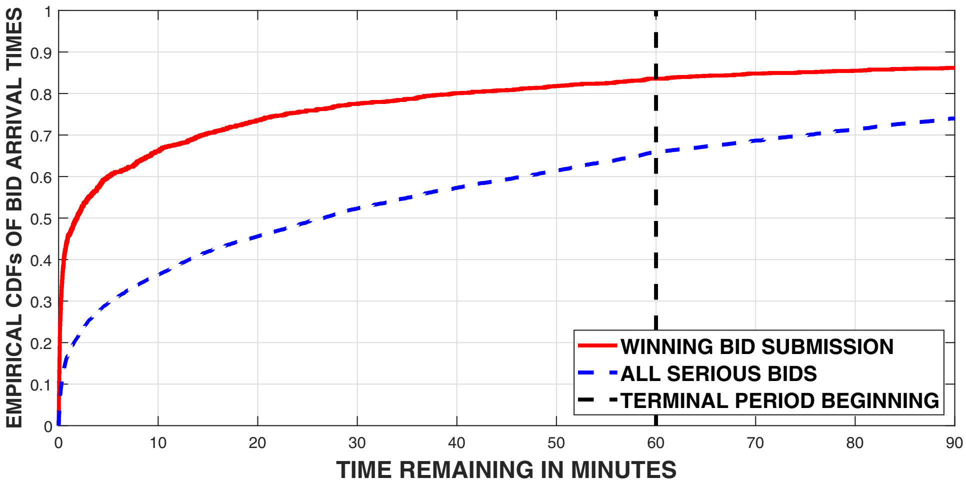

FIGURE 3

Empirical distributions: time remaining when bids are submitted

for interpreting eBay data is a large number of bids that fall too far below realistic transaction prices to be taken seriously. Many bidders place repeated bids, often within a few dollars or cents of each other, and then become inactive long before the price approaches a reasonable level. The question of intra-auction dynamics is broad, complicated, and beyond the scope of this work. In our case, inter-auction dynamics are the primary concern for answering our research questions on allocative efficiency and market design. To deal with observed early low bids, we adopt the approach of Bajari and Hortaçsu (2003) by partitioning individual auctions into two stages. During the first phase bidders may submit cheap-talk bids that are viewed as uninformative of the final bids or the final sale price. The second stage is treated as a sealed-bid auction as per our model of Sections 2 and 3. Finally, consistent with the previous section, the ordering of bidders' submission times is assumed to be random.

This requires us to take a stand on differentiating between bids that are a meaningful part of competition and those that are superfluous. We define a *serious bid* as one that affects the price path within the second stage. Note that our definition of serious bidding also counts the top two submissions from the first stage of the auction as these bids fix the price at the start of the second stage. This allows us to avoid drawing too sharp a distinction between the two stages of the auction, since some serious bidders' submission times may still occur early in the life of the auction. Likewise, a *serious bidder* is one who is observed to submit at least one serious bid. Of course, the possibility always exists that some bidders who are determined to be non-serious by the above criterion had intent to compete for the item, but were priced out before submitting their planned, serious bid during the terminal stage. This is, however, part of the problem that our model of the adjusted filter process solves (*i.e.* observed participation by serious bidders is a lower bound on actual participation).

We specify the terminal period as the last 60 min of an auction, during which we see an average of 4.01 observed serious bidders per auction. Figure 3 shows the empirical distribution of time remaining when the winning bid was submitted, which occurs within the final 60 min in roughly 85% of auctions in the sample. The figure also shows the empirical distribution for time remaining across all serious bid submissions in the sample. These figures are not sensitive to alternate specifications of the terminal period cutoff. If it is chosen as 80 min the mean number

TABLE 1
Descriptive statistics

Variable	Mean	Median	St. Dev.	Min	Max	# Obs
Time remaining (minutes)						
Winning bid submission:	111.46	1.85	638.21	0.00	9,888.30	1,461
High loser bid submission:	46.85	0.50	332.53	0.00	9,728.20	1,398
Observed participation						
$\tilde{N}$ (serious bidders only):	4.01	4	1.82	0	12	1,463
Monetary outcomes						
Sale price:	$124.92	$125.00	$17.80	$66.00	$190.00	1,461
Highest losing bid:	$123.80	$124.50	$17.41	$65.00	$189.50	1,398
Seller reserve price:	$33.58	$0.99	$45.26	$0.99	$175.00	1,463

of serious bidders becomes 4.25, and if it is chosen as 40 min the mean number of serious bidders becomes 3.67.

