

Covariance Matrix Estimation via Macroeconomic Factor Modeling

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Abstract

This report compares the performance of two asset allocation strategies namely; Risk Parity and Mean Variance during periods of volatility in South Africa. Using South African financial data, the risk adjusted and information ratio of a unlevered Risk Parity Index portfolio and Tangency Index portfolio are compared. The research paper found that the Mean Variance Optimisation approach is an superior asset allocation strategy during periods of high volatility in South Africa.

Keywords: Factor Modeling, Macroeconomic Factors, Asset Returns, Covariance Matrix

1. Introduction

Macroeconomic factor models use observable economic time series to explain the common variation in asset returns, with the unexplained, idiosyncratic, variance being asset-specific.

Overall investment risk is typically classified as being either systematic (market), affecting the broader market, or idiosyncratic risk, which is unique to individual assets. While the latter can, in effect, be diversified away, the former affects a large portion of assets. Factor models can be used to explain the common (systematic) variation in asset returns, with the remaining variance, not explained by the factors, being unique to each individual security.

2. Literature Review

The use of modern, multivariate, factor models can be seen as an extension of the seminal work of [Sharpe \(1964\)](#)'s Capital Asset Pricing Model (CAPM), which used a single factor- the market portfolio- to estimate asset returns. In most cases, however, there are potentially, infinitely, many factors which can influence asset returns. Modern factor models allow one to address the curse of dimensionality by assuming stock returns are driven by a, more, limited set of factors. Factor models thus decompose

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asset returns into two components: one driven by the common variation between stocks, due to the factors, and another idiosyncratic component, unique to each asset. Using a factor model can therefore be a useful way of reducing the number of possible parameters which can, possibly, drive asset returns; assuming that what is not explained by the, common, factors is idiosyncratic risk unique to each asset.

Factor models have gained in prominence since [Sharpe \(1964\)](#), and have been extended to include more than one factor. In particular, this class of models can generally be split into three types ([Connor, 1995](#)); macroeconomic, statistical, and fundamental factor models. Whilst statistical factor models use only the input (data) to determine the relevant factors for asset returns, macroeconomic and fundamental factor models rely on the researcher to supplement the asset returns data with factors. One of the most prominent fundamental factor model was developed by [Fama & French \(1992\)](#), who use firm-specific variables including relevant firm financial ratios, firm-size, and market beta in order to explain the variation in asset returns.

In using macroeconomic factor models to study the behavior of asset returns, [Chen, Roll & Ross \(1986\)](#) propose a set of candidate economic variables as factors to explain systematic asset risk. They found that industrial production, changes in risk premium, some measures of inflation, and changes in the yield curve were all significant in explaining expected asset returns. In a similar vein, [Kim & Wu \(1987\)](#) find that an economic factor model, generally, performs better than a multivariate CAPM. The factors that they found to be significant were split into three categories: general, economy-wide, variables, the second category focused on the monetary side of the economy and included the interest rate and money supply. And the final factor comprised of labor market variables.

3. Exploratory Analysis

This section aims to provide a, brief, exploration of the data, justifying the choice of macroeconomic variables used as factors, as well as the treatment of the relevant assets.

3.1. Data and Descriptive Statistics

Table 3.1: Macroeconomic Factors

Name	Description	Source
Bcom_Index	Bloomberg Commodities Index	N. Katzke
Inflation	Inflation (Consumer Prices)	IMF International Financial Statistics
MM.Rate	SA Money Market Rate	IMF International Financial Statistics
Real.GDP	SA Real Gross Domestic Product	IMF International Financial Statistics
Real.INV	SA Real Gross Fixed Capital Formation	IMF International Financial Statistics
US_10Yr	US Long-Term Bond Yields	N. Katzke
USDZAR	USD/ZAR Spot Price	N. Katzke
VIX	CBOE Volatility Index	N. Katzke

Table 3.2: Summary Statistics: Macroeconomic Factors

	US_10Yr	Bcom_Index	VIX	USDZAR	MM.Rate	Real.GDP	Real.INV	Inflation
median	1.250	0.003	-0.027	0.003	2.019	0.009	0.011	0.805
mean	1.274	-0.005	0.008	0.013	1.988	0.004	0.003	0.786
SE.mean	0.038	0.012	0.040	0.008	0.029	0.004	0.005	0.038
CI.mean.0.95	0.077	0.024	0.080	0.017	0.058	0.007	0.011	0.076
var	0.098	0.010	0.107	0.005	0.058	0.001	0.002	0.096
std.dev	0.314	0.099	0.326	0.069	0.240	0.030	0.044	0.310
coef.var	0.246	-18.243	41.716	5.248	0.121	6.827	13.362	0.394

Tables 3.1 and 3.2, respectively, provide a description of the factors included along with their sources, and summary statistics for the factors. Macroeconomic factors for South Africa were chosen on the basis of their possible influence in driving asset returns in South Africa. US 10 Year bond yields were used to quantify the influence of US Monetary Policy decisions in driving global liquidity. This is based on the direct relationship between bond yields and interest rates, whereby lower interest rates depress bond yields and these thus become a less attractive investment option; potentially leading investors to seek higher returns in domestic and foreign asset markets. The Bloomberg commodities index was used to measure the influence that changing commodity prices may have on asset returns, given that the South African economy is still influenced to a large extent by fluctuating commodity prices. The CBOE VIX volatility index was included to account for the effect that changing risk perceptions may have on domestic asset returns. Similarly to changes in US long term bond yields, lower risk perceptions may lead to capital flowing more towards developing (periphery) and could thus influence South African asset returns. The USDZAR spot rate was also included as changes in the price of foreign exchange can influence asset returns through, for example, increasing the cost of

financing outstanding debt, or increasing input costs. The money market rate was used to represent the domestic monetary policy stance, this can affect the cost of borrowing for businesses, as well as changing investor incentives when it comes to investing in asset markets; both of which can influence returns. Real gross fixed capital formation was used to account for the level of investment in the economy for any given quarter. Finally, inflation was measured using the quarterly growth in the Consumer Price Index (CPI).

