Matrix Combination ⇒ 5 ways

1th ways

$$\underset{A}{\begin{bmatrix} \quad & & \\ & \text{row } 3 & \\ & & \end{bmatrix}}\underset{B}{\begin{bmatrix} & \text{column } 4 & \\ & & \\ & & \end{bmatrix}} = \underset{C=AB}{\begin{bmatrix} & C_{34} & \\ & & \\ & & \end{bmatrix}}$$

$m \times n \qquad n \times p \quad \Rightarrow \quad m \times p$

C_{34} = (row 3 of A)(column 4 of B)

$= a_{31}b_{14} + a_{32}b_{24} \cdots\cdots$

$= \sum_{k=1}^{n} a_{3k}b_{k4}$

2nd ways

$$\underset{A}{\begin{bmatrix} \quad & & \\ & & \\ & & \end{bmatrix}}\underset{B}{\begin{bmatrix} \text{column } 1 & & \\ & & \\ & & \end{bmatrix}} = \underset{P}{\begin{bmatrix} A(\text{col } 1) & & \\ & & \\ & & \end{bmatrix}}$$

Col & C are combination of column of C

$$\underset{A}{\begin{bmatrix} \text{row} \\ \\ \\ \end{bmatrix}}\underset{B}{\begin{bmatrix} \text{row} \\ \text{row} \\ \text{row} \\ \text{row} \end{bmatrix}} = \underset{P}{\begin{bmatrix} \\ A\text{col} \\ \\ \end{bmatrix}}$$

rows of C are combination of rows of B

3rd ways

Column of A × rows of B

$m \times 1$ $\quad\quad$ $1 \times p$ $\quad \Rightarrow \quad m \times p$

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix}$$

4th ways

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

AB = sum of [col of A] × [rows of B]

5th ways ⇒ Blocks

$$\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right] \left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right] = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

$A_1B_1 + A_2B_3$

Inverse (square matrices)

$A^{-1}A = AA^{-1} = I$

if A^{-1} exist $\Rightarrow$ invertible / no singular

singular case: no inverse

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Does exist a vector $X \neq 0$ with $AX = 0$

$$\Rightarrow \begin{bmatrix} 3 \\ -1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow A \times$ col j of A^{-1} = col j of I

Gauss - Jordan (solve 2 equation at once)

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

⇒ if boths have solve

⇒ inverse.

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

eliminated.

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

But why?

Reason:

$$E's \begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

$\Rightarrow [EA = I]$ tell us $E = A^{-1}$

→ Gauss - Jordan Elimination

Q.E.D. #.