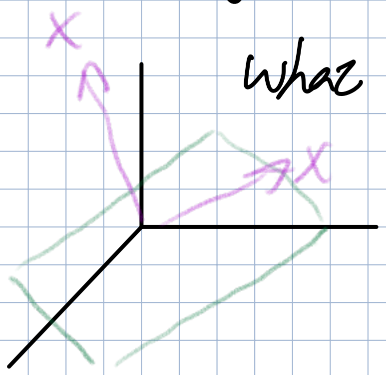
Eigenvalues - Eigenvectors

(Eigenvectors)

Ax parallel to x $\boxed{Ax = \lambda x}$

If A is singular, $\lambda = 0$ is an eigenvalues.



what are x's and λ's for projection matrix?

Any x in plane: $Px = x$ $\lambda = 1$.

Any $x \perp$ plane: $Px = 0$ $\lambda = 0$.

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $= c_1$ $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $Ax = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\lambda = 1$ $Ax = x$

$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $Ax = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\lambda = -1$ } c_2

$c_1 = c_2 =$ trace

Fact: sum of λ's: $a_{11} + a_{22} + \dots + a_{nn}$

How to solve $Ax = \lambda x$

Rewrite: $\underbrace{(A - \lambda I)}_{\text{singular}} x = 0$.

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$\det(A - \lambda I) = 0$

FIND landa

compute $\det(A - \lambda I)$

$$= \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 1 = \lambda^2 - 6\lambda + 8$$

det.

trace.

$$= (\lambda-4)(\lambda-2) \quad \lambda_1 = 4 \quad \lambda_2 = 2.$$

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$Ax_1 = \lambda_1 x_1$

$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda_1 = 4.$

$A - 4I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$\lambda_2 = 2 \; x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$A - 2I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

ex.

$90°$ rotation $Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

trace: or $0 = \lambda_1 + \lambda_2$

det $= 1 = \lambda_1 \lambda_2$

$$\det(Q - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0.$$

$\lambda_1 = i$

$\lambda_2 = -i$

★ degenerate Matrix.

$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \quad \det(A - \lambda I)$

$$= \begin{vmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(3-\lambda)$$

$\lambda_1 = 3$

$\lambda_2 = 3.$

$$(A - \lambda I)x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

x_2 = No 2nd indep. x.