Tutorial on: Digital Topology, Geometry and Applications

4th talk: Skeleton computation and use for shape analysis

Gabriella Sanniti di Baja
Istituto di Cibernetica "E.Caianiello", CNR
Pozzuoli, Naples, Italy

g.sannitidibaja@cib.na.cnr.it

Outline

- Notions
- Object representation: the skeleton
- Shape analysis via object decomposition

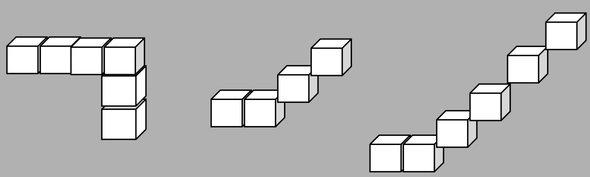
Notions

- Paths and path-based distance
- Distance transform
- Balls and Centers of Maximal Balls

Paths and path-based distance

A path of length *n* from a voxel *p* to a voxel *q* is a sequence of voxels $p = p_0, p_1, \ldots, p_n = q$, such that p_i is a (face, edge, vertex) neighbor of p_{i-1} for $1 \leq i \leq n$

6-, 18- and 26-path



The distance between a voxel *p* and a voxel *q* is the lenght of a minimal 6-, 18-, or 26-path linking *p* to *q*.

The distance can be measured by weighting the unit moves along the path with weights that take into account the different Euclidean length of the different moves. A good selection is $w_f = 3$ $w_e = 4$ $w_v = 5$

Distance transform

- The distance transform is a replica of the image where all object elements are labeled with their distance from the background.
- The distance can be computed by using any distance function (including the Euclidean distance).

Balls, maximal balls and centers of maximal balls

- Each voxel v in the distance transform of an object can be interpreted as the center of a ball whose radius is given by the distance of v from the background.
- A ball is maximal if it is not included by any other single ball in the object. The centers of the maximal balls of the object, CMBs, have a key role. In fact, the object can be recovered by the union of its maximal balls.

Centers of maximal balls

Since distance values of voxels in DT correspond to the radii of the associated balls, to check whether the ball centered on p is maximal it is enough to compare the radius of that ball with the radii of the balls centered on the voxels that are neighbors of p.

If a weighted distance is used, the weights should be taken into account while performing the radii comparison.

A CMB is a voxel p for which the following condition holds for each of its neighbors n_i

$$n_i < p + w_i$$

where w_i is 3, 4 or 5 depending on whether n_i is face-, edge- or vertex-neighbor of p.

Object representation: the skeleton

The skeleton

provides rich geometric information, giving simultaneously locational, orientational, and metric (size) description of an object;

provides a basis for description at multiple spatial scales;

provides descriptions of objects that are intuitive to non-mathematical users.

Desired features

- Stability and robustness;
- Dimensionality reduction;
- Resemblance to the object;
- Correspondence to the object;
- Centrality;
- Level of detail;
- Reversibilty;
- Topological equivalence.

Skeletonization approaches

Medial Axis Transform (MAT);
Grassfire;
Voronoi Diagrams;
Topological Thinning.

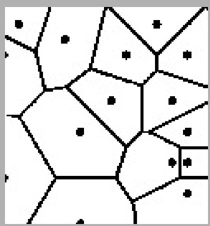
MAT

The medial axis transform, MAT, defined by Blum is the locus of the symmetry points, i.e., the object points placed midway between two sections of the object boundary. Each point of the MAT is center of a ball touching the object boundary in at least two different points and is associated with the radius of the ball.

The grassfire paradigm

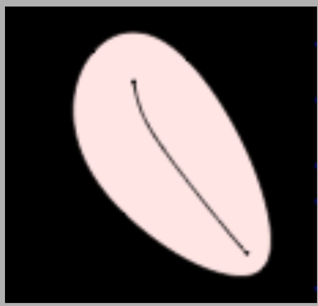
Suppose that all points on the boundary of a dry grass field are set on fire at the same instant of time. In the absence of wind, the fire propagates at constant speed in every direction toward the innermost part of the field. Fire dies out at the quenching points, i.e., the points reached at the same instant of time by fire fronts originated at different parts of the boundary of the field. Of course, the quenching points are symmetrically placed within the field and the instant of time at which they are reached depends on their distance from the boundary of the field. Thus, the grassfire transform has the same properties as the MAT.

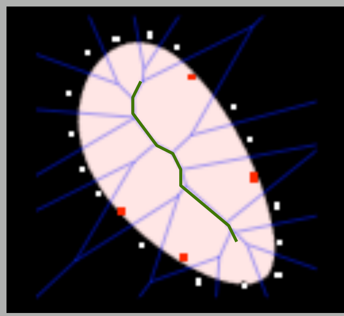
Voronoi Diagram



In the continuous plane/space, given a finite set of points S, the Voronoi cell associated to a point p of S is defined as the subset of the plane/space with the points closer to p than to any other point q in S. For each point in the plane/space there exist at least one point of S to which it is closer. A point can have the same distance from more than one point of S and, if this is the case, the point belongs to a Voronoi edge separating two or more Voronoi cells. The union of Voronoi cells of all points in S is a partition of the plane/space called Voronoi Diagram of the set S.

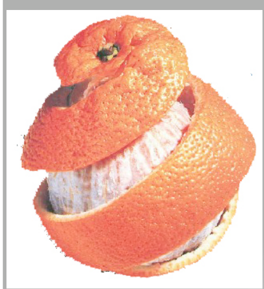
Voronoi Diagram & MAT





- Main problems:
- 1. Boundary sampling (if the density of the boundary points constituting S goes to infinity, the Voronoi diagram of S converges to the to the continuous medial locus)
- 2. Pruning (when the density increases, the number of Voronoi edges also increases).

