Matrix Combination. ⇒ 5 ways

1th ways

$$\underset{A}{\begin{bmatrix} & \\ & \end{bmatrix}}\underset{B}{\begin{bmatrix} & \\ & \end{bmatrix}}=\underset{C=AB}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

$$C_{34} = (\text{row}3 \text{ of } A)(\text{column}4 \text{ of } B)$$
$$= a_{31}b_{14} + a_{32}b_{24} \cdots\cdots$$
$$= \sum_{k=1}^{n} a_{3k}b_{k4}$$

2nd ways

$$\underset{A}{\begin{bmatrix} & \\ & \end{bmatrix}}\underset{B}{\begin{bmatrix} & \\ & \end{bmatrix}}=\underset{P}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

Col & C are combination of column of C

$$\underset{A}{\begin{bmatrix} & \\ & \end{bmatrix}}\underset{B}{\begin{bmatrix} & \\ & \end{bmatrix}}=\underset{P}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

rows of c are combination of rows of B

3th ways

Column of A × rows of B

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix}$$

4th ways

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

AB = sum of [col of A] × [rows of B]

5th ways → Blocks

$$\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right] \left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right] = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

Inverse (square matrices)

$A^{-1}A = AA^{-1} = I$

if A^{-1} exist $\Rightarrow$ invertible / no singular

singular case: no inverse

$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Does exist a vector $X \neq 0$ with $AX = 0$

$\Rightarrow \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow A \times$ col j of A^{-1} = col j of I

Gauss-Jordan (solve 2 equation at once)

$$\begin{bmatrix}1&3\\2&7\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}1&3\\2&7\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

⇒ if boths have solve

⇒ inverse.

$$\left[\begin{array}{cc|cc}1&3&1&0\\2&7&0&1\end{array}\right]\Rightarrow\left[\begin{array}{cc|cc}1&3&1&0\\0&1&-2&1\end{array}\right]$$

eliminated.

$$\left[\begin{array}{cc|cc}1&0&7&-3\\0&1&-2&1\end{array}\right]$$

But why?

Reason:

$$E\text{'s}\begin{bmatrix}A & I\end{bmatrix}=\begin{bmatrix}I & A^{-1}\end{bmatrix}$$

$\Rightarrow [EA = I]$ tell us $E = A^{-1}$

⇒ Gauss-Jordan Elimination

Q.E.D. #