Table 3.3: T40 Constituents: Financials

Ticker	Constituent
ABG	ABSA GROUP LTD
CPI	CAPITEC BANK HOL
DSY	DISCOVERY LTD
FSR	FIRSTRAND LTD
GRT	GROWTHPOINT PROP
INL	INVESTEC LTD
INP	INVESTEC PLC
NED	NEDBANK GROUP
NRP	NEPI ROCKCASTLE
OMU	OLD MUTUAL LTD
RNI	Reinet Investments SCA
SLM	SANLAM LTD
SBK	STANDARD BANK GR

Table 3.4: T40 Constituents: Industrials

Ticker	Constituent
APN	ASPEN PHARMACARE
BID	BID CORP LTD
BVT	BIDVEST GROUP
BTI	BRIT AMER TOBACC
CLS	CLICKS GROUP LTD
MNP	MONDI PLC
MRP	MR PRICE GROUP
MTN	MTN GROUP LTD
MCG	MULTICHOICE GROU
NPN	NASPERS LTD-N
PRX	PROSUS NV
CFR	RICHEMONT-DR
SHP	SHOPRITE HLDGS
SPP	SPAR GRP LTD/THE
VOD	VODACOM GROUP
WHL	WOOLWORTHS HLDGS

Table 3.5: T40 Constituents: Resources

Ticker	Constituent
AGL	ANGLO AMER PLC
AMS	ANGLO AMERICAN P
ANG	ANGLOGOLD ASHANT
BHP	BHP GROUP PLC
EXX	EXXARO RESOURCES
GLN	GLENCORE PLC
GFI	GOLD FIELDS LTD
IMP	IMPALA PLATINUM
NPH	NORTHAM PLATINUM
SOL	SASOL LTD
SSW	SIBANYE STILLWAT

For this analysis, only variables in the Top 40 stocks in the JSE with the highest market capitalization were included. This was done to account for the fact that larger companies will be more exposed to global macroeconomic factors, relative to those with smaller market caps. The variables chosen as part

of this “T40” are based on the FTSE Russell classification, as of the end of 2021 Q3 (see ([“Factsheet \(ZAR\) FTSE/JSE top 40 index”, n.d.](#))). After subsetting the Top 40 stocks, these were stratified into their respective super-industries. The stocks included, as well as their relevant industries, can be found in Tables 3.3-3.5 above. In order to estimate the covariance matrix, there could be no missing values in the asset returns data. Firstly, only those securities which had more than 70% of observations were considered, this led to five assets dropping out: NRP, OMU, RNI, MCG, and PRX. The former three were in financials, while the latter two were classified as industrials. And secondly, returns were imputed according to their own probability distribution. After this, there were no more missing values in the asset returns data.

3.2. Data Visualization

This section presents figures for the relevant macroeconomic factors, and the asset returns (disaggregated) by industry. All macro factors entered the model in log terms, and, in addition, non-stationary variables were taken as quarterly changes.