Topological Thinning: the peeling scheme



Reduction of dimensionality can be achieved by sequentially removing, border after border, all border pixels/voxels whose presence is not necessary to preserve topology of both the object and the background

Topological Thinning

Repeat
remove "deletable" boundary points
from the current object
Until no points are removed

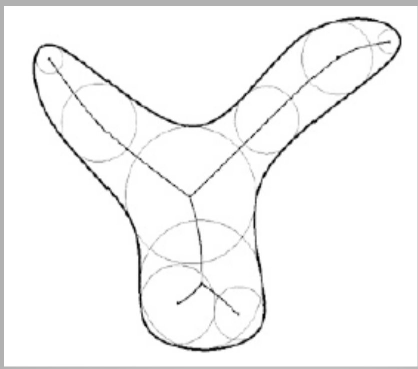
$\Big\}$ One Iteration step

Which points can be regarded as "deletable"?

Simple point

Topology preserving local removal operations (based on the notion of "simple point") can be used, since it has been proved that:
if removal of a voxel p does not alter topology in the 3×3×3 neighborhood of p, then sequential removal does not alter topology globally.

Shape preservation



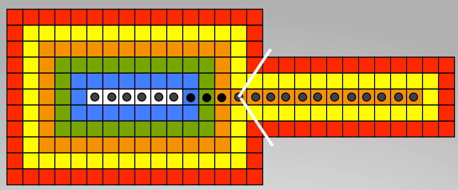
End points

MAT & Distance Transform

A discrete approximation of the MAT is given by the *centers of maximal balls* (CMB) in the *distance transform* (DT) of the object.

MAT & Distance Transform

Drawback 1: the discrete MAT and the represented object may have different topology.



drawback 1

Other object elements are necessary, besides the CMB, to obtain a topologically correct skeleton.

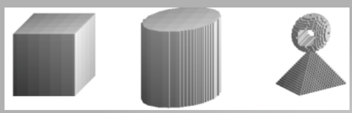
The skeleton can be computed by applying topology preserving removal operations to object elements that are not CMB.

Simple Voxels

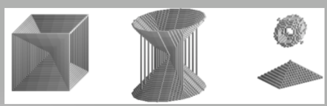
- A voxel v is **simple** if its removal does not change the topology
- Consider only voxels v having at least a face-neighbor in the background
- Count 26-connected object components in the 26-neighbourhood of v, N^{26}
- Count 6-connected background components that are face-adjacent to v in its 18-neighbourhood, N_f^{18}
- A simple voxel has N^{26} = 1 and N_f^{18} = 1

MAT & Distance Transform

Drawback 2: the structure of the discrete MAT depends on the metric used to compute the DT, which may originate a MAT that is not invariant under object rotation.



D^6



D^{26}

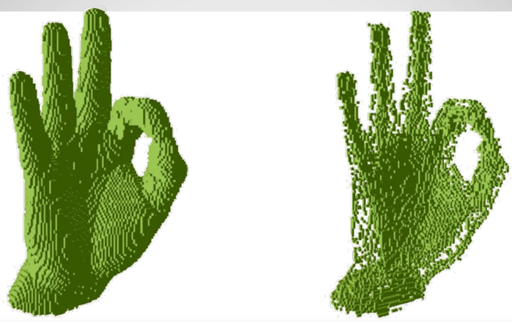
drawback 2

The skeleton may change under object rotation.

A distance function providing a good approximation of the Euclidean distance has to be used to originate a skeleton stable under object rotation.

MAT & Distance Transform

Drawback 3: whichever distance is used the CMB are generally very many, especially for natural shapes.



drawback 3

The structure of the skeleton would be too complex to effectively use it for shape analysis.

Only a suitable subset of CMB has to be taken into account, so as to obtain a skeleton with a structure easy to manage and still adequately representing the object.

Skeletonization algorithm: phase 1

From the object P to a subset PS, consisting of surfaces and curves

-<3,4,5>DT of P
- Anchor point selection (subet of CMB)
- Voxel removal

Skeletonization algorithm: phase 2

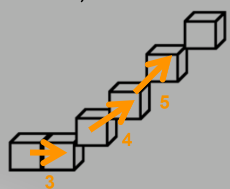
From PS to the skeleton S, consisting exclusively of curves

- Classification of the voxels of PS
- <3,4,5>DT of PS
- Anchor point selection
- Voxel removal

phase 1 – DT computation

We use the <3,4,5> weighted path-based distance, where the unit moves from a voxel to any face-neighbor, edge-neighbor, and vertex-neighbor are weighted 3, 4 and 5 respectvely.

This choice allows us to combine the simplicity of path-based distances with a good approximation to the Euclidean distance.



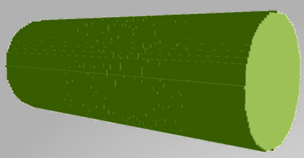
The standard 2-raster scan DT algorithm is used.

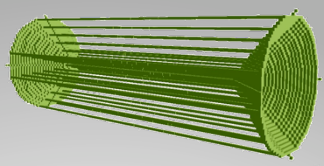
phase 1 – CMB computation

A CMB is a voxel p for which the following condition holds for each of its neighbors n_i

$$n_i<p+w_i$$

where w_i is 3, 4 or 5 depending on whether n_i is face-, edge- or vertex-neighbor of p.





			3
	3	3	4
	3	6	7
3	4	7	10
3	6	8	11
3	6	9	12

phase 1 –Anchor point selection

Due to the CMB rule ($n_i<p+w_i$), a CMB p may have neighbors with distance values larger than p.

If the neighbors of p are *small*, p is a stronger CMB.

Thus, we select as anchor points only the CMB satisfying, for each n_i, the stricter condition:

$$n_i - p < w_i - 1$$

phase 1 – Voxel removal

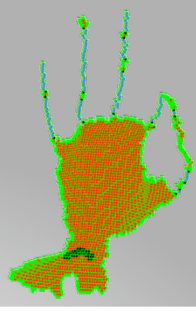
Voxels in DT are accessed in increasing distance order and are sequentially removed provided that they are simple points and are not anchor points.

