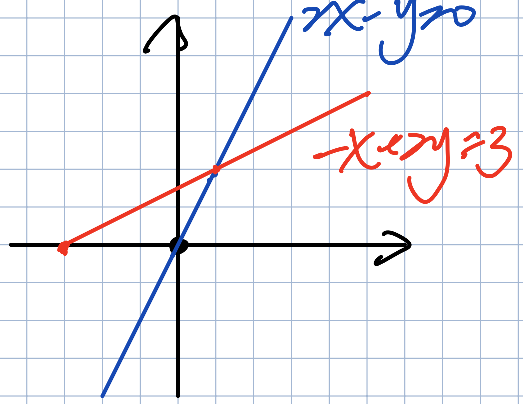
Row picture

$2x - y = 0$

$-x + 2y = 3$ $\Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

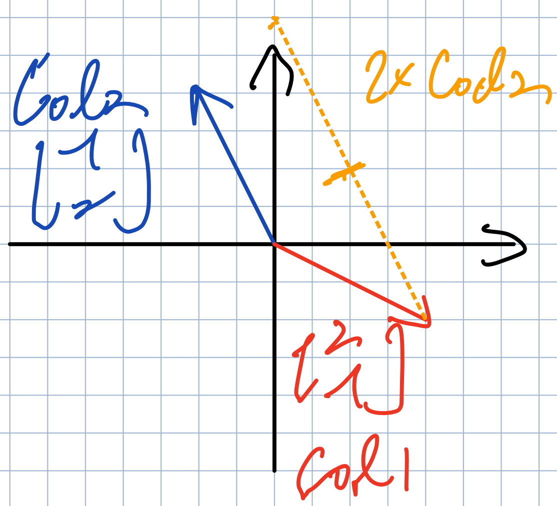
$A \quad \mathcal{X} = b$



Column picture

$\begin{cases} 2x - y = 3 \\ -x + 2y = 3 \end{cases} \Rightarrow x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

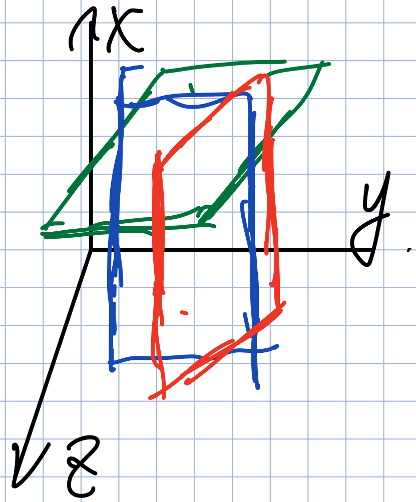
Linear combination of column



Row Picture

$$\begin{cases} 2x - y = 0 \\ -x + 2y - z = -1 \\ -3y + 4z = 4 \end{cases}$$

假想图：



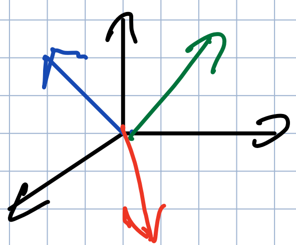
$$\Rightarrow A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Column Picture.

$$x\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

col1　col2　col3　A-col.

linear comb. $\Longrightarrow$



$(x, y, z) = (0, 0, 1)$

Can I solve $Ax = b$ for every b?

⇒ Do the linear combs. of the columns in 3D space?

Ans: For the Matrix A, it is.

A: It's a non-singular matrix ⇒ An invertible matrix

$Ax = b$ 　　(Ax is a comb. of column of A)

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$$x+2y+z=2$$
$$3x+8y+z=12$$
$$4y+z=2$$

消元法+回代法

pivot

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \xrightarrow{(2.1)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} \xrightarrow{(3.2)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{array}{cc} a & b \\ \boxed{\begin{matrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix}} & \boxed{\begin{matrix} 2 \\ 12 \\ 2 \end{matrix}} \end{array} \rightarrow \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{matrix}$$

消元法

$$\begin{array}{cc} \boxed{\begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{matrix}} & \boxed{\begin{matrix} 2 \\ 6 \\ -10 \end{matrix}} \\ c & u \end{array} \longrightarrow \begin{matrix} x+2y+z=2 \\ 2y-2z=6 \\ 5z=-10 \end{matrix}$$

回代法

$$(x,y,z)=(2,1,-2)$$

Matrix · column = column.

$$\begin{bmatrix} col1 & col2 & col3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{matrix} 3\,col1 \\ + \\ 4\,col2 \\ + \\ 5\,col3 \end{matrix}$$

Row × Matrix = Row

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} row1 \\ row2 \\ row3 \end{bmatrix} = \begin{matrix} 1 \times row1 \\ + \\ 2 \times row2 \\ + \\ 7 \times row3 \end{matrix}$$

Elimination Matrices.

step1. subtract 3 × row1 from row2

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & -2 \\ 0 & x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & -2 \\ 0 & x & 1 \end{bmatrix}$$

E_{21} ⇒ elementary or elimination.

step 2. subtract 2 × row 2 form row 3.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E_{32}

$E_{32}(E_{21}A) = U$

$\Rightarrow (E_{32}E_{21})A = U$.

置换 permination

Exchange row 1 and 2 $L\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Exchange column 1 and column 2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}R$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

可逆 Inverse.

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E^{-1} \qquad E \qquad I$$

HW