ex.

space is R^3

one basis is standard $\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$

another basis $\begin{bmatrix}1 & 2 & 3\\1 & 2 & 3\\2 & 5 & 8\end{bmatrix}$

R^n n vectors give basis if the n×n.

4 fundamental subspaces

A is $m \times n$.

Column Space $C(A)$ in R^m

null Space $N(A)$ in R^n

row space = all comb. of rows

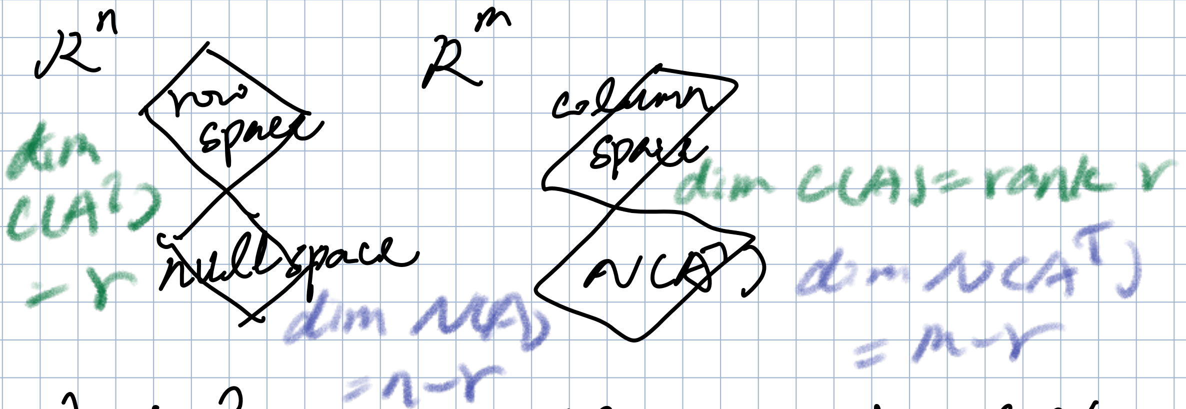
= all comb. of columns of A^T

= $C(A^T)$ in R^n

null space of A^T = $N(A^T)$

= left null space of A in R^m

4 subspaces



	$C(A)$	$N(A)$
basis? dimension?	pivot cols r	special solutions. $n-r$

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \to \begin{bmatrix} \overbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}^{I} & \overbrace{\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix}}^{F} \\ \underbrace{\begin{matrix} 0 & 0 \end{matrix}}_{} & \underbrace{\begin{matrix} 0 & 0 \end{matrix}}_{\text{zero}} \end{bmatrix} = R.$$

$C(R) \neq C(A)$ different col. spaces $\boxed{\text{Same Row space}}$

Basis for rowspace is first r rows of R

Left space $= N(A^T)$

$A^T y = 0.$ $\quad y^T A^{TT} = y^T A = 0^T$

$$[y^T]\begin{bmatrix} A \end{bmatrix} = [0]$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

$$E\,[A_{m\times n}\;\; I_{m\times m}] \rightarrow [\,R_{m\times n}\;\; E_{m\times m}]$$

$EA = R$. In chap. 2, R was I

Then E was A^{-1}

$$\underbrace{\begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{E}\underbrace{\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix}}_{A}$$