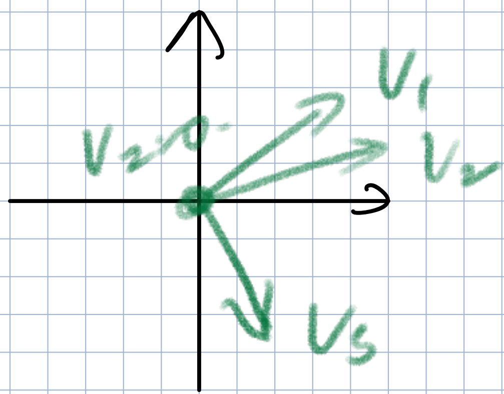
$\boxed{\text{define}}$ Independence-

Vector $x_1, x_2 \cdots x_n$ are independent
if no combination give zero vectors.
(except the zero combination) ($\sum_i c_i \neq 0$)



$$A = \begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Repeat when $v_1, v_2, \cdots v_n$ are col. of A.
There are independent if (no free variables)
nullspace of A is {zero vector}. rank $= n$.
There are dependent if (yes free variables)
$Ac = 0$ for some non-zero c
rank $< n$ $\quad N(A) \neq \{0\}$

Vector $v_1, v_2 \cdots v_l$ span a space means:
The space consist of all combs. of these vecto

$\boxed{\text{define}}$ Basis-

For a space is a sequence of vectors.
$v_1, v_2 \cdots v_d$ with 2 properties.
1. They are independent.
2. They span the space.

$\boxed{\text{define}}$

ex. Space is R^3

one basis is $\begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix}$

another basis is $\begin{bmatrix}1\\1\\2\end{bmatrix} \begin{bmatrix}2\\7\\5\end{bmatrix} \begin{bmatrix}0\\ \text{[illegible]}\\ \text{[illegible]}\end{bmatrix}$

⋮

for R^n

n vectors give basis if the $n \times n$ matrix $\underbrace{\text{with}}_{\text{those cols}}$ is inverseable

$\boxed{\text{define}}$ Demenzions of the space.

$\boxed{\text{Given a Space}}$: cols. Every basis for the space have the same number of vectors.

space is $C(A)$ $\begin{bmatrix}1 & 2 & 3 & 1\\ 1 & 1 & 2 & 1\\ 1 & 2 & 3 & 1\end{bmatrix}$

$N(A)$ $\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix} \begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$

$2 = \text{rank}(A) =$ number of pivot cols.

$=$ demenzion of $C(A)$

dim $C(A)$ = r
dim $N(A)$ = number of free variables.
= $n-r$

ex. space is R^3
one basis is standard $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\right.$

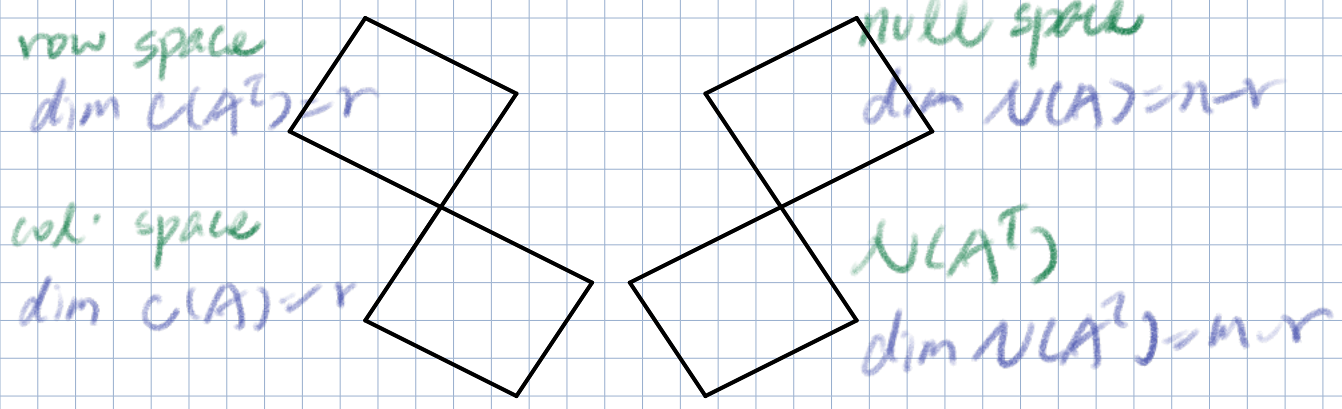
another basis $\begin{bmatrix}1&2&3\\1&2&3\\2&5&8\end{bmatrix}$

R^n n vector give basis if the $n \times n$.

