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SPONTANEOUS POTENTIAL CORRECTIONS FOR GROUNDWATER SALINITY CALCULATIONS — CARTER COUNTY, OKLAHOMA, U.S.A.

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ABSTRACT

McConnell, C.L., 1983. Spontaneous potential corrections for groundwater salinity calculations — Carter County, Oklahoma, U.S.A. J. Hydrol., 65: 363—372.

The spontaneous potential log was used to compute the thickness and distribution of groundwater with a total dissolved solids (TDS) of less than $1000 \, \text{mg} \, \text{l}^{-1}$ for Carter County, Oklahoma. To compute accurate salinities from the spontaneous potential in high-resistivity zones, empirical correction factors had to be developed and applied. For the formations present equivalent water resistivity vs. water resistivity curves were developed from chemical analyses and digitized. Water resistivity vs. TDS curves were also developed and digitized. The existing empirical thin-bed corrections were curve-fit with equations and applied to each zone. Except in beds less than 3.65 m thick they had little effect.

It was found that after applying the thin-bed corrections used by logging companies, the computed TDS was a function of resistivity of the mud filtrate at 24°C. An empirical relationship was developed between spontaneous potential and resistivity of the mud filtrate at 24°C, using well logs of beds with known water chemistry. This relationship was entered as a correction equation into a computer program developed by the author. The correction is necessary since the voltage drop in a freshwater formation is greater than predicted by earlier electric analog modeling, and the spontaneous potential is less than the static spontaneous potential. Testing of the correction factors shows that in a formation with TDS of $\sim 750 \, \text{mg} \, \text{l}^{-1}$ the average error decreases from $\sim 37\%$ to 10%.

A data set of 704 well logs was built with data from each freshwater zone from the base of the regolith to a depth where the TDS was greater than $10,000 \, \text{mg} \, \text{l}^{-1}$. The TDS was computed for each zone. Two maps were prepared from the results; an isopach of the formations with TDS of less than $1000 \, \text{mg} \, \text{l}^{-1}$, and a depth to the base of the water with TDS of $1000 \, \text{mg} \, \text{l}^{-1}$.

INTRODUCTION

Carter County, Oklahoma, is a mature and heavily drilled oil region in which most fields are ready for tertiary recovery. Much of the enhanced oil recovery is by chemical flooding, which requires large volumes of fresh water [i.e. $<10,000 \, \text{mg} \, \text{l}^{-1}$ total dissolved solids (TDS)]. Thousands of oil-well electric logs record the freshwater sands, but less than 15 water wells with

both electric logs and chemical analyses were available. Because of the projected demand for water for enhanced oil recovery and the pollution potential to the county's water wells, the author undertook a research project to compute the depth to the base of the potable water (<1000 mg l^{-1} TDS) and the thickness of the potable water using spontaneous potential logs.

Electric logs, in particular the spontaneous potential log, have long been used in the petroleum industry to compute formation-water resistivity of saline beds and to locate the base of the fresh water for setting surface casing. They have been used much less successfully to compute formation-water resistivity (R_w) of freshwater zones. The error, using published equations for fresh water, generally has been quite large. For this study the error was identified and corrected so that reliable data could be obtained.

THEORY

Origin of the potential

The theory of the spontaneous potential log (SP) as applied to petroleum has been discussed by many authors (Doll, 1949; Gondouin et al., 1957), and for water uses by Davis and DeWiest (1966).

The SP has two sources, an electrochemical potential (E_c) and an electrokinetic (E_k) potential. The spontaneous potential is the sum of these components. Due to small differences between head of the column of drilling mud and that of freshwater zones in this area, E_k can be assumed to be negligible, and SP $= E_c$. The electrochemical potential has two components, a membrane potential (E_M) and a liquid junction potential (E_L) (Schlumberger, 1972a).

For the membrane potential, from theoretical and experimental considerations, Gondouin et al. (1957) have shown that:

$$E_M = \frac{-CT}{F} \ln \left[\frac{(a_{Na^+} + \sqrt{a_{Ca^{2+}} + a_{Mg^{2+}}})_w}{(a_{Na^+} + \sqrt{a_{Ca^{2+}} + a_{Mg^{2+}}})_m} \right] \tag{1}$$

where C = gas constant; T = absolute temperature; F = the Faraday constant; a = activity; and w and mf refer to formation water and mud filtrate, respectively.

