Stable Length Estimate of Tube-like Shapes

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DGCI'13 Seville

Stable Length Estimate of Tube-like Shapes

DCGI '13 Seville

Edelsbrunner & Pausinger

IST Austria

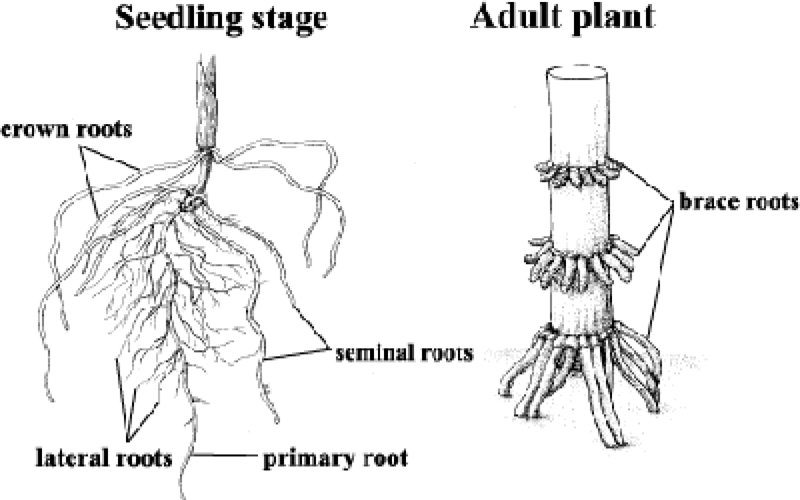
I Motivation

II Length

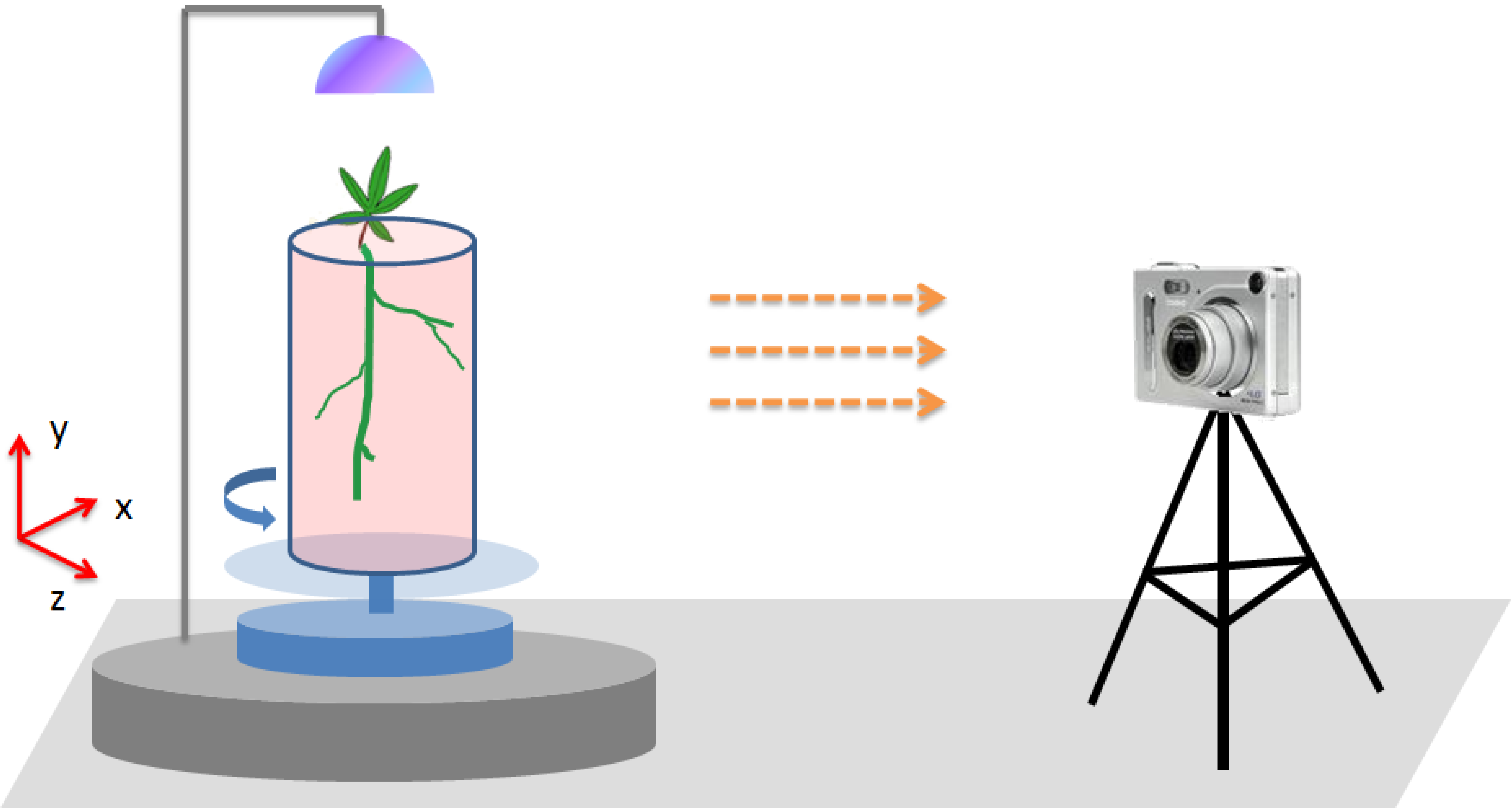
III Persistence

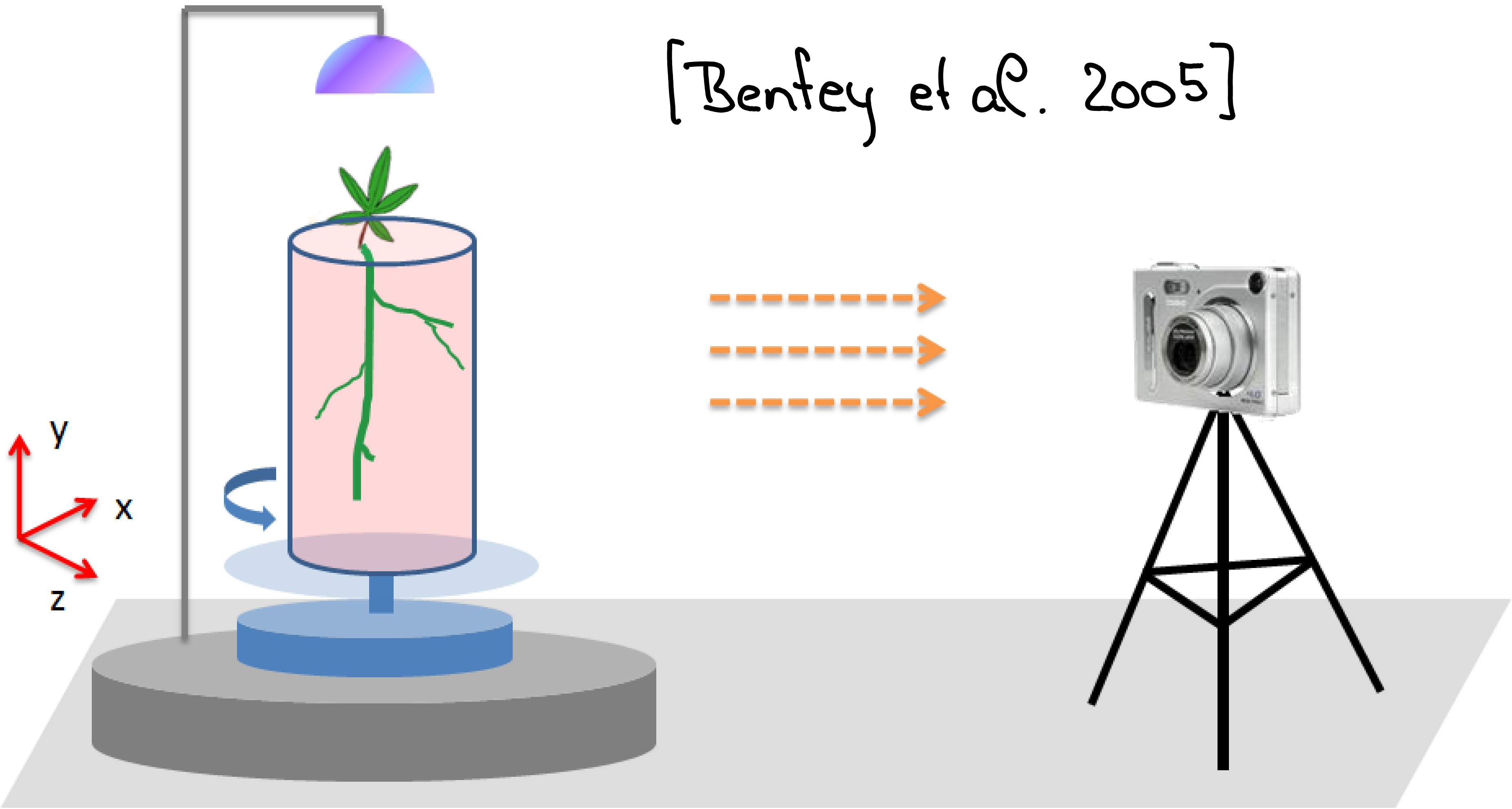
IV Experiments

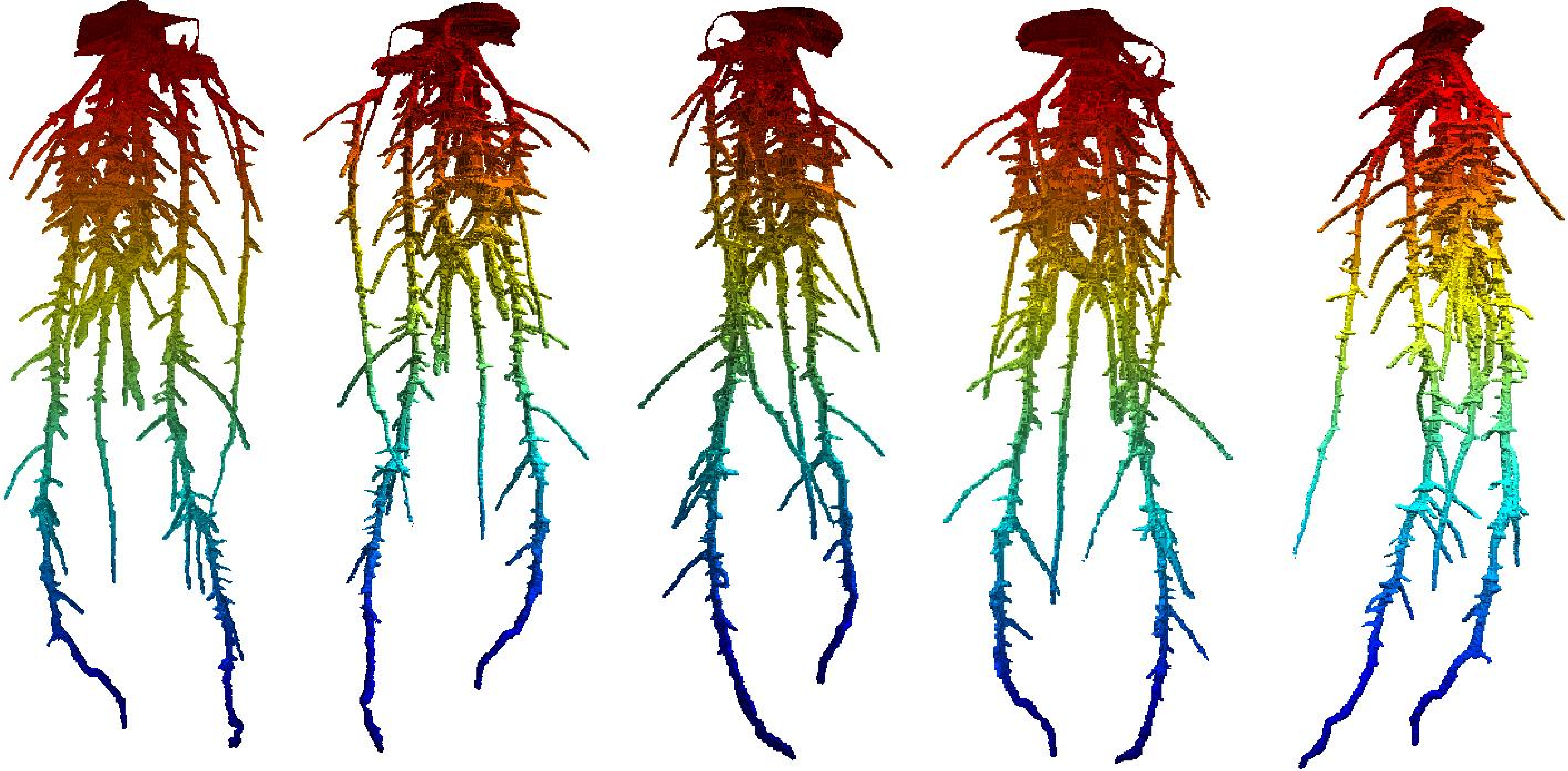
Major root types of maize

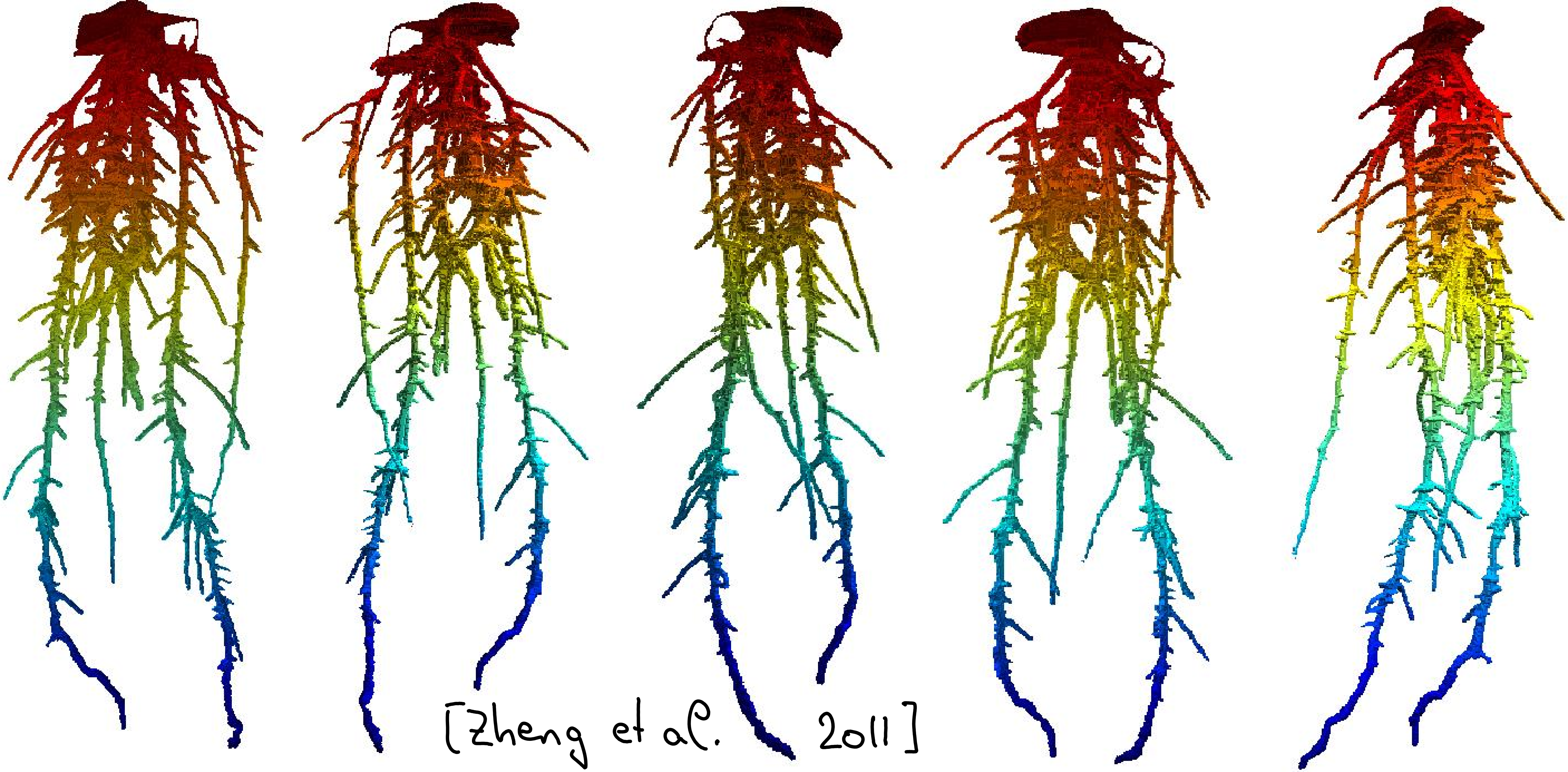


crawings by: Miwa Kojima, Schnable lab, ISU


y
x
z


[Benfey et al. 2005]
y
x
z





[Zheng et al. 2011]

PHENOTYPE - GENOTYPE

Quantify traits that make meaningful distinctions and are sensitive to details.

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Quantify traits that make meaningful distinctions and are sensitive to details.

Today: total length
stable estimate

I MOTIVATION

II LENGTH

III PERSISTENCE

IV EXPERIMENTS

II.1 Buffon's Needle

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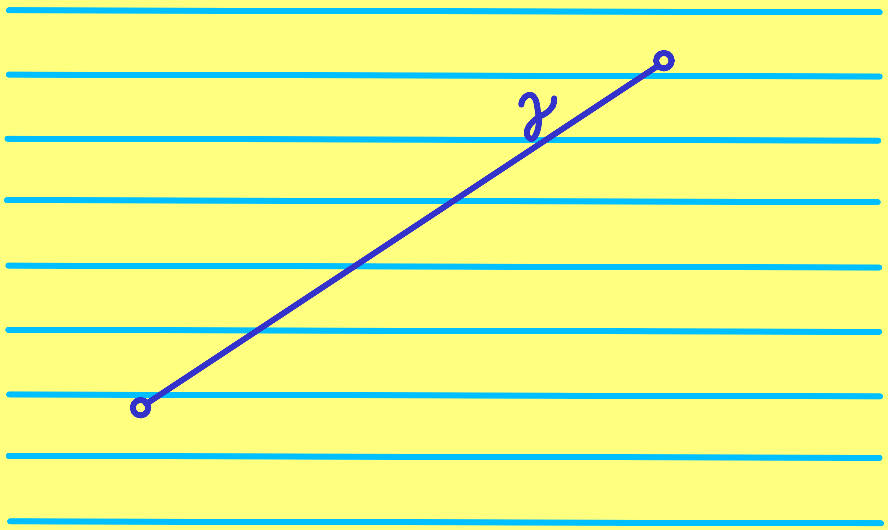

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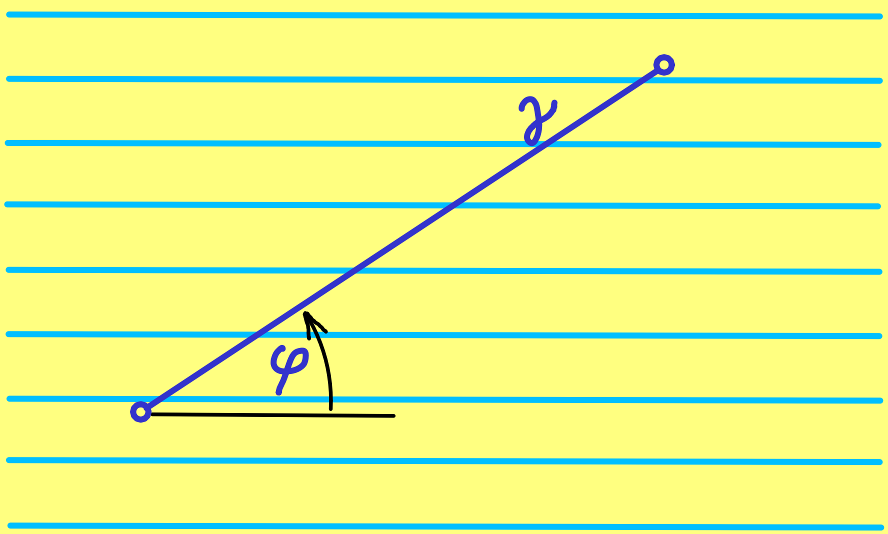

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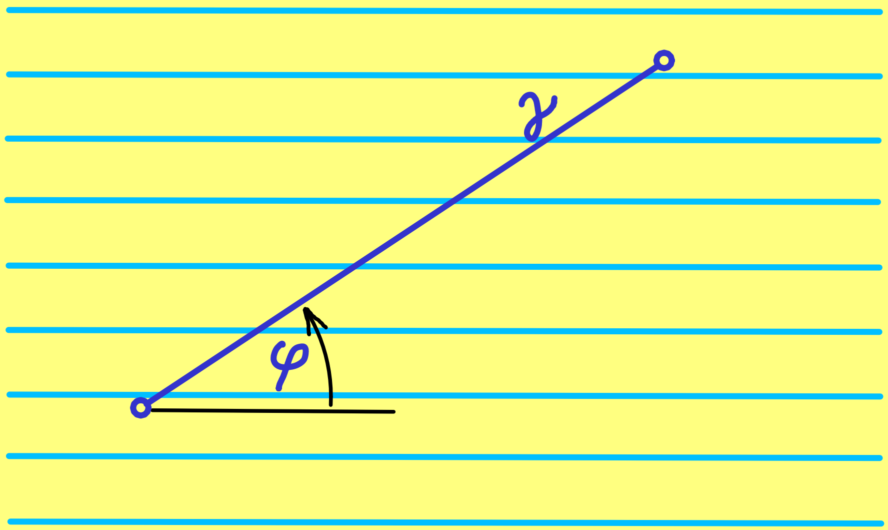