The obtained PS is likely to be 2-voxel thick at parts. Standard final thinning is used to make PS unit wide.

phase 2 – Voxel classification

By using local checks, the voxels of PS are classified as *curve* voxels (light blue), *junction voxels* (dark green), *internal* voxels (brown) and *edge* voxels (light green).

Voxel classification allows us to compute the DT of the set of internal voxels with respect to the set of the edge voxels.



phase 2 –getting the skeleton

pruning

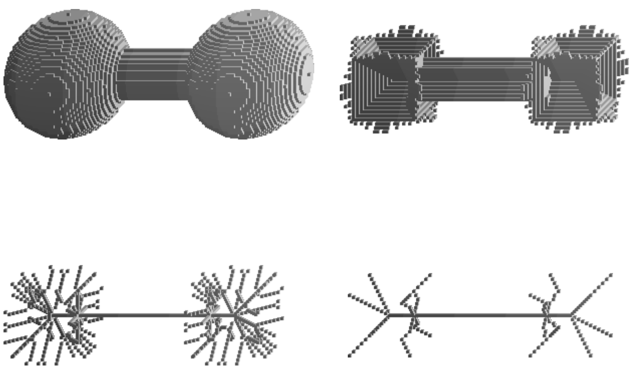
CMB and voxels classified as curve or junction voxels are taken as anchor points.

Voxels are accessed in increasing distance order and are sequentially removed if are simple points and are not anchor points.

The obtained set is likely to be 2-voxel thick. Standard final thinning is accomplished to obtain the unit wide skeleton S.

Pruning completes skeletonization.

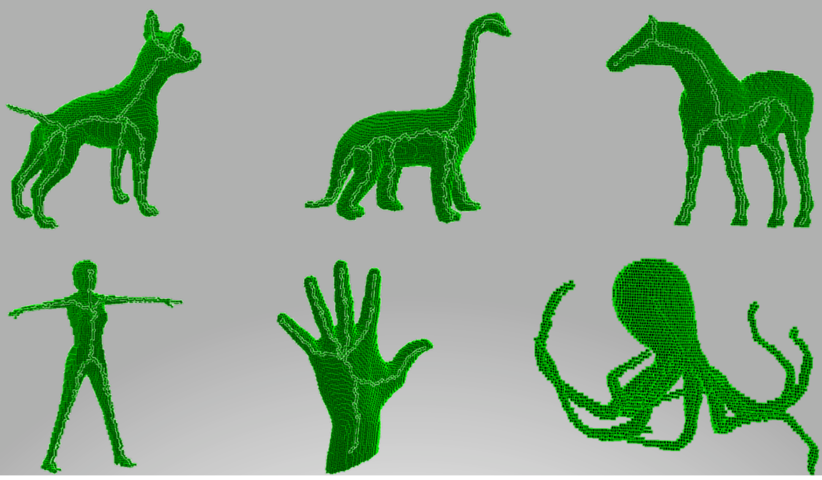
Pruning

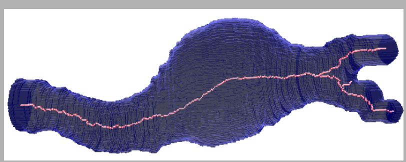


brute force

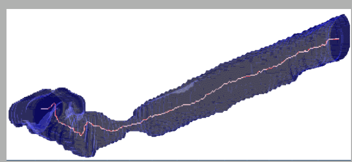
information preserving

Some examples



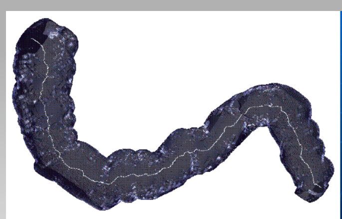


Infra-renal aortic aneurysms

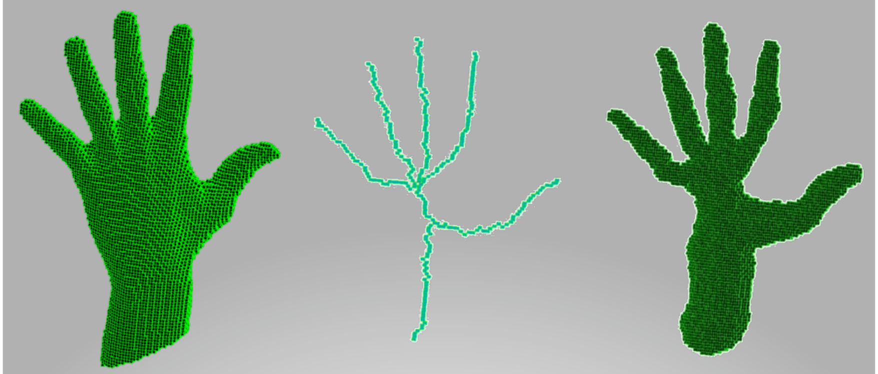


Trachealstenosis

Colonoscopy



Reconstruction

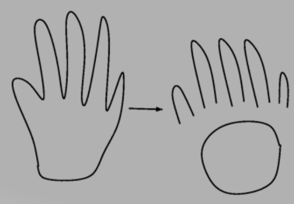


Shape analysis via object decomposition

Why decompose?

object recognition as a hierarchical process

- decompose into parts
- analyze the parts
- describe the object by a graph
  - nodes: parts
  - edges: adjacency between parts



