

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + 8y + z &= 12 \\ 4y + z &= 2 \end{aligned}$$

消元法 + 回代法

$$\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array} \xrightarrow{(2.-1)} \begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 2 \\ 0 & \textcircled{2} & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array} \xrightarrow{(3.-2)} \begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 2 \\ 0 & \textcircled{2} & -2 & 6 \\ 0 & 0 & \textcircled{5} & -10 \end{array}$$

$$\begin{array}{ccc|c} a & & b & \\ 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array} \Rightarrow \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array}$$

消元法

$$\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array}$$

c

u.

$$\begin{aligned} x + 2y + z &= 2 \\ 2y - 2z &= 6 \\ 5z &= -10 \end{aligned}$$

回代法

$$(x, y, z) = (2, 1, -2)$$

Matrix \cdot column = column.

$$[\text{col } 1 \quad \text{col } 2 \quad \text{col } 3] \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{matrix} 3 \text{ col } 1 \\ + \\ 4 \text{ col } 2 \\ + \\ 5 \text{ col } 3 \end{matrix}$$

Row \times Matrix = Row

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \end{bmatrix} = \begin{matrix} 1 \times \text{row } 1 \\ + \\ 2 \times \text{row } 2 \\ + \\ 3 \times \text{row } 3 \end{matrix}$$

Elimination Matrices.

step 1. subtract $3 \times \text{row } 1$ from row 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

$E_{21} \Rightarrow$ elementary or elimination.

Step 2. subtract $2 \times$ row 2 from row 3.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E_{32}

$$E_{32}(E_{21}A) = u$$

$$\Rightarrow (E_{32}E_{21})A = u.$$

置换 permutation

Exchange row 1 and 2 $L \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Exchange column 1 and column 2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix} R$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

逆 Inverse.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E^{-1} E I

HW

$$2.1 \Rightarrow 17.21$$

$$2.2 \Rightarrow 8.14.18.19$$

$$2.3 \Rightarrow 3.10.21$$

$$2.4 \Rightarrow 25.26.$$

$$2.5 \Rightarrow 7.24.34,$$