

$$x_1 + 2x_2 + 2x_3 + 2x_4 = b_1$$

$$2x_1 + 4x_2 + 6x_3 + 8x_4 = b_2$$

$$3x_1 + 6x_2 + 8x_3 + 10x_4 = b_3$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 & 2 & b_1 \\ 0 & 0 & 2 & 4 & b_2 - 2b_1 \\ 0 & 0 & 2 & 4 & b_3 - 3b_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 & 2 & b_1 \\ 0 & 0 & 2 & 4 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & b_3 - b_2 - b_1 \end{bmatrix}$$

OK.

$$b = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

$0 = b_3 - b_2 - b_1$

Augmented Matrix = $[A \ b]$

pivot cols.

Solvability condition on b .

$Ax = b$ solvable when b is in CCA

if a comb. of rows A gives zero rows, then the same comb. of entries of b must give 0.

Algorithm

To find complete sol'n to $Ax = b$.

Step 1. \times particular: set all free variables to zero.

plus Aug. dim. variables

$x_1 = 0$
 $x_2 = 0$

solve $AX=0$ gives variables.

(+)

$$X_1 + 2X_3 = 1$$

$$2X_3 = -3$$

$$X_1 = -2$$

$$X_3 = 3/2$$

$$x_p = c \begin{bmatrix} -2 \\ 0 \\ 3/2 \\ 0 \end{bmatrix}$$

Step 2: X nullspace.

$$X = x_p + x_n$$

$$AX_p = b$$

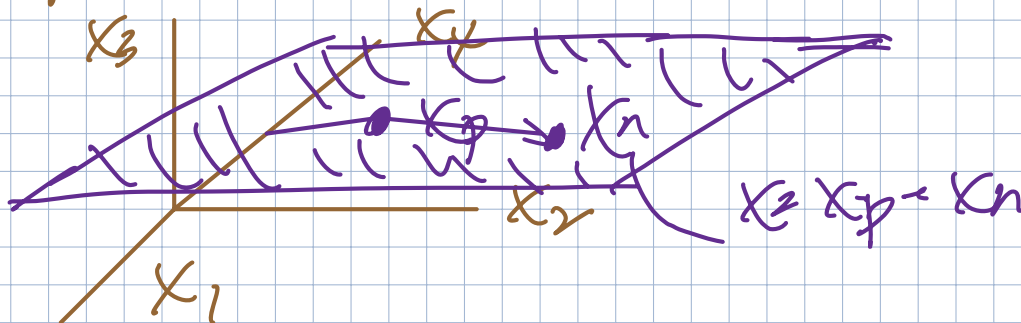
$$+ \cancel{AX_n = 0}$$

$$A(x_p + x_n) = b.$$

$$X_{\text{complete}} = \begin{bmatrix} -2 \\ 0 \\ 3/2 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

if $X_1 + 2X_2 + 2X_3 + 2X_4 = 0$. can solve?

Plot all solutions X in \mathbb{R}^4 .



m by n matrix A of rank r . ($r \leq m, r \leq n$)

Full column rank means $r = n$.

No free variables. unique solution if it exists
(0 or 1 solution)

$$N(A) = \{ \text{zero vector} \}$$

$$\text{Solution to } AX = b : X = x_p$$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 6 & 1 \\ 5 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Full row rank means $r=m$

Can I solve $ax=b$ for every b . Ex: ~~sc~~

Left with $\underbrace{n-r}_{n-m}$ free variables

$$A = \begin{bmatrix} 1 & 2 & 6 & 5 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & - & - \\ 0 & 1 & - & - \end{bmatrix}$$

$$r=m < n$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$R = I$$

$$r=m=n$$

$$R = I$$

1. solution

$$r=n < m$$

$$R = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

0 or 1 solution

$$r=m < n$$

$$R = \begin{bmatrix} I & F \end{bmatrix}$$

∞ solutions

$$r=m \quad r < n$$

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

0 or ∞ solutions