Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon

Laboratory Assignment T5: Bandpass filter using OPAMP

MEAer

Group 15

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Contents

1 Introduction

For this laboratory assignment, our objective was to implement a Bandpass Filter, using an Operational Amplifier (OPAMP). In this report, the Filter, that should have a central frequency of 1 kHz and a 40dB gain at the central frequency, will be analysed in a theoretical approach as well as using software simulation. Firstly, is it important to mention that our object of analysis involves three different stages: a high pass stage, in which the signals with high frequencies pass through and the ones with low ones are cut off, an amplification stage, where, using OPAMP, the signals are amplified, and, finally, a low pass stage, where the amplified signals from the previous stages, are allowed to pass if they're low or are cut if those frequencies are high ones.

Also, as we will present further through this report and as we did in previous lab assignments, a merit classification system was created to determine and ensure the quality of the Bandpass Filter. This system took into consideration the costs of the components used, which were given to us, the gain deviation and the central frequency deviation.

In this report, it will be presented a theoretical analysis, in section 2, where the circuit is studied using suitable theoretical models learned in class, in order to predict the gain, phase and the input and output impedances at the central frequency, as well as the frequency response (gain and phase) for a logarithmic range of frequencies. These results are compared with the experimental ones retrieved from the presential laboratory class. Then, we proceeded to a software simulation analysis, in the section 4, made by computational simulation tools, via *Ngspice*, where the OPAMP model that was provided by the professor was used: $\mu A741$ OPAMP. To sum up the report, the conclusions of this study are presented in Section 5, where the simulation results obtained in Section 4 are compared to the theoretical results obtained in Section 2. In Figure 1, the stated circuit is presented and, this time, as we had the chance to physically attend a laboratory session, we also decided to include an image, taken by an element of the group, of the final circuit achieved, that can be seen in figure 2.

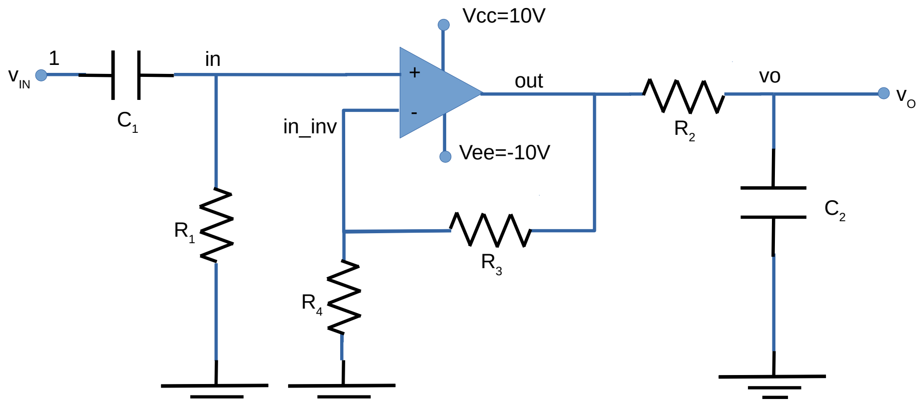


Figure 1: Circuit that will be analysed in this lab.

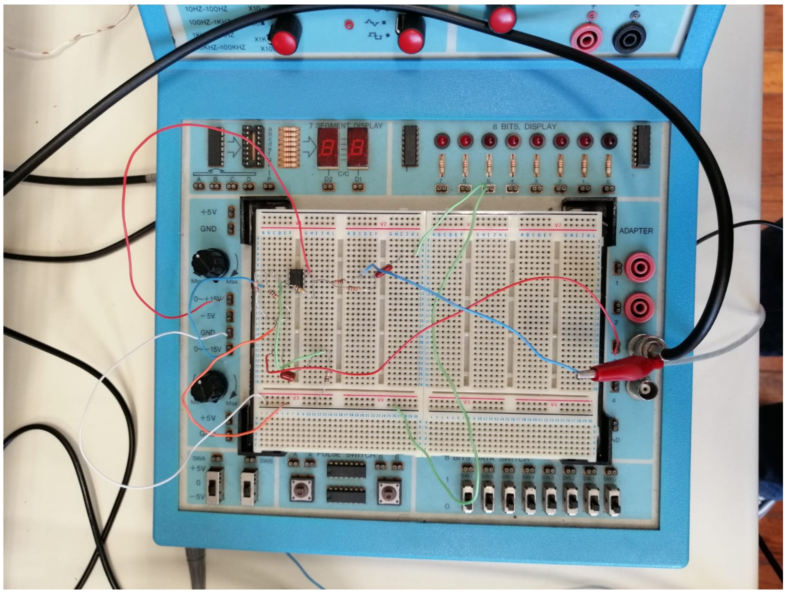


Figure 2: Montage of the Bandpass Filter circuit on the supplied breadboard.