One challenge remains: bidders may choose to submit their strategic bid to the server and make use of eBay's automated proxy bidding or incrementally raise their bid manually to the level of their strategic bid. Roughly one-third of serious bidders are observed to engage in incremental bidding. Because it is difficult to interpret gradually changing bids from a particular buyer, we assume only that the highest losing bid is fully reflective of equilibrium play. This leaves us with the three data points from each auction needed for identification: $\tilde{k}_l$, the observed number of serious bidders; r_l, the seller's reserve price; and y_l, the highest loser bid from auctions with at least two bids. After dropping `.html` pages for which our software was unable to extract data because of formatting problems, we have 1,463 total auctions, 2 of which logged no bids, and 1,398 of which had 2 or more observed bidders (so that we observed a highest losing bid). Table 1 displays descriptive statistics on bid timing, observed participation, sale prices, and highest losing bids.

4.2. *Choosing δ*

The final free parameter is the time discount factor, δ. As in many other empirical contexts, this part poses a difficult challenge. Luckily, δ does not enter Stage I estimation, so all of the necessary building blocks to compute the final structural primitives will be unaffected. Several Stage II objects are also unaffected, including the win probability $\chi(\cdot)$, the expected payment function $\rho(\cdot)$, the per-period participation cost $\hat{\kappa}$, and the exogenous, per-period measure of new agents flowing into the market $\hat{\mu}$. However, the remaining objects depend on δ. There is an intuitive reason why: $\beta(\cdot)$, $\mathcal{V}(\cdot)$, $F_V(\cdot)$, and $T_V(\cdot)$ are influenced by the opportunity cost of losing today, and δ plays a pivotal role in shaping agents' attitude toward present versus future consumption.

In lieu of taking a stand on the particular value of δ applicable to our study, we present results both here and in our counterfactual section for a range of values of δ. Where possible, we provide statistics that are stable across choices of δ. For example, instead of providing a dollar value for deadweight loss, which is sensitive to δ, we present deadweight loss as a percentage of the buyer values, which is stable across different choices of δ.

4.3. *Estimates*

Table 2 displays point estimates and standard errors for the market tightness parameters, the per-period participation cost, and the per-period measure of new agents. Figure 4 depicts point estimates for the empirical CDF of observed bidders per auction (thick, dashed line), the

TABLE 2
Estimation results

Variable:	λ_1	λ_2	κ	μ
Point estimate:	5.9100	0.2579	0.0654	0.9649
Standard error:	(0.384)	(0.058)	(0.0174)	(0.0261)

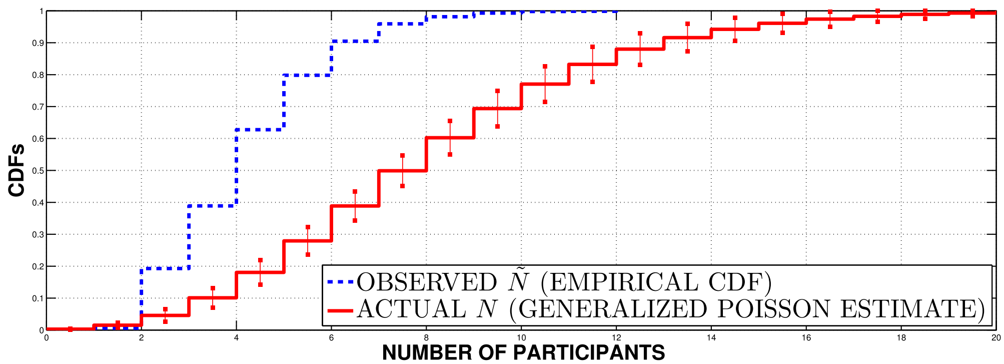

FIGURE 4
Matching process estimates

estimated distribution of total auction-level participation K (thick, solid line), and 95% pointwise confidence bounds (vertical box plots). As the figure demonstrates, failing to account for unobserved bidders would lead to a very different view of the distribution of auction participation. This substantial difference shows up in both the mean—4.07 for observed bidders per auction versus 7.96 for actual bidders per auction—and also in the variance—3.19 for observed bidders and 14.46 for actual bidders. Note also that the point estimate $\widehat{\lambda_2} > 0$ and is highly statistically significant, with a p-value of 8.7×10^{-6}. Thus, our estimates firmly reject the null hypothesis that π is standard Poisson. Supplementary Appendix D.1 contains similar plots of point estimates for $\hat{G}_B(b;\boldsymbol{\alpha}_b)$ and $\hat{G}_R(r;\boldsymbol{\alpha}_r)$.

