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Signal-to-Noise Ratio:
A Calculation from First Principle Applied to NMR in the Earth's Field

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NMR in the Earth's field is used as an example for calculating the signal-to-noise ratio from first principle. A simple loop is used as a transmitter and a receiver coil. Parameters significant in a signal-to-noise calculation are derived for this application. The influence of the receiving electronics is included in the calculations. Example spectra of NMR signals obtained in the Earth's field are given.

INTRODUCTION

Although NMR in the Earth's field has been demonstrated frequently in the past, few practical applications have been developed for low-field NMR spectroscopy or imaging. Earth's field NMR has been used in oil exploration since the mid-1960s, but the topic has received little attention in the literature and not much research has been done in this field. One reason for our work was the desire to obtain information about underground objects. If it is possible to obtain a signal from a substance that is removed to one side of the receiver coil, potential environmental hazards such as underground oil may be detected. Because NMR is an inherently insensitive method, the feasibility of a useful application relying only on the Earth's field will depend heavily on the ability to obtain a sufficiently high signal-to-noise ratio (S/N). To determine whether this is possible, we will outline how such a calculation can be performed from first principles.

In a typical NMR experiment, two magnetic fields are applied to a system: the main field B_0 and the perpendicular field B_1. In addition, the frequency of absorbed or emitted radiation corresponding to the transitions between magnetic moment energy states of the nuclei is observed. In conventional NMR, B_1 (the rotating magnetic field) is much smaller than the main field B_0, and the trend has been toward increasingly large main fields (useful B_0 fields for NMR work have now reached 15 Tesla) (*1-5*). In fact, much of the discussion of NMR — particularly analysis methods — is based on the assumption that $B_1 << B_0$. This assumption is not true for NMR phenomena in the Earth's magnetic field. Therefore, to apply NMR techniques, using the magnetic field of the Earth (approximately 50 μT) for the main field B_0, the "normal" methods

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must be reexamined (*6*). The most systematic investigation of NMR in the Earth's magnetic field was performed by Bene's group at the Jussy station in Geneva, Switzerland (*7*).

In the following section we will describe the experimental setup and conditions used in this work. Starting with a simple circular coil consisting of only one loop, we show that the extension to a coil consisting of N loops can be made by use of an ideal transformer with a ratio of $G = N$, where G is the transformation ratio of an ideal transformer at the input to the amplifier. We then look at the possibilities of connecting the receiver coil to an amplifier, designated as case A and case B. In case A, the receiver coil is directly coupled to the amplifier; thus, it is easy to remove the energy in the coil that is needed for excitation of the spin system. This allows for a shorter waiting period before acquisition can take place. However, the noise of the amplifier introduces additional noise in the coil. Unlike case B, this reactance is not compensated for by a capacitor. The amount of S/N lost will be looked at later.

In case B, the receiver coil is part of a resonance transformation at the input of the amplifier. Because any noise influence from the amplifier is compensated for by a capacitor, no amplifier noise will induce additional noise in the receiver coil. Thus the expected S/N is higher than in case A.

The resonance transformation acts as an ideal transformer with ratio $G = -iQ_1$, where Q_1 is the quality factor of the coil. Combined with the number of loops N, which also can be considered as a transformation factor, we obtain a total transformation factor of $G = -iQ_1N$. This shows that an optimal matching to the amplifiers depends not only on the number of loops N but also on the quality factor of the coil Q_1. The advantage of this method is that the coil requires fewer loops, the coil is less resistive, and the voltage necessary for excitation is smaller if a rather high Q_1 is chosen. The disadvantage of a high Q_1, however, is that more time is required for the coil to dissipate the energy supplied during excitation and a larger part of the free induction decay (FID) is therefore lost during acquisition.

In a subsequent section, we calculate the S/N at the input to the amplifier for the cases A and B. The equations show that an optimal transformation ratio $G = G_{opt}$ yields a maximum S/N. Finally, we include the influence of the Fourier transformation on S/N. Starting with a fixed measurement time T_{me}, we optimize the acquisition time T and then the repetition time T_{re}. The result is summarized in Eqs. [58] and [63] for cases A and B, respectively.

$$(\mathrm{S/N})_{ac} = G_{coil} \cdot F_{coil} \cdot \Psi_{max} \cdot \frac{B_p}{B_0} \cdot V \cdot \sqrt{T_{me} \cdot (T_2^*/T_1)} \qquad [1]$$

THE INDUCED SIGNAL

To calculate the induced signal, we assume the following:

- The free precession takes place in the Earth's field ($\vec{B}_0$).
- The receiver coil is oriented perpendicular to $\vec{B}_0$ and is used as a transmitter as well as a receiver coil (Fig. 1).
- The polarization field $\vec{B}_p$ is produced by a current I_p in the receiver coil. This field builds up the magnetization vector $\vec{M}$, and the field should be present long enough to create a maximum magnetic moment of the spins. This maximum is usually accomplished after a period of five T_1's has elapsed.
- After the polarization current I_p is turned off, the polarization field in the receiver coil decays.

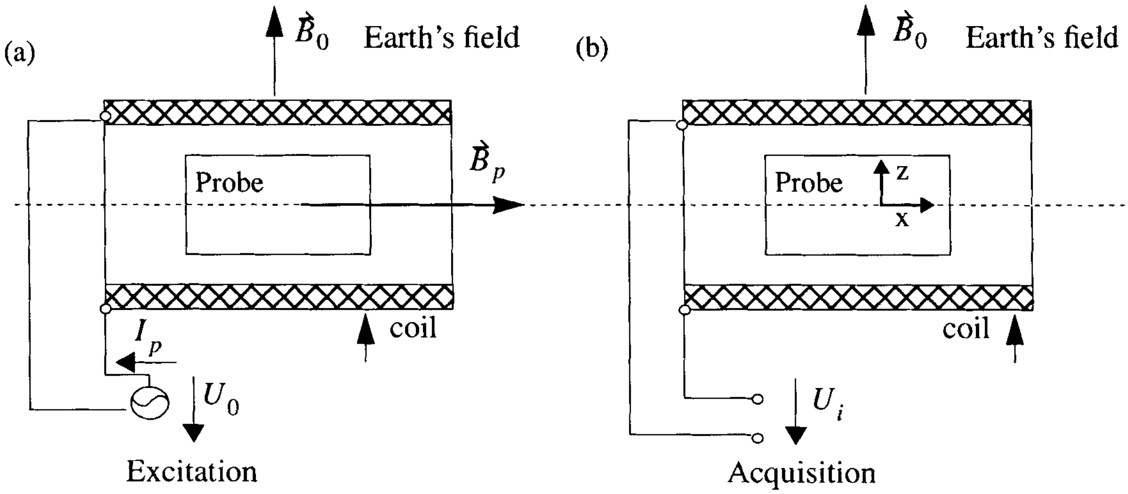

Figure 1. Geometry of acquisition and receiving experimental setup.