2 Theoretical Analysis

In this section, we shall proceed to the analysis of the circuit presented in Figure 1, using the knowledge learned in the theoretical lectures. This circuit was based in the model provided by the Professor in the presential laboratory class. To accomplish that, we are going to compute the Voltage Gain and the Input and Output Impedances of the circuit, as well as its frequency response, in terms of the Voltage Gain and the Phase of the Output Voltage. As the Voltage Gain and the Input and Output Impedances are incremental parameters, we will need to resort on Incremental Analysis to make our calculations in this section.

2.1 Circuit Design

Before starting our calculations, it is important to explain how the values relative to each component in the circuit were chosen. As the Professor explained in the presential laboratory class, the Lower and Higher Cut-Off Frequencies, according to the Time Constants Method, are, respectively, given by:

$$\omega_L = \frac{1}{R_1 C_1} \tag{1}$$

$$\omega_H = \frac{1}{R_2 C_2} \tag{2}$$

Besides, the Central Frequency for our circuit is given by the geometric average of ω_L and ω_H:

$$\omega_0 = \sqrt{\omega_L \omega_H} \tag{3}$$

From this last relation, we can compute ω_H as a function of ω_L and ω_0:

$$\omega_H = \frac{\omega_0^2}{\omega_L} \tag{4}$$

With this in mind, we chose to fix the values for each capacitor and for the resistor R_1:

- C_1 = C_2 = 220 nF;

- R_1 = 1 kΩ.

We remember that, in the laboratory, we only had 1 kΩ, 10 kΩ and 100 kΩ resistors and 220 nF and 1 μF capacitors available.

Consequently, the Lower Cut-Off (Radian) Frequency is determined by:

$$\omega_L = \frac{1}{R_1 C_1} = \frac{1}{1000 \cdot 220 \cdot 10^{-9}} = \frac{50000}{11} \approx 4545.45\ rad/s \tag{5}$$

Attending to the fact that the Central Frequency was defined as f_0 = 1 kHz, we are able to compute ω_0:

$$\omega_0 = 2\pi f_0 = 2000\pi\ rad/s \tag{6}$$

We can, now, calculate ω_H, using Equation 4:

$$\omega_H = \frac{\omega_0^2}{\omega_L} = \frac{(2000\pi)^2}{\frac{50000}{11}} \approx 8685.251873\ rad/s \tag{7}$$

Finally, we extract R_2 from Equation 2:

$$R_2 = \frac{1}{C_2 \omega_H} = \frac{1}{220 \cdot 10^{-9} \cdot 8685.251873} \approx 523.35\Omega \tag{8}$$

As it was said before, there are no resistors in the laboratory with this value. However, we chose to replace R_2 with a parallel of two 1 kΩ resistors, because its equivalent resistance is of 500 Ω, a value that is quite close to the one we obtained in Equation 8 (we remember that the equivalent resistance of a parallel of two equal resistors is half of each resistor's resistance).

For the resistor R_3, we decided to keep the resistance of 100 kΩ, as it was suggested by the Professor. However, during the laboratory class, we noticed that the suggested value of 1 kΩ for R_4 was insufficient to achieve a Voltage Gain of 40 dB, which consisted in one of the requirements for this experiment. The solution for this problem was reducing the resistance R_4, by applying, once more, a parallel of two 1 kΩ resistors, whose equivalent resistance is 500 Ω, as it was already explained. With R_4 = 500 Ω, we measured, in the laboratory class, a Voltage Gain even higher than 40 dB, which satisfies the mentioned requirement.