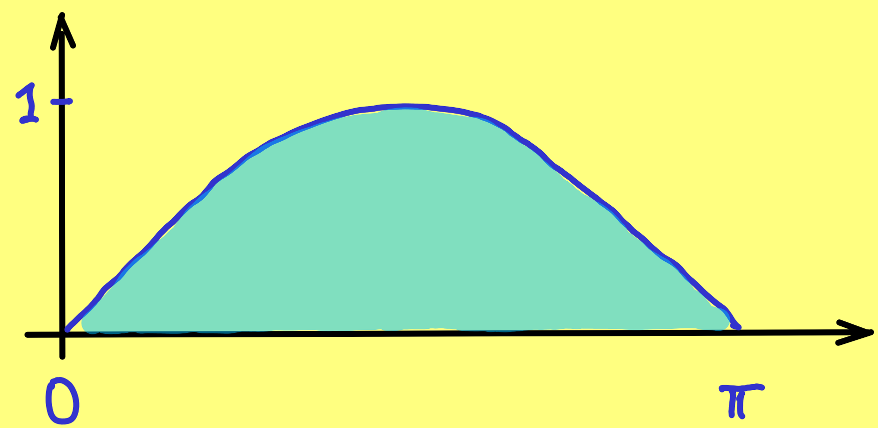

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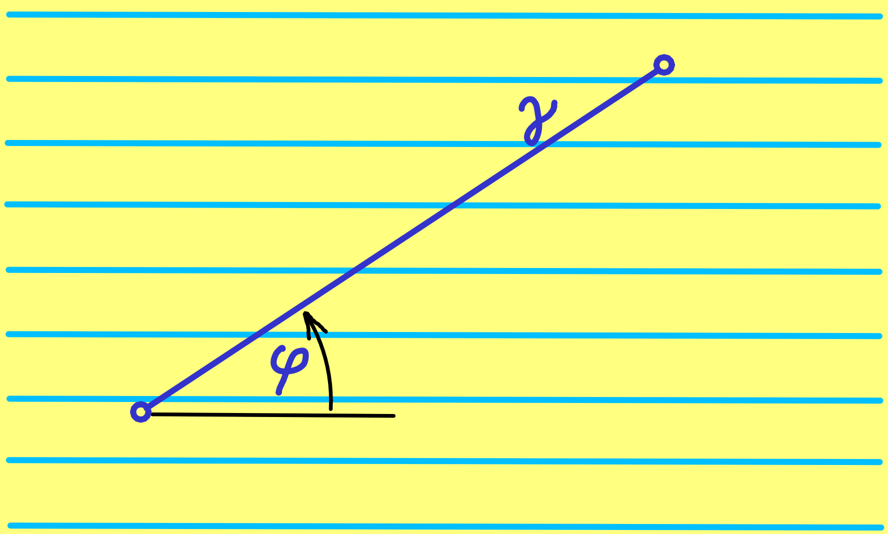

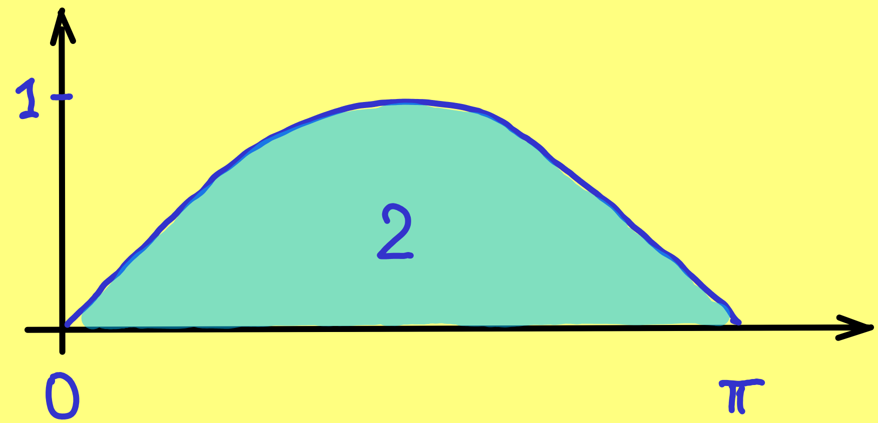

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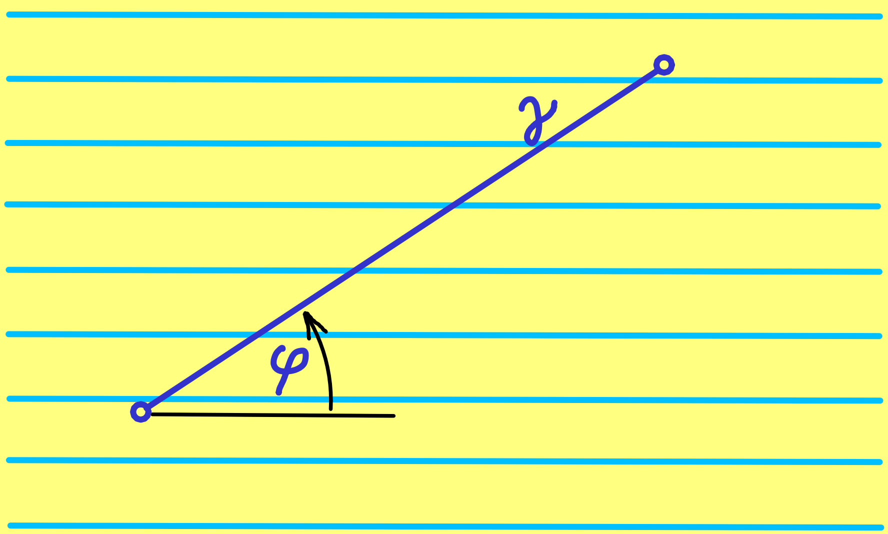
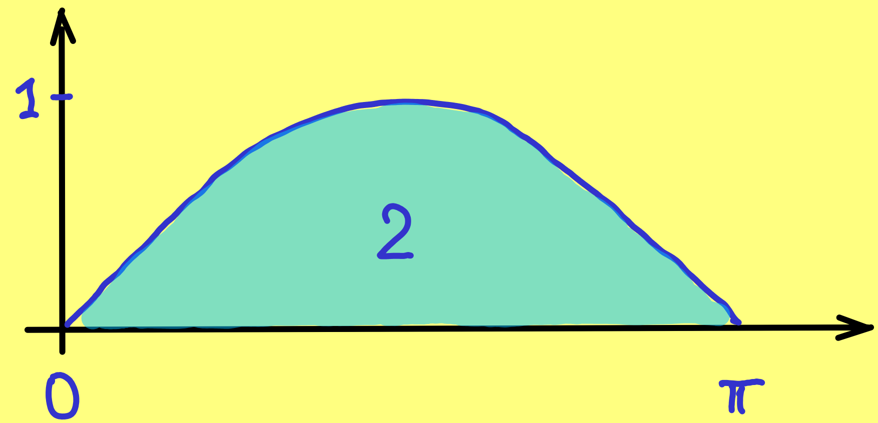
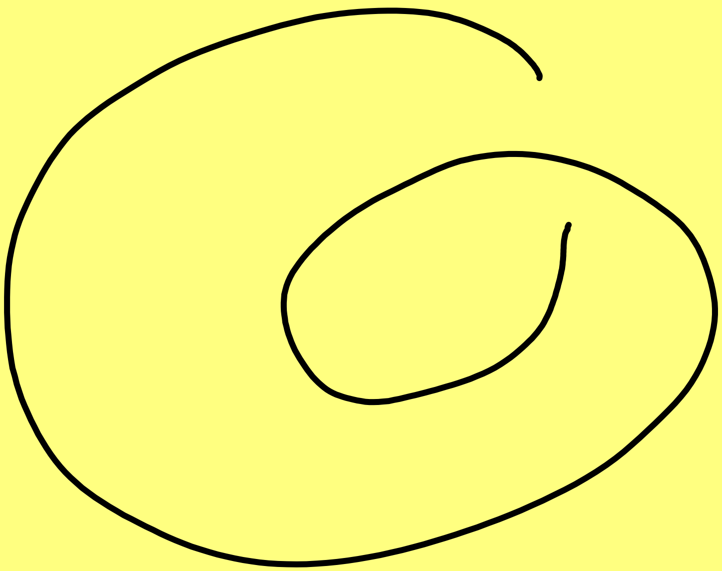




$$\text{length}(\gamma) = \frac{1}{2}\int_{\varphi=0}^{\pi} \text{height}(\gamma,\varphi)\, d\varphi$$

II.2 Tube Volume



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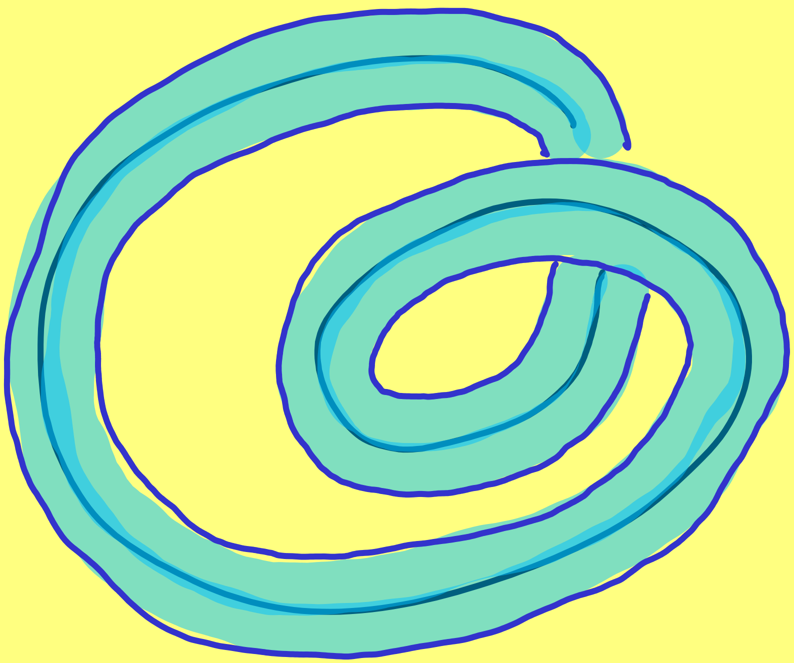

tube $\gamma_{r+\varepsilon}$

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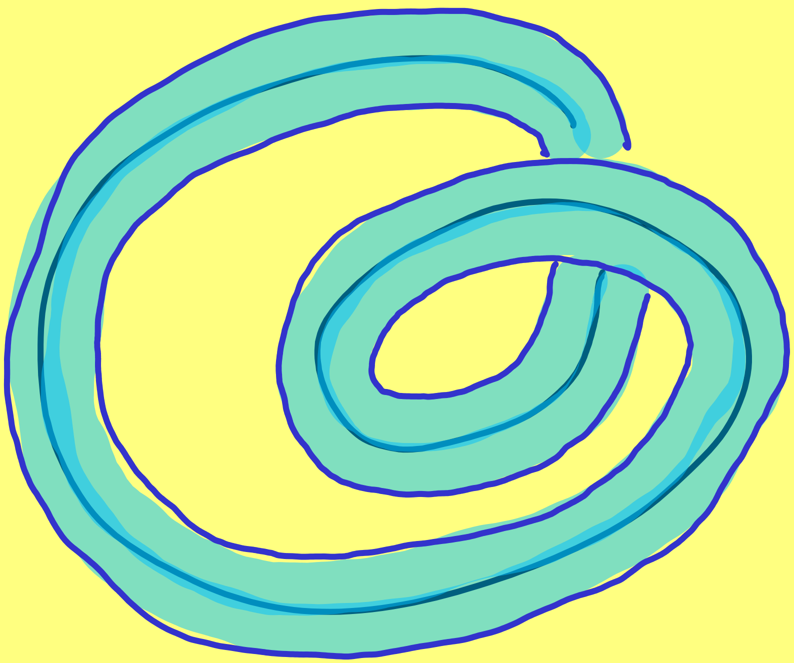

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$L = \text{length}(\gamma)$

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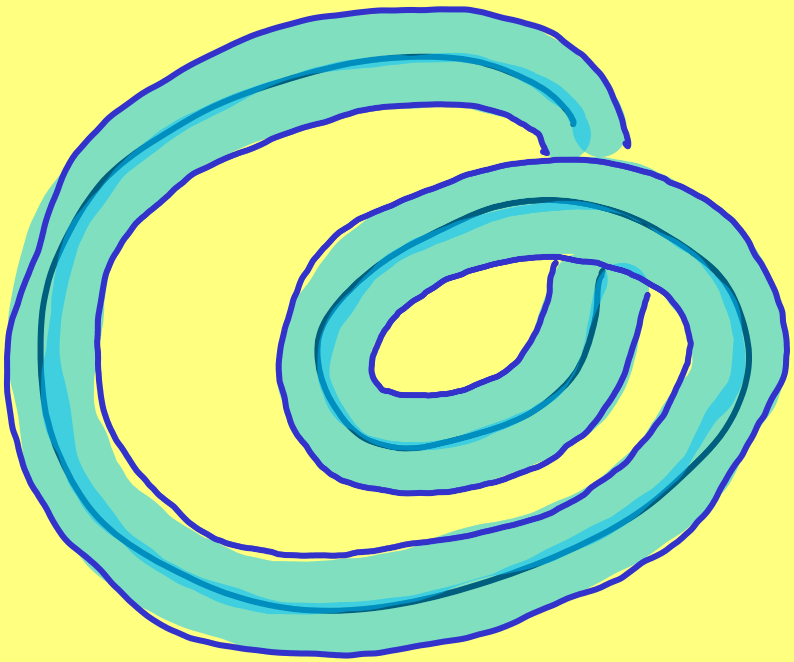
[Weyl 1939]



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$$\text{vol}(\gamma_{r+\varepsilon}) = L \cdot (r+\varepsilon)^2 \pi$$

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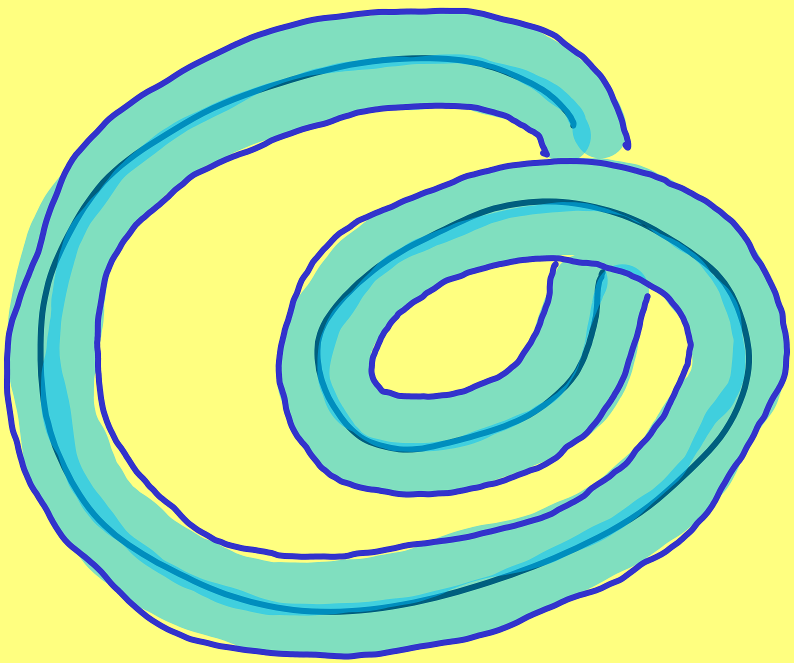

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$$\mathrm{vol}(\gamma_{r+\varepsilon}) = L \cdot (r+\varepsilon)^2 \pi$$
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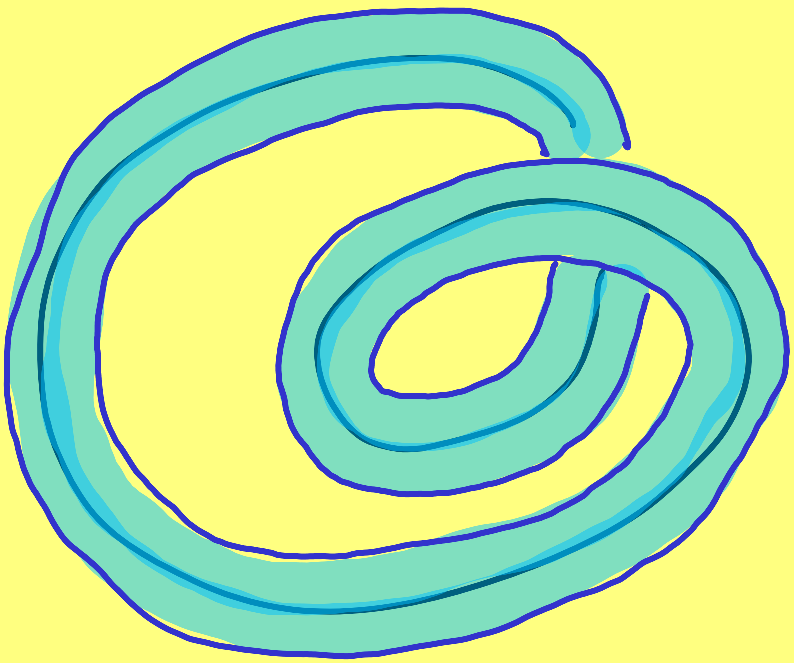

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$$\begin{aligned}\mathrm{vol}(\gamma_{r+\varepsilon}) &= L \cdot (r+\varepsilon)^2 \pi \\ &= L r^2 \pi + 2 L r \pi \varepsilon + L \pi \varepsilon^2 \\ &= Q_0 + Q_1 \varepsilon + Q_2 \varepsilon^2\end{aligned}$$

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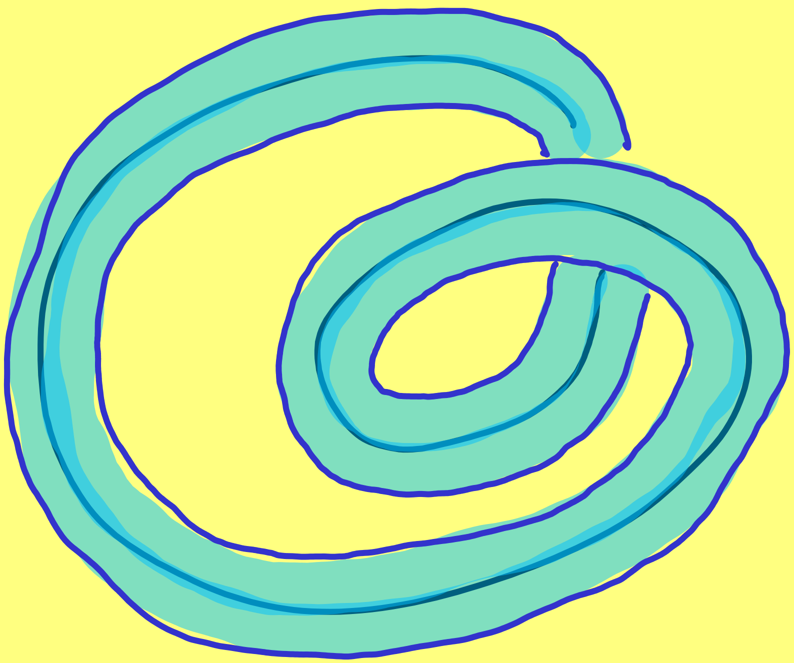

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Q_0 " volume, Q_1 " area, Q_2 " mean curv.

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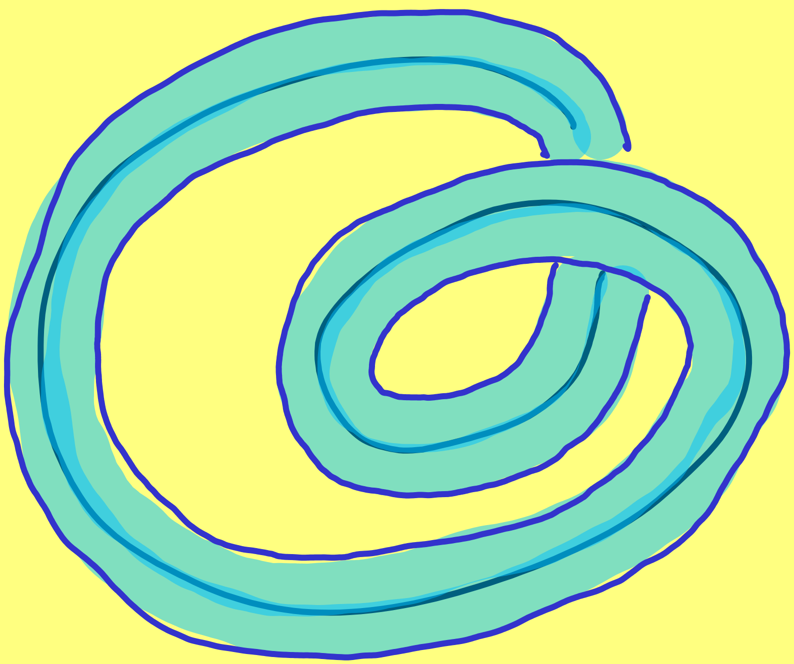

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$\Longrightarrow \quad L = Q_2/\pi.$

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solid shape $\mathbb{M}$ in $\mathbb{R}^3$

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where $$Q_i = c_i \int_{i\text{-planes } P} \chi(P \cap \mathbb{M})\, dP$$

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	$i=0$	$i=1$	$i=2$	$i=3$
Q_i				
$\int$				
c_i				

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c_i	1	$2/\pi$	1	$4\pi/3$

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$$Q_2 = \int_{\text{2-planes } P} \chi(P \cap M)\, dP$$

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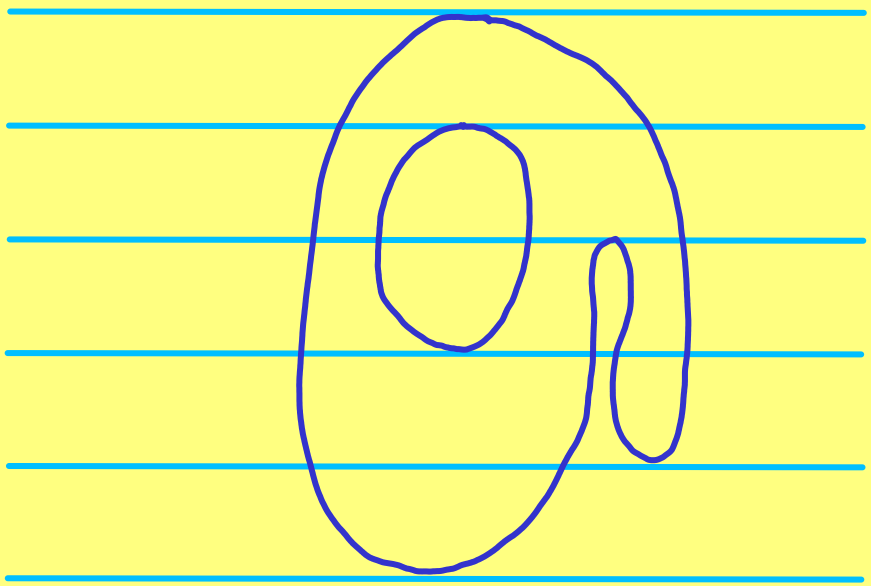

