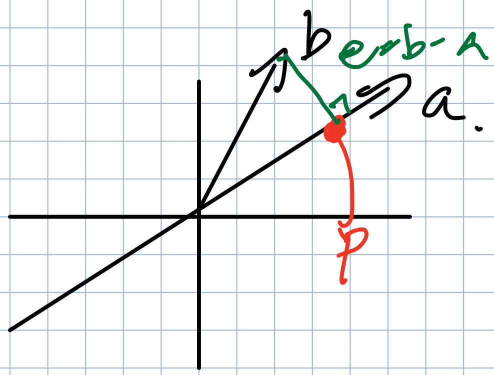


$a^T(b-xa)=0.$

$x a^T a = a^T b \qquad x=\frac{a^T b}{a^T a} \qquad p=ax.$

$p = a\frac{a^T b}{a^T a} \Rightarrow$ projection $p = Pb.$

MATRIX

$P = \frac{aa^T}{a^T a}.$ $\quad C(P) =$ line through a $\quad \text{rank}(P)=1.$

$P^T = P \qquad P^2 = P$

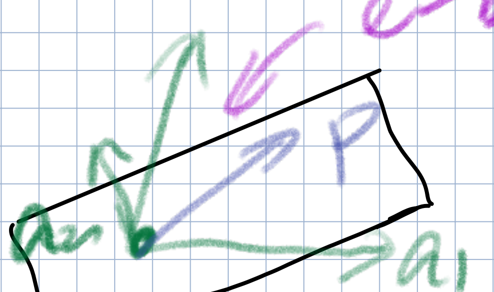
Why Project?

Because $Ax=b$ may have no solution -

Solve $Ax=p$. Instead.

proj. of b onto col space.

$e=b-p$ is prep. to plane.



plane of a_1, a_2

= col space of

$A=[a_1\ a_2]$

$p = x_1 a_1 + x_2 a_2 \qquad p = A\hat{x} \qquad$ Find $\hat{x}$

Key: $b - A\hat{x}$ is prep. to plane.

$a_1^T(b - A\hat{x}) = 0 \qquad a_2^T(b - A\hat{x}) = 0.$

$$\begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} \underbrace{(b - A\hat{x})}_{=e} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \left\{ A^T(b - A\hat{x}) = 0 \right.$$

e in $N(A^T)$

$\Rightarrow e \perp C(A)$

☆ $A^TA\hat{x} = A^Tb.$

1D $\rightarrow \dfrac{aa^T}{a^Ta}$

$\hat{x} = (A^TA)^{-1}A^Tb$

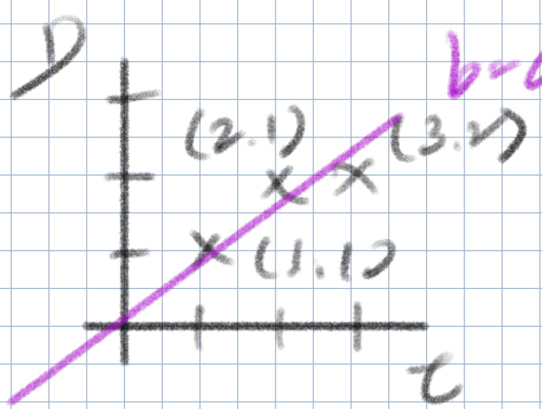
proj. $= A\hat{x} = A(A^TA)^{-1}A^Tb.$

matrix $P = \boxed{A(A^TA)^{-1}A^T}.$ nD.

$P^T = P$

$P^2 = P$

application: Least squares



$b = C + Dt.$

Fitting by a line.

$$\left\{ \begin{aligned} C + D &= 1 \\ C + 2D &= 2 \\ C + 3D &= 2 \end{aligned} \right.$$

$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{x}{\begin{bmatrix} C \\ D \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}$$