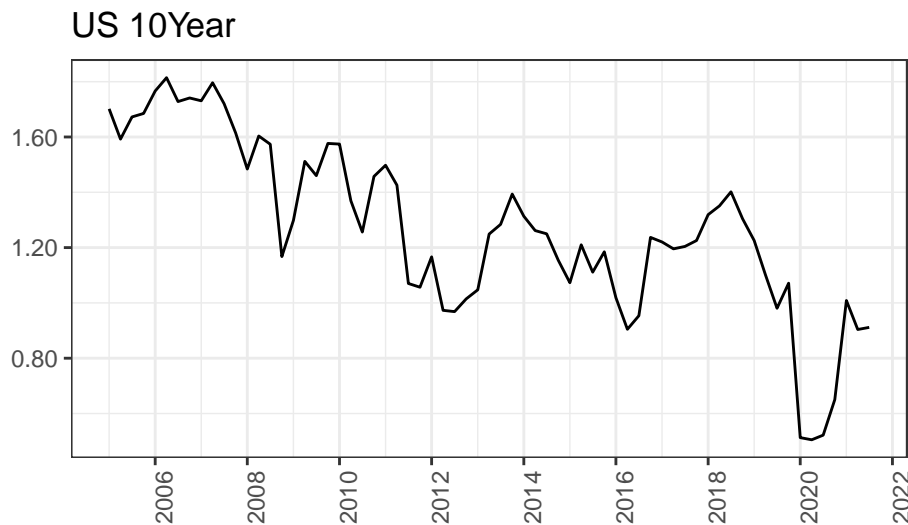


Figure 3.1: US Long-Term Bond Yields

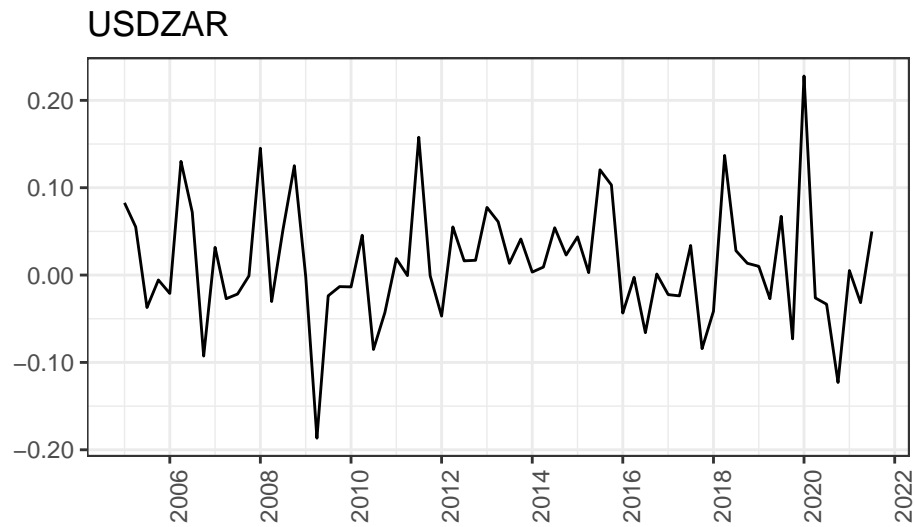


Figure 3.2: USDZAR Spot Price

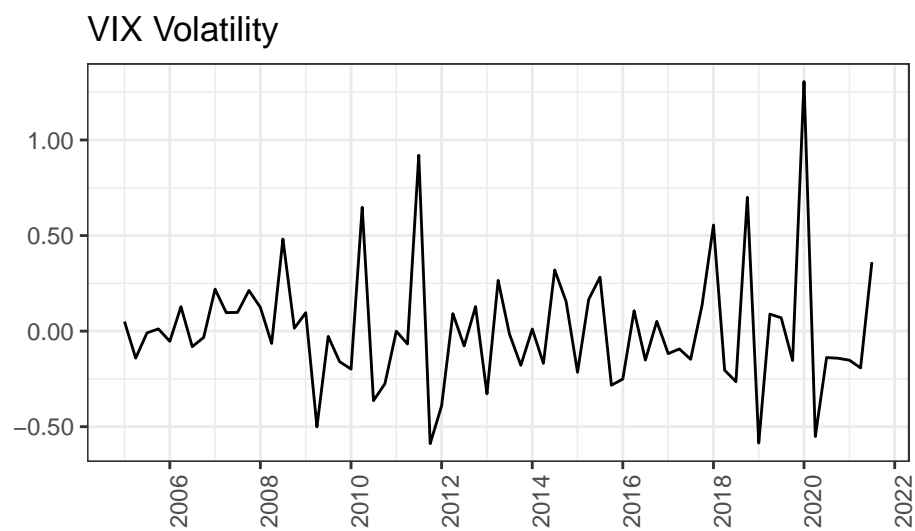


Figure 3.3: CBOE VIX Volatility Index

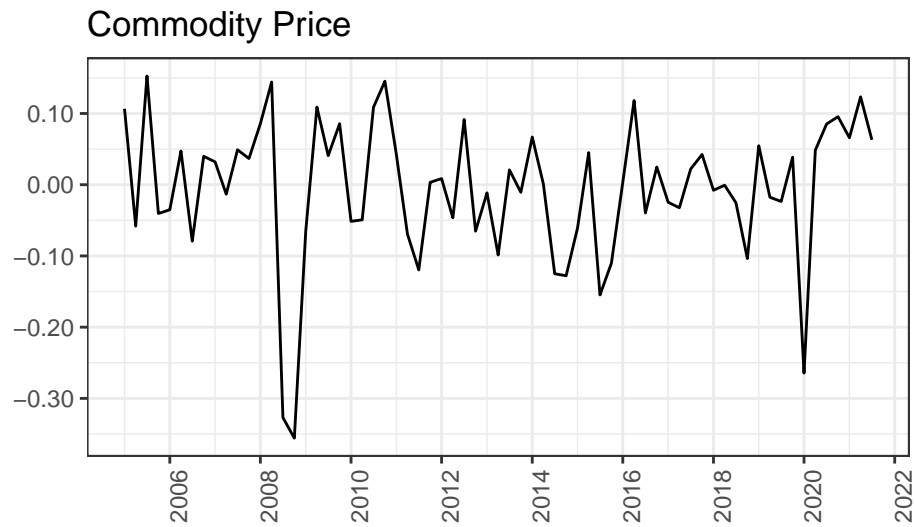


Figure 3.4: Bloomberg Commodity Price Index

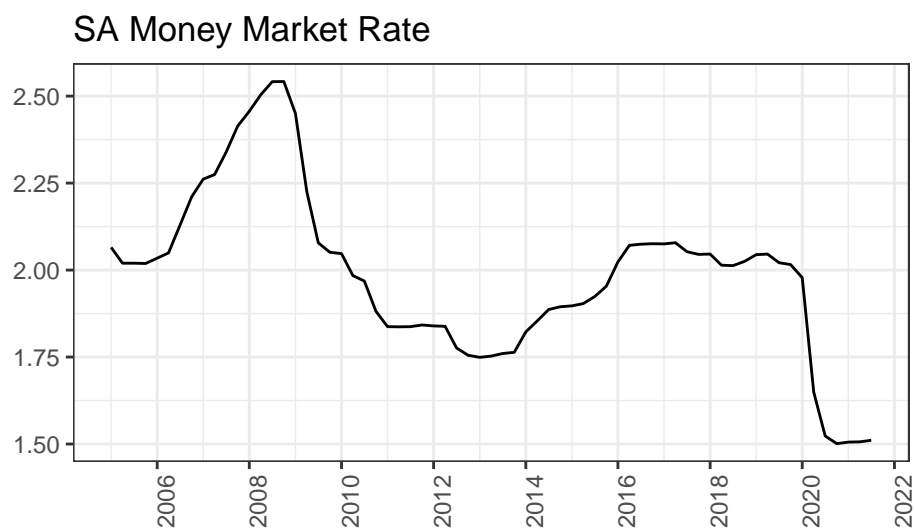


Figure 3.5: South Africa Money Market Rate



Figure 3.6: South Africa Real GDP

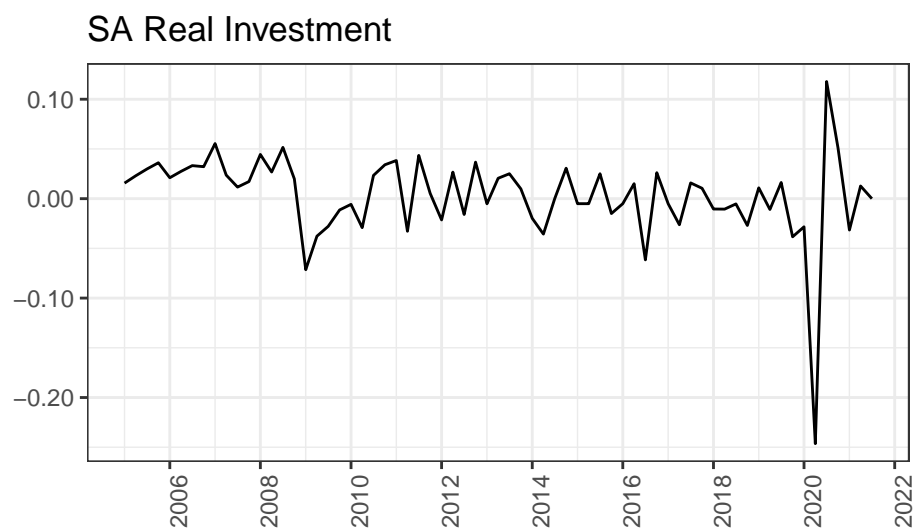


Figure 3.7: South Africa Real Gross Fixed Capital Formation

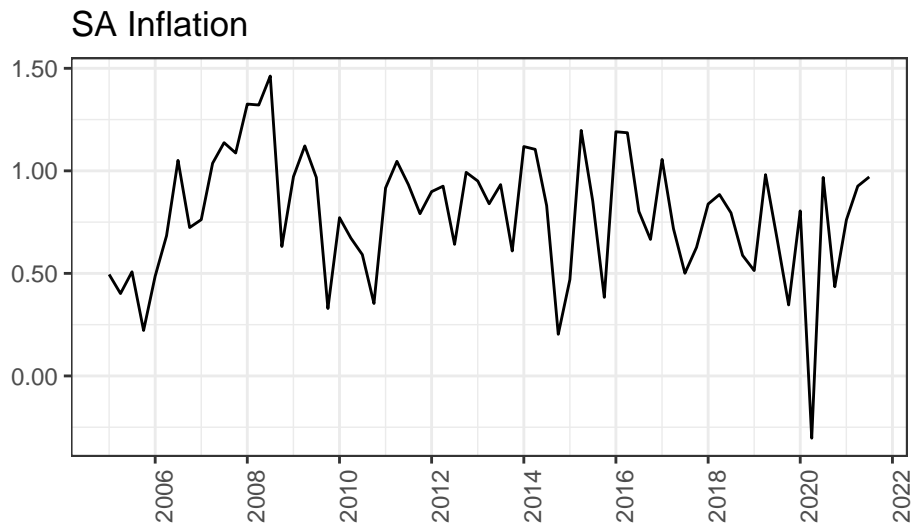


Figure 3.8: South Africa Consumer Price Inflation

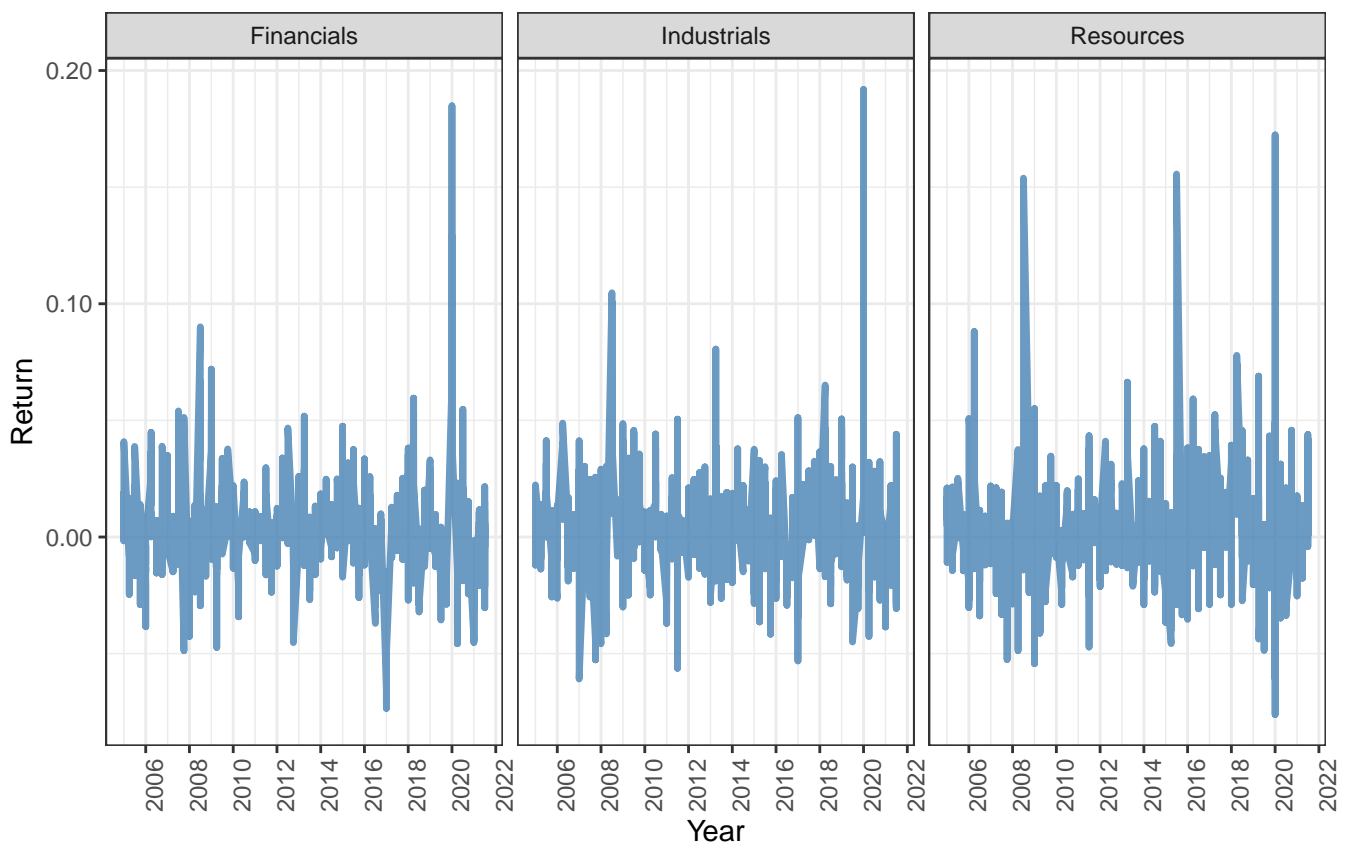


Figure 3.9: Asset Returns by Industry

4. Empirical Analysis

The model used to estimate covariance matrix parameters comprises of 8 macroeconomic factors, and 35 assets, disaggregated into their respective super-industries. The macroeconomic