Figure 5 presents the dynamic inverse bid functions $\hat{\beta}^{-1}\left(b;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b,\hat{\boldsymbol{\alpha}}_r,\delta\right)$ which we estimate for a uniform grid of values of the time discount factor δ between 0.75 and 0.98. We also include an additional value at 0.8871 taken from an experimental study by Augenblick et al. (2017) where they elicited hyperbolic time discount parameters at the daily level from college students.[28] Recall from Figure 1 that the vast majority of demand shading is driven by the option value of returning to the market in future periods if one does not win today.[29] Continuation value is primarily determined by three things: the equilibrium bid distribution G_B, the market tightness parameters $\boldsymbol{\lambda}$, and the discount factor δ. Figure 5 depicts the important role of this third piece.

Table 3 displays various descriptive statistics derived from Stage II estimates, including average private values, average private values of winners, and information rents (*i.e.* the difference

28. Another study by Burks et al. (2012) elicited daily time discounting preferences in a field experiment using real monetary incentives, and found a mean daily discount factor of 0.8921.

29. To our knowledge, ours is the first paper to make such a comparison in the context of platform markets. Procurement models also include static and dynamic incentives, although the differences in the setting (*e.g.* time-varying cost types) make a direct comparison difficult. Jofre-Bonet and Pesendorfer (2003) estimate that just over half of bid shading is due to intertemporal incentives.

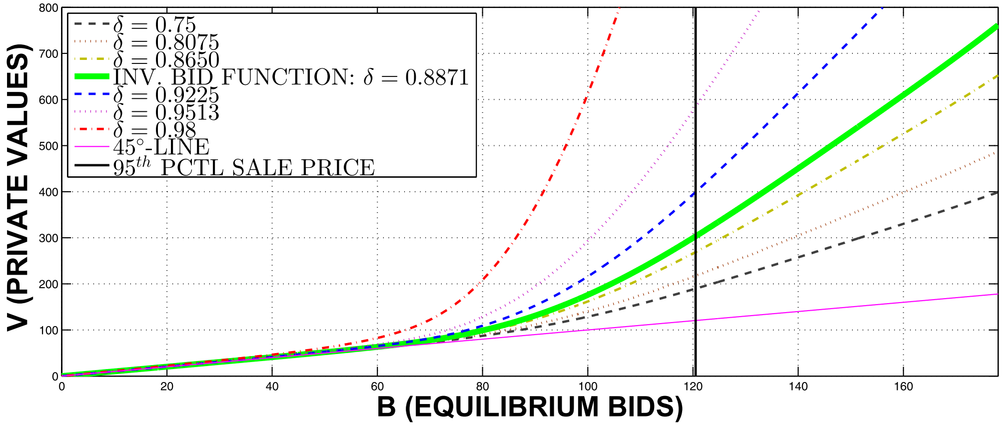

FIGURE 5

Inverse bid function estimates given various values of δ

TABLE 3

Mean private values and information rents for various δ

Discount factor δ:	**0.75**	**0.81**	**0.87**	**0.8871**	**0.93**	**0.95**	**0.98**
Mean private value:	\$48.57	\$51.29	\$56.32	\$59.63	\$68.83	\$86.15	\$153.24
Mean winner private value:	\$208.39	\$230.98	\$269.26	\$293.58	\$358.55	\$474.05	\$875.56
Mean winner information rent:	\$54.66	\$69.11	\$94.08	\$111.09	\$157.44	\$245.84	\$583.91
Mean information rent percentage:	26.23	29.92	34.94	37.84	43.91	51.86	66.69

between the winner's private value and the spot-market price). The last row of the table shows information rents as a fraction of the winner's private value, on average.

Figure 6 presents other Stage II estimates related to the distribution of buyer values. The upper pane displays the PDF of the distribution of market participants' private values in steady state under our preferred specification, $\hat{f}_V(v;\boldsymbol{\alpha}_v,\delta=0.8871)$ (dash-dot line), as well as the type distribution for new market entrants each period, $\hat{t}_V\left(v;\hat{\boldsymbol{\lambda}},\hat{\boldsymbol{\alpha}}_b;\hat{\boldsymbol{\alpha}}_r,\delta=0.8871\right)$ (solid line), with 95% point-wise confidence bounds (vertical box plots). The PDFs t_V and f_V are tied together by the win probability, χ. Since f_V depicts the type distribution for all market participants—including players remaining from previous periods—it represents a measure $\lambda_1/(1-\lambda_2)=7.96$ of agents (recall that sellers are assumed to have measure 1). On the other hand, t_V describes the type distribution of the measure $\mu=0.9649$ of new agents that enter the market each period in order to maintain the steady state. Under f_V there is a larger mass of low-value bidders who are less likely to win, and therefore they pile up in the market and remain for many periods until finally winning. t_V is a selected set of buyers who move in and out of the market at much higher frequency: they have higher private values and are much more likely to win in the spot-market in a given period.

