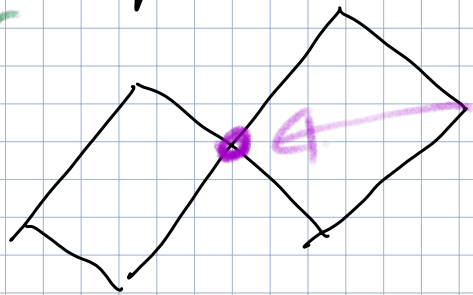


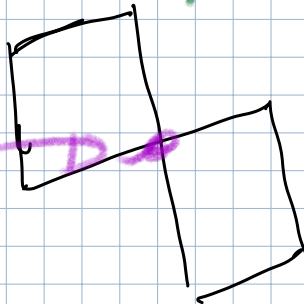
row space

$r$



col space

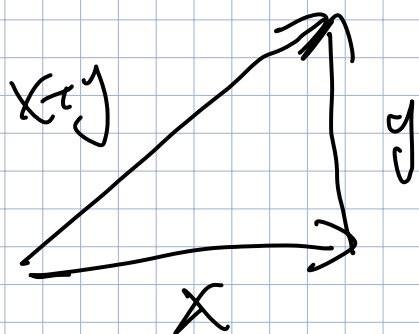
$r$



null space  $n-r$

nullspace of  $A^T$   $m-r$

orthogonal vectors.



pythagoras

$$x^T y = 0$$

$x^T x$  is positive.

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad y = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$x+y = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$\|x\|^2 = 14, \quad \|y\|^2 = 5$$

$$\|x+y\|^2 = 19.$$

$\Downarrow$

$$x^T x + y^T y = (x+y)^T (x+y) = x^T x + y^T y + x^T y + y^T x$$

Subspace  $S$  is orthogonal to subspace  $T$ .

means: every vector in  $S$  is orthogonal to every vector in  $T$ .

rowspace is orthogonal to nullspace.

why?  $Ax=0$

$$\begin{bmatrix} \text{rows 1 of } A \\ \text{rows 2} \dots \\ \text{rows } m \text{ of } A \end{bmatrix} \begin{bmatrix} x \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$$

$$\left. \begin{aligned} c_1(\text{row } 1)^T &= 0 \\ c_2(\text{row } 2)^T &= 0 \end{aligned} \right\} \Rightarrow (c_1 \text{row } 1 + c_2 \text{row } 2)^T x = 0.$$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 8 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$n=3 \quad r=1 \quad \dim N(A)=2$$

nullspace and rowspace: are orthogonal.

complements in  $\mathbb{R}^n$ .

Nullspace contains all vector  $\perp$  row space.

Coring: solve  $AX=b$  when there is no solution

$$A^T A$$

$n \times m$   $m \times n$   
 $n \times n$

symmetric.

$$(A^T A)^T = A^T A^{TT}$$

$$A^T A x = A^T b.$$

$$m=3 > n=2.$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 9 & 12 \\ 9 & 12 \end{bmatrix}$$

$$N(A^T A) = N(A)$$

rank of  $A^T A$  = rank of  $A$ .

$A^T A$  is invertible exactly if  $A$  has  
independent columns..