The Induced Potential U_i

After the spins have been polarized, the polarization field B_p is turned off. The spins continue to precess freely about the remaining field B_0. A magnetization vector **M** exists because of the influence of the polarization field. Starting from the ratio of the number of spins aligned antiparallel and parallel, one can calculate the resulting magnetic vector **M** (*8*). Because of the spins within the volume element under observation, the resulting dipole moment μ_{tot} will induce a potential in the receiver coil (*9*). In our notation this is given by

$$U_i = \gamma \cdot B_0 M_0 \cdot \left\{\frac{B_1}{I_1}\right\} V \cdot e^{-(t/T_2^*)} \sin(\omega_0 t) \tag{2}$$

For protons in water at room temperature, this becomes

$$U_i = 8.6 \cdot 10^{-13} B_0 \cdot B_p \cdot \left\{\frac{B_1}{I_1}\right\} V \cdot e^{-(t/T_2^*)} \sin(\omega_0 t) \tag{3}$$

S/N

The S/N can be calculated by first looking at the noise component and the NMR signal component separately and then forming the ratio. Numerous excellent articles have been published; a more in-depth treatment can be found in References 10-16.

RECEIVING SYSTEM S/N

Single Loop Coil

As stated above, the induced potential U_i at the beginning of the FID is given by Eq. [3], where B_0 is the induction of field during free precession — here, induction of the Earth's field (Gauss); B_p is the induction of the polarizing field (Gauss); (B_1/I_1) is the coil parameter (Gauss/A); and V is the volume of probe. Any influence due to the geometry and number of loops of the receiver coil is included in the term (B_1/I_1). I_1 is the current that induces the field B_1 at the location of the probe with volume V.

We first consider a circular coil with a single loop and calculate the ratio (B_1/I_1). (The influence of N loops will be treated later.) The geometry is illustrated in Fig. 2, where a coil is positioned flat onto the xy plane and carries a current I_1. The shaded region represents the magnetic field produced by the current in the coil, and the volume of interest is given by V.

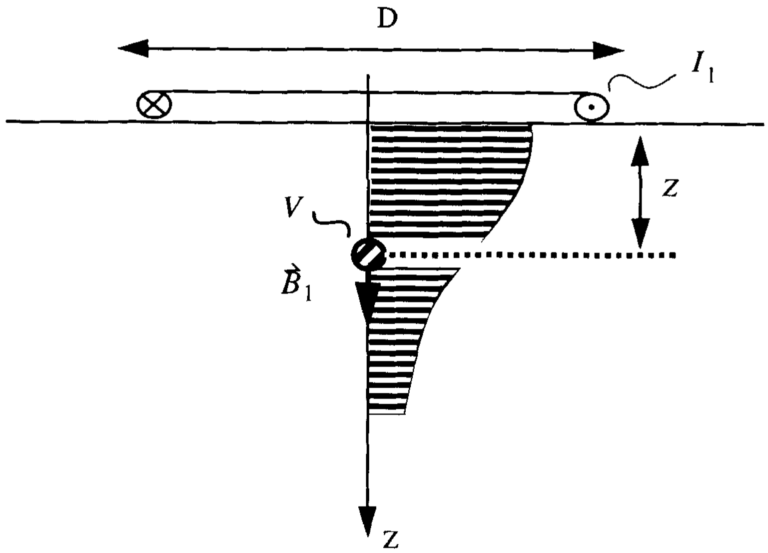

Figure 2. Field of receiver coil for the Earth's field NMR.

$$B_1 = \mu_0 \cdot \frac{I_1}{D} \cdot \frac{1}{\left[1 + \left(\frac{2z}{D}\right)^2\right]^{3/2}} \qquad [4]$$

For simplicity, we consider only the field on (or near) the z axis. In Eq. [4] $\mu_0 = (4\pi/10)(\text{Gauss} \cdot \text{cm/A})$. Defining $F_{\text{coil}} = 1/[1 + (2z/D)^2]^{3/2}$ as the field equation of the coil, we have

$$\left(\frac{B_1}{I_1}\right) = \frac{\mu_0}{D} \cdot F_{\text{coil}} \qquad [5]$$

therefore

$$U_1 = k_3 \cdot \mu_0 \frac{B_0 B_p V}{D} F_{\text{coil}} = k_3 \mu_0 B_0^2 \cdot \frac{B_p}{B_0} \frac{V}{D} F_{\text{coil}} \qquad [6]$$

where $k_3 = -8.6 \cdot 10^{-13}\,\text{VA/(Gauss} \cdot \text{cm)}^3$.

For B_0 we substitute the value of the Earth's field 0.5 Gauss to get

$$U_1 = k_4 \cdot \frac{B_p}{B_0} \frac{V}{D} F_{\text{coil}} \qquad [7]$$

where $k_4 = -2.702 \cdot 10^{-13}\,\text{V/cm}^2$.

The Circuit Equivalent of the Receiver Coil

We wish to consider a receiver with only one loop and geometric parameters D and d, as shown in Fig. 3.

Note that U is used for potentials caused by spins precessing in a volume V, whereas when an electrical engineering model is used to calculate quantities, u and u_1 are used to describe measurable quantities.

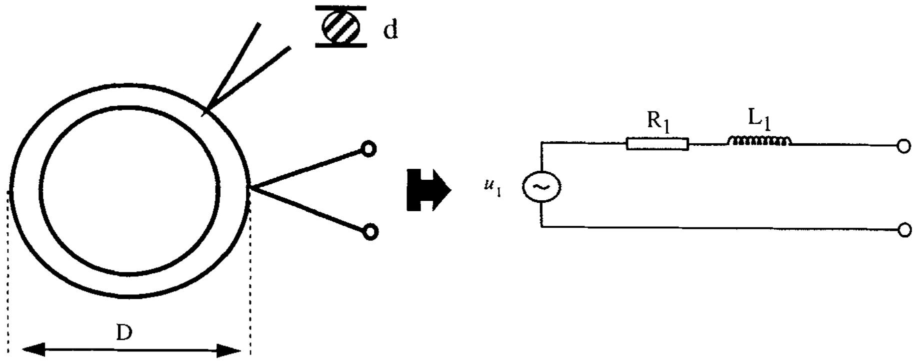

Figure 3. Electrical circuit equivalent of coil with dimensions d, D. u_1 is the induced NMR signal (only one loop), and R_1 and L_1 are the resistance and inductance, respectively, of a coil with only one loop.

On the basis of this coil, we define a new coil (Fig. 4) with the same geometric properties; however, the diameter d is due not to a single loop but to N loops.

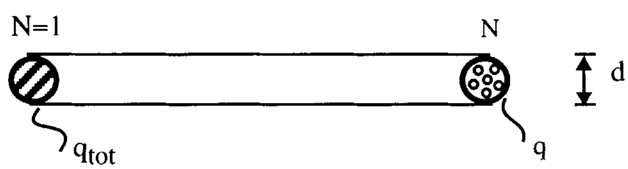

Figure 4. Coil with N loops.