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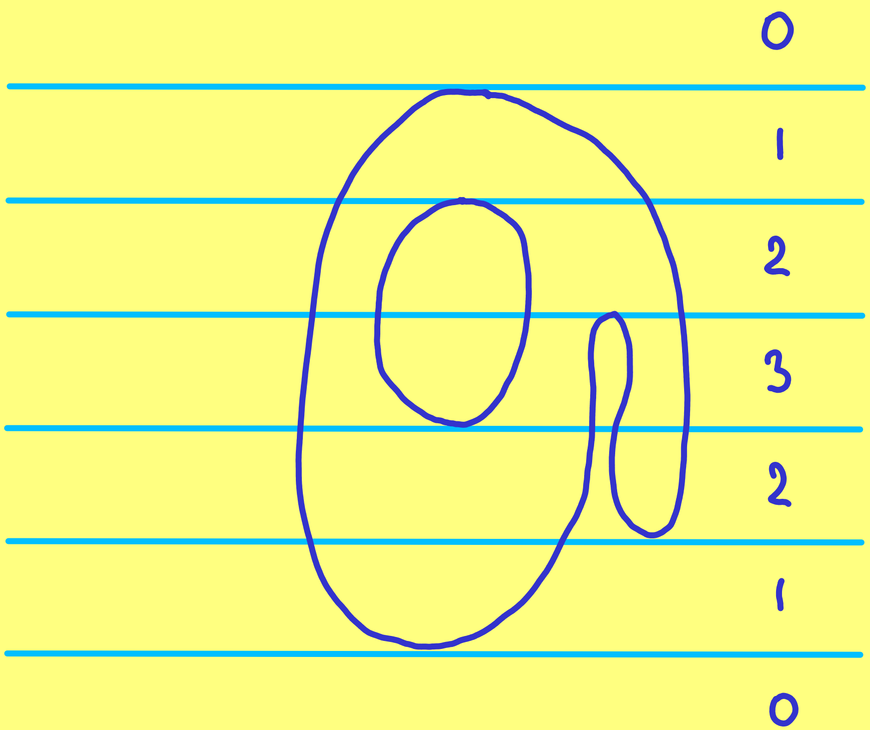

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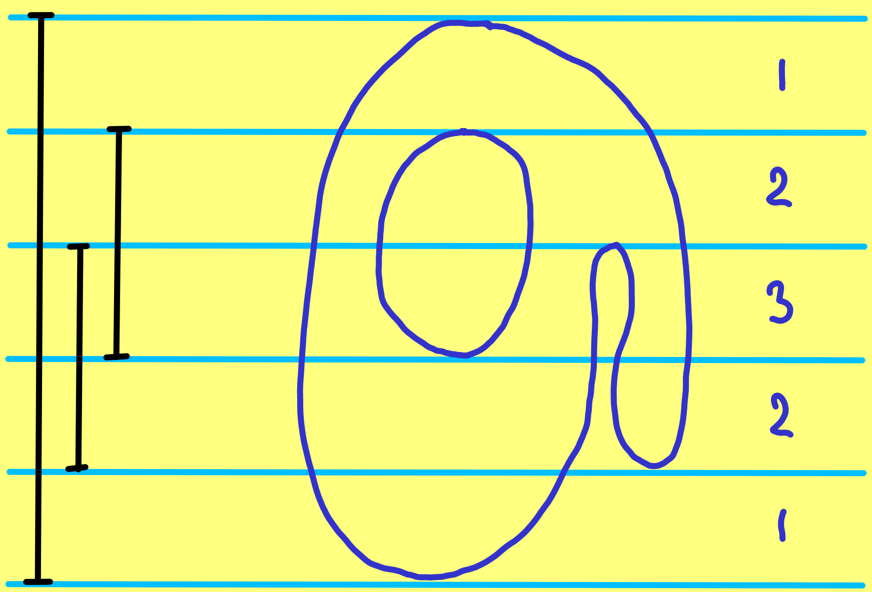

I Motivation

II Length

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IV Experiments

III.1 Homology

solid torus $\mathbb{M}$

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#components = $\beta_0 = 1$

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#components $= \beta_0 = 1$

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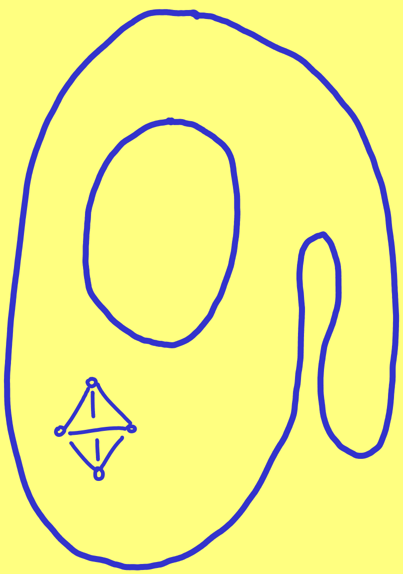
#voids $= \beta_2 = 0$

III.1 HOMOLOGY

Euler characteristic is $\chi(\mathbb{M}) = \#\text{vert} - \#\text{edg} + \#\text{tri} - \#\text{tet}$



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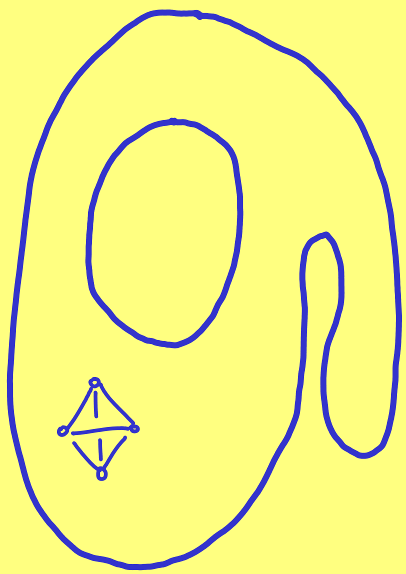

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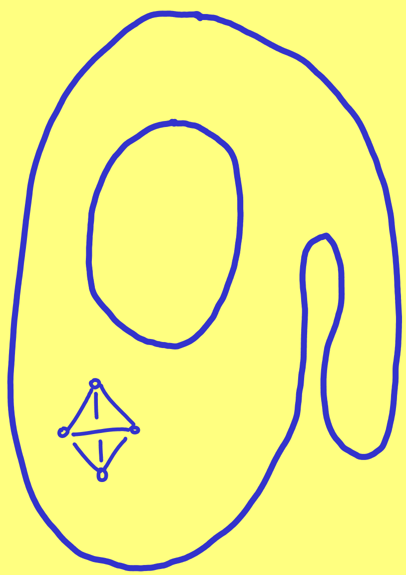
[Euler-Poincaré Formula]



solid torus $\mathbb{M}$

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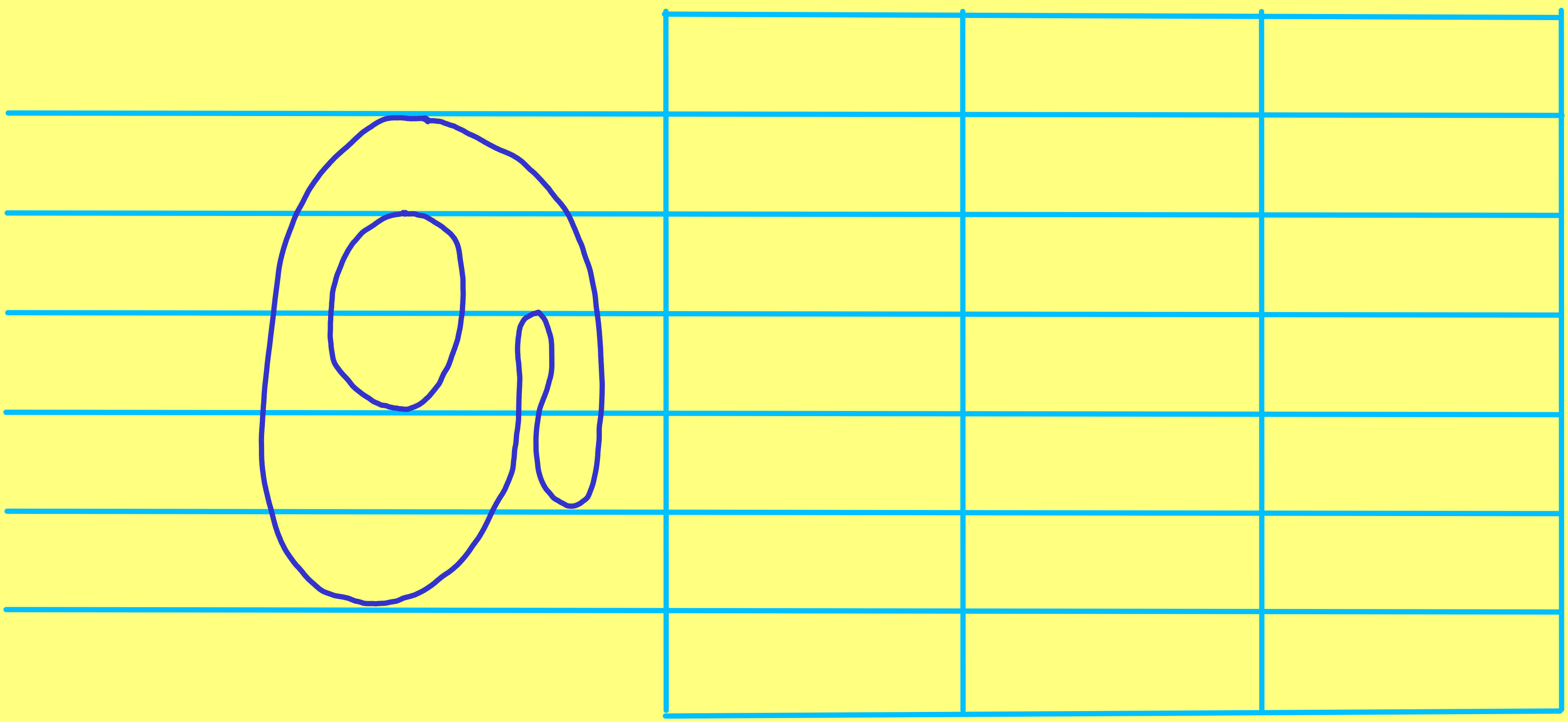
III.2 Level Sets

III.2 LEVEL SETS

level set

$M(c) = f^{-1}(c)$



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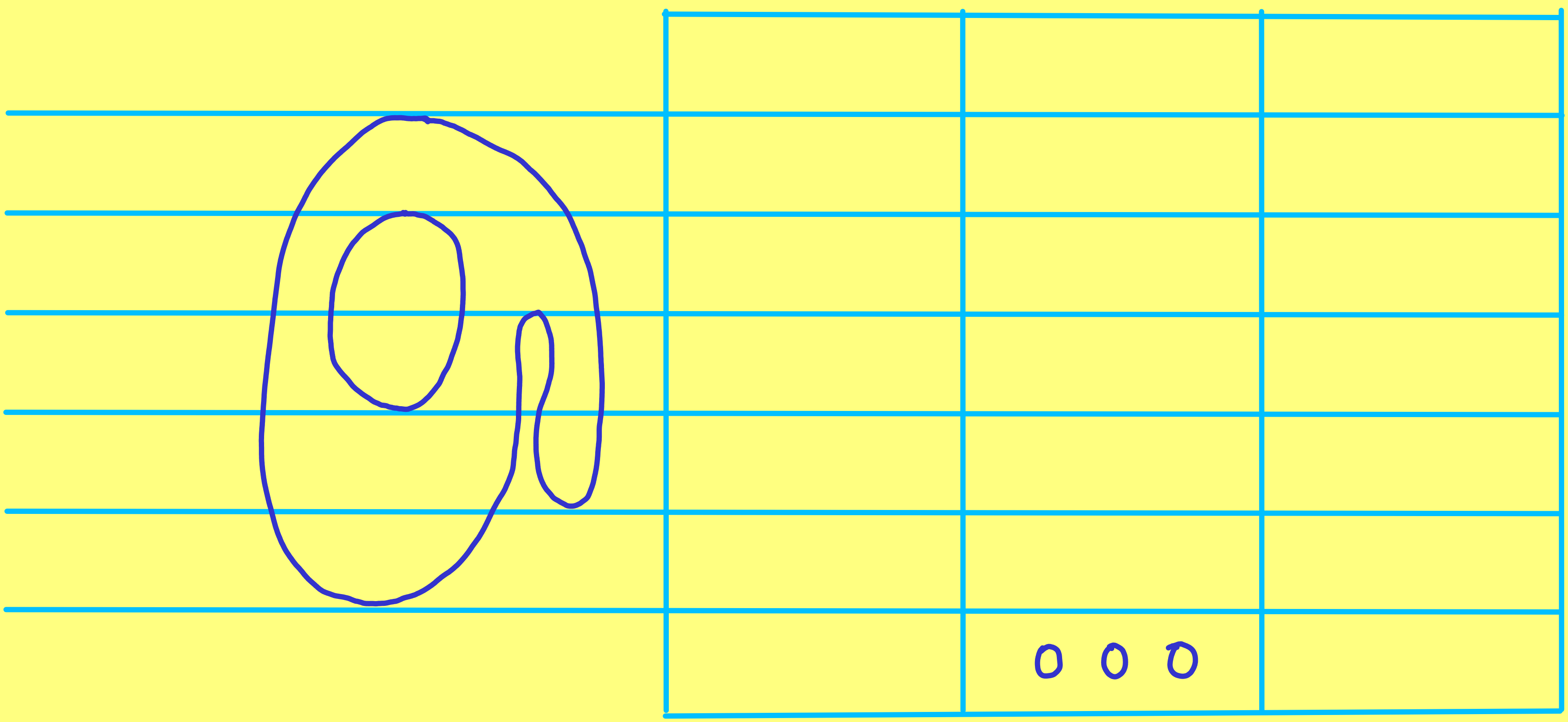

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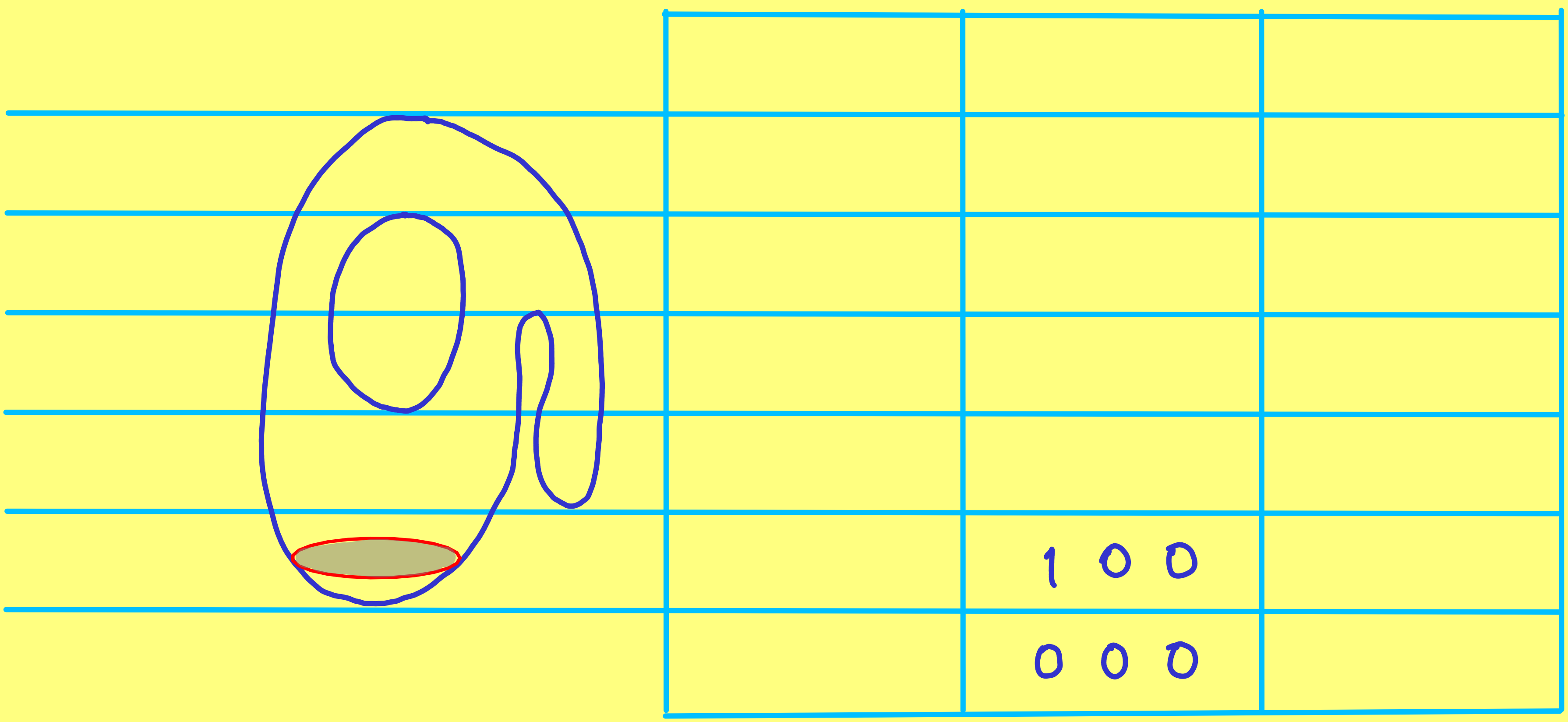

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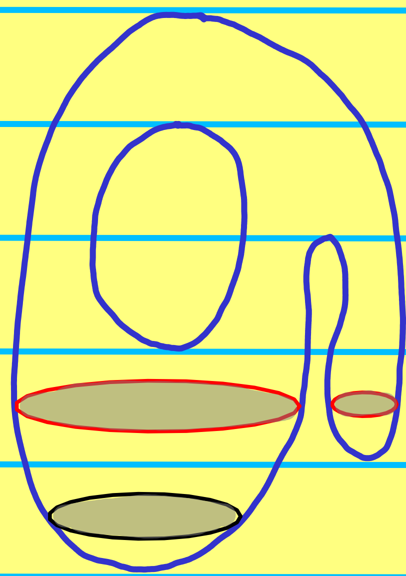

	2 0 0	
	1 0 0	
	0 0 0	

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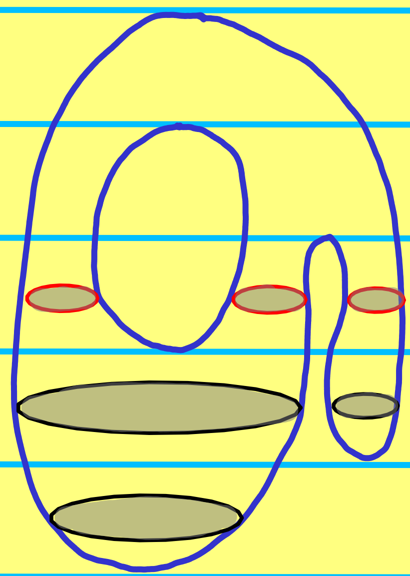

	3 0 0	
	2 0 0	
	1 0 0	
	0 0 0	

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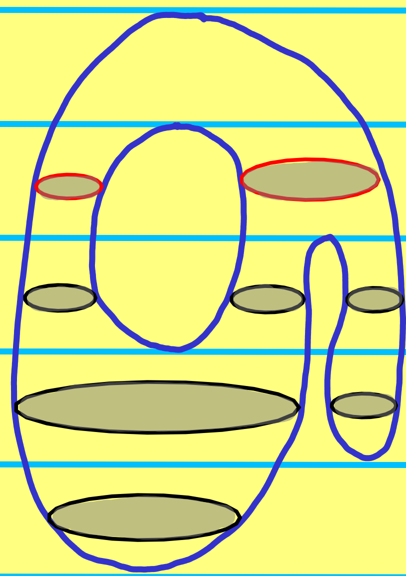

	2 0 0	
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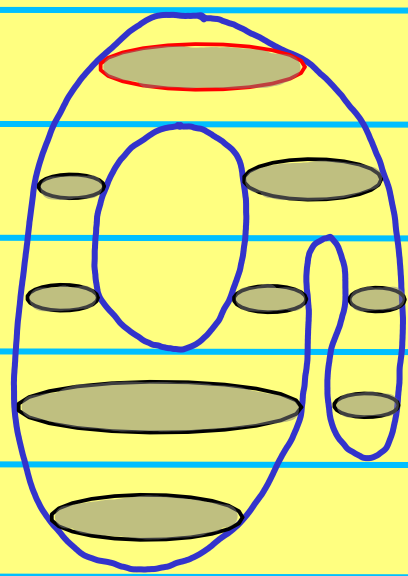

	1 0 0	
	2 0 0	
	3 0 0	
	2 0 0	
	1 0 0	
	0 0 0	

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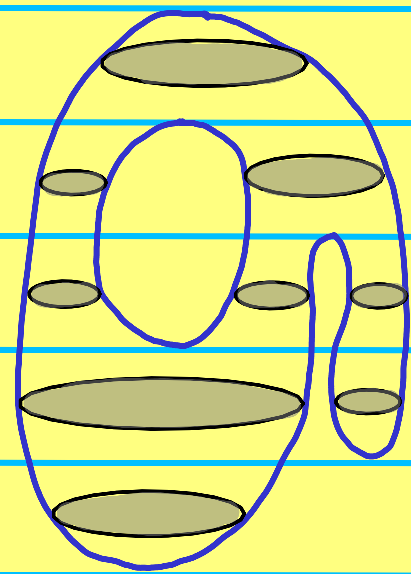

		0 0 0	
		1 0 0	
		2 0 0	
		3 0 0	
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		1 0 0	
		0 0 0	

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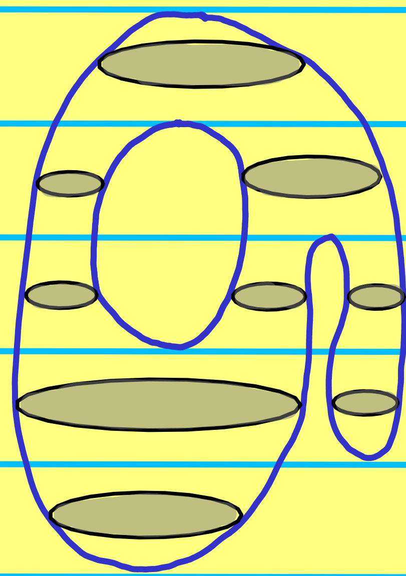

sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	
	0 0 0	
	1 0 0	
	2 0 0	
	3 0 0	
	2 0 0	
	1 0 0	
	0 0 0	