Two main approaches

- pre-defined shape primitives
- irregularities in the boundary

ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

Recognition-by-components

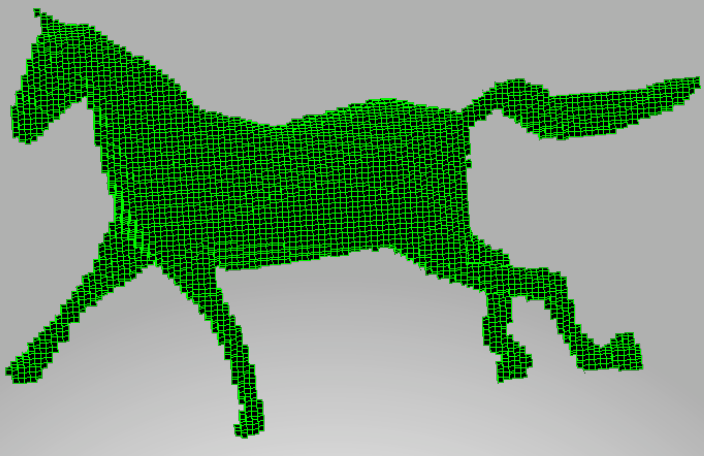
- theory of human understanding
- decomposition into shape primitives
  - generalized-cone components (geons)
  - small number

I. Biederman, *Recognition-by-Components: A Theory of Human Image Understanding*, Psycological Review, 94(2):115-147, 1987

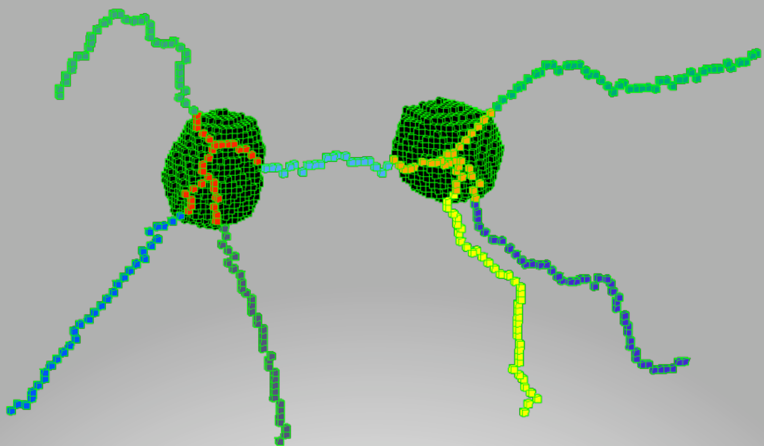
Skeleton decomposition method

- Skeleton partition
- 4D polygonal approximation
- Reconstruction of the object’s parts
- Region merging