For a NaCl solution at the membrane, eq. 1 simplifies to:

$$E = \frac{-CT}{F} \ln \frac{R_{mfe}}{R_{we}}$$

where R_{mfe} = the resistivity of the drilling mud filtrate, which is inversely proportional to the activity of an equivalent NaCl solution; R_{we} = the resistivity of the formation water, which is inversely proportional to the activity of an equivalent NaCl solution.

This equation holds when Na^+ is the only cation. If Ca^{2+} or Mg^{2+} are present eq. 1 must be used, or more commonly an empirical relationship between R_{we} and R_w (true resistivity of the formation water), and R_{mfe} and R_{mf} (true resistivity of the mud filtrate) is developed for the particular local water chemistry. This was the method used in this study and is the only feasible method where water analyses are scarce but electric logs are plentiful.

The liquid junction potential can be shown to be represented by (modified from Wyllie, 1949):

$$E_L = -11.5 \log (R_{mfe}/R_{we}) \qquad (\text{at } 25°C)$$

Since $E_c = E_L + E_M$:

$$SP = E_c = -\left(60 + 0.133[1.8\{T(°C) + 32\}]\right) \log (R_{mfe}/R_{we}) \qquad (2)$$

for any temperature T (°C) (Dresser Atlas, 1975).

These concepts have been tested in the laboratory and in the field in saline waters where Na^+ predominates; however, applications to fresh water are very limited.

Environmental effects on the SP — *The* SSP

The maximum potential which can develop at the sandstone—mud and shale—mud interface is called the static SP (SSP). It is the potential which would exist if no current were flowing. Since current is flowing from the drilling mud, to the shale, to the sandstone, then into the mud again, there is a voltage drop in each zone. The SP is the measurement of the voltage drop in the drilling mud. SSP is the voltage which reflects the true water chemistry and is the quantity which must be used to find R_w and hence TDS. Resistivity, SP, and SSP are related by (Doll, 1949):

$$SP = \frac{SSP(R_{mud}L_{mud}/A_{mud})}{(R_{mud}L_{mud}/A_{mud} + R_{sh}L_{sh}/A_{sh} + R_{sd}L_{sd}/A_{sd})} \qquad (3)$$

where L_{mud} = the length the current travels in the drilling mud; and A_{mud} = the area through which the current travels in the mud.

Since the area is a variable, correction factors must be applied when A_{sd} or R_{mud} is small. The existing correction factors were examined (Worthington and Moldan, 1958; Segesman, 1962) and applied to the freshwater zones of over 75 wells. Further environmental corrections, as will be discussed, were needed to obtain accurate R_w measurements in this study area.

R_{we}—R_w RELATIONSHIP

When eq. 2 is rearranged an expression for R_{we} is obtained, i.e.:

$$R_{we} = R_{mfe}/10^{-(SP/K)} \qquad \text{where} \qquad K = 60 + 0.133[1.8\{T(°C) + 32\}]$$

for any temperature. Therefore, if the SP, the formation temperature, and the R_{mfe} are known, one can find R_{we}. R_{we} is the resistivity of a NaCl solution and is equal to the true resistivity (R_w) when NaCl is the predominant salt. However, in the case of (eq. 1 modified):

$$SP = -K \log \frac{(a_{Na^+} + \sqrt{a_{Ca^{2+}} + a_{Mg^{2+}}})_w}{(a_{Na^+} + \sqrt{a_{Ca^{2+}} + a_{Mg^{2+}}})_{mf}}$$

when Ca^{2+} or Mg^{2+} is present in even moderate amounts in fresh waters the square root term has a large effect on the SP and R_{we} may vary greatly from R_w as measured by specific conductance. When the mud is essentially a NaCl solution the effect of having Ca^{2+} and Mg^{2+} in the formation water is to lower the SP, making the water appear saltier than it really is. Since Ca^{2+} and Mg^{2+} are common in fresh water R_w generally differs from R_{we}.