factor model was then applied to the three different industries, as these factors may affect assets in different industries differently. These three industry classes comprise of 10, 14, and 11 observations for the financial, industrial, and resources sectors, respectively. This section presents the main contribution of this paper, and the rest of it is set up as follows. Section 4.1 presents the methodology used when constructing the macroeconomic factor model, Section 4.2 presents the results from estimating the model, and finally, Section 4.3 provides a discussion of the main results.

4.1. Methodology

The equation of the model is given by:

$$R_{ji,t} = \alpha_{ji} + \beta_{ji1}F_{1t} + \dots + \beta_{jik}F_{kt} + \dots + \beta_{jiK}F_{Kt} + \epsilon_{it} \quad (4.1)$$

With F representing the 8 factors included in estimation, β_{ik} representing the sensitivity of asset i to factor k , ($k = 1, \dots, K$), for industry $j = (F, I, R)$. Factor models rely on three main assumptions. Firstly, individual factor realizations have zero expected value: $\mathbb{E}(f_k) = 0$. Secondly, asset-specific errors are uncorrelated with each of the factors: $cov(f_{kt}, \epsilon_{it}) = 0 \forall k, i, \& t$. And, finally, that error terms are serially uncorrelated: $cov(\epsilon_{it}, \epsilon_{ds}) = 0 \forall i \neq d$, and $t \neq s$. From this, one then essentially has that:

$$Var(r_i) = L\phi L' + \sigma \quad (4.2)$$

With the above implying that the variance of individual assets is given by the sum of both the systematic and idiosyncratic risk components.

4.2. Model Estimation

This section presents the results from estimating the macroeconomic factor model, with 4.1-4.3 displaying the estimated α 's and β 's for each industry, where β represents each assets sensitivity to the factors, and alpha the constant (excess) returns.