Running example

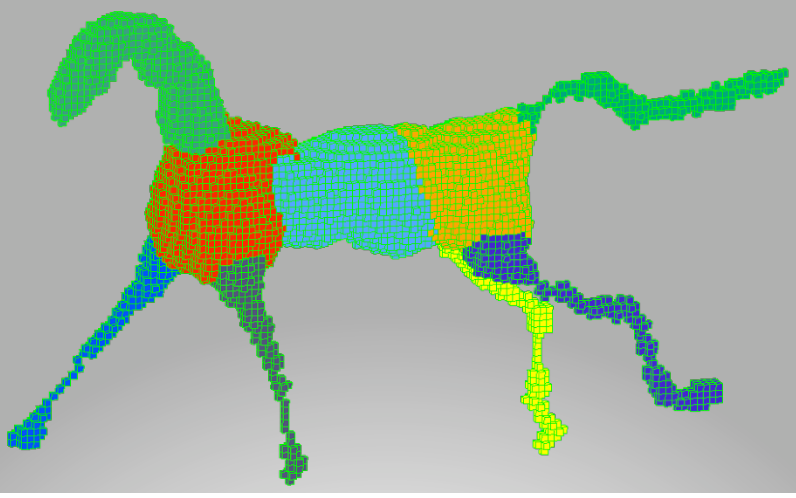


Skeleton partition

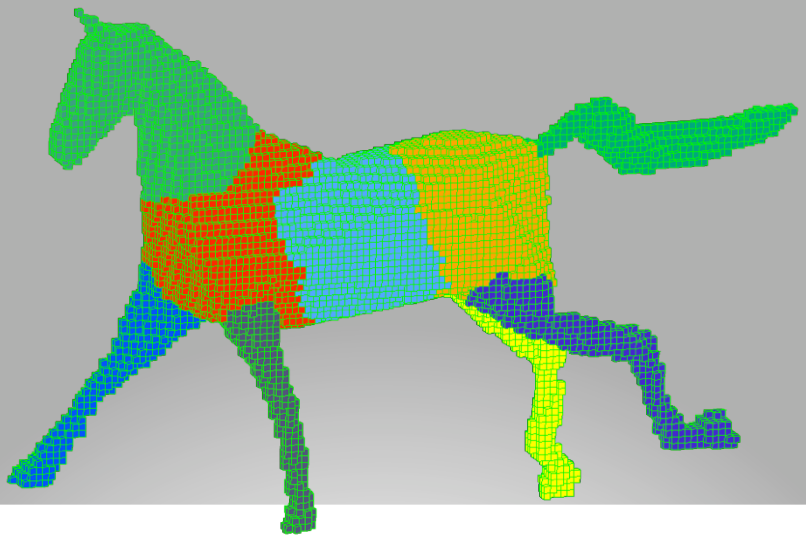


ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

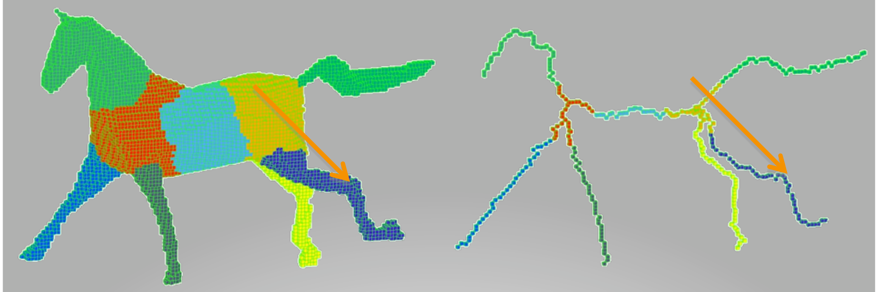
Partial reconstruction



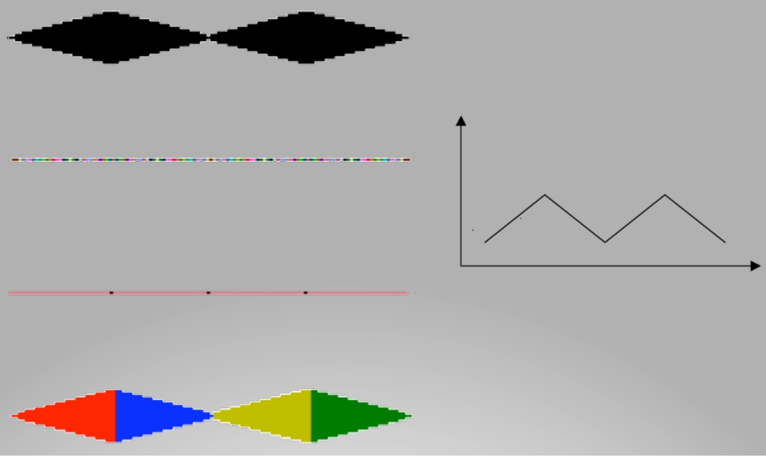
Complete reconstruction



Curvature changes along the boundary

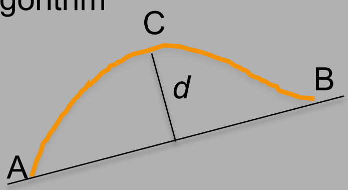


Thickness changes



4D polygonal approximation

Split type Ramer algorithm

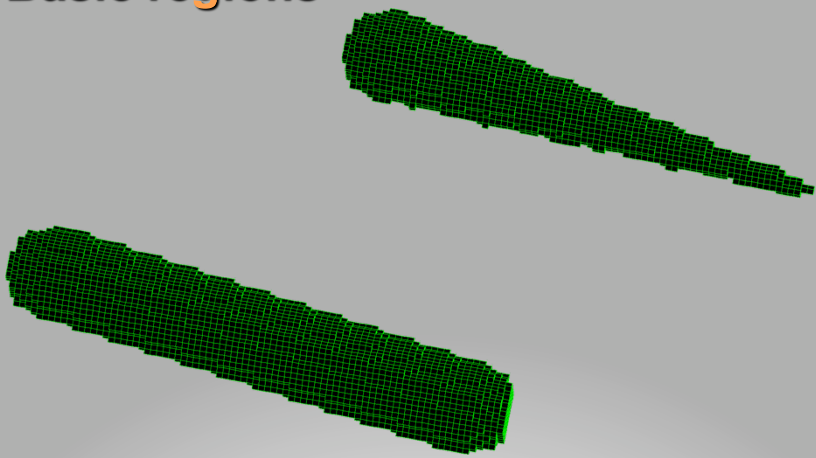


$d^2 = ||AC||^2 - P_{ABC} * P_{ABC} / ||AB||^2$

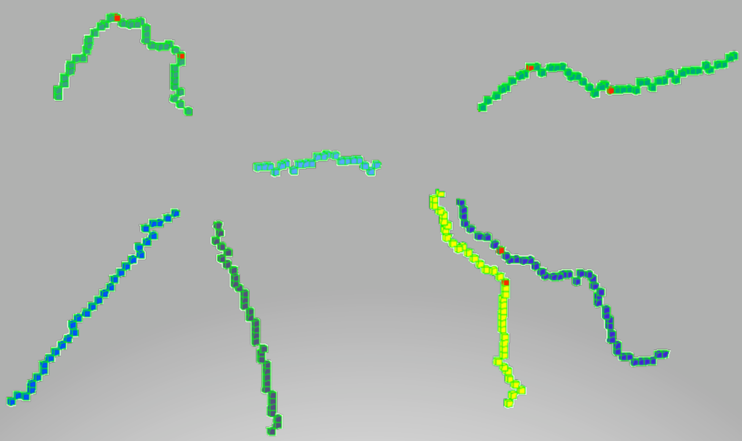
$||AB||$ is the norm of the vector AB
P_{ABC} is the scalar product between vectors AB and AC.

U.Ramer, 1972. An Iterative procedure for the polygonal approximation of plane curves, *CGIP*, 1, 244-256.

