1. 判等无穷小求极限

1) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{\sin(3x)}$

$=\dfrac{5x}{3x}=\dfrac{5}{3}$ #

2) $\lim\limits_{x\to 0}\dfrac{\sin(2x^3)}{x(1-\cos x)}$

$=\lim\limits_{x\to 0}\dfrac{2x^3}{x(\frac{x^2}{2})}=4$ #

3) $\lim\limits_{x\to 0}\dfrac{\sin x-\tan x}{x\ln(1+x^2)}$

$=\lim\limits_{x\to 0}\dfrac{\tan x(\cos x-1)}{x\cdot x^2}$

$=\lim\limits_{x\to 0}\dfrac{x\cdot(-\frac{x^2}{2})}{x^3}=-\dfrac{1}{2}$ #

4) $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x\sin x}$

$=\lim\limits_{x\to 0}\dfrac{\ln(1+\cos x-1)}{x\sin x}$

$=\lim\limits_{x\to 0}\dfrac{\ln(1-\frac{x^2}{2})}{x^2}=\dfrac{-\frac{x^2}{2}}{x^2}=-\dfrac{1}{2}$ #

5) $\lim\limits_{x\to 0}\dfrac{\sin^m x}{\sin(x^n)}$ ($m,n\in N$)

$=\lim\limits_{x\to 0}\dfrac{x^m}{x^n}$

$\begin{cases} 0, & m<n \\ 1, & m=n \\ \infty, & m>n \end{cases}$

6) $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin(2x)}{x^2\tan x}$

$=\lim\limits_{x\to 0}\dfrac{2\sin x-2\sin x\cos x}{x^2\tan x}$

$=\lim\limits_{x\to 0}\dfrac{2\sin x(1-\cos x)}{x^2\tan x}$

$=\lim\limits_{x\to 0}\dfrac{2\cdot x\cdot\frac{x^2}{2}}{x^3}=1.$

7) $\lim\limits_{x\to 1}\dfrac{1+\cos(\pi x)}{(1-x)^2}$

令 $t=1-x\to 0$. 有 $1-\cos t\sim\dfrac{[\pi t^2]^2}{2}$

原式 $=\lim\limits_{x\to 1}\dfrac{1+\cos(\pi x-\pi+\pi)}{(x-1)^2}$ $=\lim\limits_{x\to 1}\dfrac{1+\cos\pi(x-1)-\sin\pi(x-1)\sin\pi}{(x-1)^2}$

$=\lim\limits_{x\to 1}\dfrac{[\pi(x-1)]^2}{2}\cdot\dfrac{1}{(x-1)^2}=\dfrac{\pi^2}{2}$

8) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\tan x}-\cos x}{(3^x-1)\ln(1+x)}$

$=\lim\limits_{x\to 0}\dfrac{1+x\tan x-\cos^2 x}{x\ln 3\cdot x(\sqrt{1+x\tan x}-\cos x)}$

$=\lim\limits_{x\to 0}\dfrac{\sin^2 x+x\tan x}{x\ln 3\cdot x(\sqrt{1+x\tan x}-\cos x)}$

$=\lim\limits_{x\to 0}\dfrac{2x^2}{\ln 3\cdot\frac{x^2}{2}}=\dfrac{1}{\ln 3}$ #

2. 求 $\lim\limits_{x\to\infty} \dfrac{x^3}{x^2+1} \sin\dfrac{1}{x} = \lim\limits_{x\to\infty} \dfrac{x^3}{x^2+1} \cdot \dfrac{1}{x} = 1$

3. 求 $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^2(e^x-1)}$

$= \lim\limits_{x\to 0} \dfrac{1+\tan x - (1+\sin x)}{x^2(e^x-1)(\sqrt{1+\tan x}+\sqrt{1+\sin x})}$

$= \lim\limits_{x\to 0} \dfrac{\tan x(1-\cos x)}{x^2(x)\cdot(\sqrt{1}+\sqrt{1})}$

$= \lim\limits_{x\to 0} \dfrac{x\cdot\frac{x^2}{2}}{x^2\cdot x\cdot 2} = \dfrac{1}{4}$

4. 求 $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x\tan x} - \sqrt{\cos x}}{\ln \cos x}$

$= \lim\limits_{x\to 0} \dfrac{1+x\tan x - \cos x}{\ln\cos x(\sqrt{1+x\tan x}+\sqrt{\cos x})}$

$= \lim\limits_{x\to 0} \dfrac{x\tan x + (1-\cos x)}{2\ln\cos x}$

5. 证明 当 $x\to 0$ 时, $\arctan x \sim x$

令 $u = \arctan x$, $|x|<1$, 2.] $x = \tan u$