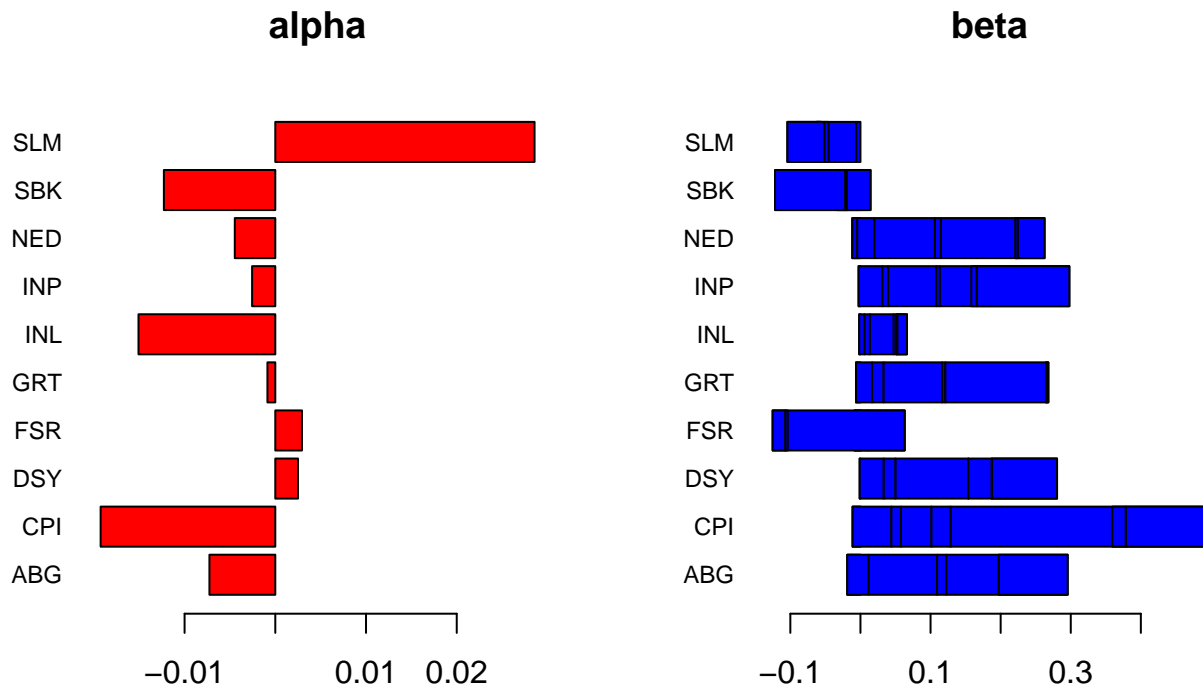


Figure 4.1: Factor Analysis: Financials

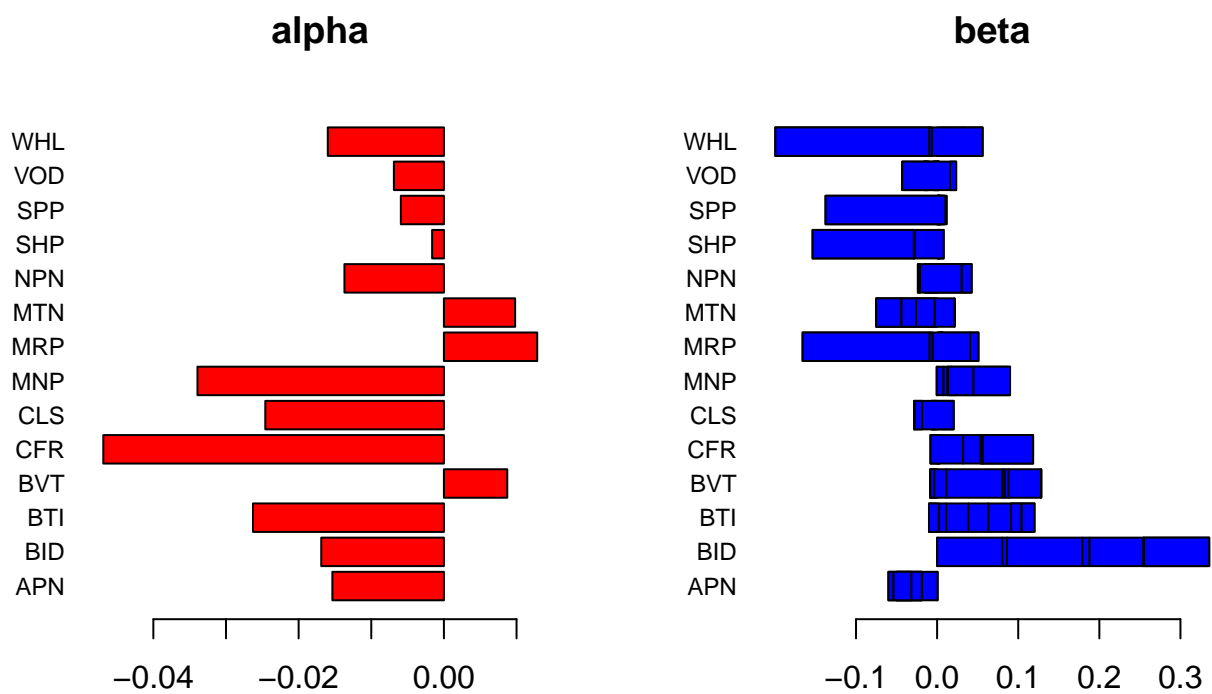


Figure 4.2: Factor Analysis: Industrial

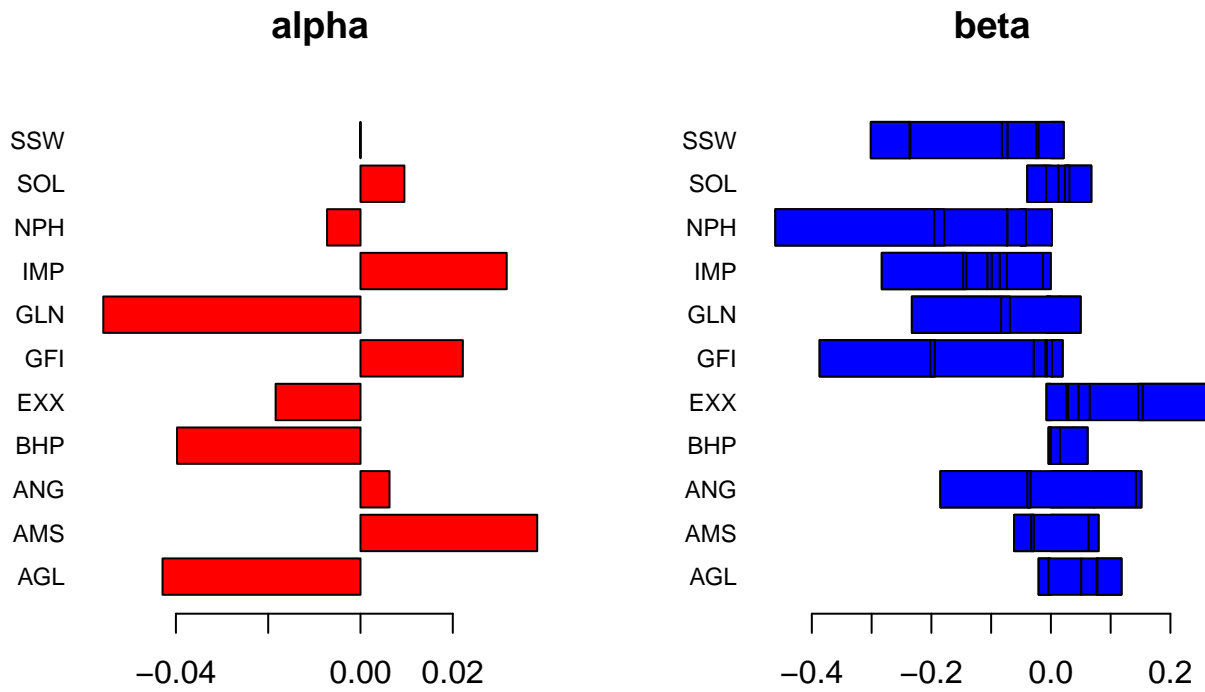


Figure 4.3: Factor Analysis: Resources

4.3. Discussion of Results

5. Conclusion

6. References

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7. Appendix: Supplementary Tables

7.1. Financial

Table 7.1: Factor Beta's: Financial

	US_10Yr	Bcom_Index	VIX	USDZAR	MM.Rate	Real.GDP	Real.INV	Inflation
ABG	-0.0133	-0.0056	0.0306	0.0976	0.0133	0.1730	-0.0981	0.0000
CPI	-0.0113	0.0554	0.0137	0.0435	0.0275	0.3649	-0.1153	-0.0192
DSY	-0.0011	0.0344	0.0168	0.1036	0.0003	0.1265	-0.0930	-0.0001
FSR	-0.0084	0.0091	0.0097	0.0494	0.0034	-0.1884	0.0182	0.0034
GRT	-0.0063	0.0230	0.0164	0.0834	0.0047	0.1469	-0.0032	0.0006
INL	-0.0017	0.0076	0.0077	0.0339	0.0088	0.0102	-0.0150	0.0013
INP	-0.0026	0.0340	0.0083	0.0686	0.0051	0.1847	-0.1322	-0.0077
NED	-0.0117	0.0071	0.0245	0.0863	0.0086	0.1478	-0.0415	0.0035
SBK	-0.0022	-0.0311	0.0093	0.0319	0.0065	-0.1362	0.1003	0.0023
SLM	-0.0061	-0.0567	0.0035	0.0030	-0.0116	-0.0363	0.0530	0.0059

Table 7.2: Factor Alpha's: Financial

	x
ABG	-0.0073
CPI	-0.0193
DSY	0.0025
FSR	0.0029
GRT	-0.0009
INL	-0.0151
INP	-0.0026
NED	-0.0045
SBK	-0.0123
SLM	0.0286

Table 7.3: Covariance Matrix: Financial

	ABG	CPI	DSY	FSR	GRT	INL	INP	NED	SBK	SLM
ABG	9e-04	1e-04	1e-04	1e-04	1e-04	1e-04	1e-04	2e-04	1e-04	1e-04
CPI	1e-04	5e-04	0e+00	0e+00	1e-04	0e+00	0e+00	1e-04	0e+00	0e+00