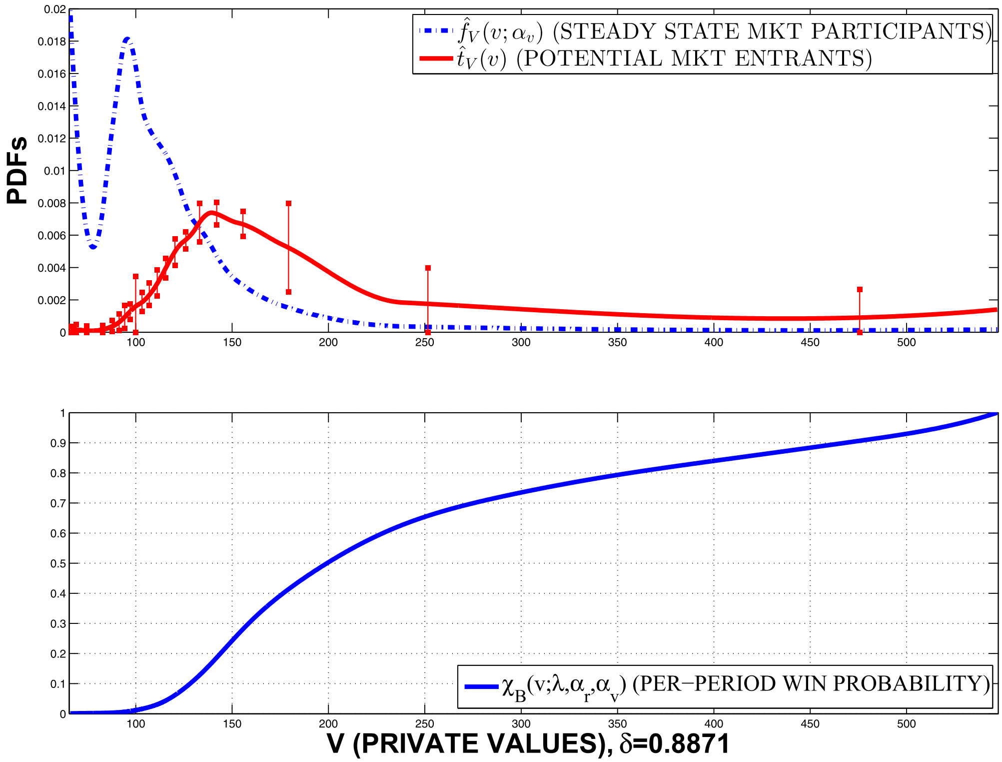

FIGURE 6

Distribution of private values (top panel) and win probability (bottom panel)

5. WELFARE COUNTERFACTUALS

We now investigate the welfare implications of our model. First we explore market efficiency and revenue. We then analyse the importance of controlling for buyer type distribution when conducting these exercises, which would be neglected in a static analysis of the market. The structural primitives of our model being held fixed in the counterfactual scenarios are μ, t_V, κ, G_R, λ_2, and the space of types $[\underline{v},\overline{v}]$. We describe how the counterfactual equilibria are computed and how we test for uniqueness in Supplementary Appendix B.

We adopt the usual notion of efficiency as the tendency for goods to be allocated to those who value them most within a given period. Even when the spot-market mechanism is efficient within an auction, auction platforms with search frictions still exhibit two related sources of inefficiency. First, there is the chance that a high-value buyer that ought to receive the good in an efficient allocation is competing against another high-value buyer, so one of them cannot receive the good. Second, an auction may fail to attract any high-value buyers, which means a low-value buyer will receive the good when she would not under an efficient outcome. The first case is one in which there is "too much" competition within the auction, while the second case is one in which there is "too little" competition.

5.1. *"Model anaemic" inefficiency calculations*

In this section, we use our Stage I estimates to bound the percentage of auctions resulting in an inefficient sale. We refer to these calculations as "model anaemic" since they do not rely on our equilibrium bidding model and thereby employ the fewest possible assumptions. These calculations rely only on our filter process model to correct for sample selection in the observed number of bidders.

To proceed, we first find the cutoff v_{eff} that separates high-value buyers that ought to receive the good in an efficient allocation from lower-value buyers that ought not. Since the buyer–seller ratio is $\lambda_1/(1-\lambda_2)$, the efficient cutoff in private value space is $v_{eff} \equiv F_V^{-1}\left(1-\frac{1-\lambda_2}{\lambda_1}\right)$. However, since quantile orderings are invariant to monotone transformations, we can re-define this cutoff in bid space (where the raw data live) as $b_{\text{eff}} \equiv G_B^{-1}\left(1-\frac{1-\lambda_2}{\lambda_1}\right)$. If the highest losing bid in a given auction exceeds b_{eff}, then the corresponding bidder would receive the good in an efficient allocation but does not win the item this period. We find that 28.47% of the auctions in our sample satisfy this criterion. For each high-value bidder who loses an auction there is a low-value bidder in some other auction who inefficiently wins, so high-value buyers losing and low-value buyers winning are two sides of the same coin. This measure is a lower bound on the frequency of inefficiency because we cannot account for auctions where two or more losing bids surpassed b_{eff} without observing all of the bids. Another disadvantage of the model-anaemic approach is that it offers no way of measuring the magnitude of unrealized gains from trade.