2.2 Voltage Gain and Phase

In this subsection, we shall determine the Voltage Gain of the Bandpass Filter circuit. We start by observing that the middle section of the circuit, which includes the Operational Amplifier (OPAMP) and the resistors R_3 and R_4, consists on a Non-Inverting Amplifier configuration, in all similar to the one studied in the theoretical classes. Considering v_+ as the voltage at the positive input of the OPAMP and v_- as the voltage at its negative input, the voltage at the output of the OPAMP is given by:

$$v_{o(OPAMP)} = A_v(v_+ - v_-) \tag{9}$$

, where A_v is the Feed-Forward Gain. Ideally, A_v tends to be infinite, which means that, in order to have a finite value of $v_{o(OPAMP)}$, $(v_+ - v_-)$ must be zero, that is, $v_+ = v_- = v_{i(OPAMP)}$. The voltage $v_{o(OPAMP)}$ can also be calculated through the following relation:

$$v_{o(OPAMP)} = v_- + R_3 i_F = v_{i(OPAMP)} + R_3 i_{i(OPAMP)} \tag{10}$$

, where the current, i_F, flowing through resistors R_3 and R_4 is computed as:

$$i_F = \frac{v_-}{R_4} \tag{11}$$

Therefore, the Voltage Gain at the output of the OPAMP is:

$$\frac{v_o}{v_i} = 1 + \frac{R_3}{R_4} \tag{12}$$

This would be the Voltage Gain of the circuit if we had not added the resistors R_1 and R_2, along with the capacitors C_1 and C_2. These components are responsible for the intended Bandpass Filter behaviour of the circuit.

The resistor R_1 and the capacitor C_1 contribute to the Voltage Gain with the following factor to the Transfer Function, derived from the Voltage Divider law ($s = j\omega$):

$$H(s) = \frac{R_1}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{\frac{R_1 C_1 j\omega + 1}{j\omega C_1}} = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \tag{13}$$

We can verify that this contribution corresponds to a High-Pass Filter, as, for low frequencies (close to 0), this factor tends to 0, and, for high frequencies (tending to infinity), this ratio tends to 1, which means this part of the circuit lets pass only the higher frequencies, cutting the low ones.

On the other hand, the resistor R_2 and the capacitor C_2 affect the Transfer Function by considering the following factor, deduced using once more the Voltage Divider law:

$$L(s) = \frac{\frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{\frac{1}{j\omega C_2}}{\frac{R_2 C_2 j\omega + 1}{j\omega C_2}} = \frac{1}{1 + R_2 C_2 s} \tag{14}$$

We can also take the conclusion that this part of the circuit consists on a Low-Pass Filter configuration, as, for low frequencies, this ratio tends to 1, and, for higher frequencies, it tends to 0, which means this section of the circuit lets pass only the low frequencies, cutting the high ones.

Adding these contributions to the one from the Voltage Gain of the Non-Inverting Amplifier section (middle section), we obtain the following Transfer Function, T(s), with $s = j\omega$ and ω being the radian frequency in study:

$$T(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \cdot \left(1 + \frac{R_3}{R_4}\right) \cdot \frac{1}{1 + R_2 C_2 s} \tag{15}$$

From this, we understood that, by decreasing resistance R_4, we would increase the Voltage Gain, thus solving the problem described at the end of Subsection 2.1.

Finally, the Voltage Gain of the circuit is just the absolute value of T(s):

$$\frac{v_o}{v_i} = |T(s)| \tag{16}$$

Hence, the Voltage Gain for the Central Frequency of 1 kHz corresponds to ω = $2\pi f_0$ = 2000π rad/s:

$$\frac{v_o}{v_i}(f_0 = 1000Hz) = 133.97 = 42.540\ dB \tag{17}$$

The Voltage Phase relative to f_0 = 1000 Hz is the angle of T(s) when s = $2000\pi \cdot j$:

$$Phase = arctan\left(\frac{Im(T(j\omega_0))}{Re(T(j\omega_0))}\right) = 1.2329^o \tag{18}$$

2.3 Input and Output Impedances

In this subsection, we intend to determine the Input and Output Impedances of the circuit.

Starting by the Input Impedance, we note that, by shutting down the independent sources in the circuit, the equivalent impedance as seen by the input source corresponds to the series of the resistor R_1 and the capacitor C_1. Therefore, the Input Impedance of the Bandpass Filter circuit is given by:

$$Z_{in} = R_1 + Z_{C_1} = R_1 + \frac{1}{j\omega C_1} \tag{19}$$

For the Central Frequency, ω_0 = 2000π rad/s; hence, the Input Impedance for the Central Frequency is:

$$Z_{in}(\omega_0) = R_1 + \frac{1}{j \cdot 2000\pi \cdot C_1} = 1000.00 - 723.43j[\Omega] \tag{20}$$

, whose absolute value is 1234.2 Ω.