DSY	1e-04	0e+00	4e-04	0e+00	1e-04	0e+00	0e+00	1e-04	0e+00	0e+00
FSR	1e-04	0e+00	0e+00	5e-04	0e+00	0e+00	0e+00	1e-04	0e+00	0e+00
GRT	1e-04	1e-04	1e-04	0e+00	4e-04	0e+00	0e+00	1e-04	1e-04	0e+00
INL	1e-04	0e+00	0e+00	0e+00	0e+00	3e-04	0e+00	1e-04	0e+00	0e+00
INP	1e-04	0e+00	0e+00	0e+00	0e+00	0e+00	4e-04	0e+00	0e+00	0e+00
NED	2e-04	1e-04	1e-04	1e-04	1e-04	1e-04	0e+00	6e-04	1e-04	1e-04
SBK	1e-04	0e+00	0e+00	0e+00	1e-04	0e+00	0e+00	1e-04	6e-04	0e+00
SLM	1e-04	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	1e-04	0e+00	4e-04

7.2. Industrial

Table 7.4: Factor Beta's: Industrial

	US_10Yr	Bcom_Index	VIX	USDZAR	MM.Rate	Real.GDP	Real.INV	Inflation
APN	0.0005	-0.0506	0.0028	0.0265	0.0018	-0.0411	0.0063	0.0220
BID	0.0001	0.0802	0.0059	0.0928	0.0088	0.1476	-0.0809	0.0003
BTI	-0.0100	0.0125	0.0086	0.0274	0.0247	0.0569	-0.0162	-0.0127
BVT	-0.0087	0.0050	0.0150	0.0690	0.0030	0.0046	0.0404	0.0001
CFR	0.0029	-0.0017	-0.0094	0.0403	0.0212	0.0651	-0.0622	-0.0011
CLS	-0.0029	-0.0040	0.0004	0.0153	0.0116	-0.0488	0.0009	0.0089
MNP	-0.0005	0.0083	0.0101	0.0532	0.0188	-0.0453	-0.0316	0.0002
MRP	0.0069	-0.0046	0.0045	0.0441	-0.0104	-0.2065	0.1566	0.0037
MTN	-0.0051	-0.0701	0.0312	0.0190	-0.0010	0.0230	0.0244	0.0001
NPN	0.0005	-0.0068	-0.0086	0.0493	0.0081	-0.0125	-0.0537	0.0027
SHP	0.0026	-0.0013	0.0016	0.0038	0.0013	-0.1616	0.1257	-0.0009
SPP	0.0048	-0.0038	0.0081	-0.0091	0.0023	-0.1399	0.1479	0.0016
VOD	0.0024	-0.0065	0.0059	-0.0181	0.0055	-0.0323	0.0666	-0.0075
WHL	0.0082	0.0223	0.0030	0.0183	0.0042	-0.2556	0.1899	0.0032

