Gaussian Elimination

Key Point : $A = LU$.

let A.B inverse-able

$(AB)(B^{-1}A^{-1}) = I$

$(B^{-1}A^{-1})(AB) = I$

transport

$AA^{-1} = I$

$\boxed{(A^{-1})^T} A^T = I$

This is $(A^{-1})^T$, inverse of A^T

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}.$$

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

In 2x2 matrix

E_{21} same as L

But 3x3 matrix

having Big Difference.

$E_{32}E_{31}E_{21}A = U$ (no row exchange)

$A = \underbrace{E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}}_{}U = \boxed{L}U$

Typical Case:

$$\underset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}} \underset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix} = E$$

Left of A.

$\begin{bmatrix} EA = U \\ A = LU \end{bmatrix}$

Reverse Derection.

Inverse (Reverse Order)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = L$$

$A = LU$.

if no row exchange.

multiplies go directly into L.

How expensive is elimination?

(How many operation we do on $n \times n$ matrix?)

⚝ Multiply + Subtract = an operation.

let $n=100$

$$\underset{100\times100}{[\quad]} \Rightarrow \underset{99\times99}{[\quad]} \Rightarrow \underset{\cdots}{[\ \cdots\ ]} \Rightarrow \underset{1\times1}{[\quad]}$$

and go on...

we can get about $\sum n^2 = \frac{1}{3}n^3$

Transposes and Permutation.

$3\times3 \Rightarrow P$'s $\qquad P^{-1}=P^T$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

if $4\times4 \Rightarrow 24$ P's

$n\times n \Rightarrow n!$ P's ~~th~~.