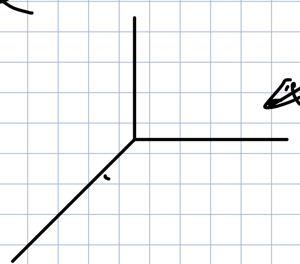
Vector space requirements $v+w$ and cv are in the space, combine $cv+dw$ are in the space.

R^3



plane through $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is subspace of R^3

2 subspaces: P and L.

$P \cup L$ = all vectors in P or L or both.

Is this a subspace? (no)

$P \cap L$ = all vectors in both P and L.

Is this a subspace? (yes)

Subspaces S and T

intersection $S \cap T$ is a subspace.

Column space of A is a subspace of R^4.

4×3

$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$: all linear combinations of cols.

$C(A)$

Does $Ax=b$ have a solution for every b? No.

4 equations, 3 unknowns.

$$Ax=\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

Which vector B allow this system to be solve?

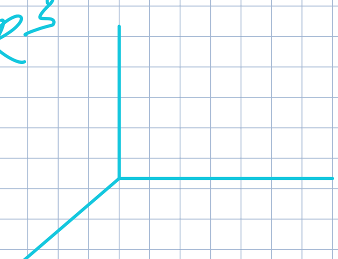
Can solve $Ax=b$ exactly when b is in $C(A)$ in R^4 #

Null space of A contains $c\begin{bmatrix}1\\1\\-1\end{bmatrix}$

= all solutions $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ to $Ax=0$ in R^3

$$A=\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

R^3



Check that solutions to $Ax=0$ always give a subspaces.

If $Av=0$ and $Aw=0$ then $A(v+w)=0$.