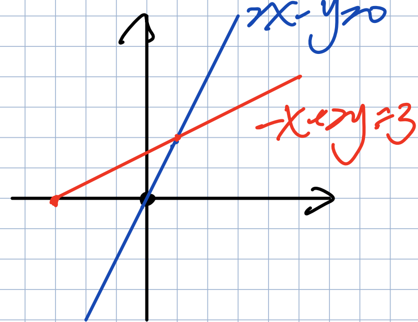
Row picture

- $2x - y = 0$
- $-x + 2y = 3 \Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

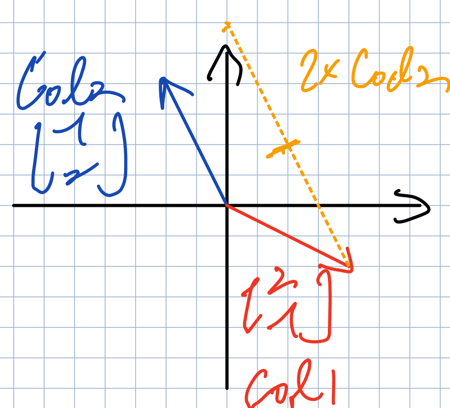
$$A \quad x = b$$



Column picture

$$\begin{cases} 2x - y = 3 \\ -x + 2y = 3 \end{cases} \Rightarrow x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

linear combination of column

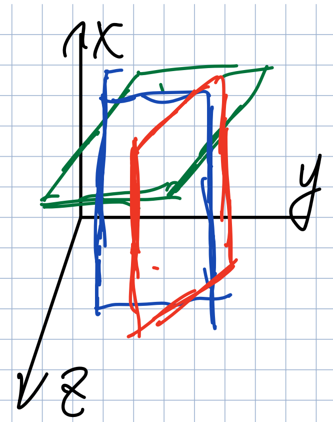


Row Picture

$$\begin{cases} 2x - y = 0 \\ -x + 2y - z = -1 \\ -3y + 4z = 4 \end{cases}$$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

假想图：

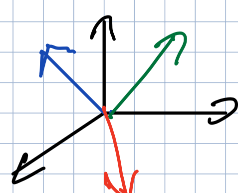


Column Picture.

$$x\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

col1 ● col2 ● col3 ● A-col.

linear comb. $\Longrightarrow$



$(x.y.z) = (0.0.1)$

Can I solve $Ax=b$ for every b?

$\Rightarrow$ Do the linear combs. of the columns in 3D space?

Ans: for the Matrix A, it is.

A: It's a non-singular matrix $\Rightarrow$ An invertible matrix

$Ax=b$ (Ax is a comb. of column of A)

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$x+2y+z=2$

$3x+8y+z=12$

$4y+z=2$

消元法+回代法

pivot

$$\begin{matrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix} \xrightarrow[(2,1)]{} \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{matrix} \xrightarrow[(3,2)]{} \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{matrix}$$

$$\begin{matrix} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{matrix} \rightarrow \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{matrix}$$

消元法

$$\begin{matrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & 10 \end{matrix} \rightarrow \begin{matrix} x+2y+z=2 \\ 2y-2z=6 \\ 5z=10 \end{matrix}$$

回代法

$(x,y,z)=(2,1,-2)$

Matrix · column = column.

$$\begin{bmatrix} col1 & col2 & col3 \end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix} = \begin{matrix}3\,col1\\+\\4\,col2\\+\\5\,col3\end{matrix}$$

Row × Matrix = Row

$$\begin{bmatrix}1 & 2 & 7\end{bmatrix}\begin{bmatrix} row1\\row2\\row3\end{bmatrix} = \begin{matrix}1\times row1\\+\\2\times row2\\+\\7\times row3\end{matrix}$$

Elimination Matrices.

step1. subtract $3\times$ row1 from row2

$$\begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix}1&2&1\\3&8&-2\\0&4&1\end{bmatrix} = \begin{bmatrix}1&2&1\\2&2&-2\\0&4&1\end{bmatrix}$$

step2. subtract 2 × row 2 form row 3.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

置换 permination

Exchange row 1 and 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Exchange column 1 and column 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

可逆 Inverse.

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E^{-1} E I