5.2. *Structural welfare calculations*

Our full Stage II structural estimates allow us to get a more complete idea of the frequency and magnitude of market inefficiency. Using equation 6, we can compute the fraction of all inefficient transactions: $\Pr[v_{\text{winner}} < v_{\text{eff}}] = \mathcal{C}\int_{\underline{v}}^{v_{\text{eff}}} \chi(\beta(s)) f_V(s)ds$. We find that 35.89% of Kindle auctions end with an inefficient outcome. Deadweight loss calculations in levels will be sensitive to the choice of δ. To address this problem, we adopt the following measure, which we refer to as the *efficiency ratio*:

$$\mathcal{E}_{u,\delta} = \frac{\mathcal{C}\int_{\underline{v}}^{\overline{v}} s\chi_u(\beta_u(s)) f_{V,u}(s)ds}{\mathcal{C}\int_{v_{eff}}^{\overline{v}} sf_{V,u}(s)ds}.$$

The numerator is the realized gains from trade in our market (within a given period), and the denominator represents gains from trade generated by a fully efficient allocation. The u subscript denotes number of units involved in each auction listing for our counterfactual centralization analysis below; for now we fix $u=1$. By expressing surplus as a fraction of total possible surplus, the separate influences of δ in the numerator and denominator largely cancel out and we get a measure that is stable with respect to different choices of δ (see Table 4). We also compute the efficiency ratio under a hypothetical lottery system, denoted $\mathcal{E}_{\text{lott},\delta}$, as the minimum efficiency benchmark (see the last row of Table 4).

Our point estimates imply that the fraction of total deadweight loss is $1-\mathcal{E}_{1,0.8871}=0.135$ under our preferred specification. To put this number into context, deadweight loss under a lottery system is estimated to be $1-\mathcal{E}_{\text{lott},0.8871}=0.53$, meaning that eBay's auction market platform achieves 76% of total gains from trade above the lottery benchmark. Note, however, that this is only a "partial equilibrium" assessment; were a social planner with complete knowledge of the bidder values to implement the efficient allocation each period, then the steady-state distribution of buyers' values and the buyer–seller ratio would change. However, we believe our figures have the benefit of giving a sense of the welfare losses while imposing minimal structural assumptions on the estimates.

TABLE 4
Counterfactual efficiency ratios $\mathcal{E}_{u,\delta}$

No. of units per listing	Discount factor $\delta=$ 0.75	0.80	0.86	0.88	0.92	0.95	0.98
1	0.89	0.88	0.87	0.86	0.85	0.84	0.82
2	0.92	0.92	0.91	0.91	0.91	0.90	0.89
4	0.94	0.94	0.94	0.94	0.94	0.93	0.93
8	0.95	0.95	0.95	0.95	0.95	0.95	0.95
Lottery	0.58	0.54	0.49	0.47	0.41	0.35	0.26

5.3. *Counterfactual market centralization*

We now consider the extent to which inefficiencies can be mitigated by changing the market structure to one in which the same number of Kindles are allocated each period, but using fewer u-unit, uniform-price auctions with $u \geq 2$. Since new Kindles are relatively homogeneous, we think it is reasonable to assume that buyers view them as near perfect substitutes; therefore, considering multi-unit auctions is reasonable. For products that are not perfect substitutes (*e.g.* used cars), the implications of selling disparate products in a multi-unit auction become much more difficult to formalize. However, our estimates provide a sense of the efficiency loss due to search frictions when selling items through decentralized, single-unit auctions.

Several aspects of our model need to be adjusted in the multi-unit auction setting. We subscript the endogenous quantities with u,δ to denote the degree of centralization and the time discount factor. First, each u-unit auction attracts a number of bidders K_u distributed as a generalized Poisson random variable where

$$\mathrm{E}[K_u]=\frac{\lambda_{1,u}}{1-\lambda_2}=u\mathcal{C}. \tag{35}$$

Equation 35 mandates that the (endogenous) measure $\mathcal{C}$ of bidders be exactly assigned to the $1/u$ measure of u-unit auctions, much as in part 1 of Assumption 2.1. We assume λ_2, the dispersion parameter, is fixed at the estimated value and allow $\lambda_{1,u}$, the size parameter, to adjust so equation (35) is satisfied in our counterfactuals.