In Fig. 4, N is the number of loops, q_{tot} is the total area of the coil, and q is the area of a single loop, $q = q_{tot}/N = (d^2\pi/4)(1/N)$. This is an approximation for a coil in which we have ignored any "empty" space on the right side of Fig. 4.

The resistance R_N of a coil with N loops is

$$R_N = \rho_{Cu}\frac{Nl}{q} = \rho_{Cu}\cdot\frac{ND\pi}{q_{tot}/N} = N^2\left[\rho_{Cu}\frac{D\pi}{d^2\pi/4}\right] = N^2R_1 \qquad [8]$$

ρ_{Cu} is the specific resistivity of copper and has dimensions of [Ω][cm], and 1 is the length of a loop of wire given in [cm].

The inductance L_N of a coil with N loops is

$$L_N = N^2L_1 \qquad [9]$$

and the induced potential u_N of a coil with N loops is

$$u_N = N\cdot u_1 \qquad [10]$$

with the circuit equivalent shown in Fig. 5.

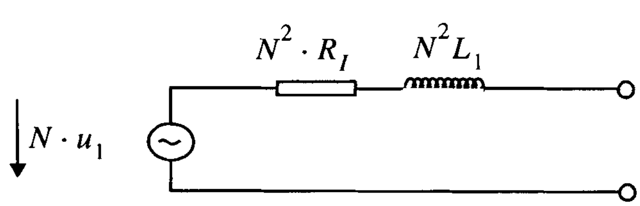

Figure 5. Circuit equivalent of coil with N loops.

This illustrates that the influence of N loops can be replaced by an ideal transformer with a ratio of N. Figure 6 shows the final circuit equivalent.

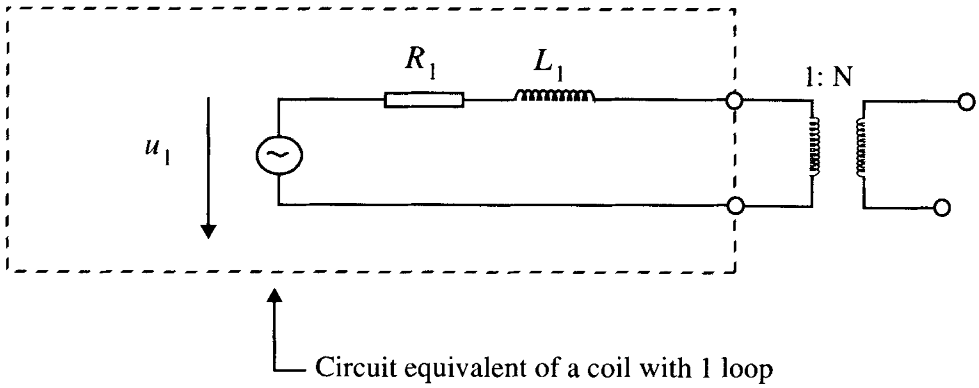

Figure 6. Circuit equivalent of a coil with N loops described by an ideal transformer.

Receiver Coil and Amplifier

The induction of the receiver coil shall not be matched by the use of capacitors. This simplifies elimination of the energy stored in the coil after excitation. To minimize losses of the received signal, it is useful to choose a high input resistance to the preamplifier (e.g., the positive input of a reverse coupled operational amplifier). The equivalent electrical circuit at the input of the amplifier is shown in Fig. 7.

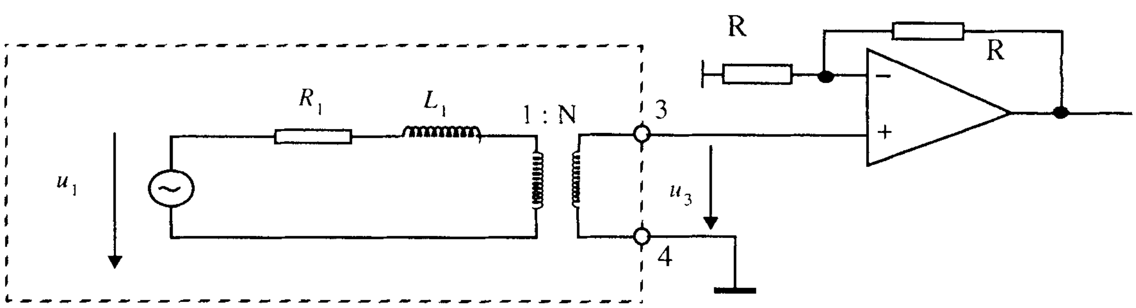

Figure 7. Circuit equivalent at the input to the amplifier.

The receiver coil can be matched to the amplifier by using a transformer. This guarantees the highest possible S/N because losses from capacitive matching do not appear. The resonance transformation is accomplished by using C_1, as shown in Fig. 8.

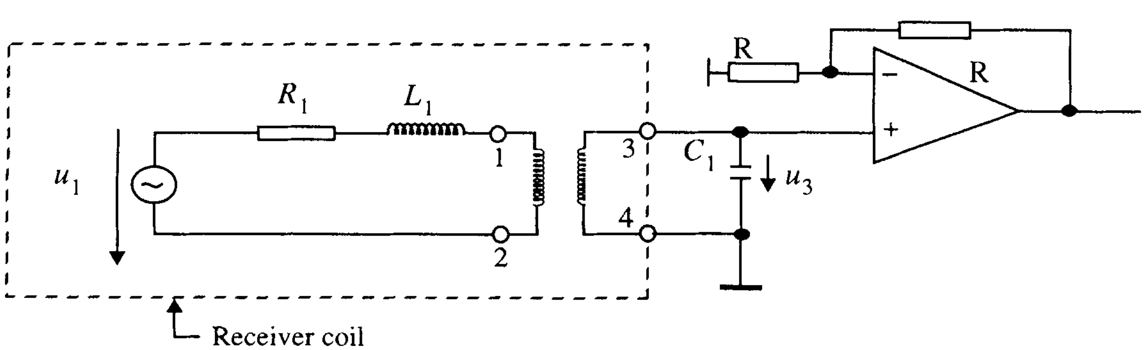

Figure 8. Receiver coil matched to amplifier by a transformer.

The left side of the ideal transformer can be replaced by Fig. 9.

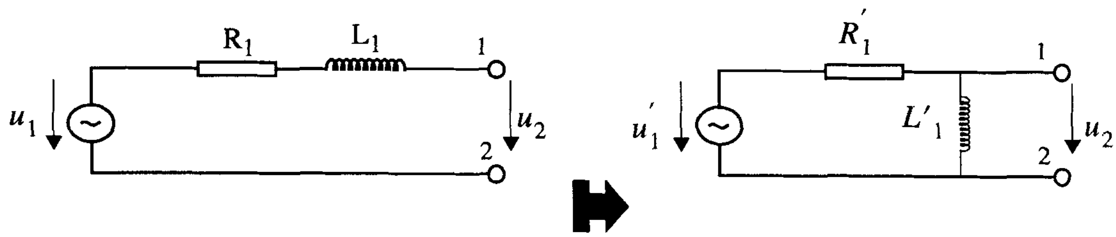

Figure 9. Electrical equivalent of receiver coil from Fig. 8.

