$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & x & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & x \\ 0 & 0 & 2 & x \end{bmatrix}$$

$$\xrightarrow[\text{echelon}]{} \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & x \\ 0 & 0 & 0 & 0 \end{bmatrix} = u$$

rank of A.

= # of pivots = 2.

$$X = c \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$x_1 + 2x_2 + 2x_3 + 2x_4 = 0$

$2x_3 + xx_4 = 0$

r = 2 # of variable

n - r = 4 - 2   free variable.

R = reduced row echelon form.

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & x \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 2 & x \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

zero above & below pivots.

= 1

$$\begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R = rref(A)$$

notice $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

in the pivot rows/cols

$x_1 + 2x_2 \qquad -2x_4 = 0$

$x_3 + 2x_4 = 0$

$Rx = 0$

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ pivot cols

F $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ free cols

rref form

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

nullspace matrix (column 3 = special sols.) ✓

$$N = \begin{bmatrix} -F \\ I \end{bmatrix} \qquad Rx = 0 \quad x_{pivot} = -F x_{free}$$

$$[I \ F] \begin{bmatrix} x_{pivot} \\ x_{free} \end{bmatrix} = 0.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 6 & 8 \\ 2 & 8 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = U.$$

$$x_1 + 2x_2 + 3x_3 = 0$$
$$2x_2 + 2x_3 = 0$$

$$x = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = c \begin{bmatrix} -F \\ 2 \\ 3 \end{bmatrix}$$

$$\leftarrow N.$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = R.$$