Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon

Laboratory Assignment T1

MEAer

Group 15

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1 Introduction

For this laboratory assignment, our objective was to study a circuit containing multiple components, such as resistors, independent voltage and current sources, V_a and I_d, respectively, a current dependent voltage source, V_c, and a voltage dependent current source, I_b, using a structured approach, as explained in the theory classes. This circuit is shown in Figure 1.

In Section 2, we present an analysis of the circuit following two different methods: the Mesh Method, in Section 2.1, and the Nodal Method in Section 2.2. In Section 3, we run a simulation of the circuit, using NGSpice, whose results are then compared to the theoretical results obtained in Section 2. Finally, the conclusions of this study are presented in Section 4.

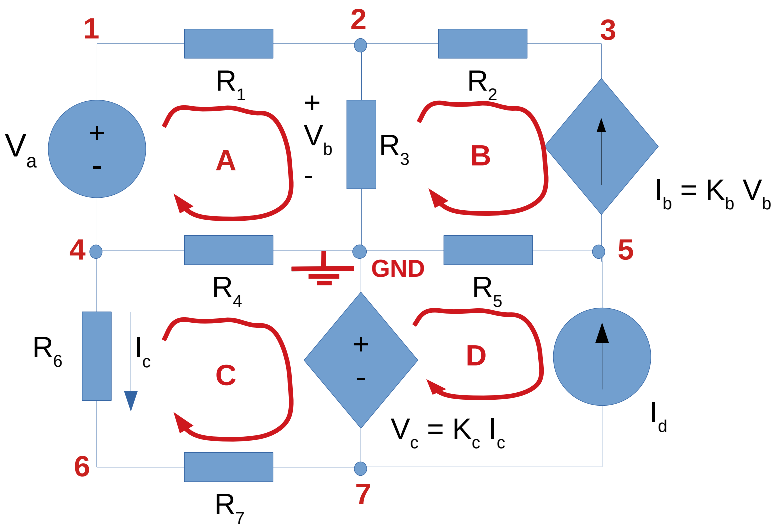

Figure 1: Circuit that will be analysed in this lab.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically using the Mesh and Nodal methods.

2.1 Circuit Analysis using Mesh Method

In this section we will proceed to the analysis of the given circuit through the Mesh Method. Briefly explaining, this method uses both Kirchhoff's Voltage Law (KVL) and Ohm's Law to define a set of equations guaranteed to be solvable if the circuit has a solution, therefore determining unknown currents (and indirectly the voltages) at any place within a circuit. Mesh analysis works by arbitrarily assigning currents to each mesh in the circuit. A mesh is a loop that does not contain any other loops. To do so, we have chosen to go through each mesh in the clockwise direction, and considered, for the currents' directions, the ones depicted in the supplied figure (we add that the direction of I_a is from node 1 to node 2). This choice is, however, entirely arbitrary. It is also important to note that we can only apply KVL to meshes that don't contain current sources, which is the case of meshes A and C. We proceeded, then, to the analysis of these meshes.

KVL equations:
Mesh A

$$-V_a + R_1 I_a + R_3 I_3 - R_4 I_4 = 0 \tag{1}$$

Mesh C

$$R_4 I_4 + V_c - R_7 I_c - R_6 I_c = 0 \Leftrightarrow \tag{2}$$

$$\Leftrightarrow R_4 I_4 + K_c I_c - R_7 I_c - R_6 I_c = 0 \tag{3}$$

Now, using Kirchhoff's Current Law, we deduced the equations of the following nodes, attempting to simplify the meshes' equations and solve them in order to the mesh currents:

Node 2

$$I_a + I_b = I_3 \tag{4}$$

Node 5

$$I_d = I_5 + I_b \tag{5}$$

Node 4

$$I_a + I_c + I_4 = 0 \Leftrightarrow \tag{6}$$

$$\Leftrightarrow I_4 = -(I_a + I_c) \tag{7}$$

We can now use the previous relations to compute I_3 and I_4 in order to I_a, I_b and I_c. Going back to the meshes' study, we obtained:
Mesh A

$$R_1 I_a + R_3(I_a + I_b) - R_4 I_4 = V_a \Leftrightarrow \tag{8}$$

$$\Leftrightarrow (R_1 + R_3 + R_4) I_a + R_3 I_b + R_4 I_c = V_a \tag{9}$$

Mesh C

$$-R_4(I_a + I_c) + (K_c - R_7 - R_6) I_c = 0 \Leftrightarrow \tag{10}$$

$$\Leftrightarrow -R_4 I_a + (K_c - R_4 - R_6 - R_7) I_c = 0 \tag{11}$$

Since I_d is given, we only need 3 equations. The last one is deduced from the condition defining the dependent current source:

$$I_b = K_b V_b = K_b R_3 I_3 = K_b R_3 (I_a + I_b) \Leftrightarrow \tag{12}$$

$$\Leftrightarrow K_b R_3 I_a + (K_b R_3 - 1) I_b = 0 \tag{13}$$

Finally, we rewrote the referenced three equations in matricial form:

