Workshop #3 Exercises

1. Find the critical points of the function $f(x, y) = x^2 + y^4 - 4xy$ by using first partial derivatives. Then use second partial derivatives to establish whether each critical point is a minimum, a maximum, or a saddle point.

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2. Using the Gradient Descent Method with initial approximation $x_0 = (0, 0)$, find the minimum point and the minimum value of the function $g(x, y) = (1 - x + x^2) \cdot e^{y^2} + (1 - y + y^2) \cdot e^{x^2}$.

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3. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45 \cdot \cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate $P(t) = 100$ tons per hour.

a) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

b) Is the amount of unprocessed gravel at the end of the workday ($t = 8$) greater or smaller than amount of gravel at the beginning of the workday?

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4. Solve the system of equations

$$\begin{cases} x - 2y + 3z = -1 \\ 3x + 2y - 5z = 3 \\ 2x - 5y + 2z = 0 \end{cases}$$

using Gradient Descent Method applied to a function of the kind $f(\mathbf{x}) = \|A\mathbf{x} - b\|_2^2$ where $A\mathbf{x} = b$ is the matrix equation that corresponds to the system, and $\| \cdot \|_2$ is the Euclidean norm.