These two circuits are equal only if the impedances and the potentials are the same. The equality of source impedance is

$$R_1 + i\omega L_1 = \frac{R_1' \cdot i\omega L_1'}{R_1' + i\omega L_1'}$$

which can be written as:

$$R_1 R_1' + i\omega L_1' R_1 + i\omega L_1 R_1' - \omega^2 L_1 L_1' = i\omega L_1' R_1' \qquad [11]$$

This gives two equations for the real and imaginary parts of Eq. [11].

$$R_1 R_1' - \omega^2 L_1 L_1' = 0 \qquad [12]$$

and

$$\omega L_1' R_1 + \omega L_1 R_1' = \omega L_1' R_1' \qquad [13]$$

From Eq. [13] we have

$$\frac{R_1'}{\omega L_1'} = \frac{\omega L_1}{R_1} = Q(\omega) \qquad [14]$$

$Q(\omega)$ is the quality factor of a coil at the frequency ω. This factor is usually >50, so $Q^2(\omega) >> 1$ is valid.

For $\omega L_1'$, it is true that

$$\omega L_1' = \frac{R_1'}{Q(\omega)} \qquad [15]$$

Substituting Eq. [15] into Eq. [13] gives

$$\frac{1}{Q(\omega)} R_1' R_1 + Q(\omega) R_1' R_1 = \frac{1}{Q(\omega)} R_1'^2 \qquad [16]$$

so that

$$R_1' = R_1 Q^2(\omega) \left[1 + \frac{1}{Q(\omega)^2} \right] \qquad [17]$$

From Eq. [12] we also have

$$\omega L_1' = \frac{R_1 R_1'}{\omega L_1} = \omega L_1 \cdot \left[\frac{R_1}{\omega L_1} \right]^2 \frac{R_1'}{R_1} \qquad [18]$$

with the result that

$$L_1' = L_1\left[1 + \frac{1}{Q(\omega)^2}\right] \qquad [19]$$

The equality of u_1 is

$$u_1 = u_1' \cdot \frac{i\omega L_1'}{R_1' + i\omega L_1'} = u_1' \frac{1}{1 + (R_1'/i\omega L_1')} \qquad [20]$$

Substituting Eq. [15] yields

$$u_1' = u_1[-iQ(\omega)]\sqrt{1 + \frac{1}{Q(\omega)^2}} \cdot \exp\{i \cdot \arctan[1/Q(\omega)]\} \qquad [21]$$

For $\omega = \omega_0$ and $Q^2(\omega) >> 1$, we finally obtain

$$Q(\omega) = \frac{\omega_0 L_1}{R_1} = Q_1 \qquad [22]$$

$$u_1' = u_1 \cdot e^{i/Q_1} \cdot (-iQ_1) \qquad [23]$$

$$R_1' = R_1 Q_1^2 = R_1(-iQ_1)^2 \qquad [24]$$

and

$$L_1' = L_1 \qquad [25]$$

From these calculations we have the following circuit equivalents (Fig. 10):

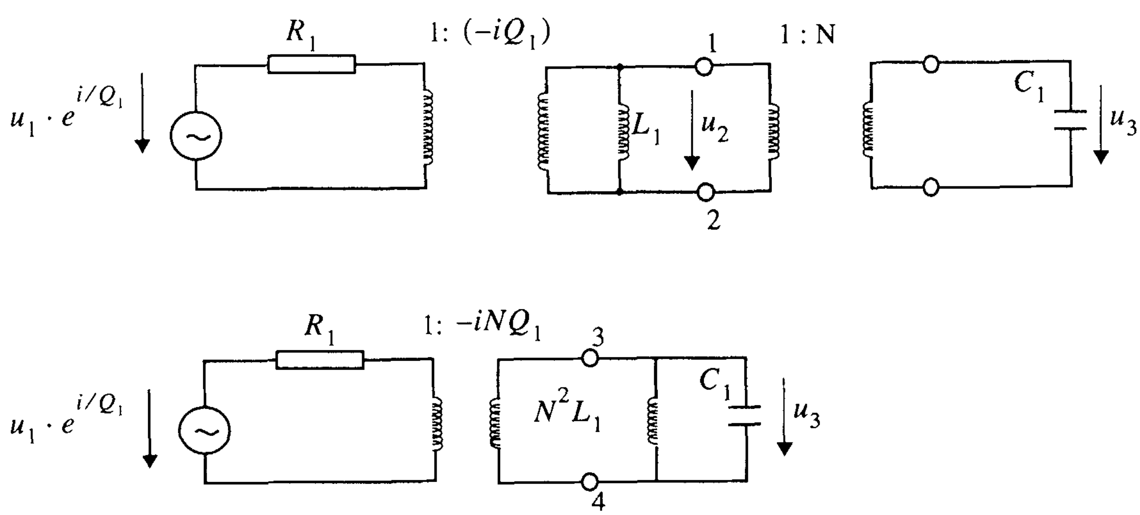

Figure 10. Resulting circuit equivalents. $\omega = \omega_0$; $Q_1 = \omega_0 L_1/R_1$; $Q_1^2 >> 1$.

The resonance transformer acts as an ideal transformer with imaginary ratio $(-iQ_1)$. The ratio is imaginary because the potential u_3 is across the capacitance C_1, which rotates the phase by -90°. To achieve an optimal resonance transformation, the induction N^2L_1 must be in resonance with C_1 at $\omega = \omega_0$. This results in an infinite impedance for these two elements (at $\omega = \omega_0$) and therefore vanishes.

The factor $\exp(-i/Q_1)$ rotates the phase of the input signal u_1 by a small amount $1/Q_1$ radian. This has no significance and can be ignored. The final circuit equivalent at the input of the amplifier is therefore simplified, as shown in Fig. 11.

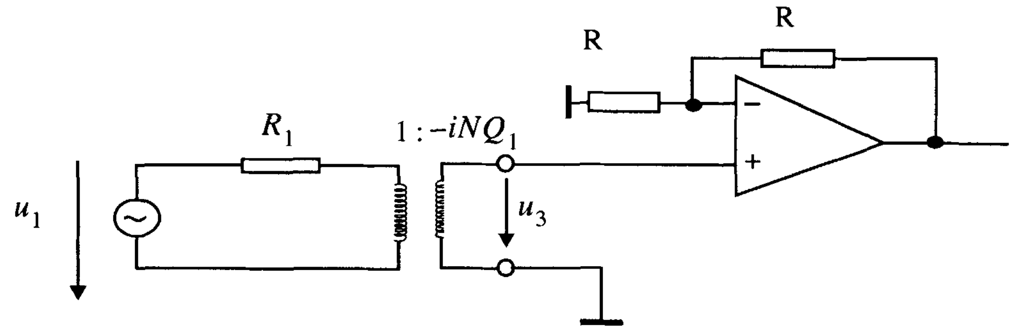

Figure 11. Final simplified circuit equivalent at the input to the amplifier. $\omega = \omega_0$; $Q_1 = \omega_0 L_1/R_1$; $Q_1^2 \gg 1$; $\omega_0^2 N^2 L_1 C_1 = 1$.

