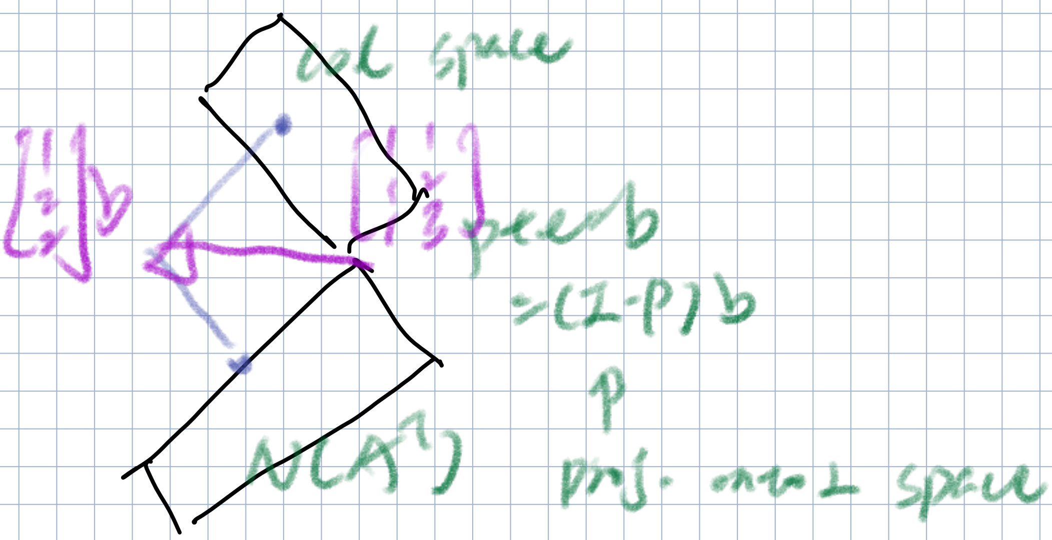
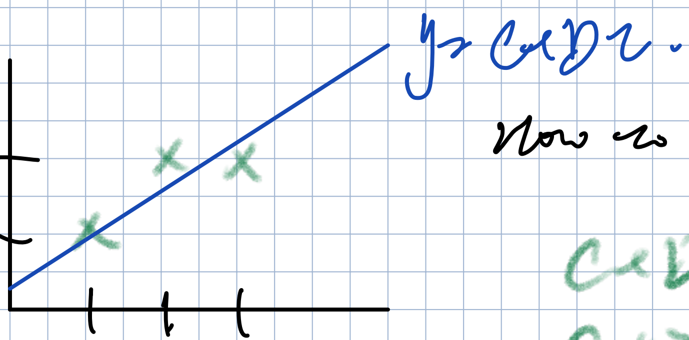
Proj. matrix.

$P = A(A^TA)^{-1}A^T$

if b in col. space $Pb = b$

if $b \perp$ col. space $Pb = 0$.





How to pick the best straight line

$C + D = 1$

$C + 2D = 2$.

$C + 3D = 2$.

$Ax = b$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Minimize $\|Ax - b\|^2 = \|e\|^2$

$$= e_1^2 + e_2^2 + e_3^2 = (C+D-1)^2 + (C+2D-2)^2 + (C+3D-2)$$

Find $\hat{x} = \begin{bmatrix}\hat{C} \\ \hat{D}\end{bmatrix}$, p

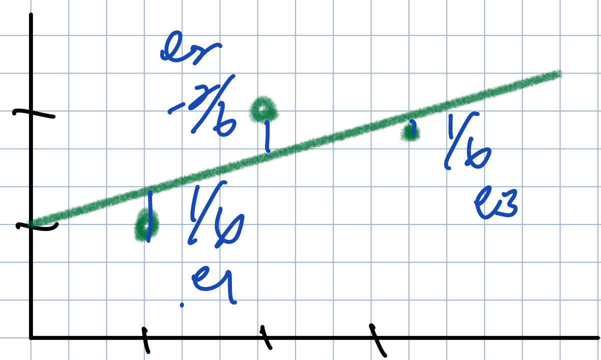
$A^TA\hat{x} = A^Tb$ normal eqns

$$\begin{bmatrix}1 & 1 & 1 \\ 1 & 2 & 3\end{bmatrix}\left[\begin{array}{cc|c}1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 2\end{array}\right] = \left[\begin{array}{cc|c}3 & 6 & 5 \\ 6 & 14 & 11\end{array}\right]$$

$$3C + 6D = 5$$
$$3C + 14D = 11$$

$$2D = 1$$
$$D = 1/2$$
$$C = 2/3$$

$y = \frac{2}{3} + \frac{1}{2}t$



$e_1 = -1/6$
$e_2 = 2/6$
$e_3 = -1/6$

$$b = p + e$$
$$\begin{bmatrix}1 \\ 2 \\ 2\end{bmatrix} = \begin{bmatrix}7/6 \\ 10/6 \\ 13/6\end{bmatrix} + \begin{bmatrix}-1/6 \\ 2/6 \\ -1/6\end{bmatrix}$$

$$\begin{cases} A^TA\hat{x} = A^Tb \\ p = A\hat{x} \end{cases}$$

If A has indep. col. then A^TA is invertible.

Suppose $A^TAx = 0$. $\Rightarrow$ To prove x must be 0.

$\boxed{\text{IDEA}}$ $x^TA^TAx = 0 = (Ax)^TAx \Rightarrow Ax = 0$ (square)

A has indep. cols.
$\Rightarrow x = 0$.

简易知識

Case: Col. definitely indep. if they are perp. unit vectors.

orthonormal vectors.