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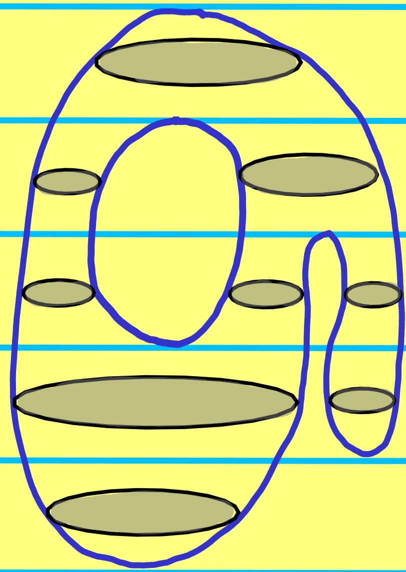

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	0 0 0	
	1 0 0	
	2 0 0	
	3 0 0	
	2 0 0	
	1 0 0	
0 0 0	0 0 0	

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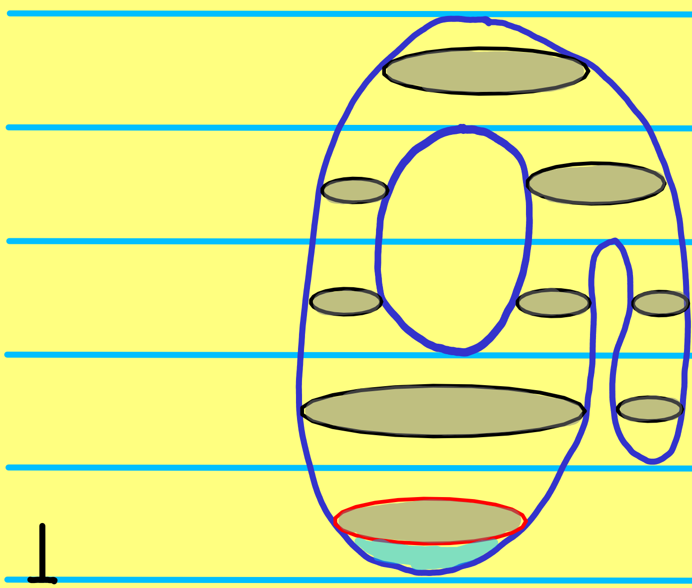

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	0 0 0	
	1 0 0	
	2 0 0	
	3 0 0	
	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

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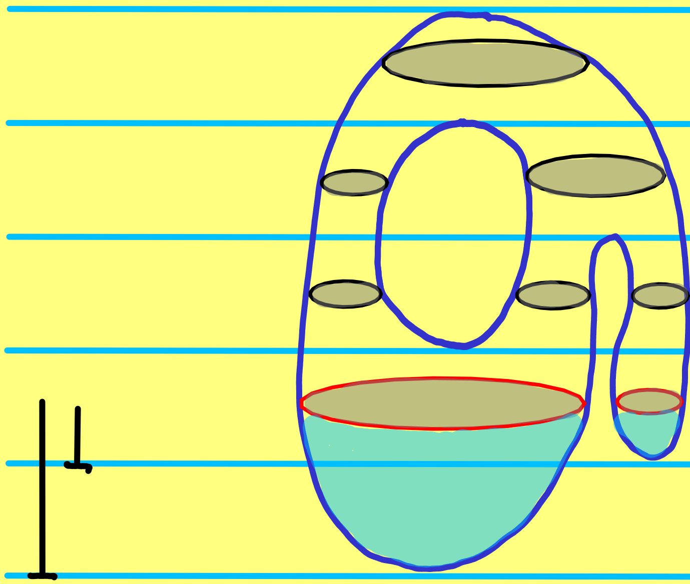

	0 0 0	
	1 0 0	
	2 0 0	
	3 0 0	
2 0 0	2 0 0	
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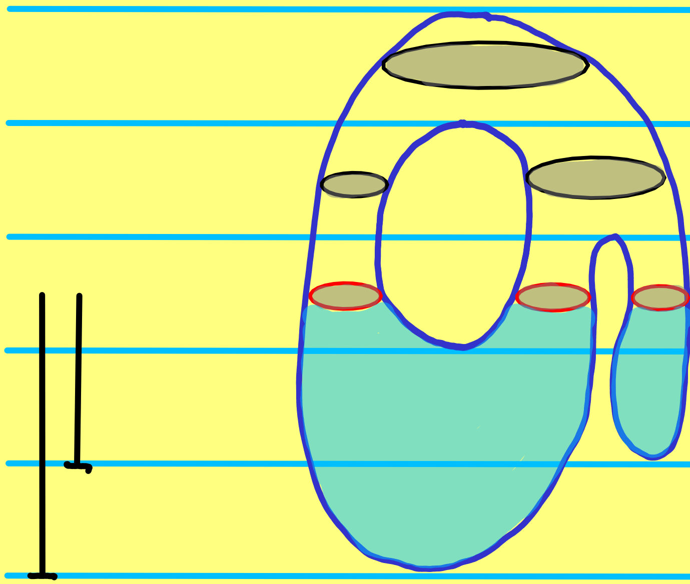
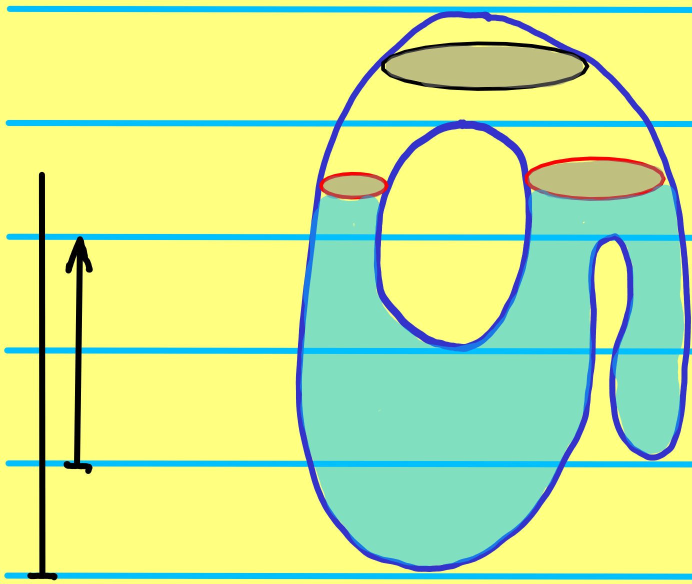
0 0 0	0 0 0	

III.2 Level Sets



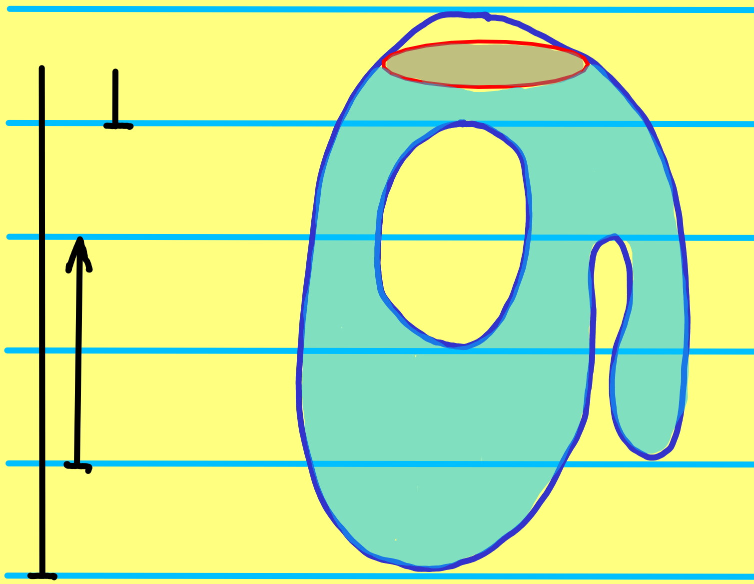
	sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	
		0 0 0	
		1 0 0	
		2 0 0	
	2 0 0	3 0 0	
	2 0 0	2 0 0	
	1 0 0	1 0 0	
	0 0 0	0 0 0	

III.2 Level Sets



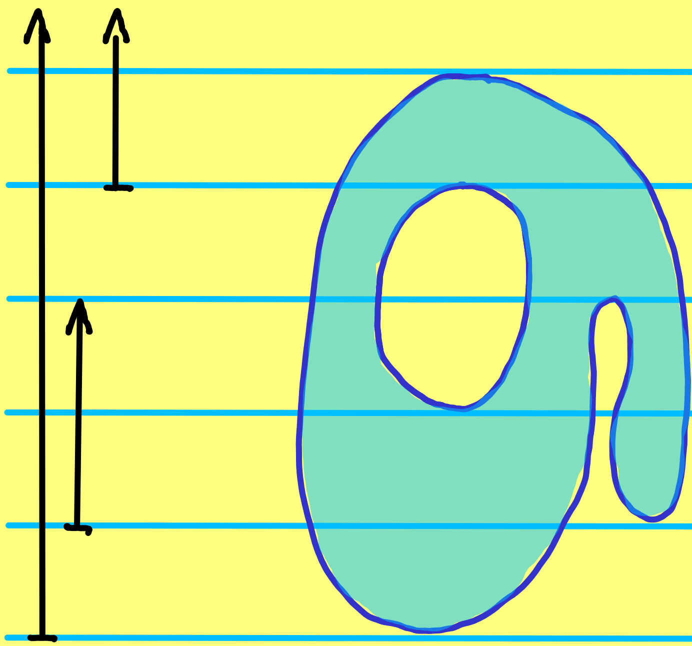
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	
	0 0 0	
	1 0 0	
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



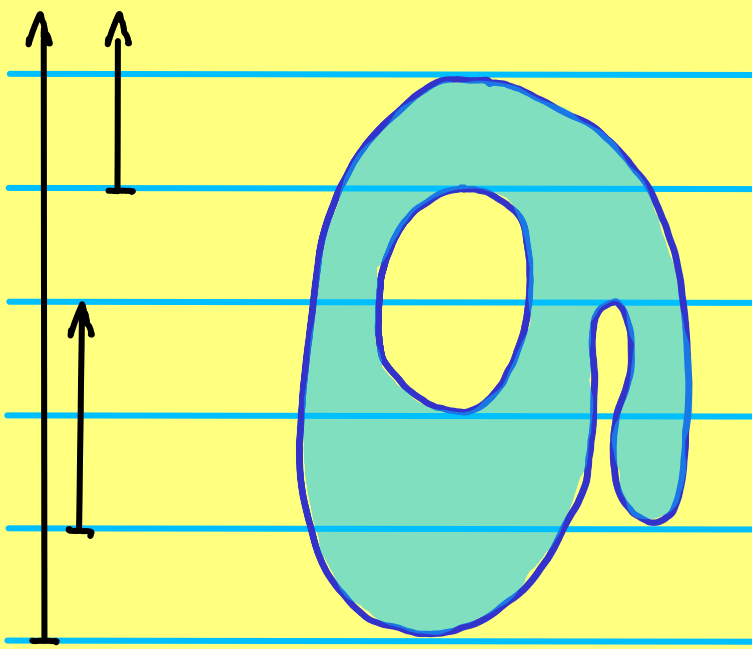
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	
	0 0 0	
1 1 0	1 0 0	
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



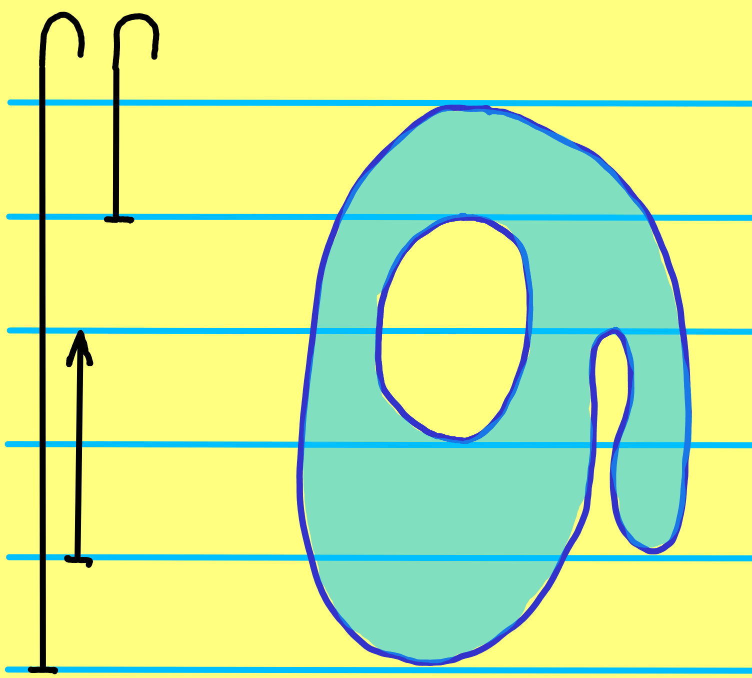
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	
1 1 0	0 0 0	
1 1 0	1 0 0	
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



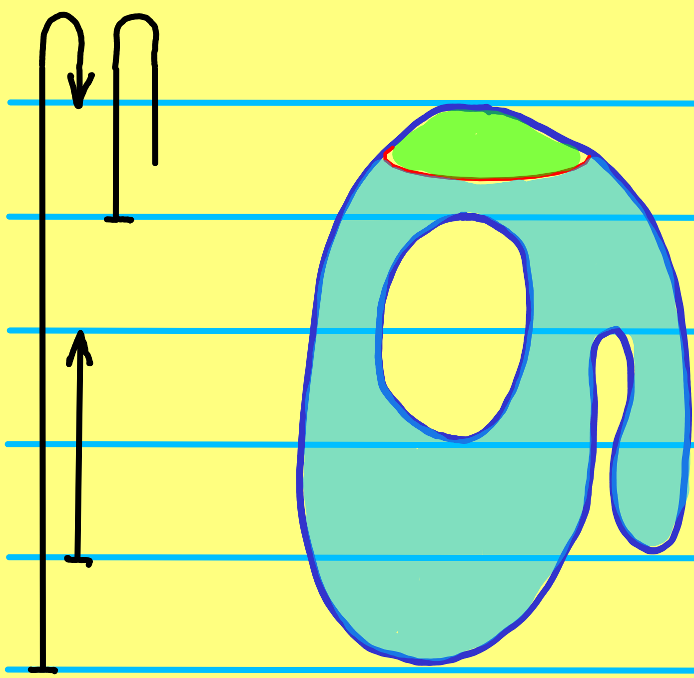
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	
1 1 0	1 0 0	
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



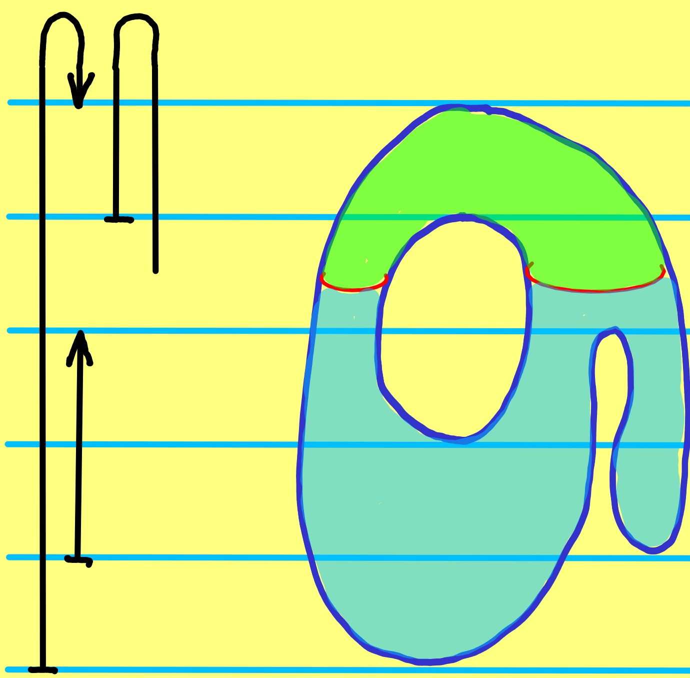
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



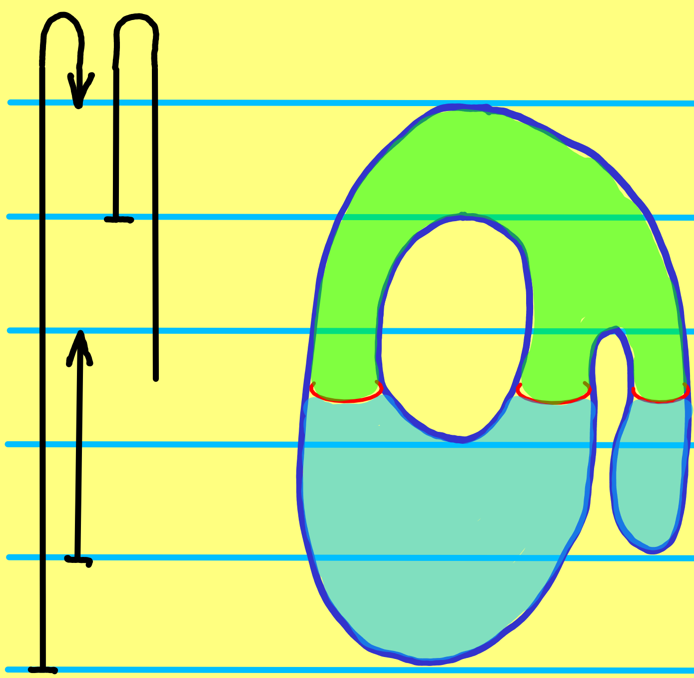
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



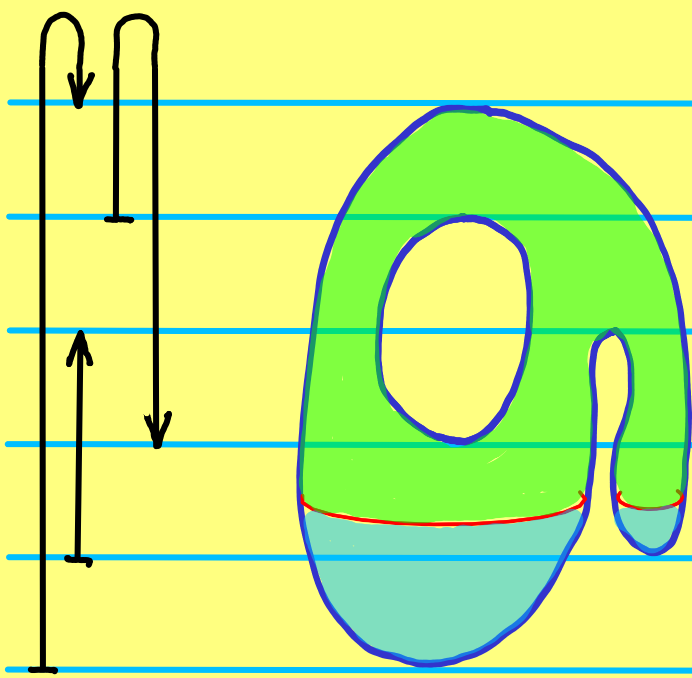
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	0 1 0
2 0 0	3 0 0	
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



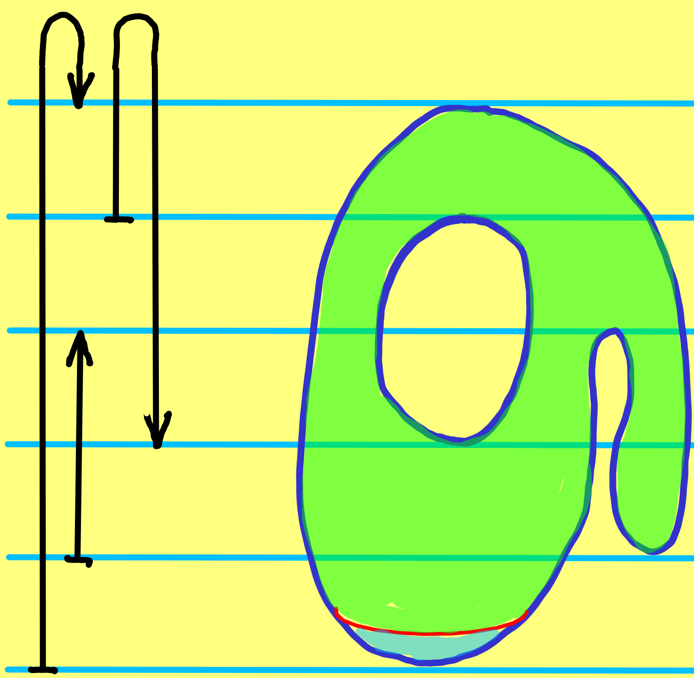
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	0 1 0
2 0 0	3 0 0	0 1 0
2 0 0	2 0 0	
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



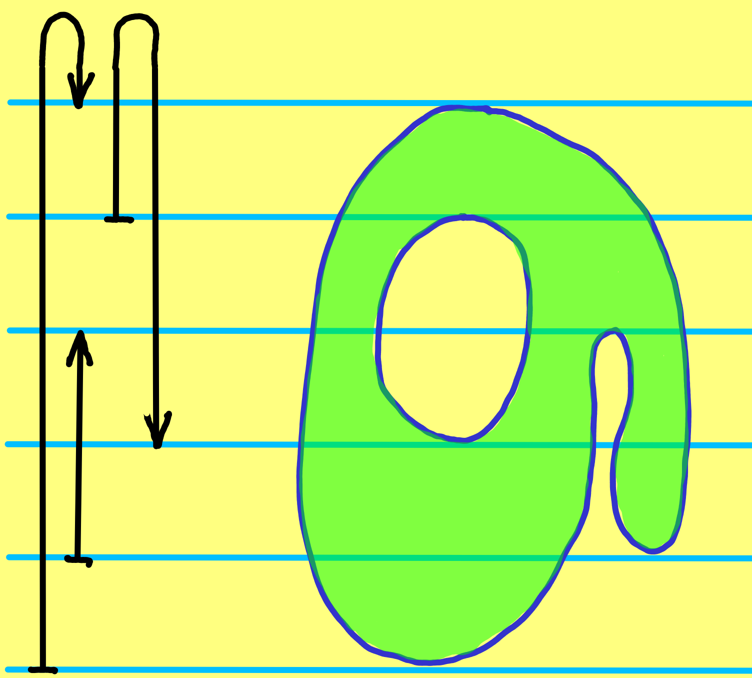
sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	0 1 0
2 0 0	3 0 0	0 1 0
2 0 0	2 0 0	0 0 0
1 0 0	1 0 0	
0 0 0	0 0 0	