OPTIMIZING THE S/N

Four main sources of noise influence the NMR signal: electrical equipment, the resistivity of the receiver coil, the applied current of the amplifier, and the potential of the amplifier. Electrical equipment such as main power lines, machinery, and electrical devices disturb from the outside and induce mainly 50- or 60-Hz noise and harmonics. They can be partly compensated for by using a second receiver coil parallel to the first. This coil is connected to compensate the induced noise in the receiver coil (mainly far-field effects) and must be sufficiently removed to prevent the desired NMR signal from diminishing. Even though a second coil will provide additional noise because of its own resistance, practical experiments have shown that the advantages outweigh the additional noise.

Some noise is due to the resistivity of the receiver coil ($R_1 = \sqrt{\overline{u_{R_1}^2}}$).

Some noise is due to the applied current $\sqrt{\overline{i_r^2}}$ of the amplifier. This noise is produced across the source impedance at the input of the amplifier. Finally, some noise is due to the potential $\sqrt{\overline{u_r^2}}$ of the amplifier.

Aside from using a compensation coil, we cannot control noise from electrical equipment. Therefore, we wish to concentrate on the other points.

Figure 12 illustrates the electrical equivalent for the input to the amplifier, which shows all relevant cases of interest.

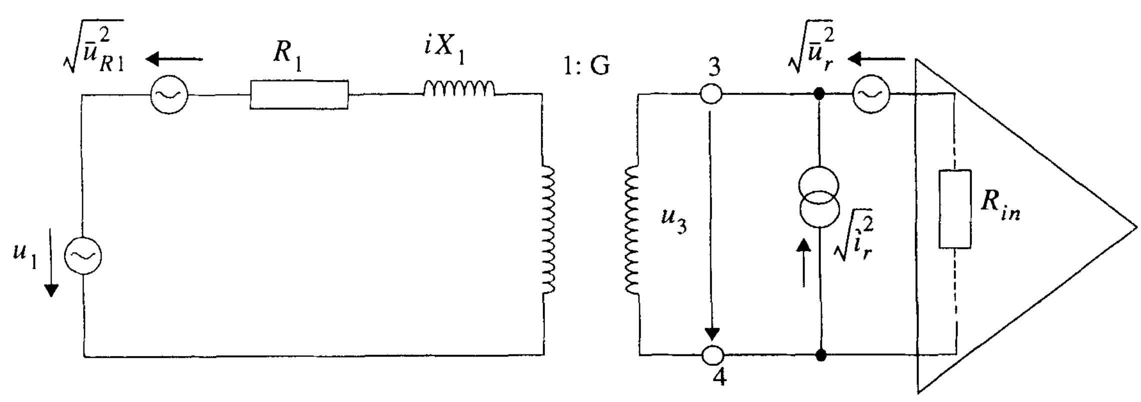

Figure 12. Circuit equivalent for all relevant cases of interest.

$G = 1$ is the ratio of an ideal transformer at the input to the preamplifier, where

$$\sqrt{\bar{u}_{R_1}^2} = \sqrt{4kT_{R_1}\Delta f \cdot R_1} \qquad [26]$$

We define the ratio $\sqrt{\bar{u}_r^2}/\sqrt{\bar{i}_r^2}$ as the resistance R_{opt}. Thus for $\sqrt{\bar{i}^2}$ we have

$$\sqrt{\bar{i}_r^2} = \frac{\sqrt{\bar{u}_r^2}}{R_{\mathrm{opt}}} \qquad [27]$$

Calculation of the total noise potential at the input of the amplifier gives

$$\bar{u}_{r3}^2 = \left[\left|\frac{R_{\mathrm{in}} \cdot G^2(R_1 + iX_1)}{R_{\mathrm{in}} + G^2(R_1 + iX_1)}\right|\right]^2 \frac{\bar{u}_r^2}{R_{\mathrm{opt}}^2} + \left[\left|\frac{R_{\mathrm{in}}}{R_{\mathrm{in}} + G^2(R_1 + iX_1)}\right|\right]^2 (\bar{u}_r^2 + G^2 \cdot \bar{u}_{R_1}^2) \qquad [28]$$

$$\bar{u}_{r3}^2 = \left|\frac{R_{\mathrm{in}}}{R_{\mathrm{in}} + G^2(R_1 + iX_1)}\right|^2 \left[\left|\frac{G^2(R_1 + iX_1)}{R_{\mathrm{opt}}}\right|^2 \cdot \bar{u}_r^2 + \bar{u}_r^2 + G^2 \cdot \bar{u}_{R_1}^2\right] \qquad [29]$$

Calculation of the potential u_3 at the input to the amplifier gives

$$u_3 = \frac{R_{\mathrm{in}}}{R_{\mathrm{in}} + G^2(R_1 + iX_1)} \cdot G \cdot u_1 \qquad [30]$$

Calculation of the S/N gives

$$\mathrm{S/N} = \frac{|u_3|}{\sqrt{\bar{u}_{r3}^2}} = \frac{G \cdot u_1}{\sqrt{\left[1 + \left(\frac{G^2|R_1 + iX_1|}{R_{\mathrm{opt}}}\right)^2\right] \cdot \bar{u}_r^2 + G^2 \cdot \bar{u}_{R_1}^2}} \qquad [31]$$

$$= \frac{|u_1|}{\left(\left[\frac{R_{\mathrm{opt}}}{G^2\sqrt{R_1^2 + X_1^2}} + \frac{G^2\sqrt{R_1^2 + X_1^2}}{R_{\mathrm{opt}}}\right] \frac{\sqrt{R_1^2 + X_1^2}}{R_{\mathrm{opt}}}\, \bar{u}_r^2 + \bar{u}_{R_1}^2\right)^{1/2}} \qquad [32]$$

$$= \frac{|u_1|}{\left(\eta\sqrt{R_1^2 + X_1^2}\, \frac{\bar{u}_r^2}{R_{\mathrm{opt}}} + \bar{u}_{R_1}^2\right)^{1/2}} \qquad [33]$$

where

$$\eta = \frac{R_{\mathrm{opt}}}{G^2(R_1^2 + X_1^2)^{1/2}} + \frac{G^2\sqrt{(R_1^2 + X_1^2)}}{R_{\mathrm{opt}}} \qquad [34]$$

With $\bar{u}_{R_1}^2 = 4kT_{R_1}\Delta f \cdot R_1$, for the S/N we have

$$\mathrm{S/N} = \frac{|u_1|/\sqrt{4kT_{R_1}\Delta f R_1}}{\sqrt{1 + \eta\sqrt{1 + \left(\frac{x_1}{R_1}\right)^2}\, \frac{\bar{u}_r^2/4R_{\mathrm{opt}}}{kT_{R_1}\Delta f}}} \qquad [35]$$

Equation [35] gives the S/N, which has a maximum value when

$$\frac{R_{opt}}{G^2\sqrt{R_1^2 + X_1^2}} = 1 \tag{36}$$

Equation [36] can be interpreted as follows:

$$G_{opt} = \sqrt{\frac{R_{opt}}{\sqrt{R_1 + X_1^2}}} \tag{37}$$

The absolute value of the source impedance, transformed to the input of the amplifier, must equal the optimum source resistance R_{opt} of the amplifier.