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 & R_4 \\ -R_4 & 0 & K_c - R_4 - R_6 - R_7 \\ K_b R_3 & K_b R_3 - 1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

The solutions we obtained by solving the matricial equation shown above, with the help of Octave software, as recomended, are presented in the following table (1).

Name	Value [A or V]
I_a	0.0002242195
I_b	-0.0002346776
I_c	0.0009813848
I_d	0.0010417825
V_b	-0.0326067730
V_c	8.0258356267
V_1	0.1926947401
V_2	-0.0326067730
V_3	-0.5096738697
V_4	-4.9552367516
V_5	4.0033486476
V_6	-7.0136099164
V_7	-8.0258356267

Table 1: Solutions given by the Mesh Method. *I* stands for currents, which are expressed in Ampere; *V* represents voltages, which are expressed in Volt.

2.2 Circuit Analysis using Nodal Method

In this subsection, we are going to analyze the given circuit using the Nodal Method, as explained in the course lectures. The aim is the same: if the circuit is solvable, we shall, then, find several equations that would allow to determine the behavior of every component in the circuit, namely the voltage and the current on each one. The method consists, then, in applying the Kirchhoff Current Law (KCL) to the nodes that are not connected to voltage sources, using additional equations for nodes related by voltage sources. As a result, we obtain the voltage values on each node, which will permit to calculate the currents on each branch, by applying Ohm's Law. In this analysis, we kept the same choice for the directions of the currents flowing through the circuit.

With this in mind, we obtained the following equations:

KCL equations:

Node 2:

$$I_a + I_b = I_3 \Leftrightarrow \tag{15}$$

$$\Leftrightarrow G_1V_1 + (-G_1 - G_2 - G_3)V_2 + G_2V_3 = 0 \tag{16}$$

Node 3:

$$I_b = I_2 \Leftrightarrow \tag{17}$$

$$\Leftrightarrow (K_b + G_2)V_2 - G_2V_3 = 0 \tag{18}$$

Node 5:

$$I_d = I_5 + I_b \Leftrightarrow \tag{19}$$

$$\Leftrightarrow K_bV_2 + G_5V_5 = I_d \tag{20}$$

Node 6

$$I_c = I_7 \Leftrightarrow \tag{21}$$

$$\Leftrightarrow G_6V_4 + (-G_6 - G_7)V_6 + G_7V_7 = 0 \tag{22}$$

Additional equations:

There are seven nodes and, therefore, there must be seven equations (we note that none of the nodal voltages is given). In order to acquire two more equations, we applied the Kirchhoff Voltage Law (KVL) to the meshes A and C, as followed:

Mesh A:

$$-V_a + (V_1 - V_2) + (V_2 - 0) - (V_4 - 0) = 0 \Leftrightarrow \tag{23}$$

$$\Leftrightarrow V_1 - V_4 = V_a \tag{24}$$

Mesh C:

$$(V_4 - 0) + V_c - (V_6 - V_7) - (V_4 - V_6) = 0 \Leftrightarrow \tag{25}$$

$$\Leftrightarrow K_cG_6V_4 - K_cG_6V_6 + V_7 = 0 \tag{26}$$

For the final equation, we made use of a supernode, which consists on the fusion of multiple nodes. We have chosen the supernode connecting the nodes 2, 0 and 7, to which we applied the Kirchhoff Current Law, as shown in the following equation:

Supernode 2-0-7:

$$I_a + I_b + I_4 + I_5 + I_c = I_d \Leftrightarrow \tag{27}$$

$$\Leftrightarrow G_1V_1 + (-G_1 - G_2)V_2 + G_2V_3 + (G_4 + G_6)V_4 + G_5V_5 - G_6V_6 = I_d \tag{28}$$