---

4 fundemental Subspaces. $\boxed{A \text{ is } m \times n}$
column space $C(A)$ in R^m
row space = all combs. of rows
= all combs. of cols of A^T
= $C(A^T)$ in R^n
null space $N(A)$ in R^n
null space of A^T = $N(A^T)$
= left null space of A in R^3

$\mathbb{R}^n$ $\qquad\qquad$ $\mathbb{R}^m$



$C(A)$ $\qquad\qquad$ $N(A)$

pivot cols $\qquad\qquad$ special sols.

rank $=r$ $\qquad\qquad$ rank $=n-r$

$$A\begin{bmatrix}1&2&3&1\\1&1&2&1\\1&2&3&1\end{bmatrix}\rightarrow\begin{bmatrix}\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{I}&\underbrace{\begin{matrix}1&1\\1&0\end{matrix}}_{F}\\ \underbrace{\begin{matrix}0&0\end{matrix}}&\underbrace{\begin{matrix}0&0\end{matrix}}_{\text{zero}}\end{bmatrix}=R$$

$C(R)\neq C(A)$ diff. col. spaces

$\boxed{\text{same Row space}}$

Basis for rowspace is first r rows of R

$A^Ty=0 \qquad y^TA^{TT}=y^TA=0^T$

$$[y^T][A]=[0]$$

$$A=\begin{bmatrix}1&2&3&1\\1&1&2&1\\1&2&3&1\end{bmatrix}$$

M = all 3x3 matrices

vector space → M = all 3x3 (dim M = 9)

→ matrices upper triangular (dim S = 6)

→ symmetric 3x3 (dim U = 6)

basis for M = all 3x3's

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Int

$S \cap U$ = symm and upper triangular

= diagonal 3x3's $\qquad \dim(S \cap U) = 3$

Sum

$S + U$ = any element of S + any element of U

= all 3x3's $\qquad \dim(S + U) = 9$

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$$

$\dim C(A) = \text{rank} = \dim C(A^T)$

Rank 1 matrix $\quad A = uv^T$

M = all (3×3) matrices

Subset of rank 1 matrices not a subspace.

In R^4 $\quad$ S = all v in R^4 with

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \qquad v_1 + v_2 + v_3 + v_4 = 0.$$

= null space of $A = [1\ 1\ 1\ 1]$

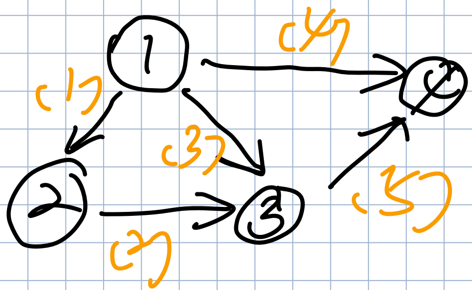
rank = 1 = r

dim N(A) = n − r

Basis for S

$$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{array}{l} C(A) = R^1 \\ N(A^T) = \{0\} \end{array}$$

Graph: Nodes. Edges.



n = 4 nodes

m = 5 edges

Incidence Matrix

$$A = \begin{array}{c} \text{node} \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \text{edge} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$A^T y = 0 \quad \dim N(A^T) = m - r = 5 - 3 = 2.$

$\downarrow$

$n \times m = 4 \times 5$

$$\begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Ax = 0 = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_4 - x_3 \\ x_4 - x_1 \\ x_4 - x_5 \end{bmatrix}$$

$x = x_1 . x_2 . x_3 . x_4$ potentials at nodes.

$e = Ax$

$x_2 - x_1$, etc.

potential diff.

$y = Ce$

Ohm's Law on edges.

currents $y_1 \ y_2 \ y_3 \ y_4 \ y_5$

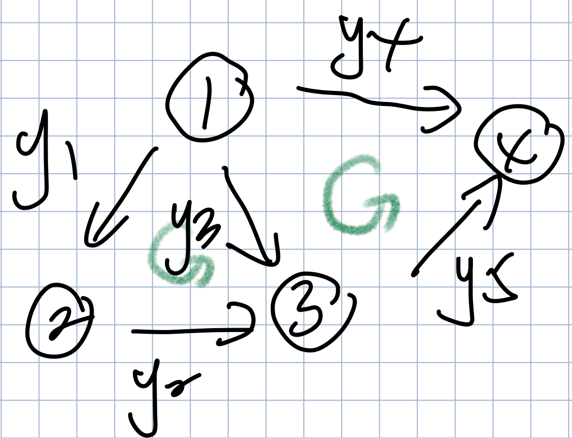
A^T

$A^T y = 0$ Kirchehhofs C2.

$$x = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$\dim N(A) = 1$

Rank $= 3$



$-y_1 - y_3 - y_4 = 0$

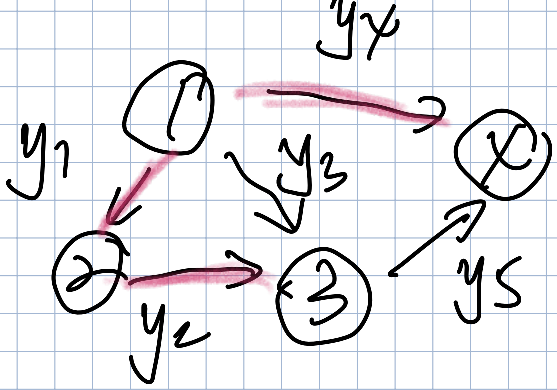
$y_1 - y_2 = 0$

$y_2 + y_3 - y_5 = 0$

$y_4 + y_5 = 0.$

Basis for $N(A^T)$

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$



TREE: no loop.

$\dim N(A^T) = m - r$

nums. loops = nums. edges − (nums. node − 1)

(rank = n − 1)

nums nodes − nums. edges + nums. loops = 1