The parameter η can be written as

$$\eta = \frac{G_{opt}^2}{G^2} + \frac{G^2}{G_{opt}^2} \tag{38}$$

For optimal conditions $G = G_{opt}$ we have

$$\eta = \eta_{opt} = 2 \tag{39}$$

$$(\mathrm{S/N})_{max} = \frac{|u_1 X| / \sqrt{4kT_{R_1}\Delta f R_1}}{\left\{1 + 2\sqrt{1 + \left[\frac{X_1}{R_1}\right]^2 \frac{\overline{u_r^2}/4R_{opt}}{kT_{R_1}\Delta f}}\right\}^{1/2}} \tag{40}$$

A further improvement in S/N is feasible if the following conditions are met.

- The reactance of X_1, the source impedance, should be compensated to 0. If X_1 is the induction of the receiver coil, the compensation can be accomplished by adding a capacitance. We are then dealing with a resonant circuit that is in resonance at the wanted frequency.
- The available noise power $\overline{u_r^2}/4R_{opt}$ of the amplifier should be small relative to the available noise power $kT_{R_1}\Delta f$ of the source resistance R_1.
- The S/N $= |u_1| / \sqrt{4 \cdot k \cdot T_{R_1} \cdot \Delta f \cdot R_1}$ of the source should be as large as possible. Unfortunately, not a lot can be done to accomplish this. One possibility is to lower the temperature of the receiver coil and therefore lower the source resistance.

Equations [35] through [40] can be applied to the case treated earlier in this section (Case A).

CASE A

Receiver Coil Directly Coupled to Amplifier, without Resonance Transformation

Figure 7 becomes Fig. 12 with the substitutions $X_1 = \omega_0 L_1$ and $G = N$ applied to Fig. 7. It is also true that $(\omega_0 L_1)^2 >> R_1^2$. Substituting these relations in Eqs. [35] through [40] we get

$$\mathrm{S/N} = \frac{|u_1| / \sqrt{4kT_{R_1}\Delta f \cdot R_1}}{\sqrt{1 + \eta \frac{\omega_0 L_1}{R_1} \frac{\overline{u_r^2}/4R_{opt}}{kT_{R_1}\Delta f}}} \tag{41}$$

$$N_{opt} = \sqrt{R_{opt}/\omega_0 L_1} \qquad [42]$$

$$(S/N)_{max} = \frac{|u_1|/\sqrt{4kT_{R_1}\Delta f \cdot R_1}}{\sqrt{1 + 2\,\dfrac{\omega_0 L_1}{R_1}\,\dfrac{u_r^{-2}/4R_{opt}}{kT_{R_1}\Delta f}}} \qquad [43]$$

CASE B

Receiver Coil as Part of a Resonance Transformer at the Input of the Amplifier

Figure 8 becomes Fig. 12 with the substitutions $X_1 = 0$ and $G = -iNQ_1 = -iN\omega_0 L_1/R_1$ applied to Fig. 8. Substituting these relations in Eqs. [35] through [40] we have

$$S/N = \frac{|u_1|/\sqrt{4kT_{R_1}\Delta f \cdot R_1}}{\sqrt{1 + \eta\,\dfrac{u_r^{-2}/4R_{opt}}{kT_{R_1}\Delta f}}} \qquad [44]$$

$$N_{opt} = \sqrt{R_{opt} R_1/\omega_0^2 L_1^2} \qquad [45]$$

$$(S/N)_{max} = \frac{|u_1|/\sqrt{4kT_{R_1}\Delta f \cdot R_1}}{\sqrt{1 + 2\,\dfrac{u_r^{-2}/4R_{opt}}{kT_{R_1}\Delta f}}} \qquad [46]$$

S/N after Fourier Transformation

The S/N after Fourier transformation is given by

$$(S/N)_\omega = (S/N)\sqrt{\frac{\Delta f \cdot T_2^*}{2}} \qquad [47]$$

This assumes an optimal exponential multiplication applied in the time domain.

When multiple FIDs are accumulated, it is useful to define a total measurement time T_{me}. Within this time we wish to accumulate n_{ac} FIDs. The repetition time T_{re} is in general five times larger than the spin-lattice relaxation time T_1 to allow the spins to relax to their equilibrium position. This means that the spin system has fully relaxed before the next excitation; however, there are long time periods between the end of one acquisition and the start of the next. If we do not insist on a fully relaxed spin system and accumulate more FIDs in a given time period, we will obtain a lower S/N per FID; however, the S/N within T_{me} will be better. This qualitative description can be calculated rigorously by looking at the exact angle of the spin system after excitation. Calculations resulting in the well-known Ernst angle have been published by R. Ernst (*13*, *14*). D. Traficante (*16*) has calculated and provided an excellent comparison for different excitation angles.

Figure 13 shows the timing of the relevant quantities. The factor β gives the magnitude of the magnetic vector in the direction of B_p at the beginning of the acquisition period T, relative to the maximum value M_p (direction of B_p). The S/N of a single FID is reduced by this factor β, but the total S/N, $(S/N)_{ac}$, after the accumulation will be increased by $\sqrt{n_{ac}}$.