In our status quo model, we assume each seller draws an independent starting price from G_R. For comparability to the status quo, we assume that a single starting price is drawn from G_R for each u-unit auction, and that starting price applies to all u units being allocated in that auction. Each bidder submits a bid to the auction to which she is matched, and the u highest bids that are larger than the auction's starting price win an item. Each winning bidder then pays a sum equal to the larger of the $(u+1)$th highest bid and the starting price. If $v=\beta^{-1}(b)$, the probability of winning in a u-unit auction is:

$$\chi_u(b)=G_R(b)\left[\sum_{m=0}^{u-1}\pi_M(m)+\sum_{m=u}^{\infty}\pi_M(m)\sum_{i=0}^{u-2}\binom{m}{m-i}F_V(v)^{m-i}(1-F_V(v))^i\right]. \tag{36}$$

One of the general takeaways from our research is that understanding the impact of platform market design on participation decisions is crucial. The social planner's welfare calculus will be strongly influenced by changes in entry behaviour (*e.g.* how many low-value buyers leave the market?) and the steady state-distribution of private values for market participants (*e.g.* how many low-value bidders accumulate in the market when they are less likely to win an item?). We find that e increases as the market becomes more centralized (*i.e.* as u grows) since player types with very low values will see a decrease in their probability of winning as market allocations become more efficient.

TABLE 5
Counterfactual mean auction revenues

No. of units per listing	**Discount factor $\delta=$** 0.75	0.80	0.86	0.88	0.92	0.95	0.98
1	\$115.05	\$114.90	\$114.71	\$114.59	\$114.37	\$114.10	\$113.65
2	\$112.79	\$112.33	\$112.97	\$112.72	\$112.26	\$111.96	\$112.28
4	\$111.73	\$111.21	\$111.09	\$110.84	\$110.40	\$110.31	\$111.30
8	\$110.05	\$109.54	\$109.12	\$108.94	\$108.64	\$108.81	\$109.75

Table 4 provides results for counterfactual efficiency ratio statistics for $u \in \{1,2,4,8\}$. Recall from above that the efficiency ratio compares gains from trade in a single period of a u-unit model with the welfare generated by the efficient allocation from clearing the market once per period with a single, large, multi-unit, uniform-price auction. Note that the efficiency ratios are remarkably stable across different specifications of the time discount factor δ. Also, the majority of possible gains from centralization can be realized by 2- or 4-unit uniform-price auctions, so there is little need to shift to a fully centralized market.

One might expect that if eBay could re-design their platform market to increase allocative efficiency, then it should be able to benefit by capturing some of the increased gains from trade.[30] From a static perspective of a single auction, the revenue equivalence theorem implies that the choice of payment rule has no effect on the revenues of the seller. If the revenues change with the altered auction format it must be due to a combination of (1) the new allocation rule and (2) the change in the measure and value distribution of the bidders. An examination of the moving parts within the model indicates that the sign of these effects on revenue is ambiguous. On the one hand, a bidder with a value above the new participation cutoff e_u faces fewer competitors in the market. On the other hand, her remaining competitors also value the object more highly on average. This combination of effects make it difficult to derive a priori predictions on bidding behaviour and revenue. Table 5 demonstrates that the average sale price actually falls as u increases.

To explain why revenue drops as efficiency rises, Figure 7 plots the probability of winning and equilibrium bids for each agent type in the $u=1$ (solid line) and $u=8$ (dashed line) cases. The left panel reveals that for most agents (especially those most likely to win), greater efficiency raises the probability that they will win in a given period. This raises their continuation values, and therefore the opportunity cost of winning an auction today. This in turn promotes further demand shading as shown in the second panel of Figure 7. Reduced bids translate into decreased lower for both sellers and eBay, which currently charges sellers a percentage commission on auction revenue. This highlights an interesting point: what is good for market welfare is not necessarily good for platform market designers like eBay.

In Supplementary Appendix F.2, we analyse the effect of centralization on the participation costs paid by the agents. Centralizing from single-unit auctions to 8-unit auctions reduces the participation costs paid by roughly 60%.

5.4. *The importance of platform composition*

One of the main findings of Section 5.3 is that the revenue of the sellers decreases as the efficiency of the within-period allocation increases. In this section, we try to understand whether these effects are due more to the change in the spot-market mechanism or to the change in the composition

30. There are examples where this general intuition does not hold; for example, the literature on optimal auctions suggests that efficiency-reducing reservation prices can increase revenue.