In order to find R_w from R_{we} an empirical relationship must be established. This necessitated collecting all available chemical analyses of water from the two aquifers present, the Oscar Group and the Garber Sandstone, over a wide range of TDS. Sources of data included published reports (Hart, 1974; O.W.R.B., 1980), the Water Resources Board files, Oklahoma State Health Department, unpublished U.S. Bureau of Mines analyses, oil companies, and in one case having a new analysis run.

When the R_w was not available from specific conductance measurements it was calculated using the Dunlap method (Dunlap and Hawthorne, 1951). R_{we} was calculated by first converting the analyses in mg l^{-1} to activities. Then the sum $\sqrt{a_{Mg^{2+}} + a_{Ca^{2+}}} + a_{Na^+}$ was entered in a published a_{Na^+} vs. R_{we} graph (Alger, 1966), and R_{we} was obtained.

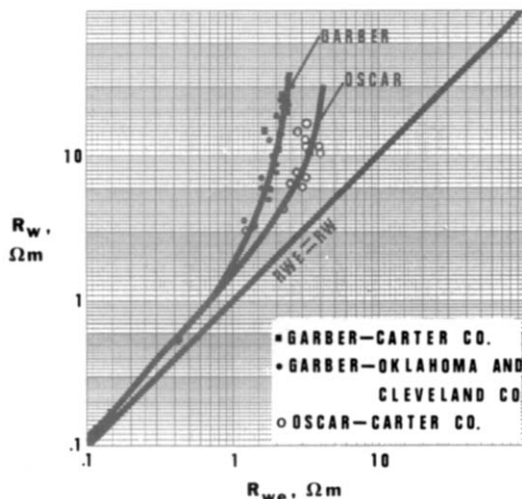Fig. 1 is the R_{we} vs. R_w relationship on log-log paper plotted from the

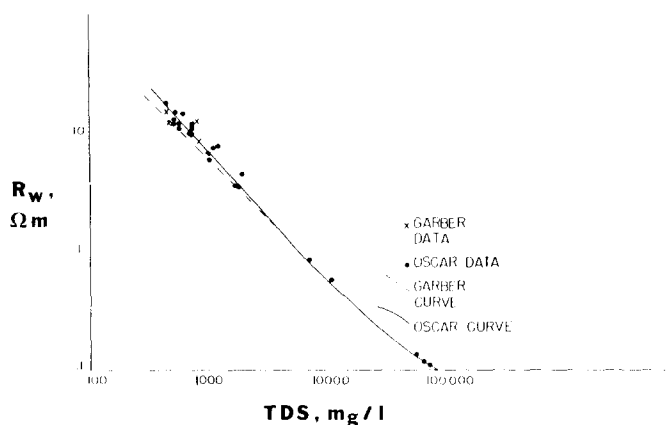Fig. 1. R_{we}–R_w relationship.

Fig. 2. TDS—R_w relationship.

chemical analyses. Data from both aquifers are shown. The straight line is the $R_{we} = R_w$ line (i.e. if the only cation were Na^+). Low resistivity values plot near the $R_{we} = R_w$ line since the a_{Na^+} predominates over the $\sqrt{a_{Mg^{2+}} + a_{Ca^{2+}}}$ value in waters of high TDS.

A curve was hand-fitted to the Oscar data since the unusual shape of the data would not allow least squares to fit an accurate equation. The Garber Sandstone data for Carter Co. was too sparse to fit a curve. To calculate a meaningful R_{we} vs. R_w curve for the Garber, data from the Garber Sandstone in Cleveland and Oklahoma Counties, ~160 km north (Wood and Burton, 1968) were examined. Data from wells deeper than 60 m were selected and the R_{we} vs. R_w relationship was determined. This was the closest Garber Sandstone water analysis available and since the Garber was volumetrically small, compared to the Oscar Group, this was considered sufficient.

The data from Carter County were combined with the portion of data from Cleveland and Oklahoma Counties for which they most nearly fit. The curve for these data was also hand-fit.

The TDS vs. R_w relationship is shown in Fig. 2. Because of the scarcity of data for the Garber Sandstone the curve was made to conform to the Oklahoma and Cleveland County data.