Now, we proceed to calculate the Output Impedance of the circuit. We observe that the equivalent impedance, as seen from the output, is, approximately, the parallel of the resistor R_2 and the capacitor C_2 (for medium frequencies). The resistances R_3 and R_4 are neglectable because they are in parallel with the Output Impedance of the OPAMP, which is approximately null (short-circuit).

Then, the Output Impedance of the Bandpass Filter circuit is determined as:

$$Z_{out} = R_2||C_2 = \frac{1}{\frac{1}{R_2} + j\omega C_2} \tag{21}$$

Therefore, the Output Impedance for the Central Frequency is:

$$Z_{out} = R_2||C_2 = \frac{1}{\frac{1}{R_2} + j \cdot 2000\pi C_2} = 338.37 - 233.86j \tag{22}$$

Its absolute value is 411.32 Ω.

2.4 Frequency Response

In this subsection, we will study how the behaviour of the circuit varies with the source frequency, f, in particular how the Voltage Gain and Phase are affected by it. Therefore, we are going to solve the circuit for a frequency vector in logarithmic scale with 10 points per decade, from 10 Hz to 100 MHz, applying the equations 15, 16 and 18, referred in Subsection 2.2. This procedure allows us to plot the Voltage Gain and Phase in function of the source frequency, as presented in Figures 3 and 4.

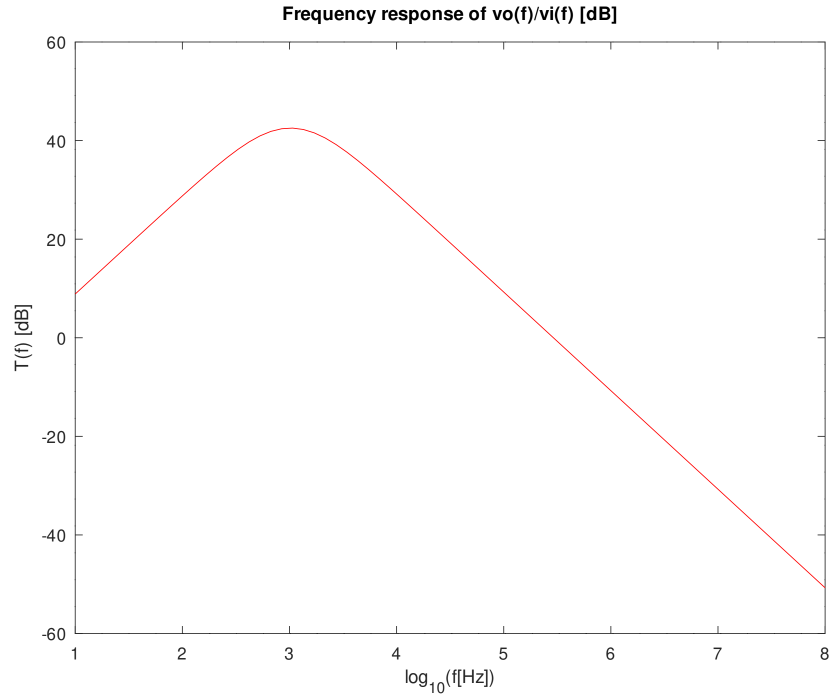


Figure 3: Voltage Gain [dB] vs Source Frequency [Hz]