The previous equations led to the following matricial equation:

$$\begin{bmatrix} G_1 & -G_1 - G_2 - G_3 & G_2 & 0 & 0 & 0 & 0 \\ 0 & K_b + G_2 & -G_2 & 0 & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & G_5 & 0 & 0 \\ 0 & 0 & 0 & G_6 & 0 & -G_6 - G_7 & G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_cG_6 & 0 & -K_cG_6 & 1 \\ G_1 & -G_1 - G_2 & G_2 & G_4 + G_6 & G_5 & -G_6 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_d \\ 0 \\ V_a \\ 0 \\ I_d \end{bmatrix} \tag{29}$$

By solving this matricial equation, using Octave software, as recommended, we obtained the results presented in the next table:

Name	Value [A or V]
V_1	0.1926947401
V_2	-0.0326067730
V_3	-0.5096738697
V_4	-4.9552367516
V_5	4.0033486475
V_6	-7.0136099164
V_7	-8.0258356266
V_b	-0.0326067730
V_c	8.0258356266
I_a	0.0002242195
I_b	-0.0002346776
I_c	0.0009813848
I_d	0.0010417825

Table 2: Solutions given by the Nodal Method. *I* stands for currents, which are expressed in Ampere; *V* represents voltages, which are expressed in Volt.

3 Simulation Analysis

For the simulation process, we defined each component of the circuit in NGSpice, with the proper synthax, having inserted all the given values (that we extracted from the supplied Python script). We used the same node numbers as the ones in Figure 1. The results we obtained are shown in Table 3.

Name	Value [A or V]
@gb[i]	-2.34678e-04
@id[current]	1.041782e-03
@r1[i]	2.242195e-04
@r2[i]	-2.34678e-04
@r3[i]	-1.04581e-05
@r4[i]	-1.20560e-03
@r5[i]	1.276460e-03
@r6[i]	9.813848e-04
@r7[i]	9.813848e-04
v(1)	1.926947e-01
v(2)	-3.26068e-02
v(3)	-5.09674e-01
v(4)	-4.95524e+00
v(5)	4.003349e+00
v(6)	-7.01361e+00
v(7)	-8.02584e+00
f6	-7.01361e+00

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

First of all, we should point out what each name in the table represents:

By NGSpice default:
- *@rn[i]* is the current flowing through R_n
- *v(n)* is the voltage V_n(voltage in node n)
- *@id[current]* is the current I_d

By the analysis of the circuit in Figure 1, the unknown variables are the following:
- V_b is *v(2)*
- V_c is *-v(7)*
- I_b is *@gb[i]*
- I_c is *@r6[i]* or *@r7[i]*

Note: *f6* represents a fictitious node between R_6 and R_7, created in Ngspice in order to properly define the current dependent voltage source V_c (as required by NGSpice synthax). It is supposed to be in the same node as *v(6)*; therefore, they have the same voltage value.

If we compare the values of V_b, V_c, I_b and I_c in Tables 1 and 2 with their respective representative value in Table 3, we notice that the results are the same.

Also, the values of voltage in each node obtained in Table 2 are equal to the simulated ones.

Finally, in addition to the relations stated above, by analyzing the circuit we see that I_a is *@r1[i]*, as intended. Comparing both Table 1 and Table 3, we also realize that the results are equivalent.

4 Conclusion

In this laboratory assignment, the objective was to analyze the circuit in Figure 1, using two different structured methods: the Nodal and the Mesh Methods.

The calculations were solved theoretically by using Octave, and the circuit simulation was done using the NGSpice tool.

As stated in Section 3, the simulation results matched the theoretical results precisely, as we expected, given that the studied circuit only contained linear components; therefore, the theoretical and simulation models shouldn't differ.

However, we cannot say that the models we used are correct and represent the reality. In fact, if we had worked with real components (instead of ideal ones, as assumed by the used model), we would be most likely to obtain results different from the theoretical ones, as each component, and even the measurement equipment and the wiring cables, is related to uncertainties and passible to origin errors that would contaminate the extracted values. All we can conclude from this lab is that the two theoretical methods are in alignment because they gave us the same results, and we can also state that NGSpice probably does its calculations following the same theoretical background as the Nodal and the Mesh Methods, given that the results generated by the program were equal to the theoretical ones. This laboratory assignment was also important from the perspective that it was our first contact with some tools of invaluable utility for any engineer, such as GitHub, LaTeX and, of course, NGSpice.