R_{mfe} vs. R_{mf} and environment corrections

Referring to eq. 3, a consolidated aquifer which is thin and resistive may have an SP which differs considerably from the SSP. Established thin-bed correction curves by Dresser Atlas (1975) were curve-fit and applied to all data. Except for beds less than 3.65 m thick this correction had little effect.

Over 74 well logs were carefully correlated in the freshwater zones from four different areas in the county. In each of the four areas the R_{we} was known (three by chemical analysis and one by estimating the TDS from

knowledge of the water usage in the area). The $R_{we} \sim 3.3\,\Omega\,m$ for all the areas. As the zones were each $\sim 6\,m$ thick, the thin-bed correction previously discussed was insignificant. Accordingly R_{mf} and SP were read from the logs and the R_{mf} was corrected to $24°C$.

The data were then used in a SAS (Statistical Analysis System) data set and run on the computer. The observations are plotted in Fig. 3. The best-fit equation was cubic, i.e.

$$SP \; = \; 58.311 - 66.265\,R_{mf} + 37.642(R_{mf})^2 - 7.333(R_{mf})^3 \, .$$

Shown also is a plot of the theoretical SP vs. R_{mf} at $24°C$ for:

$$SP \; = \; - \left(60 + 0.133[1.8\{24(°C) + 32\}] \right) \log(R_{mf}/R_{we})$$

$$= \; -70 \log(R_{mf}/3.3)$$

Only where the two curves intersect is no correction of the theoretical values needed. For values of R_{mf} less than $1.8\,\Omega\,m$, SP units need to be subtracted to match the theoretical values.

The difference between the least-squares curve and the theoretical curve in Fig. 3 is the combined correction factor for the environmental effects on the SP and the R_{mfe} vs. R_{mf} relation. This difference, as a function of R_{mf}, was put into a data set and an equation was fit. The data are plotted in Fig. 4. The resultant equation given below is used in the data processing:

$$SPCOR2 \; = \; 25.163 - 1.147\,R_{mf} - 14.061(R_{mf})^2 + 3.868(R_{mf})^3$$

This correction factor is added to the measured SP before R_{we} is computed.

The author attributes the variation between the field and theoretical data to two reasons. Below an R_{mf} of $1.7\,\Omega\,m$ the voltage drop in the formation is greater than predicted by the earlier Worthington—Moldan and Segesman electric analog modeling. Between R_{mf} of 1.7 and $2.0\,\Omega\,m$ this effect vanishes since the voltage drop in the mud becomes large. Above $2\,\Omega\,m$ the R_{mfe}
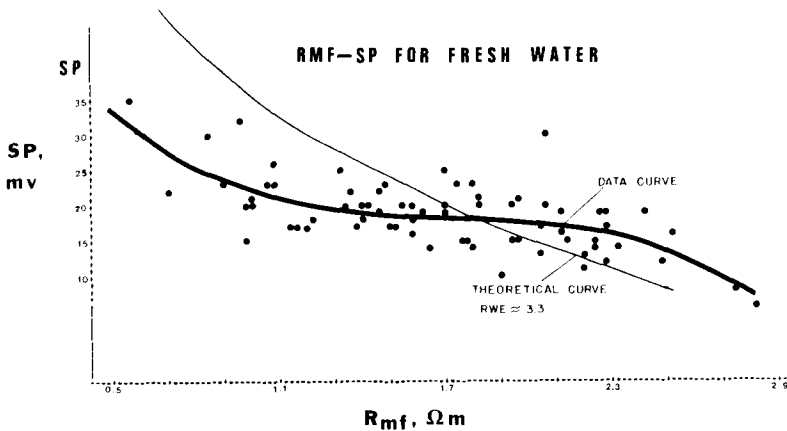

Fig. 3. Field and theoretical curves for R_{mf} vs. SP.
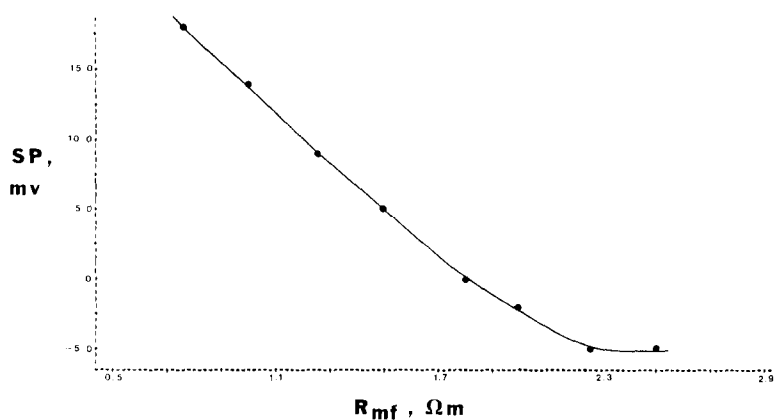
Fig. 4. Curve for combined correction factor.