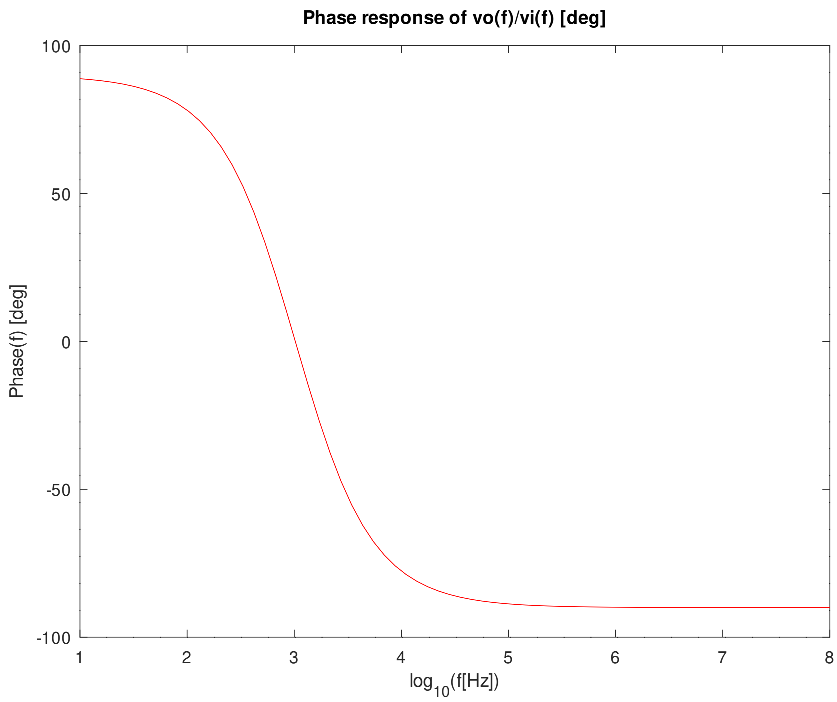


Figure 4: Voltage Phase [^{o}] vs Source Frequency [Hz]

In theory, the Voltage Gain would be stable in a passband around the Central Frequency f_0 = 1 kHz. We can limit this passband reasonably by computing the Lower and Higher Cut-Off Frequencies from their respective radian frequencies. As the radian frequency is given by:

$$\omega = 2\pi f \tag{23}$$

, the Lower and Higher Cut-Off Frequencies (f_L and f_H, respectively) are simply given by:

$$f_L = \frac{\omega_L}{2\pi} \approx 723.43Hz \tag{24}$$

$$f_H = \frac{\omega_H}{2\pi} \approx 1382.30Hz \tag{25}$$

By inspection of Figure 3, we verify that the Voltage Gain between these two frequencies is very close to 40 dB, as intended.

Focusing now on the Voltage Phase, the plot starts at 90 degrees, as the Transfer Function has a zero at the origin (we note that the numerator of T(s) is a linear function of s). Then, the plot has a slope of 90 degrees per decade, as the Transfer Function has two poles (we add that the denominator of T(s) has a quadratic dependence on s). After that, the Voltage Phase tends to -90 degrees, in the region of the higher frequencies.

3 Experimental Analysis

This section is dedicated to the analysis of the experimental results, measured in the presential laboratory class. The montage of the Bandpass Filter implemented in the presential laboratory class has already been presented in Figure 2.

Now, we are computing the Voltage Gain, but using the experimental values. For the Central Frequency of 1 kHz, the amplitude of the Output Voltage signal was $v_o = 13.5V$, as presented in Figure 5.

Figure 5: Experimental Results for f = 1 kHz

As the Input Voltage amplitude was $v_i = 0.111V$, the Voltage Gain is determined from:

$$\frac{v_o}{v_i}(f = 1000Hz) = \frac{13.5}{0.111} = 121.62 = 41.700\ dB \tag{26}$$

, which is a higher value than the requested 40 dB, and rather close to the theoretical value: 42.540 dB. As we will explain further, this small difference can be explained, among other factors, by the fact that each resistor or capacitor has an associated tolerance, along with the possibility of the wires having a non-neglectable resistance.

In order to verify that the circuit is in the Cut-Off Region for low and high frequencies, we also measured the amplitude of the Output Voltage for f = 100 Hz and f = 10000 Hz. The results are presented in Figures 6 and 7.

Figure 6: Experimental Results for f = 100 Hz

Figure 7: Experimental Results for f = 10000 Hz

The Voltage Gain values for this frequencies are:

$$\frac{v_o}{v_i}(f = 100Hz) = \frac{3.4}{0.115} = 29.565 = 29.416\ dB \tag{27}$$

$$\frac{v_o}{v_i}(f = 10000Hz) = \frac{0.860}{0.113} = 7.6106 = 17.628\ dB \tag{28}$$

As the Voltage Gain values obtained for this frequencies are way lower than $(40 - 3) = 37$ dB, we conclude that this frequencies belong, indeed, to the Cut-Off Region.

4 Simulation Analysis

In this section we used the NGSpice script provided, specifically, we maintained the original OPAMP model, and altered the circuit to get the one presented in Figure 1.

The values we used are presented in Table 1.

Component	Value ([Ω] or [F])
R1	1000
R2	500
R3	100000
R4	500
C1	2.2E-07
C2	2.2E-07

Table 1: Circuit Component's values

To get the resistance values of 500 Ω we used two resistors of 1kΩ in parallel, as it was already explained in Section 2.