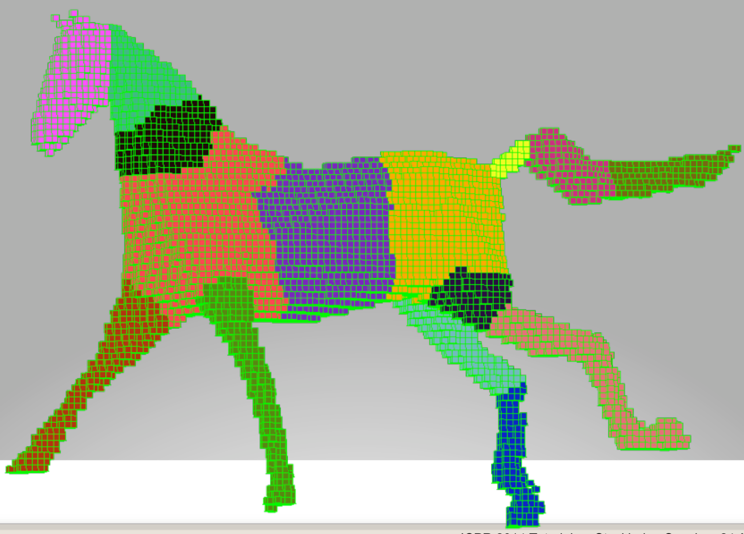
Basic regions



Approximating the simple curves



Decomposition into kernels and simple regions, possibly split into basic regions

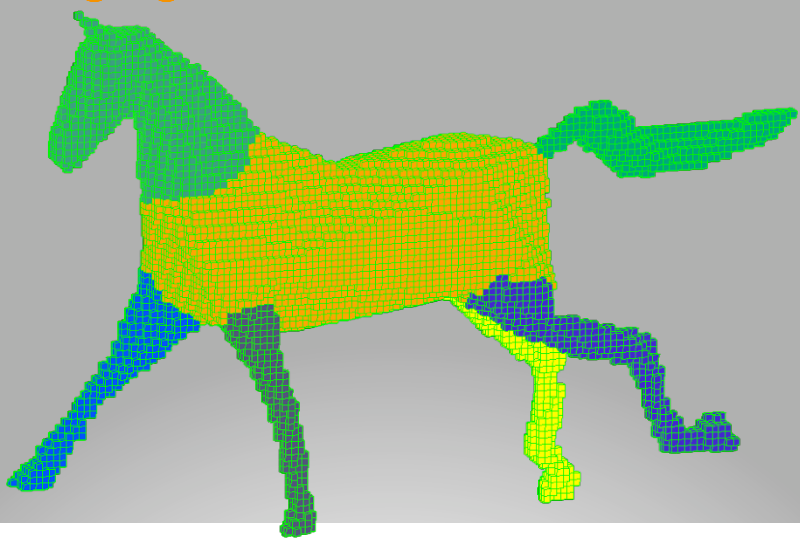


Merging

Consider for merging only simple-regions in between kernels

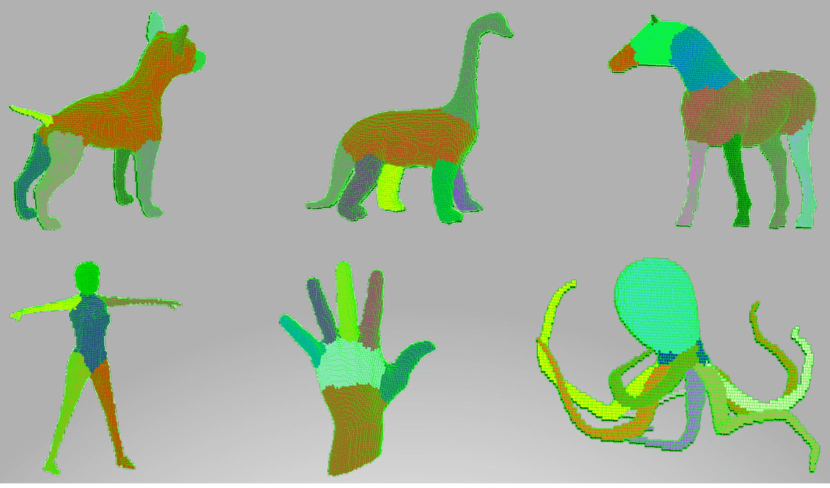
Use the ”visibility criterion” to merge scarcely elongated simple-regions, i.e., regions whose ”visible” boundary part is smaller than the non-visible boundary part

Merging



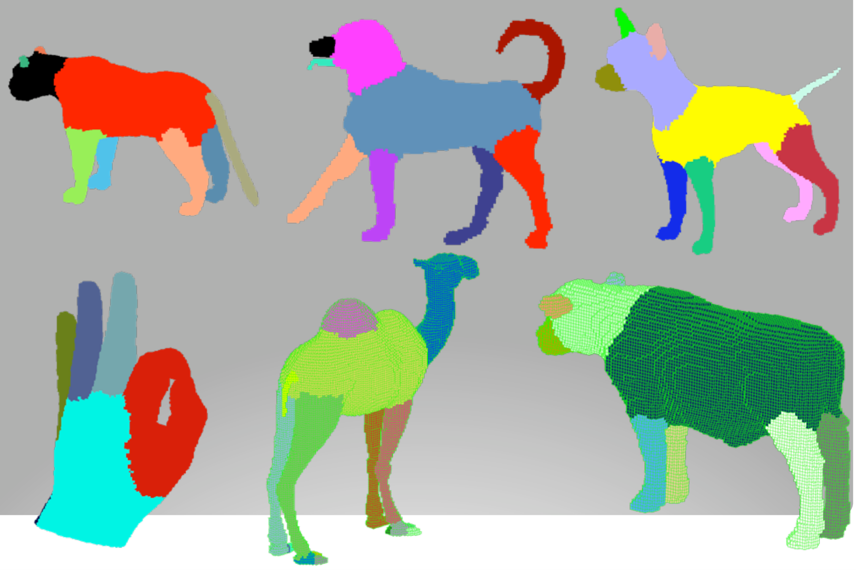
ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

Some results



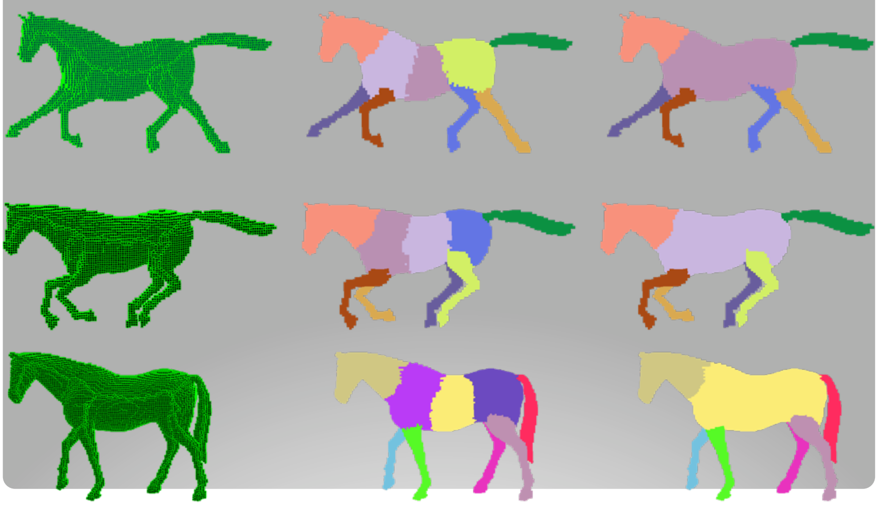
ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

