M = all 3 by 3 matrices

Vector space
- $\to M$ = all 3 by 3 — $\dim M = 9$.
- $\to$ matrices upper triangular — $\dim S = 6$.
- $\to$ symmetric 3×3 — $\dim U = 6$

Basis for M = all 3×3's

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \ldots, \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

Int

$S\cap U$ = symm. and upper triangular.

Sum = diagonal 3×3's $\quad \dim(S\cap U) = 3$

$S+U$ = any element of S + any element of U.

= all 3×3's $\quad \dim(S+U) = 9$.

Q: $\frac{d^2y}{dx^2} + y = 0 \quad y = \cos x, \sin x$ basis

complete solution: $\quad \dim(\text{solution space}) = 2$

$y = c_1\cos x + c_2 \sin x$

$\dim C(A) = \text{rank} = \dim C(A^T)$

$$\underset{2\times 3}{A} = \begin{bmatrix}1&4&5\\2&8&10\end{bmatrix} = \underset{2\times 1}{\begin{bmatrix}1\\2\end{bmatrix}} \underset{1\times 3}{\begin{bmatrix}1&4&5\end{bmatrix}}$$

Rank 1 matrix

$A = uv^T$

M: all 5×17 matrices

Subset of rank 1 matrices are a subspace.

$Av=0$

In R^4

$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ S = all v in R^4 with $v_1+v_2+v_3+v_4=0$.

= null space of $A = [1\ 1\ 1\ 1]$

rank = 1 = r

$\dim N(A) = n - r$

Basis for S

$C(A) = R^1 \quad N(A^T) = \{0\}$

$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Graph = {nodes, edges}