Right away, we made an AC Analysis to study the circuit's behavior according to the frequency. Starting of with the circuit's gain: $\frac{v_o(f)}{v_i(f)}$, we plotted the following graph in dB, so that it would be easier to compare with the theoretical results later on.

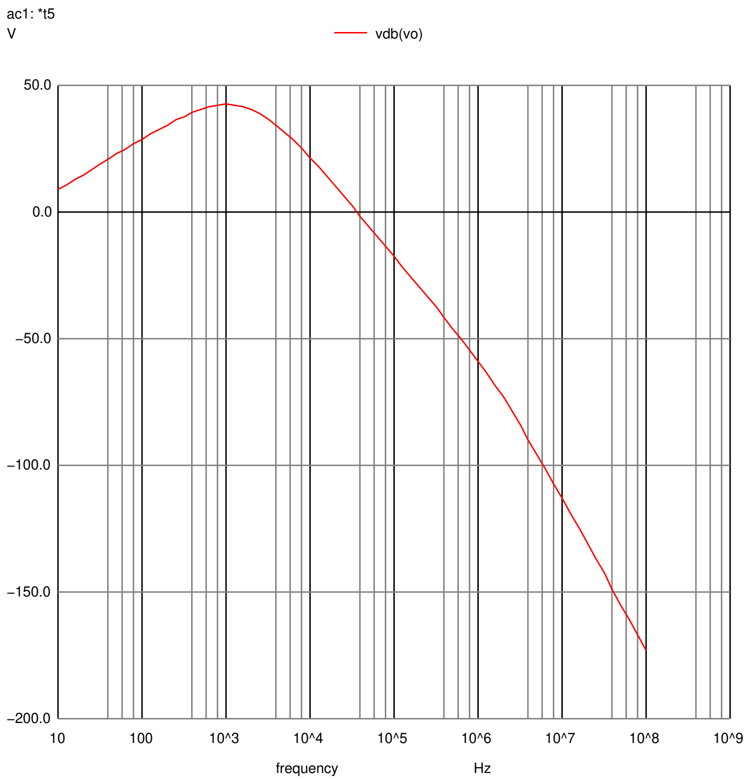


Figure 8: $v_o(f)/v_i(f)$ Gain [dB] - Frequency Analysis

We can also see the phase correspondent to the Transfer Function of the circuit.

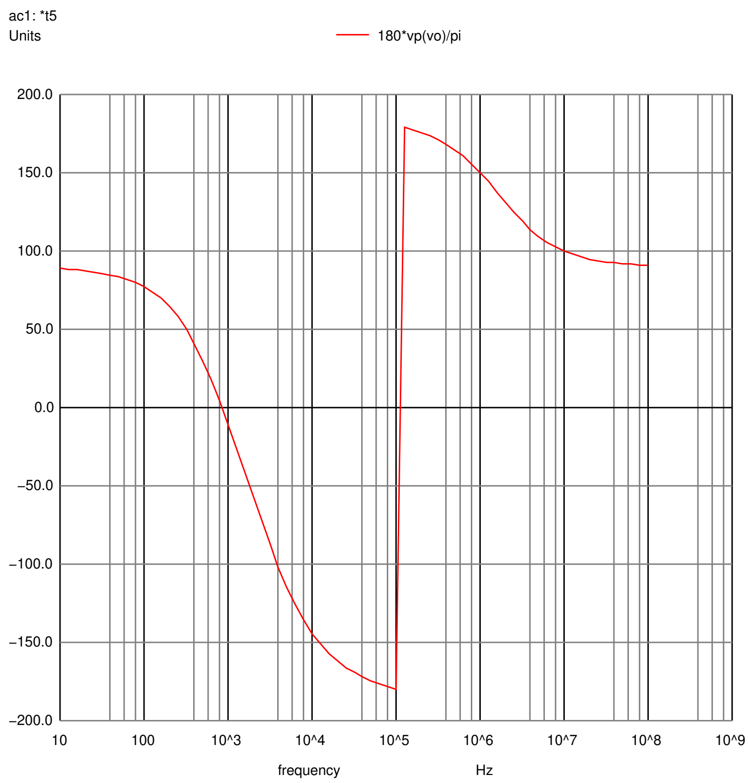


Figure 9: $v_o(f)/v_i(f)$ Phase [Deg] - Frequency Analysis

Also, given that the NGSpice model of the OPAMP considered the existence of many components, specially two extra capacitors, two poles will be added to the circuit's Transfer Function resulting in a total slope of -180 degrees per decade (for medium frequencies).