Table 7.5: Factor Alpha's: Industrial

	x
APN	-0.0154
BID	-0.0169
BTI	-0.0263
BVT	0.0087
CFR	-0.0469
CLS	-0.0246
MNP	-0.0339
MRP	0.0128
MTN	0.0098
NPN	-0.0137
SHP	-0.0016
SPP	-0.0059
VOD	-0.0069
WHL	-0.0160

Table 7.6: Covariance Matrix: Industrial

	APN	BID	BTI	BVT	CFR	CLS	MNP	MRP	MTN	NPN	SHP	SPP
APN	5e-04	0e+00	0e+00	1e-04	0e+00	0e+00	0e+00	0e+00	0.0001	0e+00	0e+00	0e+00
BID	0e+00	5e-04	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	0e+00
BTI	0e+00	0e+00	2e-04	0e+00	0e+00	0e+00	0e+00	0e+00	0.0001	0e+00	0e+00	0e+00
BVT	1e-04	0e+00	0e+00	4e-04	0e+00	0e+00	0e+00	0e+00	0.0001	0e+00	0e+00	0e+00
CFR	0e+00	0e+00	0e+00	0e+00	3e-04	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	0e+00
CLS	0e+00	0e+00	0e+00	0e+00	0e+00	5e-04	0e+00	0e+00	0.0000	0e+00	0e+00	0e+00
MNP	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	3e-04	0e+00	0.0001	0e+00	0e+00	0e+00
MRP	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	4e-04	0.0001	0e+00	0e+00	0e+00
MTN	1e-04	0e+00	1e-04	1e-04	0e+00	0e+00	1e-04	1e-04	0.0011	0e+00	0e+00	0e+00
NPN	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	4e-04	0e+00	0e+00
SHP	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	3e-04	0e+00
SPP	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	2e-04
VOD	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	0e+00
WHL	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	0e+00

7.3. Resources

Table 7.7: Factor Beta's: Resources

	US_10Yr	Bcom_Index	VIX	USDZAR	MM.Rate	Real.GDP	Real.INV	Inflation
AGL	-0.0206	0.0168	-0.0002	0.0548	0.0347	0.0327	-0.0416	0.0011
AMS	0.0009	0.0040	0.0093	0.0655	-0.0169	-0.1244	0.0329	-0.0044
ANG	0.0158	0.0556	0.0015	0.0786	-0.0093	-0.3271	0.1507	-0.0053
BHP	-0.0040	0.0021	-0.0007	0.0409	0.0232	-0.0461	-0.0156	-0.0022
EXX	-0.0074	0.0331	0.0033	0.0174	0.0187	0.1998	-0.1114	-0.0073
GFI	0.0197	-0.0179	-0.0084	-0.0029	-0.0194	-0.3582	0.1927	-0.0074
GLN	-0.0068	0.0015	0.0193	0.0039	0.0320	-0.2825	0.1495	0.0145
IMP	-0.0134	-0.0604	-0.0120	-0.0134	-0.0073	-0.1765	0.1362	0.0055
NPH	0.0016	-0.0527	0.0093	-0.0315	0.0003	-0.3881	0.2661	0.0167
SOL	-0.0109	-0.0287	0.0319	0.0201	0.0004	0.0549	-0.0442	0.0076
SSW	0.0214	-0.0425	-0.0038	-0.0478	-0.0094	-0.2192	0.0661	-0.0014

Table 7.8: Factor Alpha's: Resources

	x
AGL	-0.0429
AMS	0.0383
ANG	0.0063
BHP	-0.0398
EXX	-0.0184
GFI	0.0222
GLN	-0.0557
IMP	0.0317
NPH	-0.0072
SOL	0.0095
SSW	0.0000

Table 7.9: Covariance Matrix: Resources

	AGL	AMS	ANG	BHP	EXX	GFI	GLN	IMP	NPH	SOL	SSW
AGL	4e-04	0e+00	0e+00	0e+00	0e+00	0e+00	0.0001	0e+00	0e+00	0e+00	0e+00
AMS	0e+00	6e-04	0e+00	0e+00	0e+00	0e+00	0.0000	0e+00	0e+00	1e-04	0e+00
ANG	0e+00	0e+00	5e-04	0e+00	0e+00	1e-04	0.0000	0e+00	0e+00	0e+00	0e+00
BHP	0e+00	0e+00	0e+00	3e-04	0e+00	0e+00	0.0001	0e+00	0e+00	0e+00	0e+00
EXX	0e+00	0e+00	0e+00	0e+00	5e-04	0e+00	0.0000	0e+00	0e+00	0e+00	0e+00
GFI	0e+00	0e+00	1e-04	0e+00	0e+00	6e-04	0.0000	0e+00	0e+00	-1e-04	1e-04
GLN	1e-04	0e+00	0e+00	1e-04	0e+00	0e+00	0.0015	0e+00	1e-04	1e-04	0e+00
IMP	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0000	6e-04	0e+00	0e+00	0e+00
NPH	0e+00	0e+00	0e+00	0e+00	0e+00	0e+00	0.0001	0e+00	8e-04	1e-04	0e+00
SOL	0e+00	1e-04	0e+00	0e+00	0e+00	-1e-04	0.0001	0e+00	1e-04	7e-04	0e+00
SSW	0e+00	0e+00	0e+00	0e+00	0e+00	1e-04	0.0000	0e+00	0e+00	0e+00	7e-04