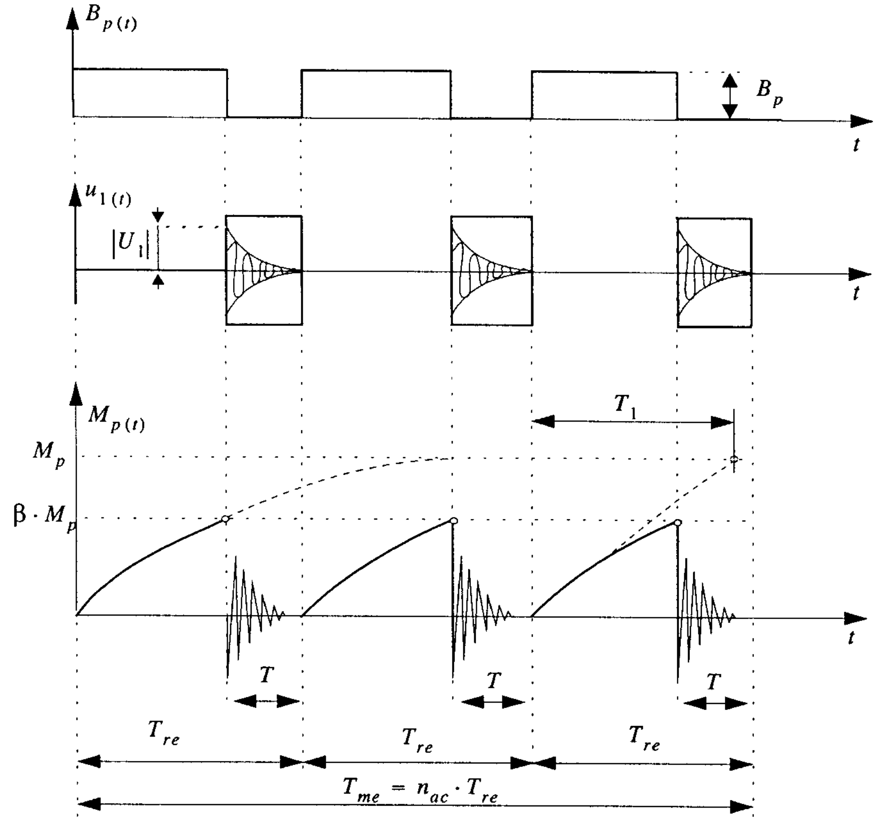

Figure 13. Time evolution of parameters during acquisition. $M_{p(t)}$ is the component of magnetic vector in direction B_p; T is the acquisition time; T_{re} is the repetition time of FID; and T_{me} is the total measurement time. T_1 is the point at which the initial slope intersects the asymptote.

$$(S/N)_{ac} = (S/N)_{\omega}\beta\sqrt{n_{ac}} = (S/N)_{\omega}\beta\sqrt{\frac{T_{me}}{T_{re}}} \qquad [48]$$

As Fig. 13 indicates

$$M_p(t) = M_p(1 - e^{-t/T_1}) \qquad [49]$$

for the interval $T_{re} - T$. In addition,

$$\beta M_p = M_p(1 - e^{-[T_{re} - T]/T_1}) \qquad [50]$$

with

$$\beta = 1 - (e^{T/T_1} \cdot e^{-T_{re}/T_1}) \qquad [51]$$

Substituting in Eq. [48] gives

$$(S/N)_{ac} = (S/N)_{\omega} \cdot \sqrt{\frac{T_{me}}{T_1}} \cdot k_5 \cdot \Psi \qquad [52]$$

where

$$\Psi = \frac{1 - (e^{T/T_1} \cdot e^{-T_{re}/T_1})}{k_5\sqrt{T_{re}/T_1}} \qquad [53]$$

and $k_5 = 0.638173$.

For every value of T/T_1 there exists an optimal value $(T_{re}/T_1)_{opt}$ for which Ψ has a maximum value Ψ_{max}. The normalizing factor k_5 in Eq. [53] is chosen such that for small values of T/T_1, $\Psi_{max} = 1$.

We now substitute Eq. [47] into Eq. [53] and choose the maximum value for Ψ:

$$(S/N)_{ac} = (S/N) \cdot k_5 \cdot \Psi_{max} \sqrt{\frac{\Delta f \cdot T_{me}(T_2^*/T_1)}{2}} \qquad [54]$$

Ψ_{max} is chosen according to Fig. 14.

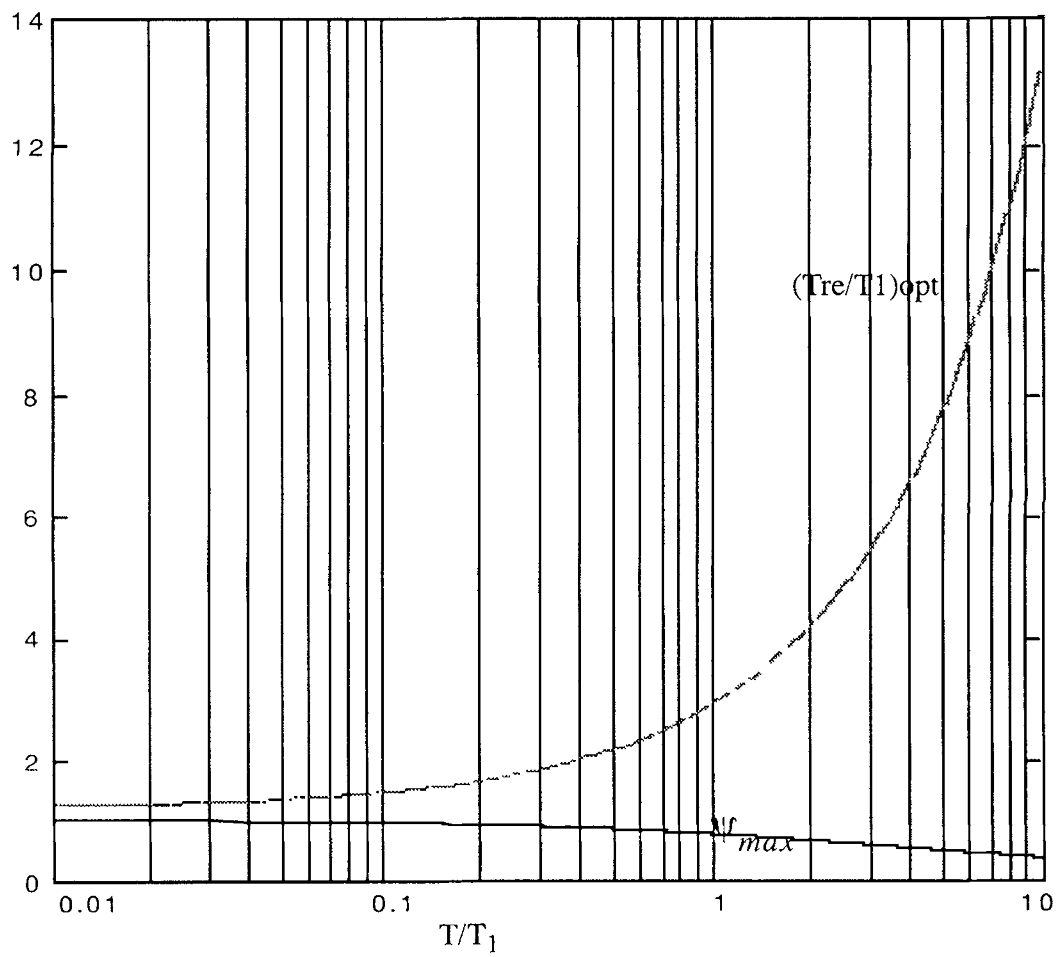

Figure 14. Ψ_{max} as a function of T/T_1.

We next consider the consequences for cases A and B discussed above.