The abrupt change in the plot happens because NGSpice plots angles in an interval of [-180;180] Degrees.

This circuit is obviously a Bandpass, and it's peak value is of about 40 dB (100 in linear value), for 1kHz which is exactly what we were looking for. The exact values of gain, central frequency, their deviations to the wanted values, and merit are presented in the Table 2. The figure of merit value was calculated using the following equation:

$$M = \frac{1}{cost(gaindev + fcdev + 10^{-6})} \tag{29}$$

	Value
cost	1.342874e+04
gain	1.299370e+02
gaindb	4.242502e+01
fc	1.000000e+03
gaindev	2.993700e+01
fcdev	0.000000e+00
merit	2.487462e-06

Table 2: Merit.

As we can see the central frequency is exactly 1kHz, so we were quite successful on that aspect. On the other hand, the gain value is slightly above intended: we obtained a gain of 42.4 dB, so the signal will be amplified a little bit more than our objective. That affected the merit, which will be better discussed in section 5.

Finally we calculated the input and output impedances, both shown in the following table.

Input Impedance	Value [Ω]
Zin	999.965 + -723.635 j
Zin - Amplitude	1234.33

(a) Input Impedance

Output Impedance	Value [Ω]
Zout	345.034 + -232.928 j
Zout - Amplitude	416.298

(b) Output Impedance

Table 3: Input and Output Impedance

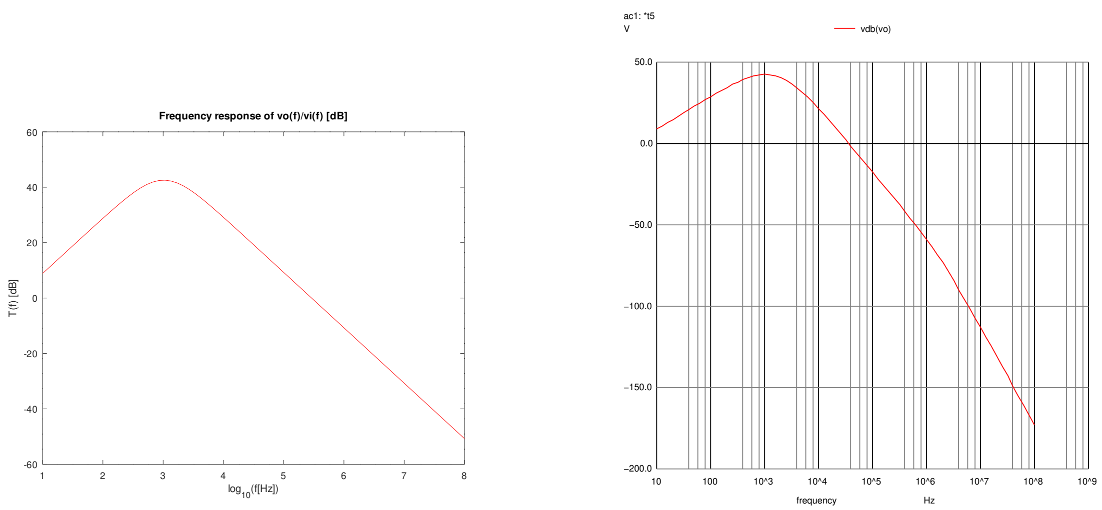
The absolute value of the input impedance is quite high, above 1kΩ, which will be necessary to amplify the received signal, without degrading it, and, thus, get a high gain value.

However, for the output impedance we would want a much smaller absolute value. This would allow the circuit to connect to other systems without losing much of the signal. The value we obtained of 416.298 Ω isn't ideal, but we weren't able to obtain better values without compromising the final merit, and overall these values of impedances are acceptable.

5 Conclusion

To start this conclusion, we will compare all the results we obtained in NGSpice and in Octave side by side.

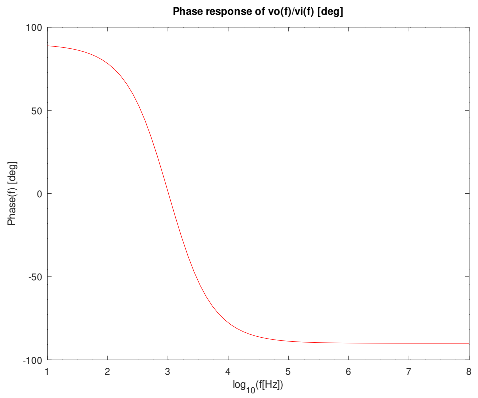
First off, we compare the plotted graphs of the frequency response (gain and phase).



