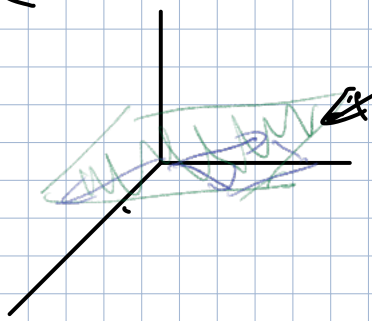
Vector space requirements $v+w$ and $cv$ are in the space, combine $cv+dw$ are in the space.

$R^3$



plane through $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is subspace of $R^3$

2 subspaces: $P$ and $L$.

$P \cup L$ : all vectors in $P$ or $L$ or both,

Is this a subspace? (no)

$P \cap L$ = all vectors in both $P$ and $L$.

Is this a subspace.? (yes)

Subspaces $S$ and $T$

intersection $S \cap T$ is a Subspace.

Column space of $A$ is a subspace of $R^4$.

$4 \times 3$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ : all linear combinations of cols.

$C(A)$

Does $Ax=b$ have a solution for every $b$? No.

4 equations, 3 unknowns.

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

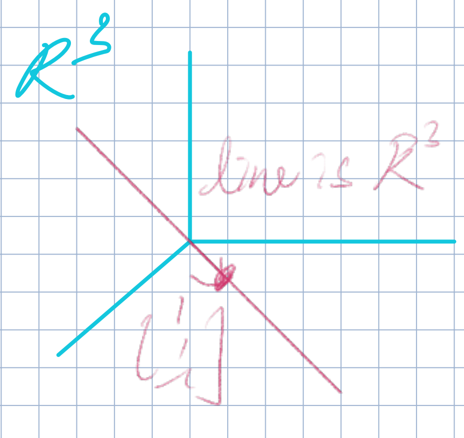
Which vector B allow this system to be solve?

Can solve $Ax=b$ exactly when $b$ is in $C(A)$ in $R^4$

Null space of A contains $c\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

= all solutions $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to $Ax=0$ in $R^3$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$




Check that solutions to $Ax=0$ always give a subspaces.

If $Av=0$ and $Aw=0$ then $A(v+w)=0$.