III.2 Level Sets



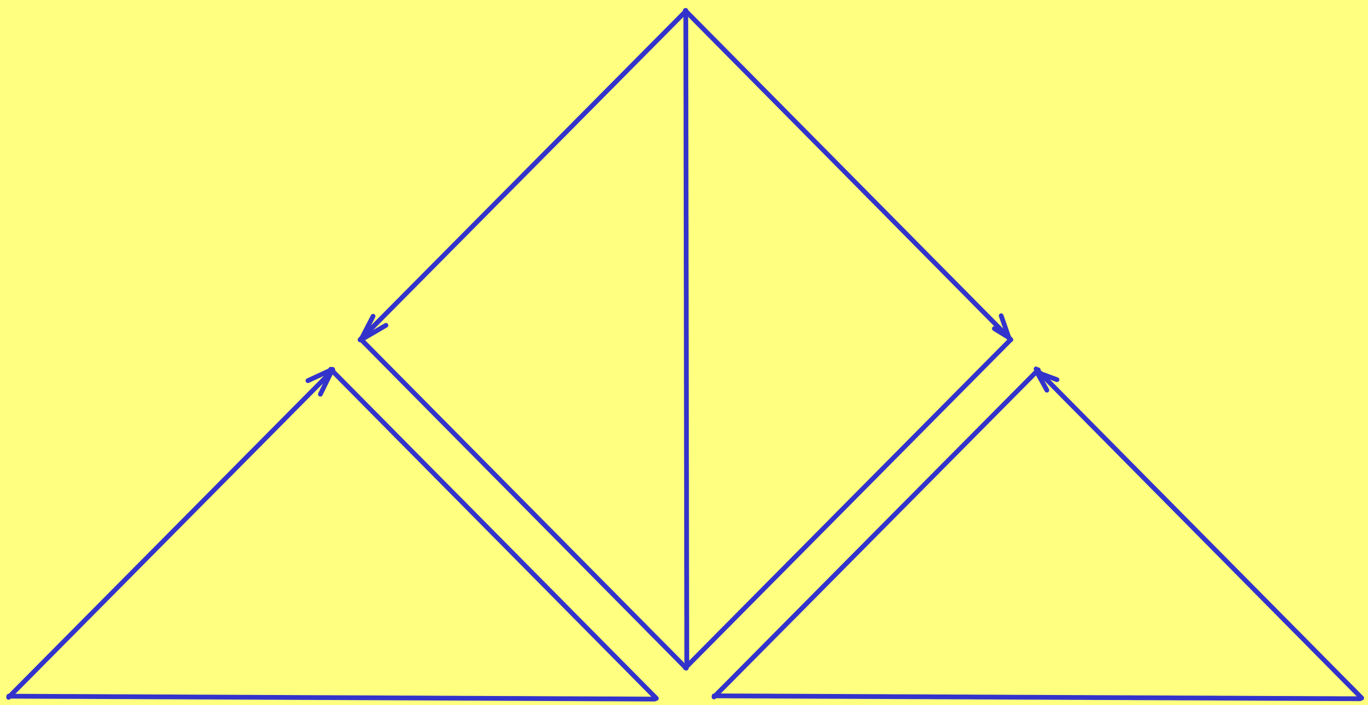
sublevel set	level set	superlevel set
$M_c = f^{-1}(-\infty, c]$	$M(c) = f^{-1}(c)$	$M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	0 1 0
2 0 0	3 0 0	0 1 0
2 0 0	2 0 0	0 0 0
1 0 0	1 0 0	0 0 0
0 0 0	0 0 0	

III.2 Level Sets



sublevel set $M_c = f^{-1}(-\infty, c]$	level set $M(c) = f^{-1}(c)$	superlevel set $M^c = f^{-1}[c, \infty)$
1 1 0	0 0 0	1 1 0
1 1 0	1 0 0	0 1 0
1 0 0	2 0 0	0 1 0
2 0 0	3 0 0	0 1 0
2 0 0	2 0 0	0 0 0
1 0 0	1 0 0	0 0 0
0 0 0	0 0 0	0 0 0

III.3 Persistent Homology [ELZ 2002]



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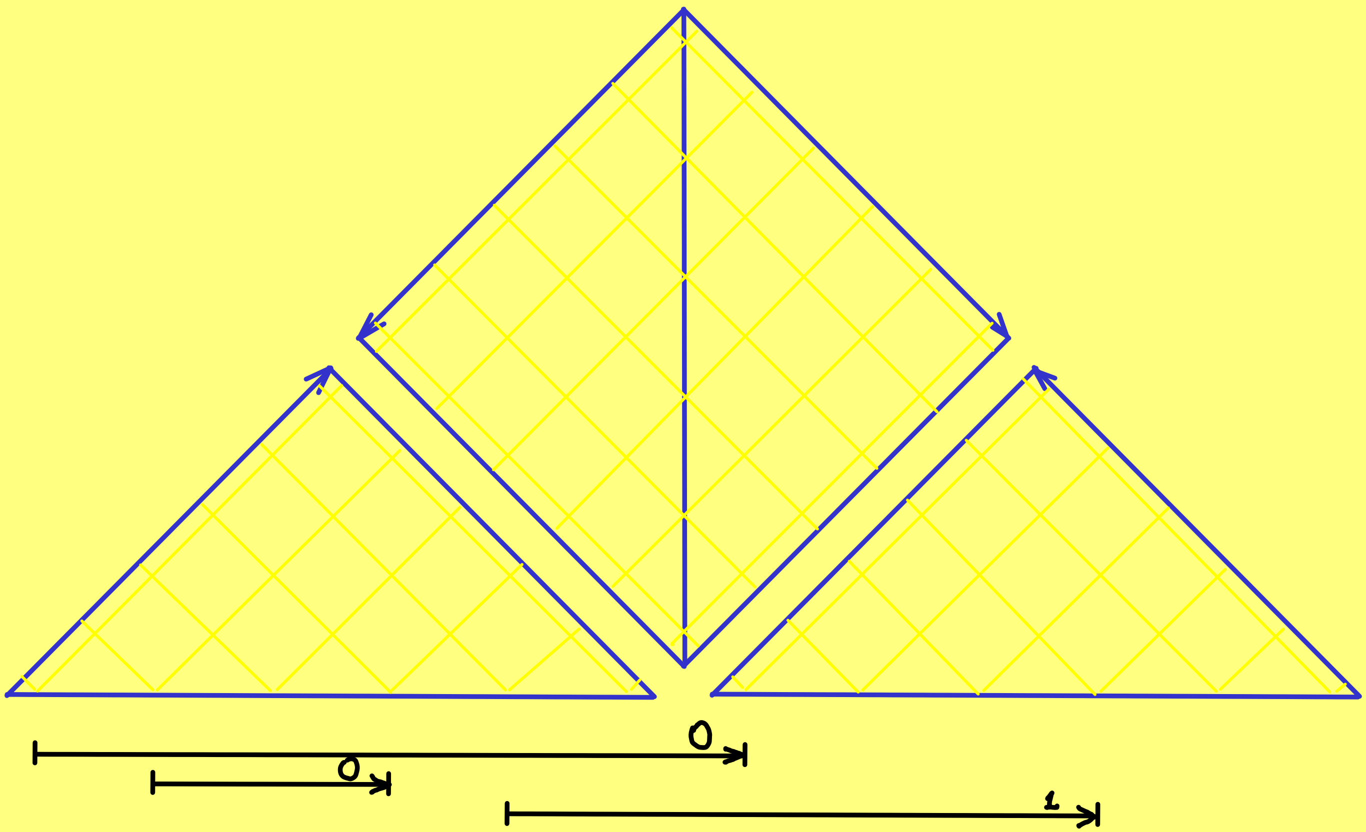
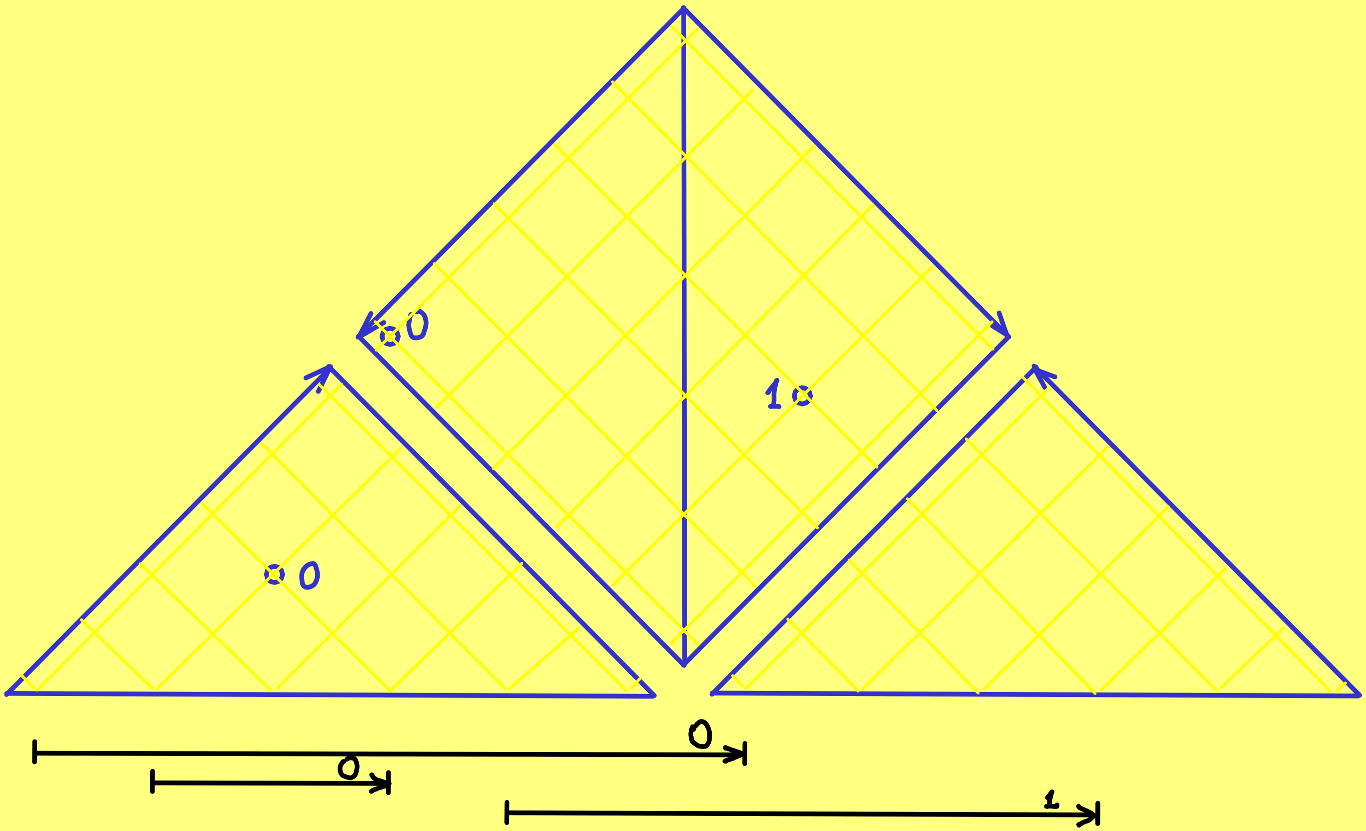


III.3 PERSISTENT HOMOLOGY [ELZ 2002]



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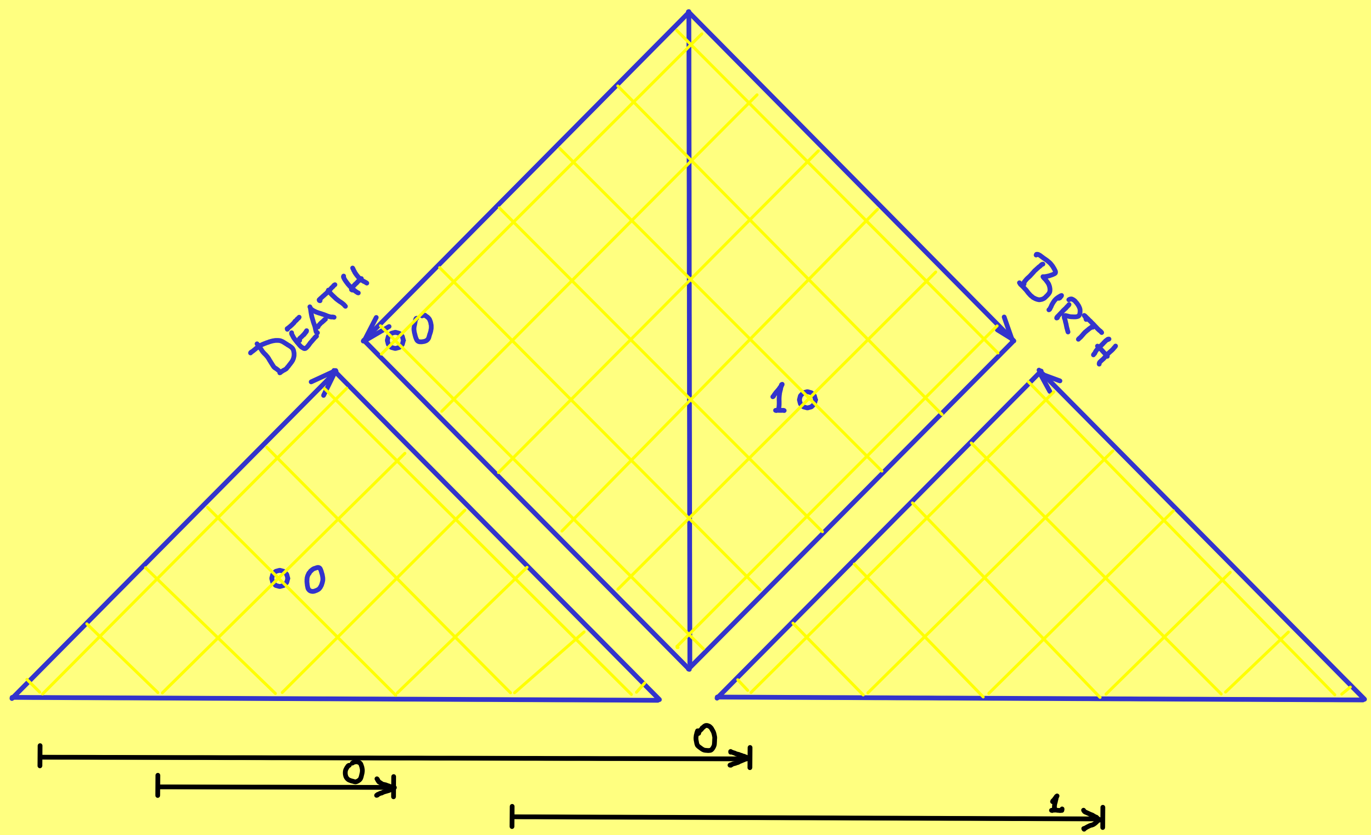

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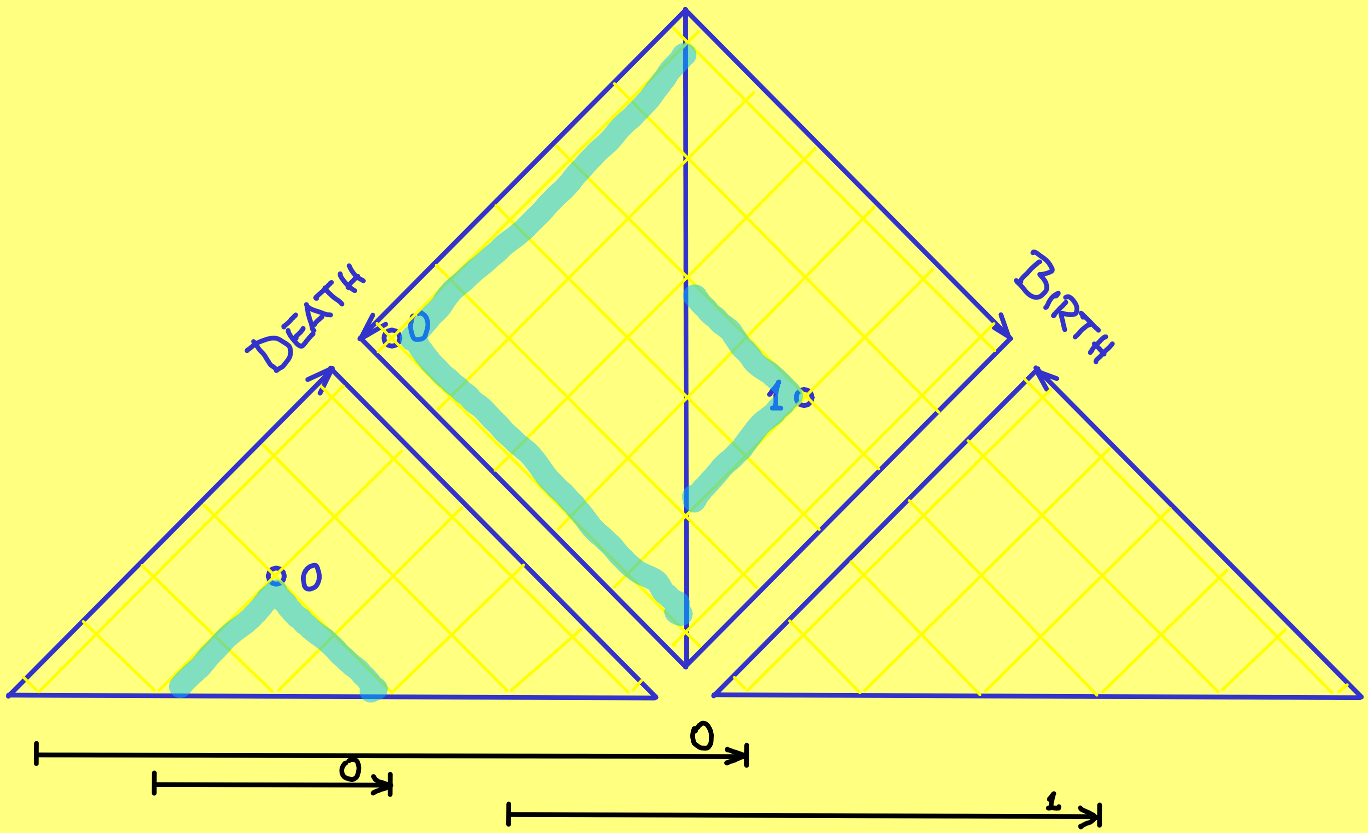

$A = (b, d)$, $\text{pers}(A) = |d - b|$

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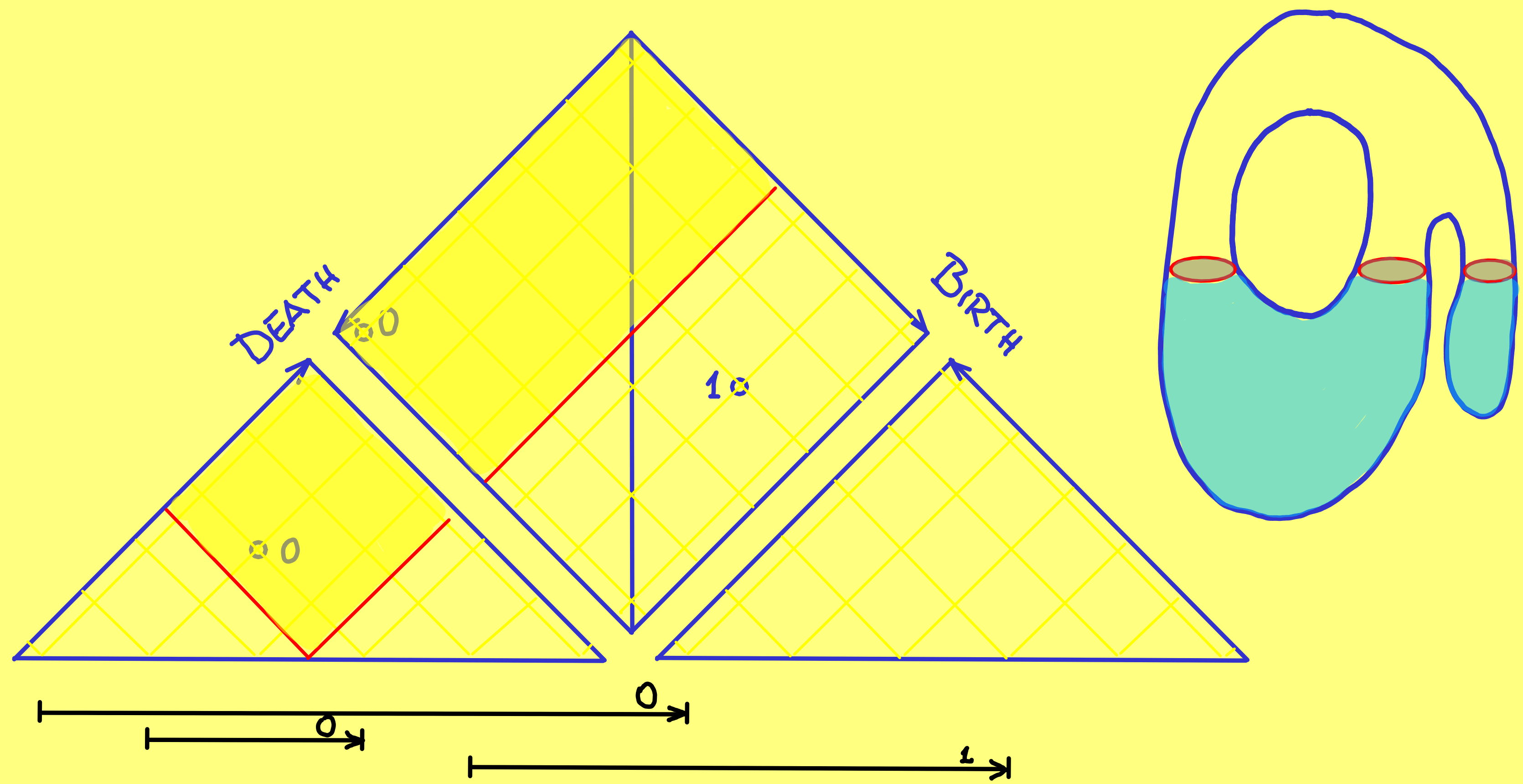