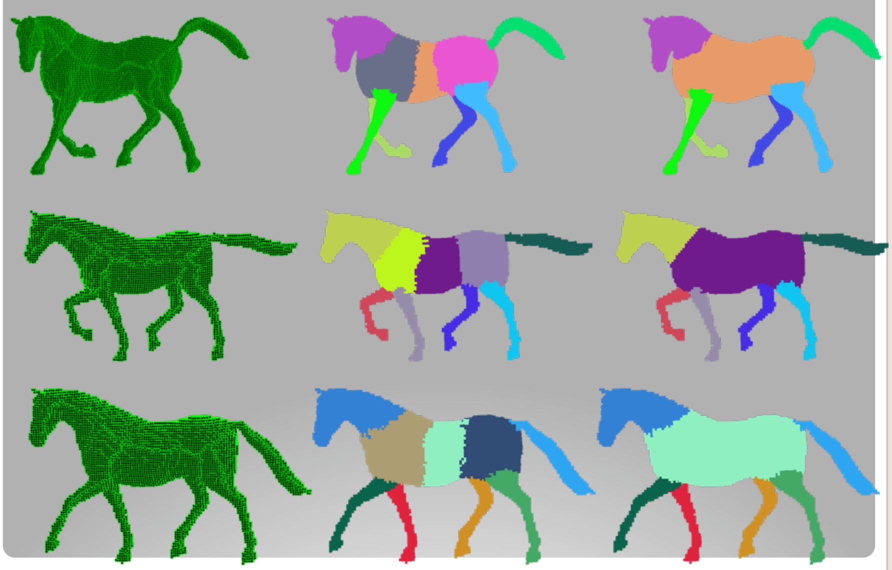
More results

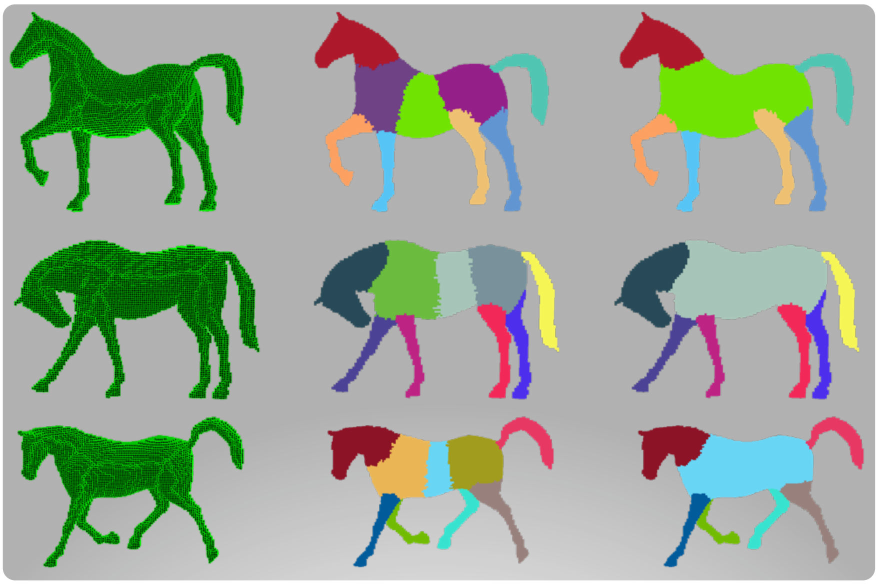


ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

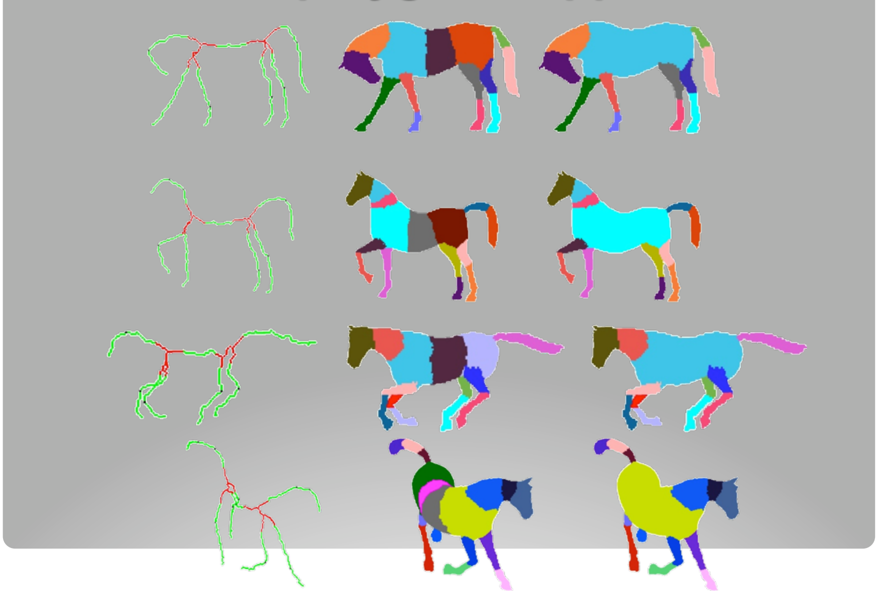
Shape distortions & results



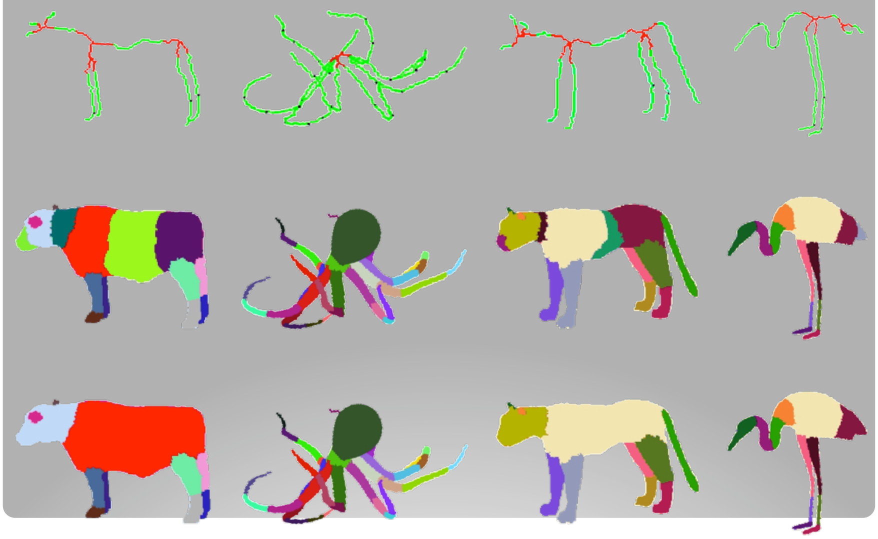




Results with polygonal approximation



Results with polygonal approximation



ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

Questions?

Thank you!

ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

Some references

- Arcelli C., Sanniti di Baja G. and Serino L., *"A parallel algorithm to skeletonize the distance transform of 3D objects"*, Image and Vision Computing **27**, 666-672 (2009).
- Arcelli C., Sanniti di Baja G. and Serino L., "The <3,4,5> curvilinear skeleton", DGCI 2009: Lecture Notes in Computer Science, Springer Publ. **Vol. 5810**, Brlek S., Reutenauer C., Provencal X. Eds., p.409-420 (2009).
- Serino L., Sanniti di Baja G. and Arcelli C., *"Object decomposition via curvilinear skeleton partition"*, 20th International Conference on Pattern Recognition ICPR 2010- August 23-26, 2010, Istanbul (Turkey), Proceedings, 4081-4084, IEEE CS Press
- Arcelli C., Sanniti di Baja G. and Serino L., *"Distance driven skeletonization in voxel images"*, IEEE Transactions on Pattern Analysis and Machine Intelligence **33 (4)**, 709-720 (2011).
- Serino L., Arcelli C., and Sanniti di Baja G. , *"On the computation of the <3,4,5> curve skeleton of 3D objects"*, Pattern Recognition Letters **32 (9)**, 1406-1414 (2011).
- Serino L., Sanniti di Baja G., and Arcelli C. , *"Using the skeleton for 3D object decomposition"*, SCIA 2011: Lecture Notes in Computer Science, Springer Publ. **Vol.6688**, Heyden A. and Kahl F. Eds, p. 447–456 (2011)
- Sanniti di Baja G., Serino L., Arcelli C, *"From linear representations to object parts"*, MCPR2012: Lecture Notes in Computer Science, Springer Publ. **Vol. 7329**, Carrasco-Ochoa J.A. et al. Eds., p. 1-12 (2012)
- Serino L., Arcelli C., and Sanniti di Baja G., *"From the zones of influence of skeleton branch points to meaningful object parts"*, DGCI2013: Lecture Notes in Computer Science, Springer Publ. **Vol. 7749**, R. Gonzalez-Diaz et al. Eds., p. 131-142 (2013).
