

1. 假设物体的运动方程为 $s(t) = 5t - \frac{1}{2}t^2$ (m)

求 1sec 时的瞬时速度?

$$v(t) = s'(t) = 5 - t \quad \text{故 } v(t) = 5 - 1 = 4 \text{ (m/s)}$$

2. 假定下列各题中 $f(x)$ 存在, 按导数定义求各极限值

$$(1) \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0)$$

$$(2) \lim_{h \rightarrow 0} \frac{f(x_0 + \alpha h) - f(x_0 + \beta h)}{h} \quad (\text{其中 } \alpha, \beta \text{ 为不等于 } 0 \text{ 的常数})$$

$$= \lim_{h \rightarrow 0} \frac{f(x_0 + \beta h + \alpha h) - f(x_0 + \beta h)}{h} = (\alpha + \beta) f'(x_0)$$

$$(3) \lim_{x \rightarrow x_0} \frac{x f(x_0) - x_0 f(x)}{x - x_0}$$

$$= \lim_{x \rightarrow x_0} \frac{x f(x_0) - x_0 f(x_0) + x_0 f(x_0) - x_0 f(x)}{x - x_0}$$

$$= \lim_{x \rightarrow x_0} \frac{(x - x_0) f(x_0) + x_0 [f(x_0) - f(x)]}{x - x_0} = f(x_0) - x_0 f'(x_0)$$

3. 已知 $f'(x_0) = 2$, 求 $\lim_{x \rightarrow 0} \frac{x}{f(x_0 + x) - f(x_0)}$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{f(x_0 + x) - f(x_0)}{x}} = \frac{1}{f'(x_0)} = \frac{1}{2}$$

4. 求曲线 $y = 3^x$ 在 (0, 1) 点处的切线方程和法线方程

$$y' = \lim_{\Delta x \rightarrow 0} \frac{3^{x+\Delta x} - 3^x}{\Delta x} = 3^x \lim_{\Delta x \rightarrow 0} \frac{3^{\Delta x} - 1}{\Delta x} = 3^x \ln 3$$

$$\text{切线方程} \Rightarrow y - 1 = 3^0 \ln 3 (x - 0) \Rightarrow y = 3^x \ln 3 + 1$$

$$\text{法线方程} \Rightarrow y = -\frac{1}{3^x \ln 3} + 1$$

5. 在抛物线 $y = x^2$ 上求一点, 使得过该点的切线平行于直线 $y = 6x + 1$. 该抛物线上该一点的切线垂直于直线 $3x + 6y - 7 = 0$

$$(1) \begin{cases} y = x^2 \\ y = 6x + 1 \end{cases} \Rightarrow \begin{cases} y = x^2 \\ y' = 6 \end{cases}$$

$$\Rightarrow x = 3, y = 9$$

$$(2) \begin{cases} y = x^2 \\ y = -2x + b \end{cases} \Rightarrow \begin{cases} y = x^2 \\ y' = -2 \end{cases}$$

$$\Rightarrow x = -1, y = 1$$

6. 已知函数 $f(x)$ 在 $x=2$ 连续, 且 $\lim_{x \rightarrow 2} \frac{f(x)}{x-2} = 4$, 求 $f'(2)$

$$f'(x) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{f(x)}{x - 2} = \lim_{x \rightarrow 2} \frac{f(x)}{x - 2} = 4 - 0 = 4.$$

7. 判断下列函数在 $x=0$ 处的连续性及可导性.

(1) $y = (\tan x)$

$x=0$ 处连续, 但不可导

(2) $y = \begin{cases} x^a \sin \frac{1}{x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

$a > 1$ 时, 连续可导

$a > 0$ 时, 连续不可导

$a \leq 0$ 时, 不连续不可导

8. 已知 $y = x^2 - x$, 计算在 $x=2$ 处 $\Delta x = 0.1$ 和 $\Delta x = 0.01$ 时的 Δy 和 dy

$$\left. \frac{\Delta y}{\Delta x} \right|_{x=2, \Delta x=0.1} = \frac{f(2+0.1) - f(2)}{0.1}$$

$$\Delta y = 0.3 \quad dy = 0.3$$

$$\left. \frac{\Delta y}{\Delta x} \right|_{x=2, \Delta x=0.01} = \frac{f(2+0.01) - f(2)}{0.01}$$

$$\Delta y = 0.0301 \quad dy = 0.03$$

9. 计算下列函数的微分:

(1) $y = \sqrt{x}$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} dx$$

(2) $y = \frac{1}{3\sqrt{x}}$

$$y' = -\frac{2}{3} x^{-\frac{5}{2}} dx$$

(3) $y = \lg x$

$$dy = \frac{1}{x \ln 10} dx$$