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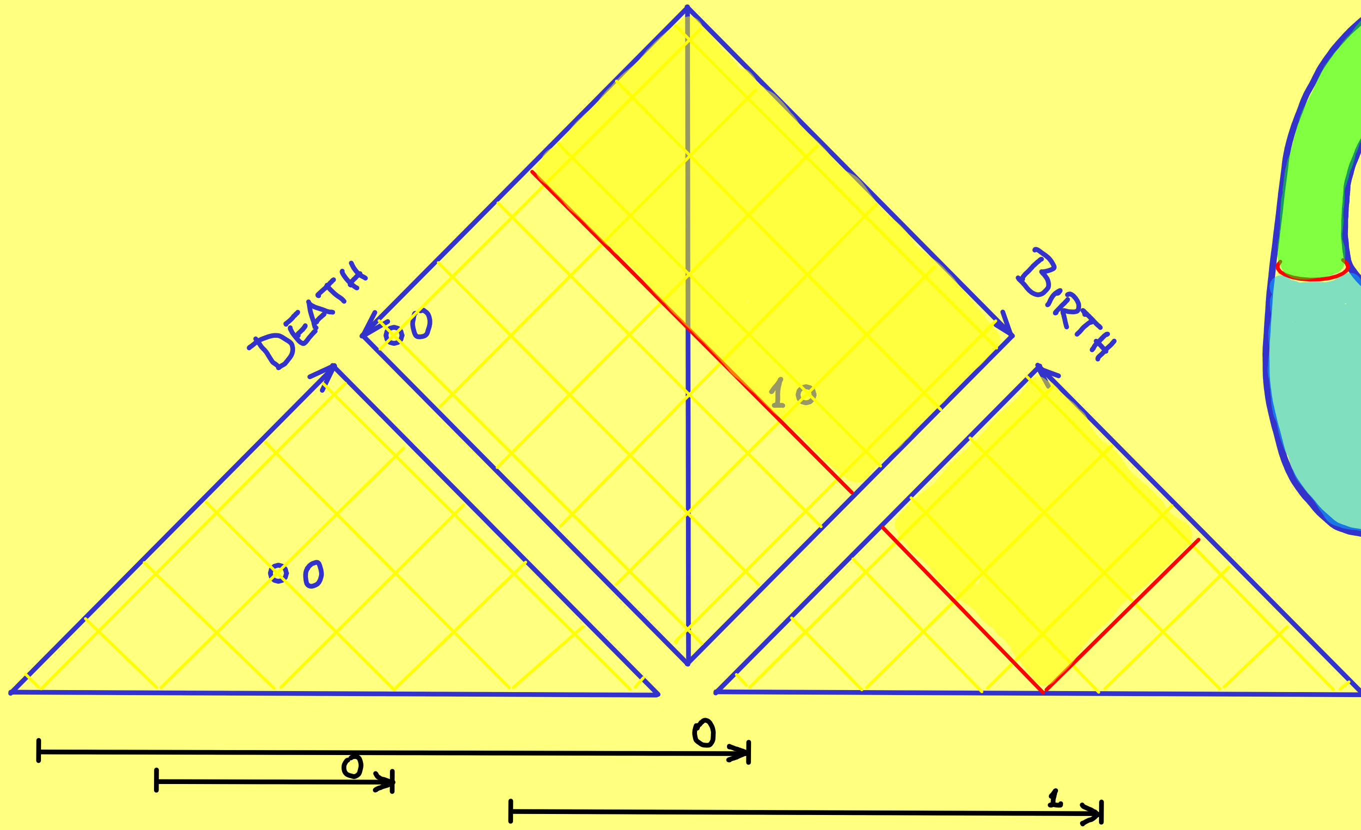

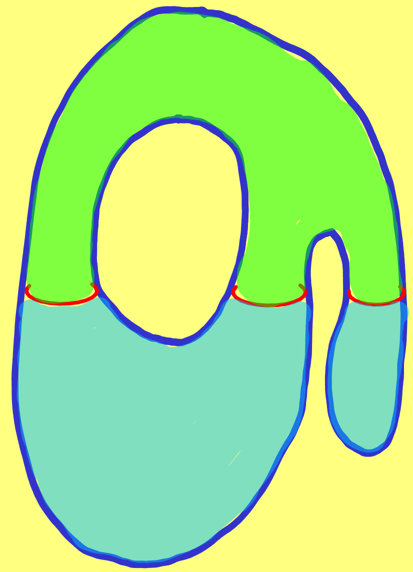

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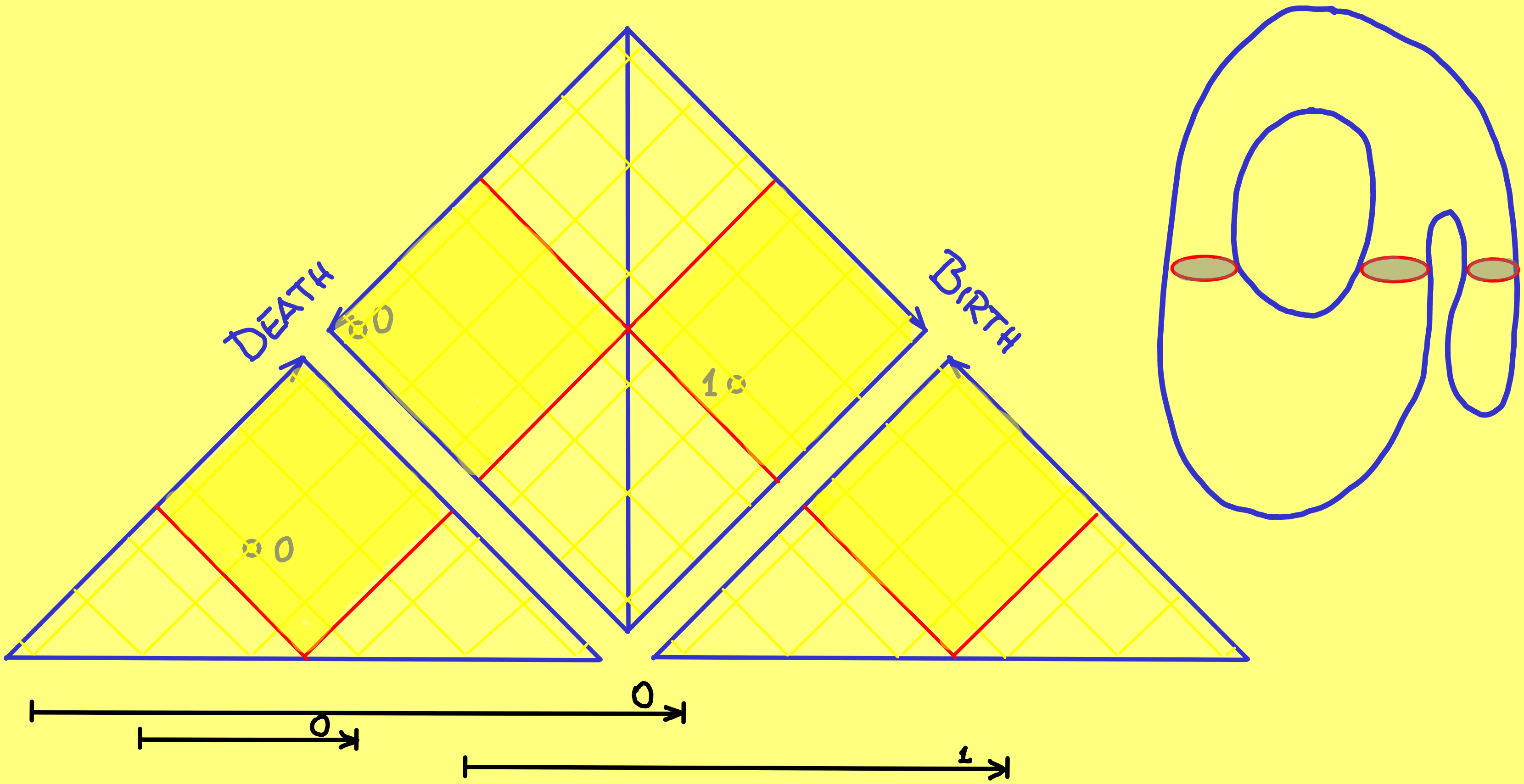

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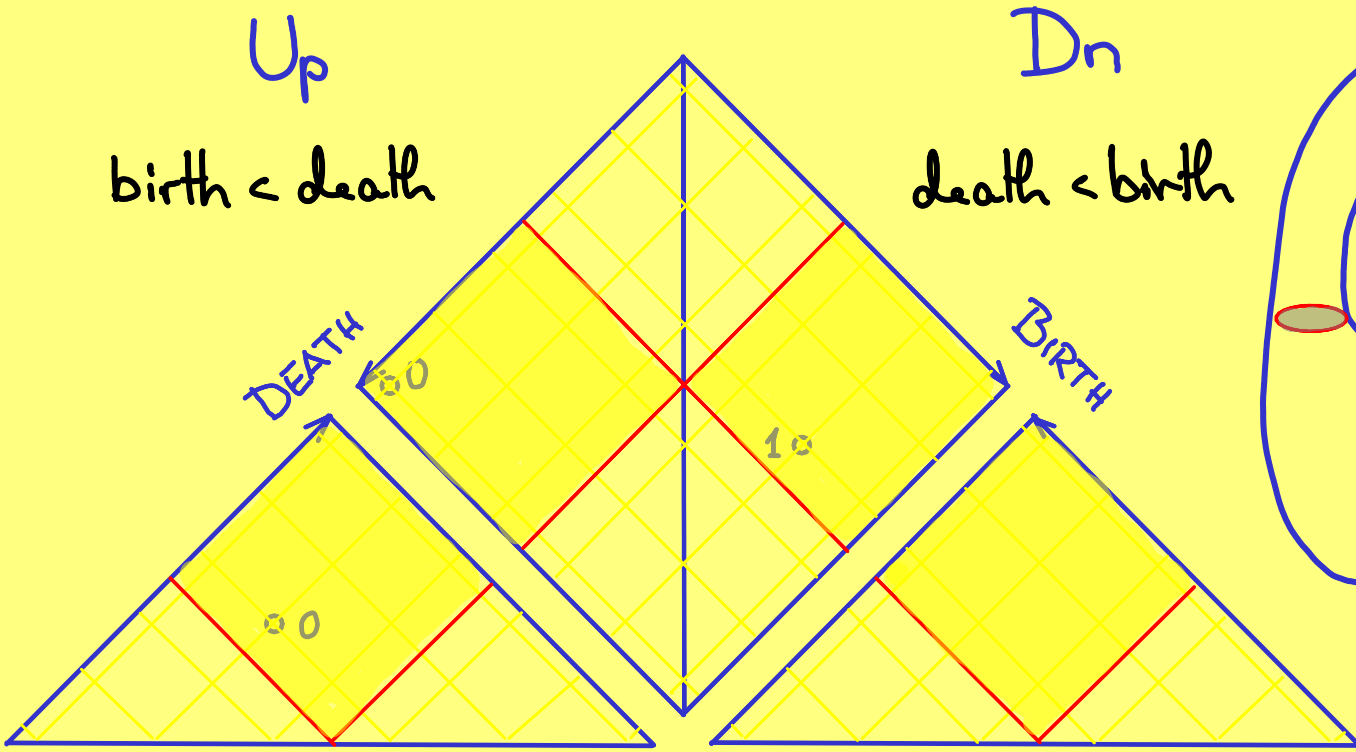
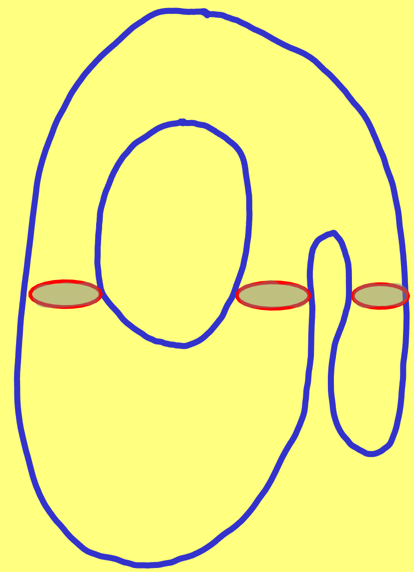
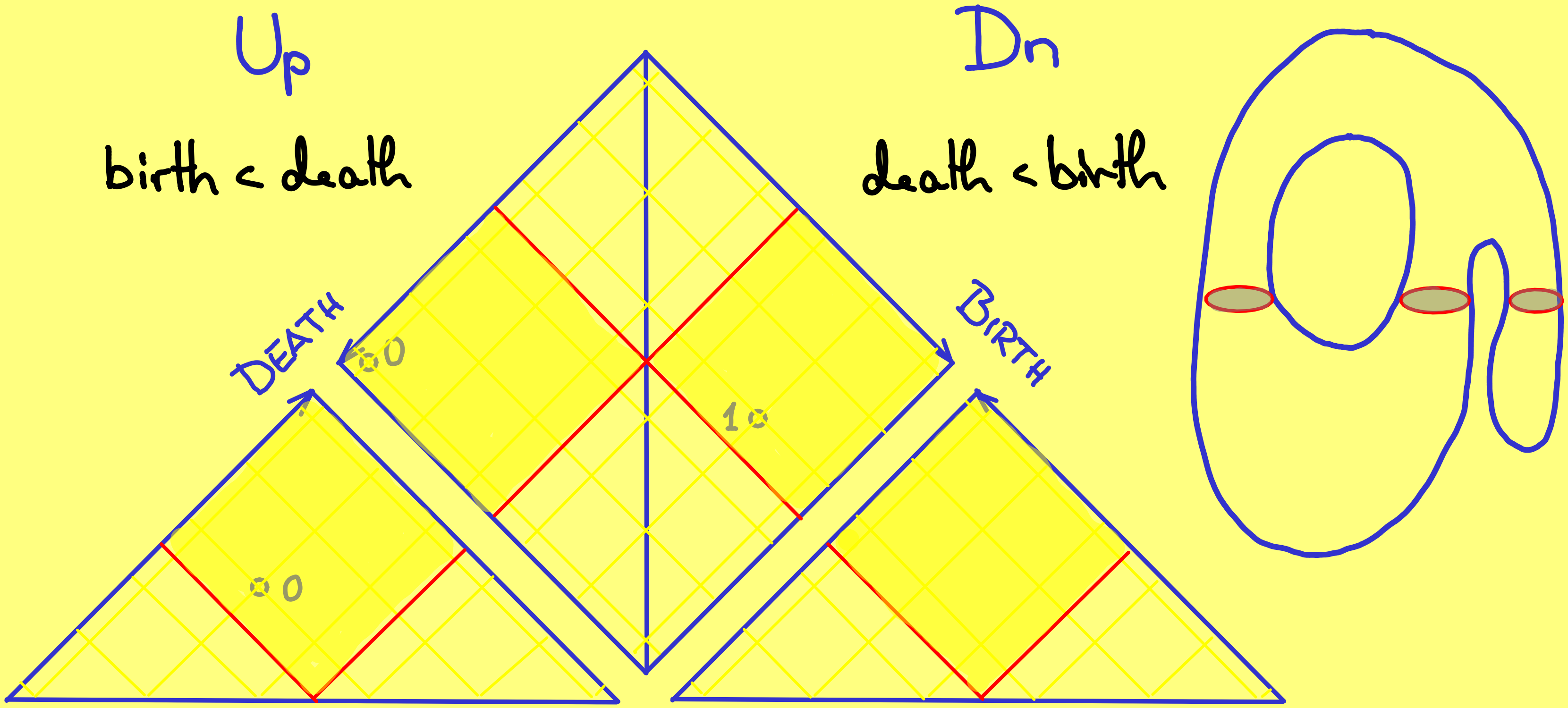




THM. [Bendich et al. 2009] The p-th Betti # of $f^{-1}(c)$ is

$$\#\{A \in Up_p(f) \mid b < c < d\} + \#\{A \in Dn_{p+1} \mid d < c < b\}.$$

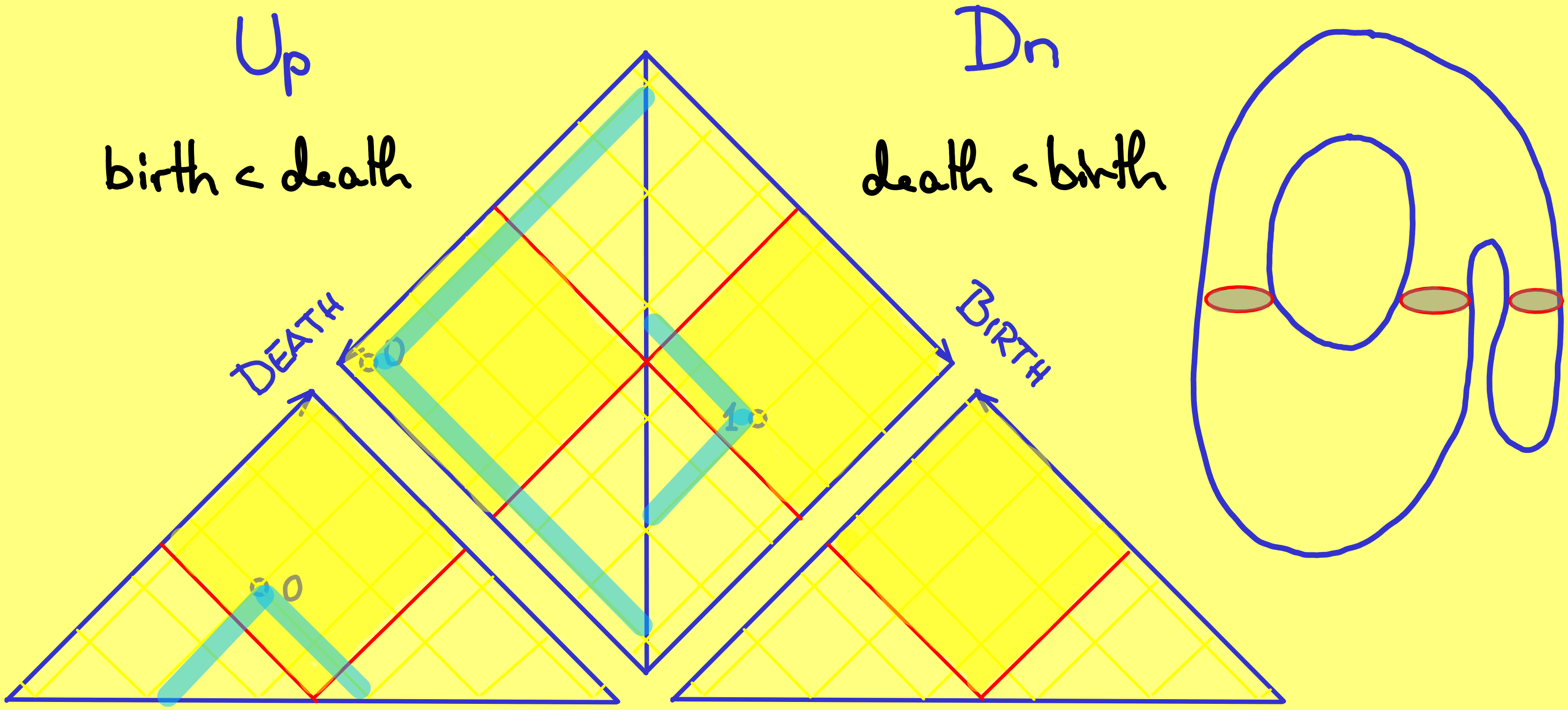
III.4 Moments



The k-th p-dim. level set moment is

$$B_p^k(f) = \sum_{A \in Up_p(f)} pers(A)^k + \sum_{A \in Dn_{p+1}(f)} pers(A)^k .$$

III.4 MOMENTS



The k-th p-dim. level set moment is

$$B_p^k(f) = \sum_{A \in Up_p(f)} \text{pers}(A)^k + \sum_{A \in Dn_{p+1}(f)} \text{pers}(A)^k .$$

III.4 MOMENTS

The total mean curvature is

$$Q_2(M) = \frac{1}{2} \int_{u \in S^2} \sum_{p \geq 0} (-1)^p B_p^1(f_u)\, du .$$

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III.4 MOMENTS

The total mean curvature is

$$Q_2(M) = \frac{1}{2} \int_{u \in \mathbb{S}^2} \sum_{p \geq 0} (-1)^p B_p^1(f_u)\, du .$$

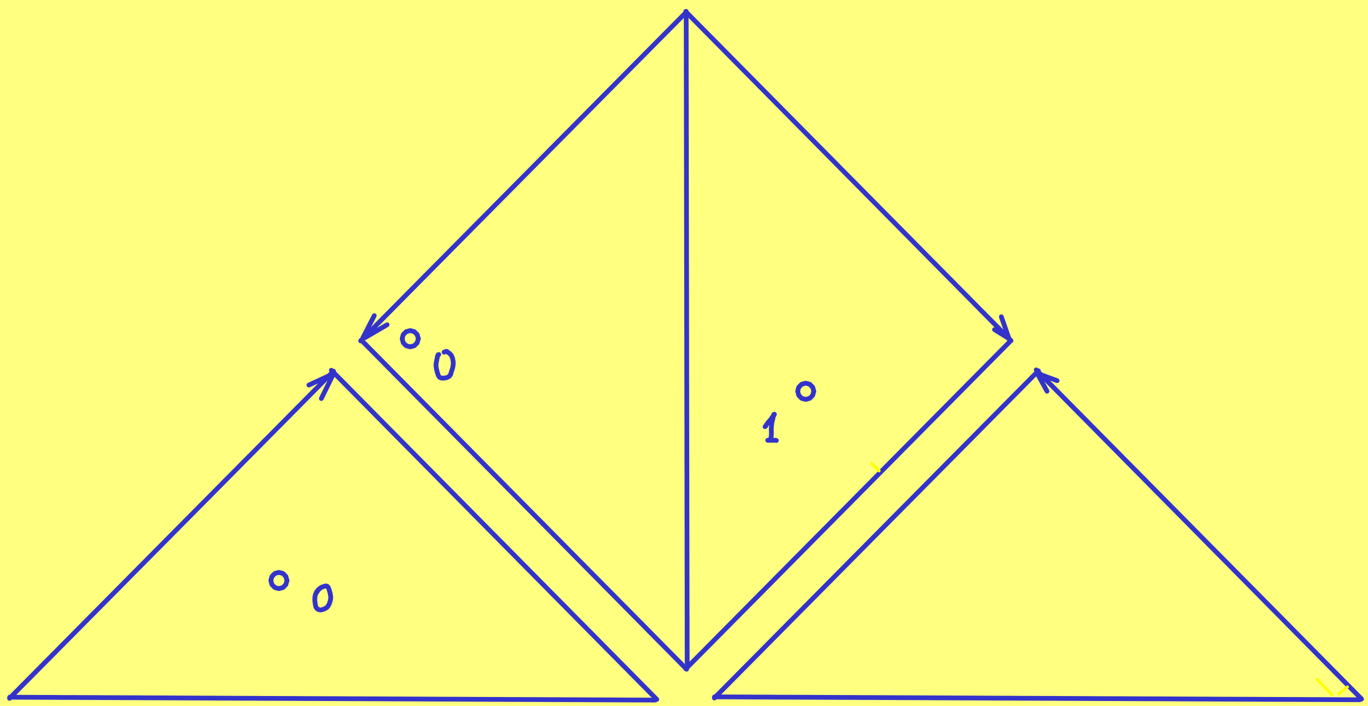
Define $X_p^k(f) = B_p^1(f, r_0^+) + \frac{1}{r_0^{k-1}} B_p^k(f, r_0^-)$

and use it to replace $B_p^1(f)$ in comput. of $Q_2(M)$.

