Gaussian Elimination

Key Point: $A = LU$.

let A.B inverse-able

$(AB)(B^{-1}A^{-1}) = I$

$(B^{-1}A^{-1})(AB) = I$

transport

$AA^{-1} = I$

$(A^{-1})^T A^T = I$

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$E_{32}E_{31}E_{21}A = U$ (no raw exchange)

$A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U = LU$

Typical Case:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}}_{E_{32}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 0 \end{bmatrix} = E$$

Reverse Derection.

Inverse (Reverse Order)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = L$$

$A = LU$.

if no row exchange.

multiplies go directly into L.

How expensive is elimination?

(How many operation we do

on $n \times n$ matrix?)

⊗ Multiply + Subtract = an operation.

let $n = 100$

$$\begin{bmatrix} & & \\ & & \cdot \\ & & \end{bmatrix}_{100\times100} \Rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{99\times99} \Rightarrow \begin{bmatrix} & \cdots & \\ & & \\ & & \end{bmatrix}_{\cdots\cdots} \Rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{1\times1}$$

and go on……

we can get about $\sum n^2 = \frac{1}{3}n^3$

Transposes and Permutation.

$3\times3 \Rightarrow P's$ $\quad P^{-1} = P^T$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

if $4\times4 \Rightarrow 24$ $P's$

$n\times n \Rightarrow n!$ $P's$ 种.