Permutation

P: execute row exchanges

becomes $PA = LU$

$\Rightarrow$ Any inverable case.

P = identity matrix with reordered rows.

$= n! = n(n-1)(n-2)\cdots 2\times 1$

counts reorders

$\Rightarrow$ all $n\times n$ permutations.

$P^{-1} = P^T$ $\quad P^T P = I$

Transposed.

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

3×2 $\quad$ 2×3

$\Rightarrow$ So Transpose $(A^T)_{ij} = (A)_{ji}$

Symmetric Matrix.

$\Rightarrow A = A^T$

ex. $\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 9 \\ 7 & 9 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}^T \leftarrow R^T$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix} \leftarrow R$$

$R^T R$ is always symmetric

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 11 & 7 \\ 11 & \square & \square \\ 7 & \square & \square \end{bmatrix}$$

Why?

We can take the transpose.

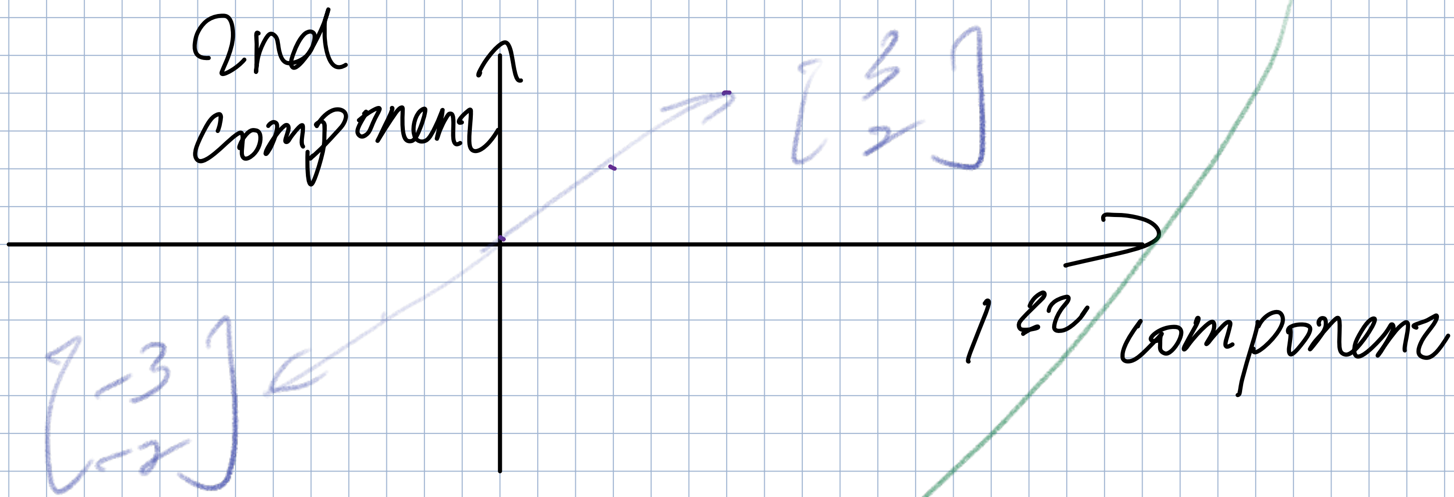
$$(R^T R)^T = R^T R^{TT}$$
$$= R^T R$$, proved #

Vector Space

ex.

R^2 = all 2-dimensional real vectors

$$\begin{bmatrix}3\\2\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\pi\\e\end{bmatrix}, \ldots$$



x-y plane
= all vectors with 3 components

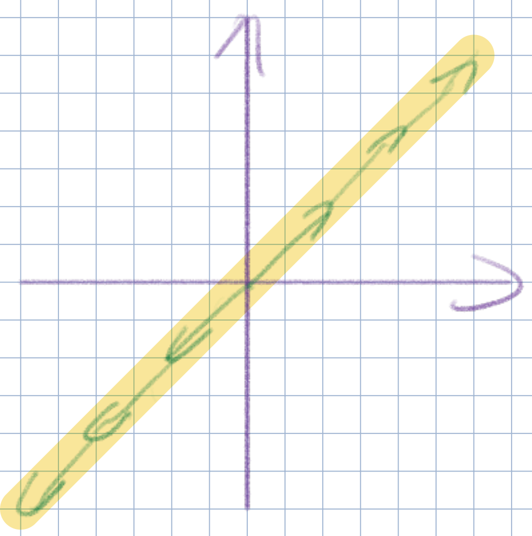
R^n = vectors with n column.
(n components)

Can we do those addition and still in the spaces?

Ans: Yes

Ex. a vector space inside R^2

$\Rightarrow$ subspace of R^2



$\Rightarrow$ line in R^2 through zero vector

① all in R^2

② any line through $\begin{bmatrix}0\\0\end{bmatrix}$

③ zero vector only "Z"

R^3: Plane / line / $Z = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

But, How they come out of Matrices

$A = \begin{bmatrix} 1 & 3 \\ 3 & 3 \\ 4 & 1 \end{bmatrix}$ column in R^3

$\Rightarrow$ all their combine for a subspace call column space $C(A)$