begins to depart from R_{mf} and the commonly applied (Schlumberger, 1972b) rule, $R_{mfe} = 0.85 R_{mf}$, is a good approximation. Below 1.7 Ω m, $R_{mfe} = R_{mf}$ at 24°C.

FIELD TESTING

Several computer runs were made to test the reproducibility and accuracy of the program. Electric logs including formations with fresh waters of known water chemistry were correlated and the SP was measured. The R_{mf} and the temperature of the mud filtrate were recorded and the TDS of the water was then calculated.

Fig. 5 shows two of the tests before and after applying the calculated correction factors. The bed thickness correction commonly used in oilfield work was applied to all the tests but had little effect. The R_{we} for all three tests was $\sim 3.3\ \Omega$ m suggesting a TDS of ~ 750 mg l^{-1}. The mean and standard
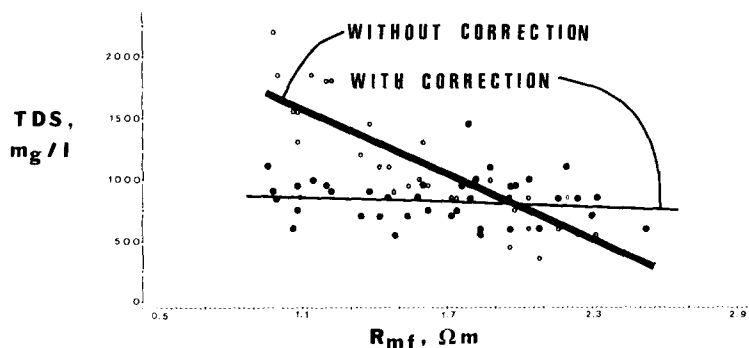

Fig. 5. TDS vs. R_{mf}: with and without correction.

deviation of all tests before and after correcting was 1030 mg l^{-1}, SD = 241 and 823 mg l^{-1}, SD = 160, respectively.

APPLICATION

A data base was compiled using 704 oil-well electric logs including data from all zones from the base of the regolith to ~10,000 mg l^{-1} TDS. The correction factors were incorporated in a computer program and the R_w and TDS were computed for each zone for both aquifers. An isopach map was prepared for the less than 1000-mg-l^{-1} zones, and a map of depth to the base of the 1000-mg-l^{-1} water was prepared. These appear as Figs. 6 and 7 in a simplified and condensed form.
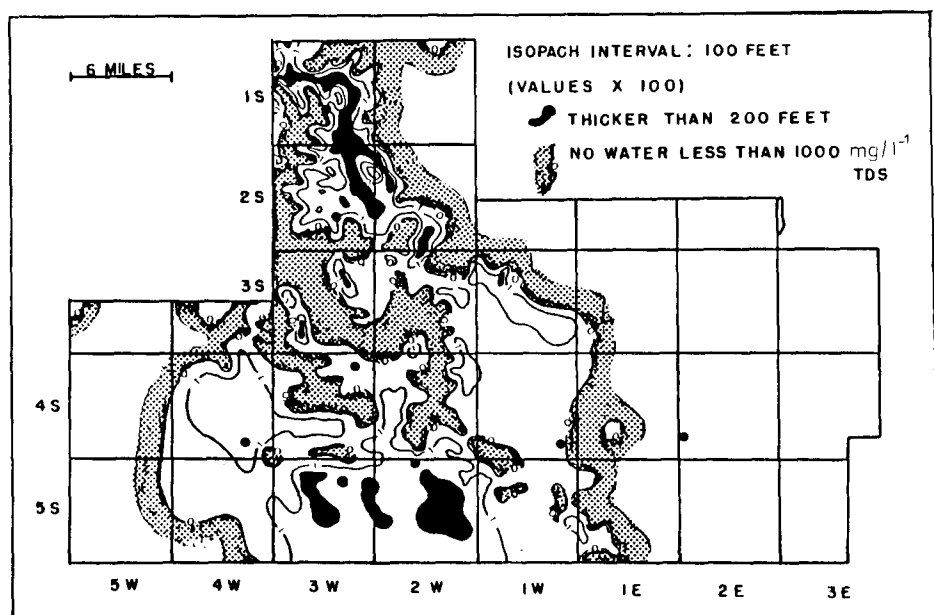