The k-th p-dim. level set moment is

$$B_p^k(f) = \sum_{A \in Up_p(f)} \mathrm{pers}(A)^k + \sum_{A \in Dn_{p+1}(f)} \mathrm{pers}(A)^k .$$

III.5 Stability

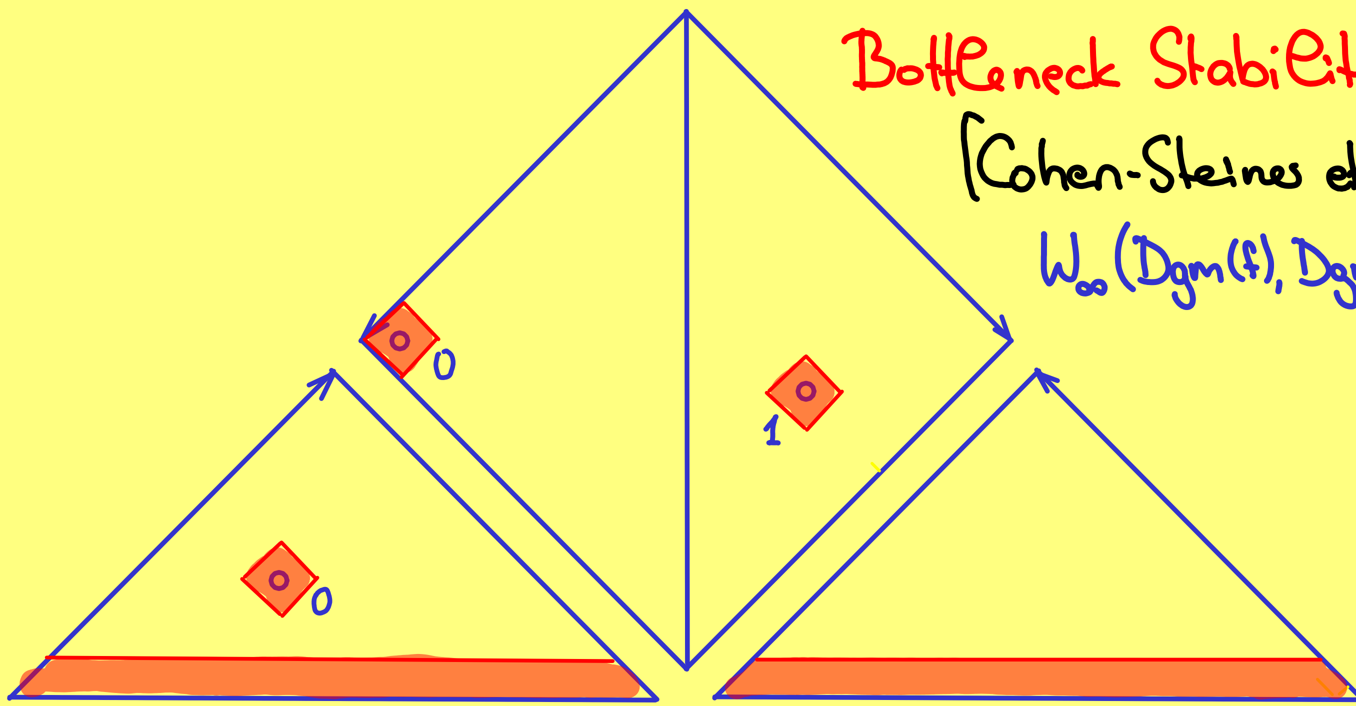


III.5 STABILITY

Bottleneck Stability Thm.

[Cohen-Steiner et al. 2007]

$W_\infty(\mathrm{Dgm}(f), \mathrm{Dgm}(g)) \leq \|f-g\|_\infty.$



III.5 STABILITY

L_k-Norm Stability Thm.

[Cohen-Steiner et al. 2010]

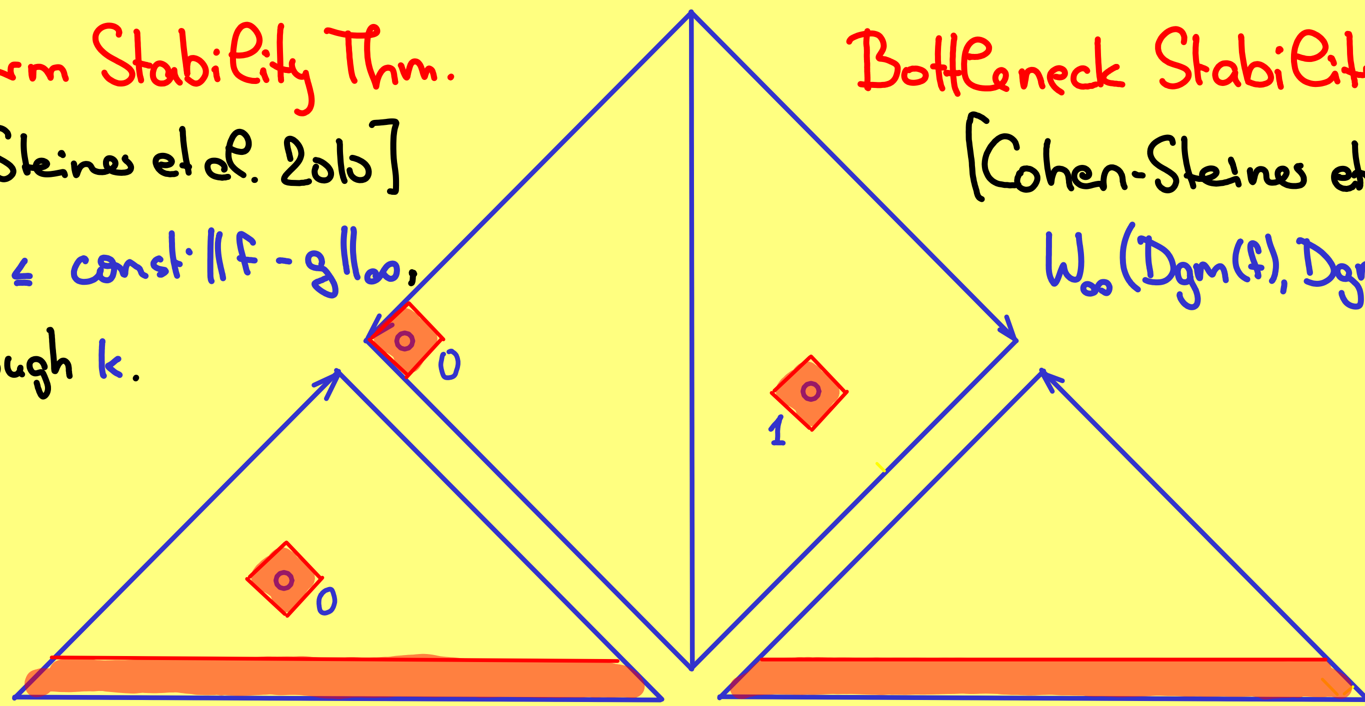
$|B^k(f) - B^k(g)| \leq \text{const} \cdot \|f-g\|_\infty$,

for large enough k.

Bottleneck Stability Thm.

[Cohen-Steiner et al. 2007]

$W_\infty(\mathrm{Dgm}(f), \mathrm{Dgm}(g)) \leq \|f-g\|_\infty$.



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$|B^k(f) - B^k(g)| \le \text{const} \cdot \|f-g\|_\infty$,

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Bottleneck Stability Thm.

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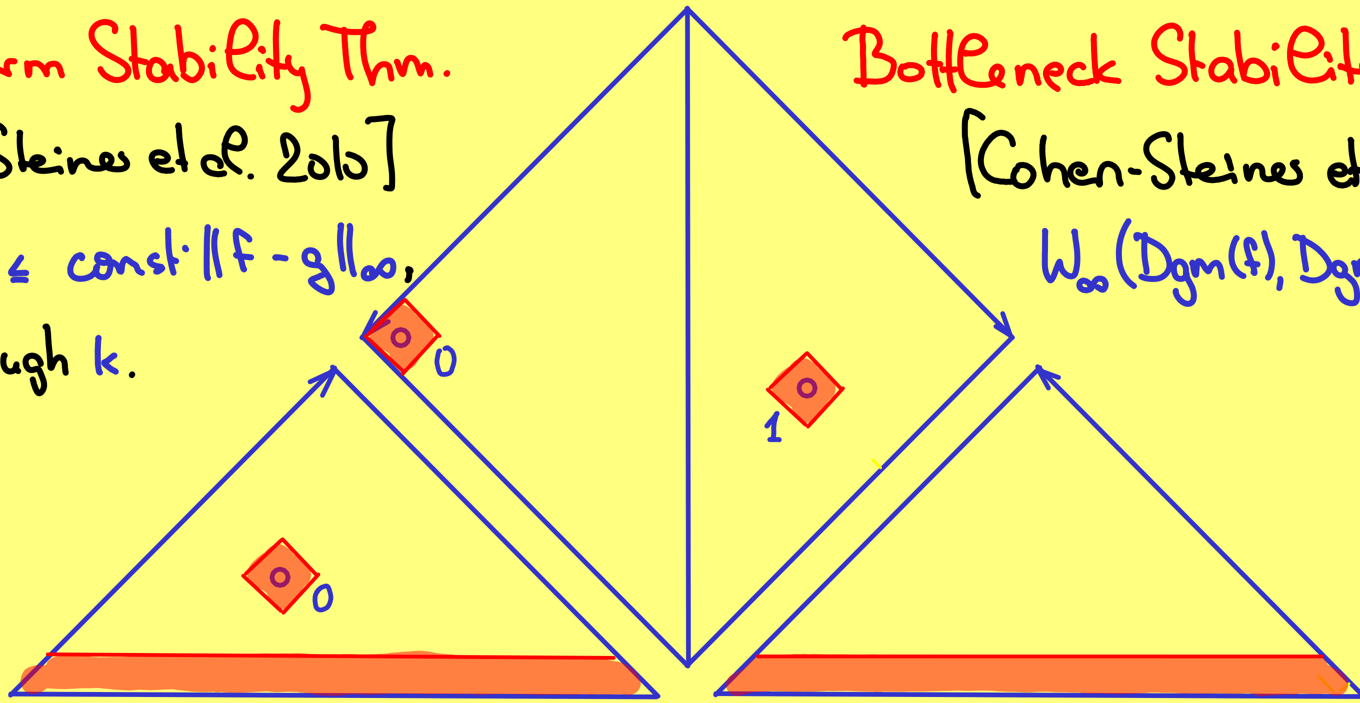
$W_\infty(\mathrm{Dgm}(f), \mathrm{Dgm}(g)) \le \|f-g\|_\infty$.



Damped Stability Thm. for Tubes

$f, g : \mathbb{M} \to \mathbb{R}$.

$$|X^k_p(f) - X^k_p(g)| \le \text{const} \cdot \frac{L^{1+\delta}}{r_0^{1+\delta}} \|f-g\|_\infty, \text{ for } k > 4.$$

I Motivation

II Length

III Persistence

IV Experiments

IV.1 Algorithms

IV.1 ALGORITHMS

DMC (discrete mean curvature):

$$Q_2 = \sum_{\text{edges}} \text{length} \cdot \text{dihedral angle} .$$

IV.1 Algorithms

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$$Q_2 = \sum_{\text{edges}} \text{length} \cdot \text{dihedral angle}.$$

PS (plane sampling) versions rPS, qPS

$$Q_2 = \int_{\text{planes } P} \chi(P \cap \mathbb{M})\, dP$$

IV.1 ALGORITHMS

DMC (discrete mean curvature):

$$Q_2 = \sum_{\text{edges}} \text{length} \cdot \text{dihedral angle}.$$

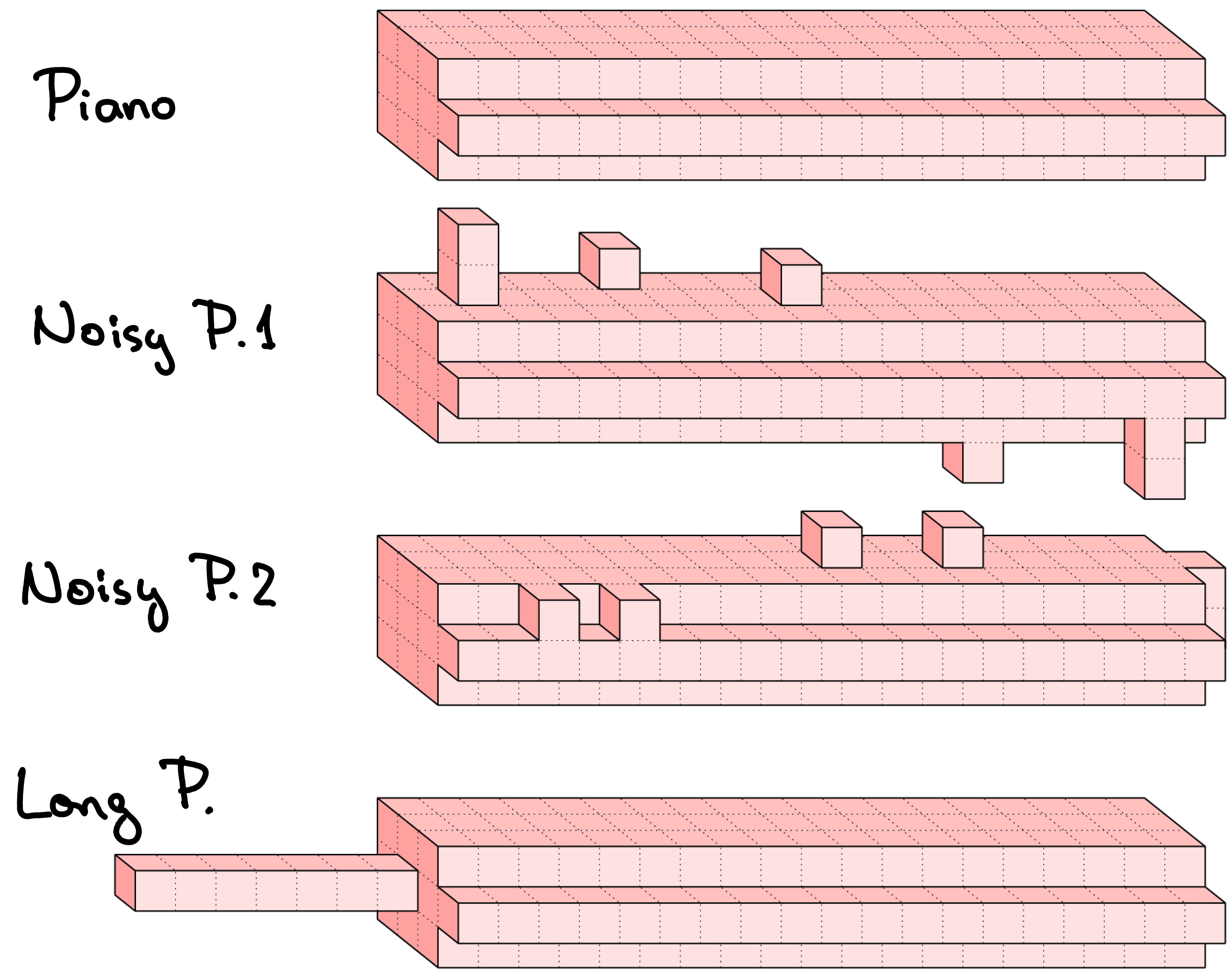
PS (plane sampling) versions rPS, qPS

$$Q_2 = \int_{\text{planes } P} \chi(P \cap \mathbb{M})\, dP$$

DS (direction sampling) versions rDS$_1$, qDS$_1$, qDS$_k$

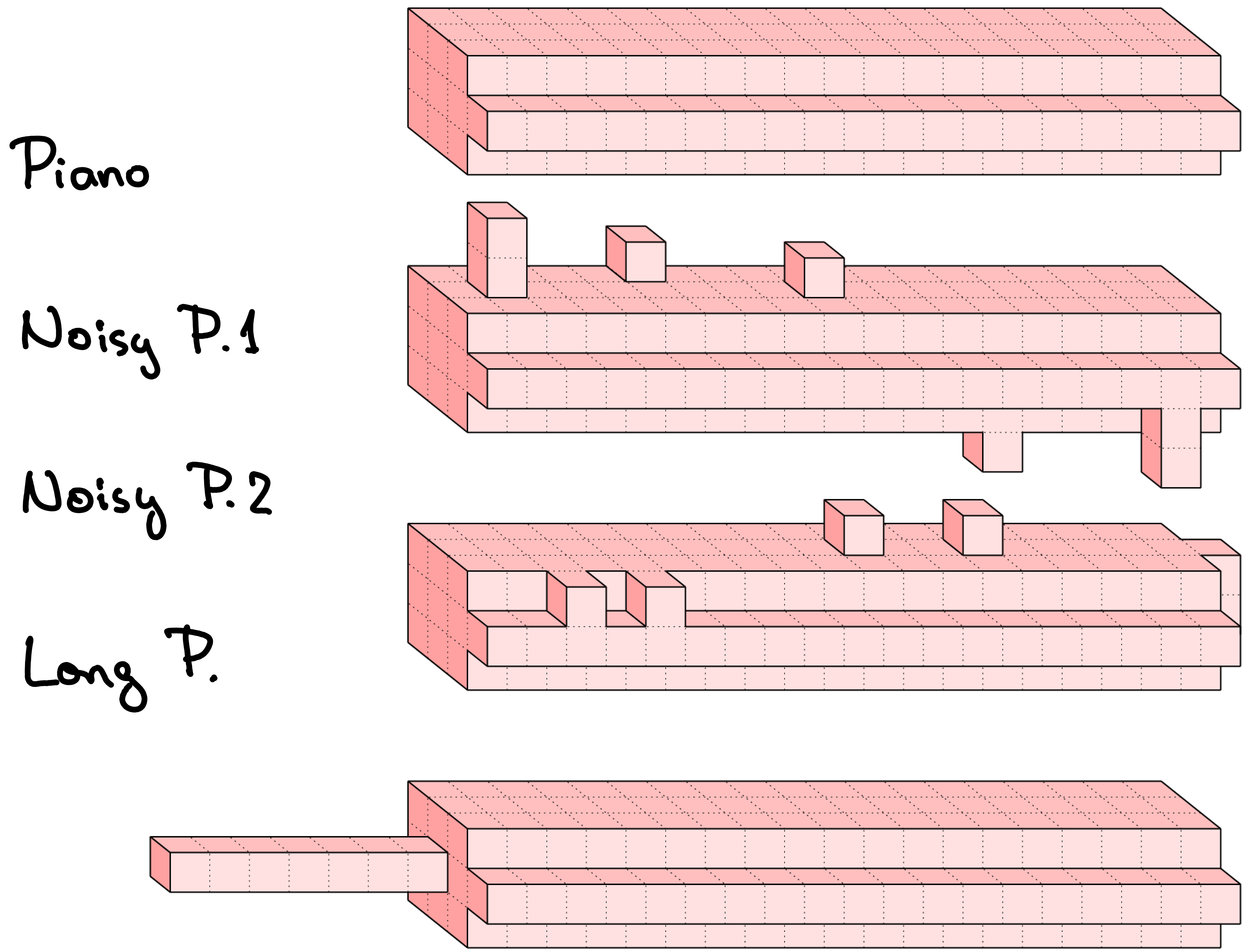
$$Q_2 = \frac{1}{2} \int_{u \in \mathbb{S}^2} \sum_{p \neq 0} (-1)^p B'_p(f_u)\, du$$

