Last Time: Cofactor Formula.

$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} \dots + a_{in}C_{in}$ product of n-1 entries

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\det A} C^T$

products of n entries

Check $AC^T = (\det A) I$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} C_{11} & \cdots & C_{n1} \\ \vdots & \ddots & \vdots \\ C_{1n} & \cdots & C_{nn} \end{bmatrix} = \begin{bmatrix} \det A & & & \\ & \det A & & \\ & & \ddots & \\ & & & \det A \end{bmatrix}$$

$A_s = \begin{bmatrix} a & b \\ a & b \end{bmatrix} \Rightarrow \det A_s = ab + b(-a) = 0.$

$Ax = b$

$x = A^{-1}b = \frac{1}{\det A} C^T b$

Cramer's Rule.

$x_1 = \frac{\det B_1}{\det A}$ $\qquad x_2 = \frac{\det B_2}{\det A}$

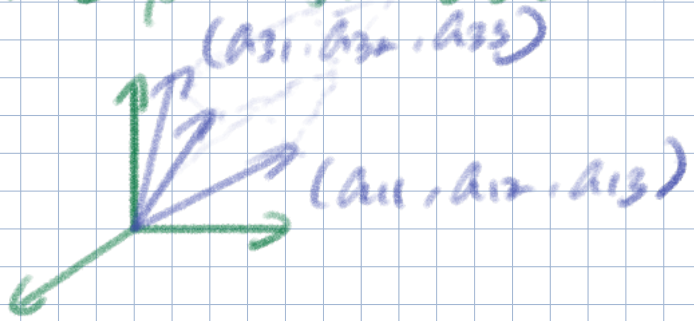
A with column 1 replaced by b

$B_1 = \begin{bmatrix} b & \begin{matrix} n-1 \text{ cols} \\ \text{of } A \end{matrix} \end{bmatrix}$

$C_{11}b_1 + C_{21}b_2 + \cdots$

B_j = A with cols j replaced by b

3×3 |det A| volumn of box.



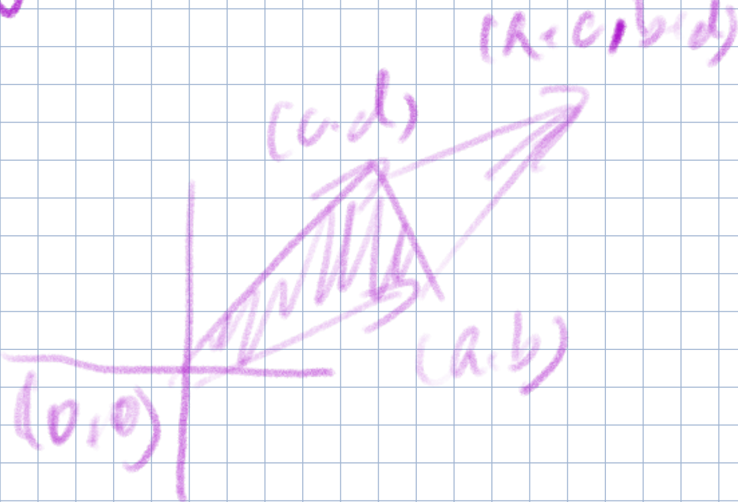
A = Q (orthog. matrix)

Proved $Q^TQ = I$

$\det Q^TQ = I$

$|Q^T||Q| = 1 \Rightarrow |Q|^2 = 1$

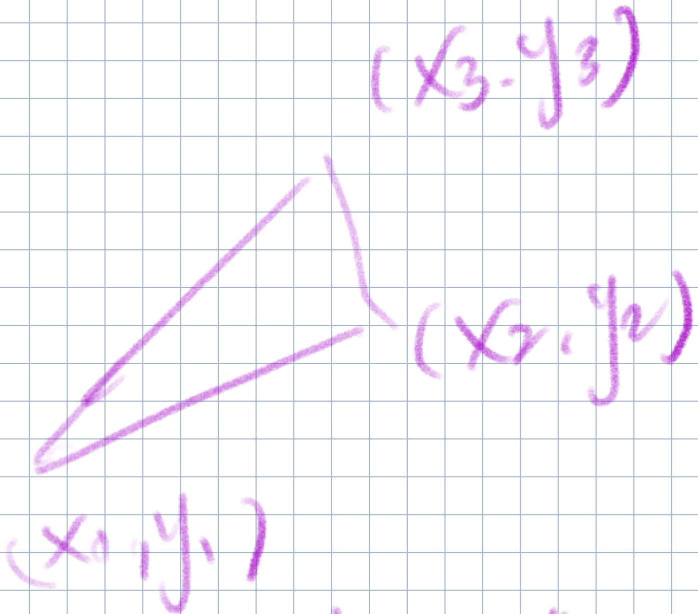
Property 1. 2. 3a. 3b.



$\text{area} = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

for parallelogon

$\frac{1}{2}(ad - bc)$ for the triangle.



$$\text{area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$