Fig. 6. Isopach of water with TDS $\leqslant$ 1000 mg l^{-1} (1 mi. = 1.609 km; 1 ft. = 0.3048 m).

CONCLUSIONS

(1) Two aquifers with different water chemistry are present in the study area, the Oscar Group and the Garber Sandstone. R_{we} vs. R_w curves were developed for both from chemical analyses.

(2) The borehole environmental correction factor was greater for salty muds than predicted by earlier electric analog models. This correction is largely independent of bed thickness between 3.65 and 7.60 m which is the normal thickness of these beds. Below an R_{mf} of 1.8 Ω m a large correction
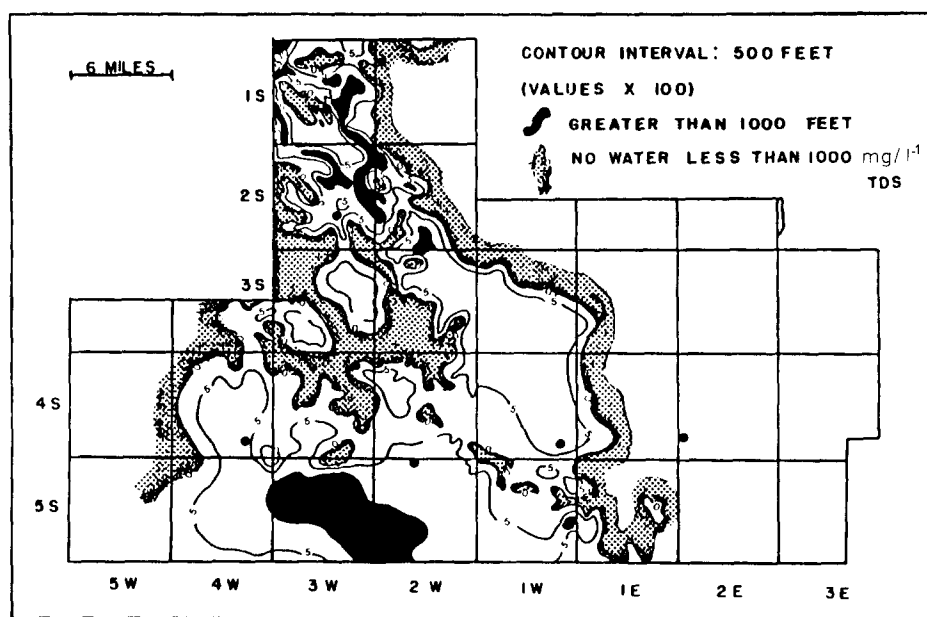
Fig. 7. Depth to the base of water with TDS $\leqslant 1000\,\text{mg}\,l^{-1}$ (1 mi. = 1.609 km; 1 ft. = 0.3048 m).

is needed due to the voltage drop in the formation. Above $1.8\,\Omega\,\text{m}$ the borehole correction was not needed as the voltage drop in the mud column dominated, and only the usual thin-bed corrections are required. At an R_{mf} at $24°\text{C}$ of greater than $2.1\,\Omega\,\text{m}$ the R_{mfe} differs from R_{mf}. The correction factor for R_{mf} at $24°\text{C}$ from 0.9 to $2.5\,\Omega\,\text{m}$ is presented.

(3) Testing the correction factors on zones of known water chemistry zones having TDS of $\sim 750\,\text{mg}\,l^{-1}$ lowered the average computed TDS from 1030 to $823\,\text{mg}\,l^{-1}$.

ACKNOWLEDGEMENTS

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