- Sanniti di Baja G, Serino L, and Arcelli C. , *"3D curve skeleton computation and use for discrete shape analysis"*, Innovations for Shape Analysis: Models and Algorithms, Springer, Breuss M., Bruckstein A., Maragos P. Eds., p. 117-136, (2013)
- Serino L, Arcelli C and Sanniti di Baja G. , *"4D polygonal approximation of the skeleton for 3D object decomposition"*, ICPRAM 2013: SCITEPRESS Science and Technology Publications (ISBN: 978-989-8565-41-9), De Marsico M., Fred A. Eds, p. 467-472 (2013)
- Serino L, Arcelli C and Sanniti di Baja G. , *"Approximating the skeleton for fine-to-coarse shape representation"*, ICIAP 2013, Part I, Lecture Notes in Computer Science, Springer Publ. Vol. **8156**, A. Petrosino, Ed. p. 111-120 (2013)
- Serino L, Arcelli C and Sanniti di Baja G. , *"Decomposing and sketching 3D objects by curve skeleton processing"*, CIARP 2013, Part I, Vol. **8258**, J. Ruiz-Shulcloper and G. Sanniti di Baja Eds, p. 25–32, (2013).
- Serino L. and Sanniti di Baja G., *"Pruning the 3D curve skeleton"*, 22th International Conference on Pattern Recognition ICPR 2014- August 24-29, 2014, Stockholm (Sweden), Proceedings, 1051-4651/14 $31.00, ©2014IEEE, 2269-2274, (2'14).

ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014

Gabriella Sanniti di Baja
Institute of Cybernetics "E.Caianiello" (ICIB),
Italian National Research Council,
Pozzuoli, Naples, Italy
<gsdb@cib.na.cnr.it>

Education:
- "laurea cum laude" in Physics,University of Naples, Italy, 1973
- PhD Honoris Causa, Uppsala University, Sweden, 2002.

Professional experience:
- Director of Research at ICIB-CNR 1990 -
- Co-Editor-in-Chief of Pattern Recognition Letters 2000 -
- International Association for Pattern Recognition (IAPR) Executive Committee member 1994-2002
- IAPR Governing Board member 2008 - 2012
- GIRPR (Group of Italian Researchers in Pattern Recognition) President 2008 – 2012
- Organization of international conferences

Professional achievements: IAPR Fellow; Foreign Member of the Royal Society of Sciences at Uppsala, Sweden.

Main research interests:
Pattern Recognition, Computer Vision, Digital Geometry and Topology, Distance Transformation and applications, Representation Systems, Shape Decomposition and Description in 2D and 3D images, Multiresolution, Segmentation.

Publications: ≈ 200 papers

ICPR 2014 Tutorials – Stockholm, Sweden, 24 August 2014