① det I = 1 .

② Exchange rows: reverse sign of det.

det P = 1 even

or

-1 odd.

① $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

② $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

3a 3b $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$

det(A+B)

= det A + det B

2 equal rows → det = 0.

exchange those rows → same matrix.

Subtract $l \times$ row l from row k.

det doesn't change.

→ $\begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix} = 0.$

Row of zeros → det A = 0.

$5 \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} = \begin{vmatrix} 5\cdot 0 & 5\cdot 0 \\ c & d \end{vmatrix}$

$$\det U = \begin{vmatrix} d_1 & * & * & * \\ 0 & d_2 & * & * \\ 0 & 0 & d_3 & * \\ 0 & 0 & 0 & \ddots \end{vmatrix}$$

$= (d_1)(d_2)(d_3)\cdots(d_n)$

product of pivots.

$\det A = 0$

when A is singular

$\det A \neq 0$

when A is invertible

$U \to D \to d_1 d_2 d \ldots n$

$$\begin{matrix} a & b \\ c & d \end{matrix} \to \begin{matrix} a & b \\ 0 & d - \frac{c}{a} b \end{matrix} \qquad ad - bc$$

⑨ $\det AB = (\det A)(\det B)$

$\det A^{-1}$ $\qquad (\det A^{-1})(\det A)$

$A^{-1} A = I$

$\det A^{2} = \det(A)^{2} = 1.$

$\det 2A = 2^{n} \det A$

$\det A^t = \det A$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$