Linear independence, Spanning a space Basis and demension.

Suppose A is m by n with $m<n$

Then there are nonzero solutions to $Ax=0$.

(More unknowns then equations)

Reason: There will be free variables.

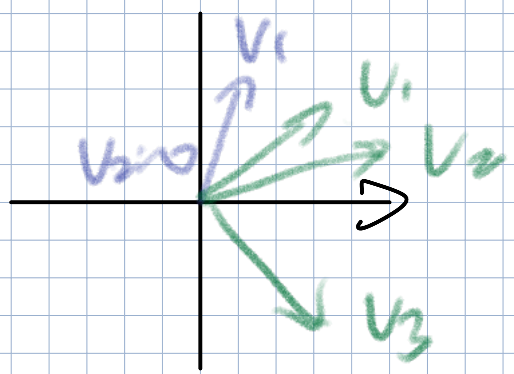
Def 1

Independence

Vectors $x_1, x_2, \dots, x_n$ are independent if

no combination give zero vectors (except the zero comb.)

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n = 0 \qquad \left(\sum_i c_i \neq 0\right)$$



$$A = \begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Repeat when $v_1, v_2 \cdots v_n$ are columns of A.

There are independent no free variables

if nullspace of A is $\{$zero vector$\}$ rank $= n$

There are dependent yes free variables.

if $Ac=0$ for some non-zero c. rank $< n$

$$N(A) = \{0\}$$

Vectors $v_1, v_2 \dots v_\ell$ span a space means:
The space consist of all combs. of these vectors.

Defn.
Basis for a space is a sequence of vectors.

$\vec{v}_1, v_2, \dots, v_d$. with 2 properties.

1. They are independent
2. They span the space.

Defn.

ex. space is $\mathbb{R}^3$

One basis is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

anothers basis is $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 7 \end{bmatrix}$

For $\mathbb{R}^n$

n vectors give basis if the $n \times n$ matrix with these cols is invertible.

Given a Space: cols

Every basis for the space have the same number of vectors.

Def'n:

Dimenzions of the space.

Space is $C(A)$ $\quad$ $N(A)$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

r = rank(A) = # pivot columns.

= demenzion of $C(A)$

$\dim C(A) = r$

$\dim N(A)$ = # free varlables

$= n - r$