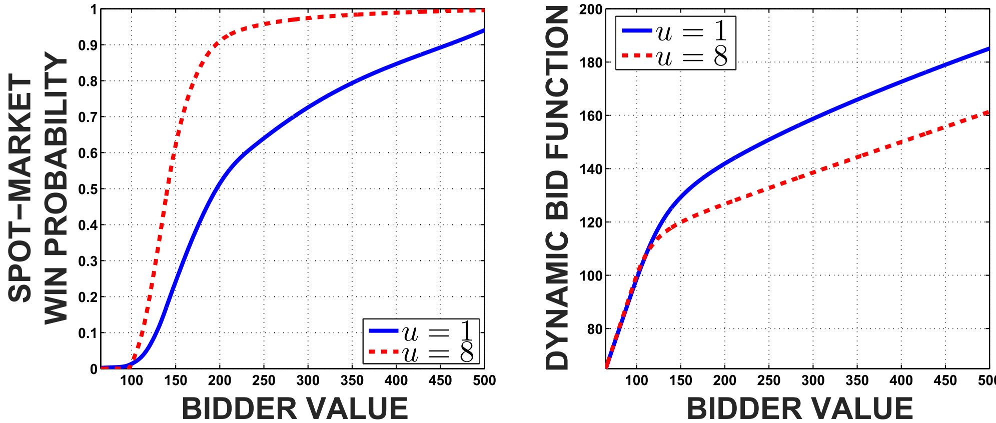

FIGURE 7

The efficiency-revenue link

of the pool of buyers using the market. In order to tease apart these effects, we recompute the welfare and revenue statistics holding the measure and type distribution of the buyers fixed at the status quo values.[31] The most natural interpretation of our exercise is that we are studying how our predictions would differ had we used a mis-specified model that assumed the measure and type-distribution of the agents are exogenous objects. As we will see, assuming the population of buyers is exogenous will lead us to over-estimate the efficiency gains from centralization. More surprisingly, we will find the mis-specified model would predict a sharp *increase* in auction revenues, which is the opposite of what the true model predicts. The primary take-away from our exercise is that it is crucial to account for the endogenous changes to the population of buyers when producing counterfactuals.

We now step through the changes to the endogenous objects (*i.e.* $\chi_u(b)$, $\mathcal{V}(v)$, and $\beta(v)$) when evaluating the centralization counterfactual holding F_V and C fixed at the status-quo values. The left pane of Figure 8 plots $\chi_1(b)$ and $\chi_8(b)$ for $\delta = 0.88$.[32]

The central pane of Figure 8 displays the value function, which is uniformly *lower* when $u = 8$ than when $u = 1$, although the difference is small for the highest value buyers. This contrasts with the results of Section 5.3 wherein the value function for the agents that remain in the market increases as u rises. The decrease in the value function causes the bids, which are displayed in the right pane of Figure 8, to increase as u increases.

If we had accounted for the changes in F_V and C that centralization causes (as in Section 5.3), we would predict that the number of buyers (*i.e.* C) drops and, in particular, the high-value buyers leave the platform quickly. The first effect reduces competition for all of the buyers, and the second effect reduces the number of high-value buyers and mutes competition between them. In contrast, there is a large population of high-value buyers in the status quo F_V. Even for buyers with very high values, they are less likely to win the good and, when they do win, the price is typically set by the bid of another high-value buyer. In other words, the high-value buyers win less frequently and pay higher prices. These effects depress the value function of all of the agents.

The out-of-equilibrium efficiency ratios and revenues (Table 6) are both higher than in equilibrium due to two effects. First, the status quo type distribution has a large population of

31. All of the analysis in this section is out-of-equilibrium since we are not accounting for how altering the auction format changes the steady-state measure and type distribution of the buyers.