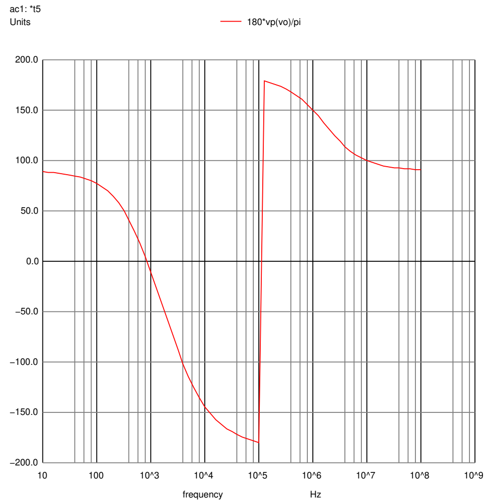
(a) Octave (b) NGSpice

Figure 10: Frequency Gain analysis - $v_o(f)/v_i(f)$

Both plots are very similar for smaller frequencies and even show approximately the same gain value for the central frequency, which is the value we are most interested in. The theoretical value for gain was 42.54 dB while the simulated value was 42.43 dB, so these values were quite close, and we can conclude that the theoretical model used to calculate the absolute gain for the central frequency is corroborated by the NGSpice model.



(a) Octave



(b) NGSpice

Figure 11: Frequency Phase analysis - $v_o(f)/v_i(f)$

Once again it's noticeable the sudden discontinuity of the NGSpice phase graph but as explained earlier, this is only because NGSpice will only show phase values between [-180;180] Degrees; while doing the theoretical analyses, we didn't have to take that into consideration.

In addition to this, as explained in Sections 2 and 4, while in the theoretical analysis, the Transfer Function had two poles, in the NGSpice model there were two extra capacitors to define the OPAMP, which led to the addition of two more poles, leaving the Transfer Function with 4 poles in the simulated analysis. As each pole adds a shift of -45 degrees per decade, that corresponds to a total slope of -180 degrees per decade, leaving the phase stable at -90-180 = -270^o (which is the same as +90 degrees if measured in the opposite direction), instead of the original -90 obtained in the theoretical analysis, and therefore the plot graphs differ.

After this, let's compare the values of the Input and Output Impedances of the complete circuit:

	Value [Ω]
Z_{IN}	1000.000000-723.431560j
$Z_{IN}(Absolute)$	1234.241962
Z_{OUT}	338.366226-233.861947j
$Z_{OUT}(Absolute)$	411.318748

Table 4: Input and Output Impendances - Theoretical

Input Impedance	**Value [Ω]**
Zin	999.965 + -723.635 j
Zin - Amplitude	1234.33

(a) Input Impedance

Output Impedance	**Value [Ω]**
Zout	345.034 + -232.928 j
Zout - Amplitude	416.298

(b) Output Impedance

Table 5: Input and Output Impedance - Simulated

These values are also quite similar, so once again we can conclude that the theoretical and simulated models are close to equivalent.

Any discrepancies between the theoretical and the simulated values may be explained by the fact that each resistor and capacitor has an associated tolerance, as well as the OPAMP has non-idealities that were not considered in the theoretical analysis.

Finally, as we saw in Table 2, our final merit value was $M = 2.4875 \cdot 10^{-6}$. Even though we got a perfect central frequency, we still got a relatively high deviation for the gain, but the most important factor was the high cost of the components used, specially the resistors that were used to define the OPAMP model. The resistances with a high value made up for a very high fraction of the total cost, having led to its high value, that otherwise would have been in the order of 10^2. But considering the values and options we had, we still consider this to be a successful outcome as we were able to obtain values very close to desired.

With this laboratory assignment, we had the opportunity to interact with real components in a laboratory environment, as well as understanding better how an Operational Amplifier, a very important encapsulated circuit, behaves.

As we finish the last lab assignment, we reflect on what these projects taught us. Thanks to them, we understood better how the theoretical knowledge learned in the classes applied in practice. They were also very important, as they consisted on our first contact with many tools that are very useful for any engineer, such as NGSpice, Octave, LaTeX, GitHub and the operating system, Linux Ubuntu, itself.