IV.2 Toy Models



IV.2 Toy Models

DMC	qPS	qDS_1	qDS_k



IV.2 Toy Models

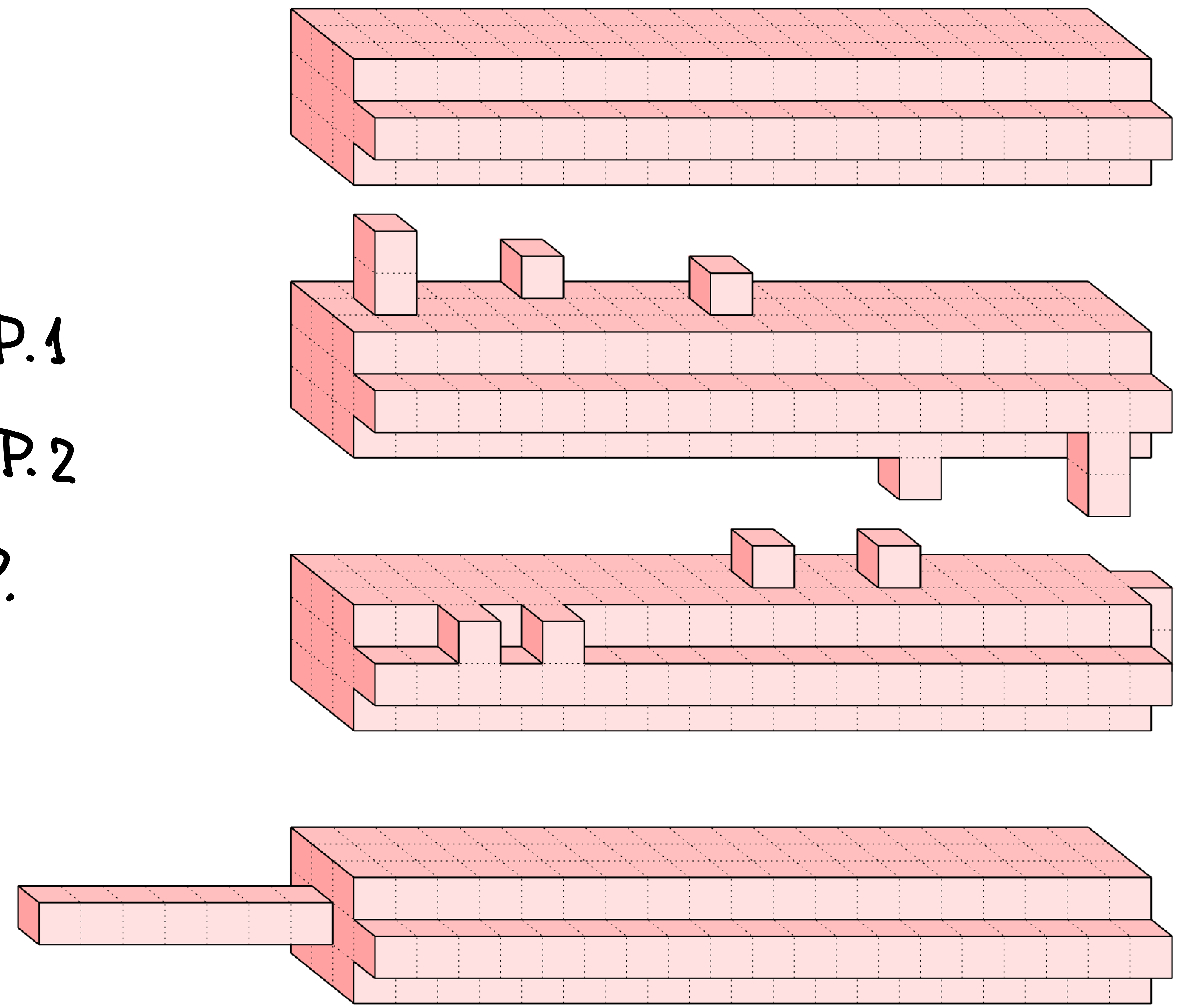
DMC	qPS	qDS_1	qDS_k
26π			
32π			
29π			
33π			

Piano

Noisy P.1

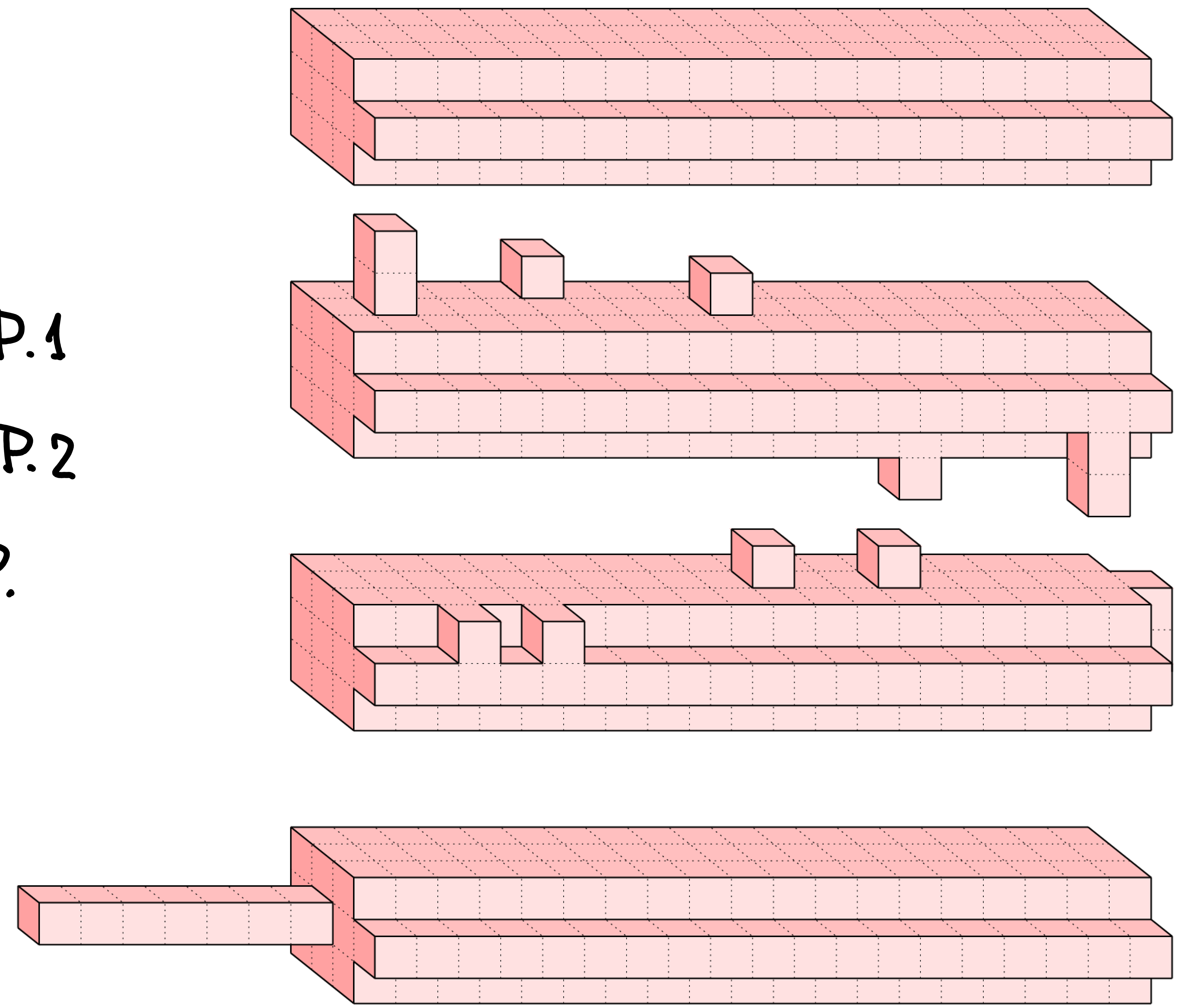
Noisy P.2

Long P.



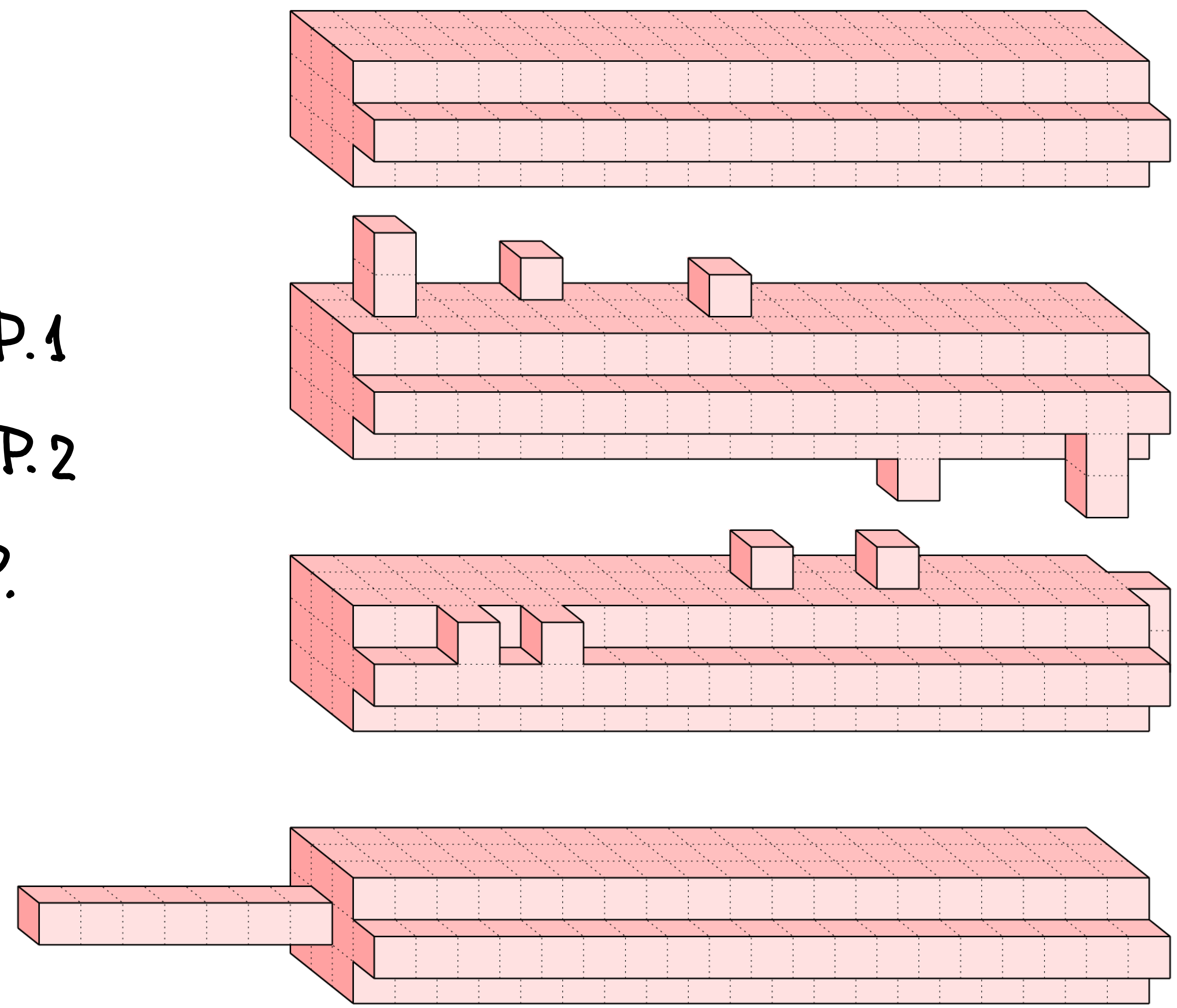
IV.2 Toy Models

DMC	qPS	qDS_1	qDS_k	
26π	25.85π			Piano
32π	32.74π			Noisy P.1
29π	29.87π			Noisy P.2
33π	32.47π			Long P.



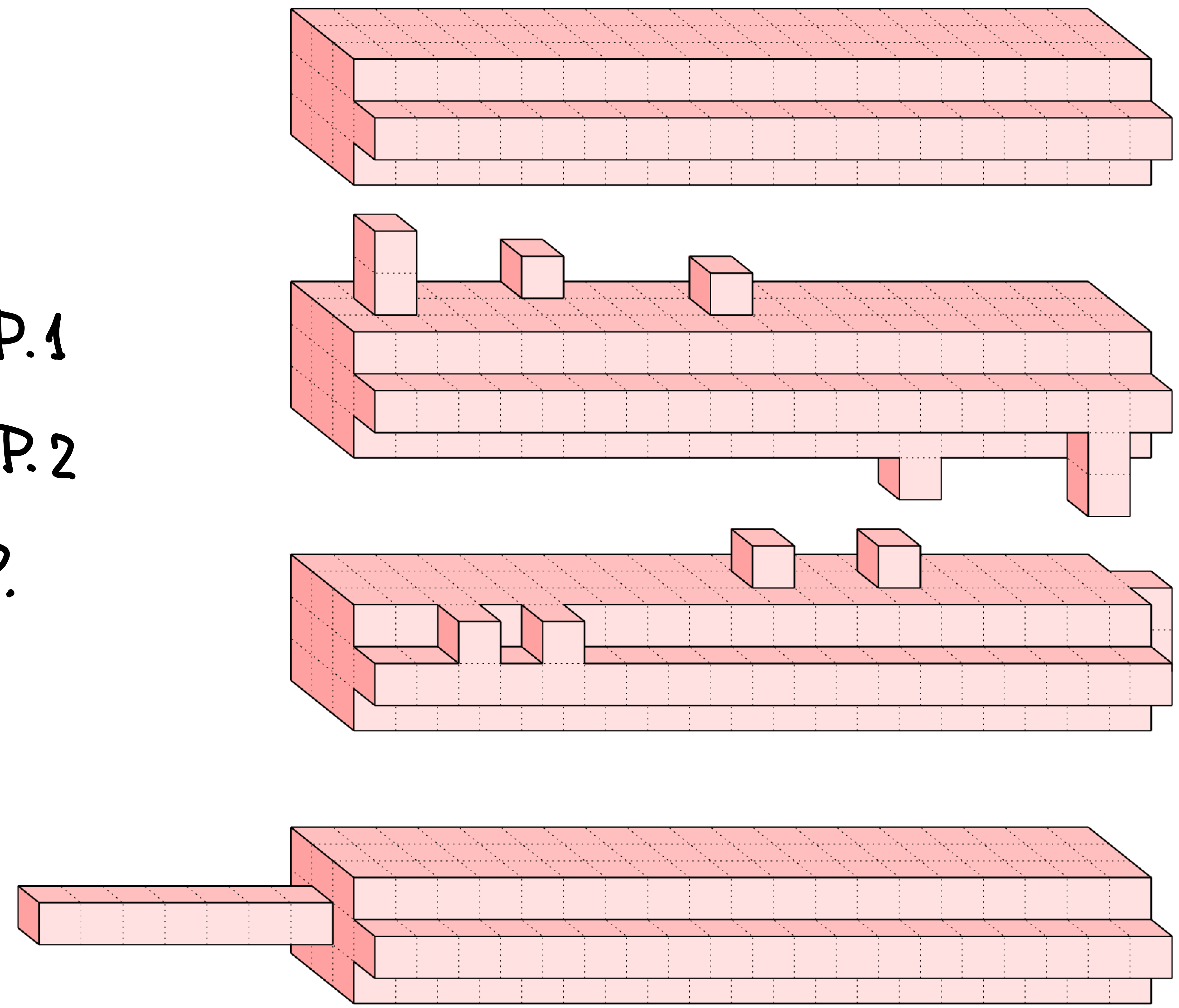
IV.2 Toy Models

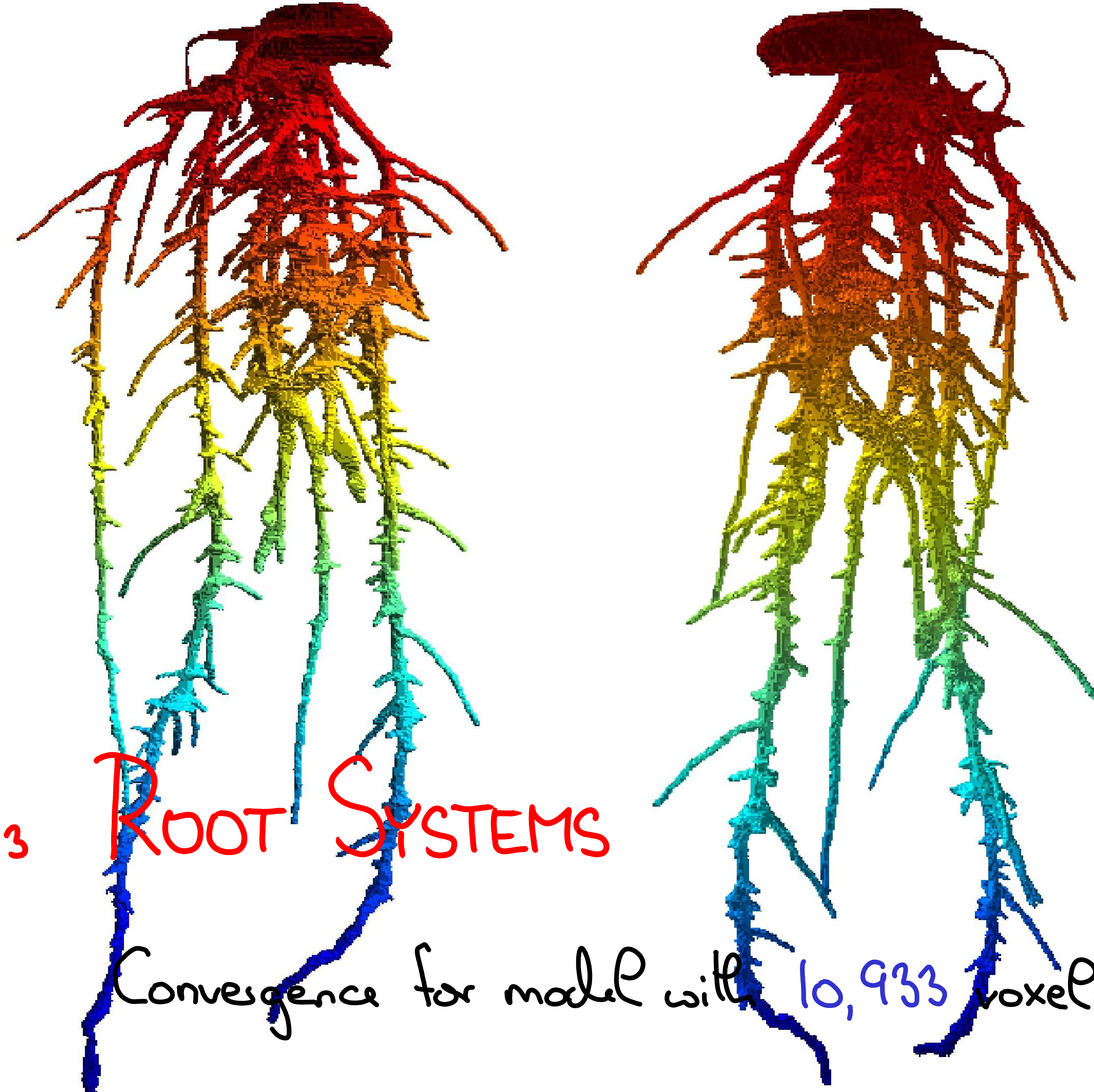
	DMC	qPS	qDS_1	qDS_k
Piano	26π	25.85π	26.00π	
Noisy P.1	32π	32.74π	32.01π	
Noisy P.2	29π	29.87π	28.98π	
Long P.	33π	32.47π	32.98π	



IV.2 Toy Models

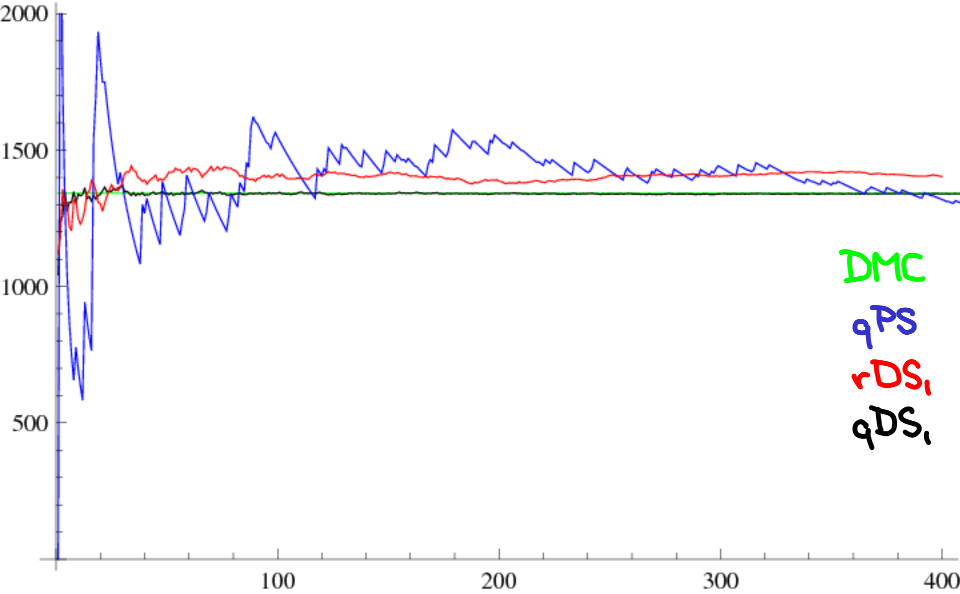
DMC	qPS	qDS_1	qDS_k	
26π	25.85π	26.00π	25.42π	Piano
32π	32.74π	32.01π	26.98π	Noisy P.1
29π	29.87π	28.98π	25.96π	Noisy P.2
33π	32.47π	32.98π	31.12π	Long P.

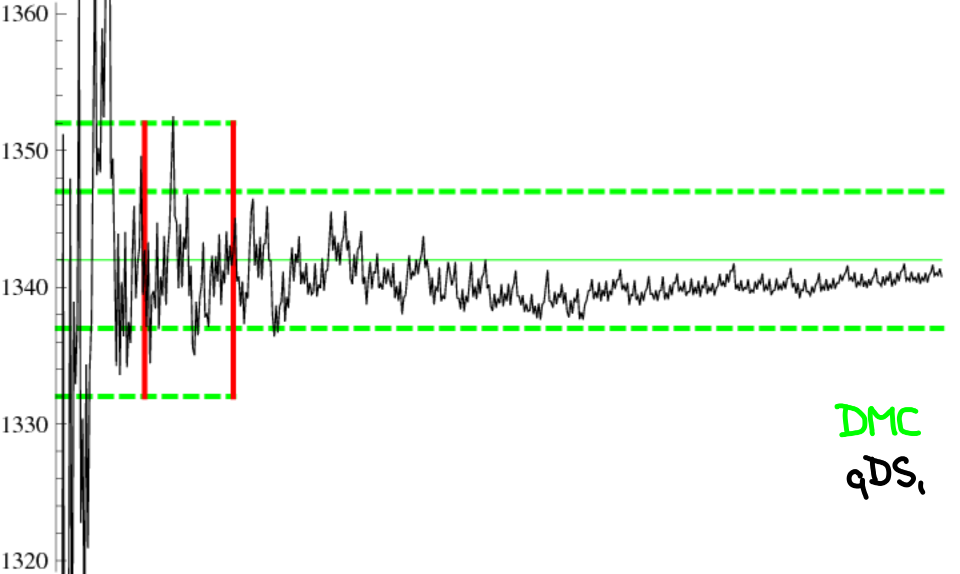




IV.3 Root Systems

Convergence for model with 10,933 voxels.


2000
1500
1000
500
100
200
300
400
DMC
qPS
rDS1
qDS1


1360
1350
1340
1330
1320
DMC
qDS,

THANK YOU