32. The results are qualitatively similar for other choices of δ.

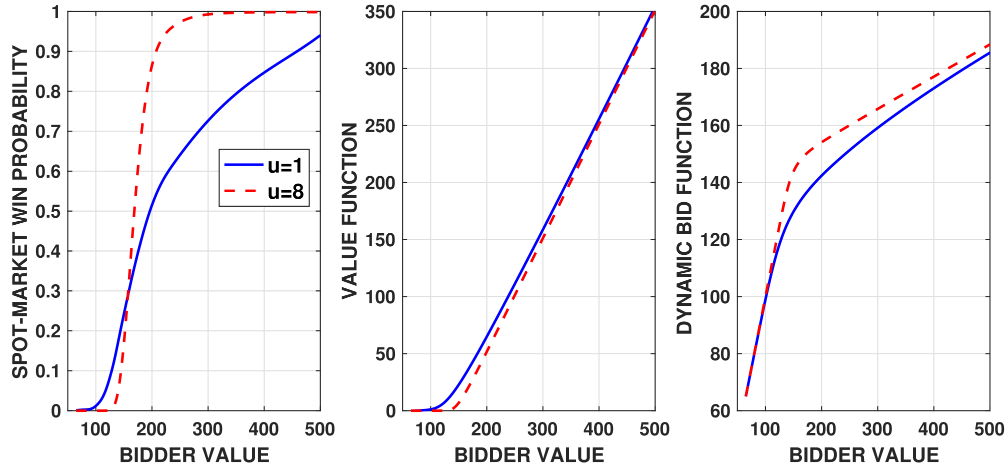

FIGURE 8

Out-of-equilibrium outcomes

TABLE 6

Mis-specified counterfactual results

Efficiency ratios, $\mathcal{E}_{u,\delta}$							
No. of units per listing	Discount factor $\delta=$ **0.75**	**0.80**	**0.86**	**0.88**	**0.92**	**0.95**	**0.98**
1	0.89	0.88	0.87	0.87	0.85	0.84	0.82
2	0.94	0.94	0.93	0.93	0.92	0.92	0.91
4	0.97	0.97	0.96	0.96	0.96	0.96	0.95
8	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Lottery	0.58	0.55	0.50	0.47	0.41	0.35	0.25

Revenue							
No. of units per listing	Discount factor $\delta=$ **0.75**	**0.80**	**0.86**	**0.88**	**0.92**	**0.95**	**0.98**
1	\$120.83	\$120.77	\$120.68	\$120.63	\$120.52	\$120.37	\$120.06
2	\$129.33	\$130.00	\$131.17	\$131.91	\$133.80	\$136.90	\$145.95
4	\$134.11	\$135.52	\$138.03	\$139.59	\$143.67	\$150.49	\$171.54
8	\$137.11	\$139.23	\$143.00	\$145.36	\$151.60	\$162.29	\$197.46

high-value buyers relative to the equilibrium type distribution for $u=8$. The mis-specified model predicts large welfare gains by more efficiently allocating goods to this group of buyers. Second, the higher bids have the effect of increasing revenue, whereas the lower bids in the $u=8$ SCE depress revenue. In Supplementary Appendix F.1, we explore further the role of platform market composition and its relation to bidding incentives. There we find that in terms of revenue and efficiency, incentives driving selection into the platform are at least as important as the incentives driving behaviour once a buyer participates. We believe these counterfactuals provide practical guidance to eBay and other online market designers regarding what issues are of most importance when considering changes to a platform.

6. CONCLUSION

Our goal has been to provide a model of a dynamic auction platform that is both rich enough to capture the salient features of the market (*e.g.* the large number of auctions concluding each day, the cost of participation) and yet remain tractable enough to facilitate empirical analysis. To accomplish this, we have developed a model with a continuum of buyers and sellers that is easy to estimate and solve, and we show that this model approximates the more realistic setting with a finite number of agents in Supplementary Appendix C. We have also demonstrated that the structural components of this model can be identified from observables that are commonly available from real-world platform markets. In constructing these identification results, we have overcome several important problems including sample selection in the number of observed spot-market competitors and allowing for pricing rules that give rise to static demand shading incentives. Finally, we have also proposed a flexible GMM estimator for the structural primitives.

Most platform markets exist in order to eliminate barriers to trade and allow for buyers and sellers to interact in a relatively low-friction environment. However, the sheer size of the markets may give rise to search frictions which prevent market outcomes from attaining the social optimum. We have estimated our model within the context of the market for Kindle Fire tablets, and we use these estimates both to compute the welfare loss under the present design and to suggest novel designs to mitigate these welfare losses. We begin by providing a "model-anaemic" analysis that relies only on the bid distribution. We find a lower bound of at least 28% of auctions that close with a highest losing bidder whose private value exceeds that of some winner from another auction on that same day.

We then use our structural estimates to compute the deadweight loss and to study alternative spot-market mechanisms that reduce the welfare loss due to search frictions. We find that over 36% of the auctions within a day allocate goods to winners with inefficiently low private values, which causes a 14% welfare loss due to the decentralized nature of the platform. This implies that the single-unit auction market attains three quarters of total possible welfare improvement over a pure lottery system. By taking small steps toward a more centralized market structure—such as running multi-unit, uniform-price auctions with as few as 4 units each—2/3 of the remaining welfare loss can be recovered. However, centralization causes seller revenue to fall.

Our final counterfactual explores the importance of controlling for the composition of the buyers on the platform, which is shaped both by the flow of new buyers into the market, the participation cost, and the choice of spot-market auction format. We recompute our centralization statistics holding the market composition, F_V and C, fixed at the status quo levels. We find that the value function *decreases* as the market becomes more centralized. Therefore, if we fail to account for endogenous changes to F_V and C, we would overestimate the welfare improvements yielded by centralization and we would predict that auctioneer revenues would *increase*.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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