CASE A

For (S/N) of Eq. [54] we use Eq. [35], and u_1 is given by Eq. [7]. The result is

$$(S/N)_{ac} = \frac{k_4 \dfrac{B_p}{B_0} \dfrac{V}{D} F_{sp} / \sqrt{4kT_{R_1}\Delta f R_1}}{\sqrt{1 + \eta \dfrac{\omega_0 L_1}{R_1} \dfrac{u_r^{-2}/4R_{opt}}{k \cdot T_{R_1} \cdot \Delta f}}} \cdot k_5 \cdot \Psi_{max} \cdot \frac{\sqrt{\Delta f T_{me}(T_2^*/T_1)}}{2} \qquad [55]$$

$$(\mathrm{S/N})_{\mathrm{ac}} = \frac{k_4 k_5 \Psi_{\max} F_{\mathrm{coil}} (B_p/B_0) V \sqrt{T_{\mathrm{me}} \cdot (T_2^*/T_1)}}{\sqrt{8D^2 \left[k T_{R_1} R_1 + \eta \omega_0 L_1 \cdot \dfrac{u_r^{-2}}{4 R_{\mathrm{opt}} \cdot \Delta f} \right]}} \qquad [56]$$

$T_{\mathrm{EM}} = T_2^*$, $T \geq 3 \cdot T_2^*$, $T_{\mathrm{re}} = (T_{\mathrm{re}}/T_1)_{\mathrm{opt}} \cdot T_1$, $B_0 = 0.5$ Gauss, $\omega_0 = 2\pi \cdot 2129$ Hz, and η is defined by Eq. [38] with $G = N$ (corresponding to case A):

$$\eta = \left[\frac{N}{N_{\mathrm{opt}}} \right]^2 + \left[\frac{N_{\mathrm{opt}}}{N} \right]^2 \qquad [57]$$

where $N_{\mathrm{opt}} = \sqrt{R_{\mathrm{opt}}/\omega_0 \cdot L_1}$ (Eq. [42]).

We now define the following parameters: $\sqrt{u_r^{-2}}/\sqrt{\Delta f} = 9 \cdot 10^{-10} V \cdot s^{1/2}$ (from amplifier data specs), $R_{\mathrm{opt}} = 900\Omega$ (from amplifier data specs), $k = 1.380 \cdot 10^{-23}$ Ws/K (Boltzmann constant), and $T_{R_1} = 300\ K$ (room temperature). For R_1 and L_1 we use the values for a circular coil with one loop: $R_1 = 6.68 \cdot 10^{-6} \cdot (D/d^2) \Omega \cdot \mathrm{cm}$ and $L_1 = 6.28 \cdot 10^{-9} \cdot D \cdot F(D/d) H \cdot \mathrm{cm}^{-1}$ with $F(D/d) = [2.303 \cdot \log_{10}(8D/d)] - 1.75$. In addition, the constants k_4, k_5 are $k_4 = -2.702 \cdot 10^{-13} V \cdot \mathrm{cm}^{-2}$ and $k_5 = 0.638173$. Substituting all these values in Eq. [56] gives the result

$$(\mathrm{S/N})_{\mathrm{ac}} = G_{\mathrm{coil}} \cdot F_{\mathrm{coil}} \cdot \Psi_{\max} \frac{B_p}{B_0} \cdot V \cdot \sqrt{T_{\mathrm{me}} \cdot (T_2^*/T_1)} \qquad [58]$$

G_{coil} is the geometric factor of coil and F_{coil} is the field function of coil. Figure 14 shows $\Psi_{\max}$.

The following conditions need to be met: $T_{\mathrm{EM}} = T_2^*$, $T \geq 3 \cdot T_2^*$, $T_{\mathrm{re}} = T_1 \cdot (T_{\mathrm{re}}/T_1)_{\mathrm{opt}}$ (see Fig. 14); $B_0 = 0.5$ Gauss; $\omega_0 = 2\pi \cdot 2129$ Hz; and the coil is circular, defined by D, d, and N.

The geometric factor is defined by

$$G_{\mathrm{coil}} = \frac{0.3666}{\sqrt{D \cdot \left[\dfrac{D}{d} \right]^2 + \eta \cdot 0.6835 \cdot D^3 \left[2.303 \cdot \log_{10} \left[\dfrac{8D}{d} \right] - 1.75 \right]}} \qquad [59]$$

$G_{\mathrm{coil}} = [\mathrm{cm}^{-3} \cdot s^{-1/2}]$, $D = [\mathrm{cm}]$, η is given by Eq. [38], and $N_{\mathrm{opt}} = \sqrt{R_{\mathrm{opt}}/\omega_0 \cdot L_1}$.

The optimal number of loops N_{opt} of the coil is given by

$$N_{\mathrm{opt}} = \frac{3273}{\sqrt{D \left[2.303 \cdot \log_{10} \left[\dfrac{8D}{d} \right] - 1.75 \right]}} \qquad [60]$$

with D in cm.

CASE B

For the quantity S/N from Eq. [54] we use Eq. [44], where u_1 is given by Eq. [7].

$$(\mathrm{S/N})_{\mathrm{ac}} = \frac{k_4 \dfrac{B_p}{B_0} \dfrac{V}{D} F_{\mathrm{coil}} / \sqrt{4 k T_{R_1} \Delta f R_1}}{\sqrt{1 + \eta \dfrac{u_r^{-2}/4R_{\mathrm{opt}}}{k T_{R_1} \Delta f}}} k_5 \Psi_{\max} \sqrt{\frac{\Delta f T_{\mathrm{me}} (T_2^*/T_1)}{2}} \qquad [61]$$

$$(\mathrm{S/N})_{\mathrm{ac}} = \frac{k_4 k_5 \Psi_{\max} G_{\mathrm{coil}}(B_p/B_0) V \sqrt{T_{\mathrm{me}} \cdot (T_2^*/T_1)}}{\sqrt{8D^2 \left[kT_{R_1} \cdot R_1 + \eta \cdot R_1 \frac{u_r^{-2}}{4R_{\mathrm{opt}} \Delta f} \right]}} \quad [62]$$

$T_{\mathrm{EM}} = T_2^*$, $T \geq 3 \cdot T_2$, $T_{\mathrm{re}} = (T_{\mathrm{re}}/T_1)_{\mathrm{opt}} \cdot T_1$, (Fig. 14); $B_0 = 0.5$ Gauss, and $\omega_0 = 2\pi \cdot 2129$ Hz; η is given by Eq. [38], with $G = -i \cdot N \cdot \omega_0 \cdot L_1/R_1$ (case B above); and N_{opt} is given by Eq. [45].

If we substitute the parameters defined for case A into Eq. [62], we get

$$(\mathrm{S/N})_{\mathrm{ac}} = G_{\mathrm{coil}} \cdot F_{\mathrm{coil}} \cdot \Psi_{\max} \frac{B_p}{B_0} \cdot V \cdot \sqrt{T_{\mathrm{me}} \cdot (T_2^*/T_1)} \quad [63]$$

G_{coil} is the geometric factor of coil, F_{coil} is the field function of coil (defined below Eq. [4]), and $\Psi_{\max}$ is given in Fig. 14. The following conditions need to be met: $T_{\mathrm{EM}} = T_2^*$, $T \geq 3 \cdot T_2^*$, $T_{re} = T_l \cdot (T_{re}/T_1)_{\mathrm{opt}}$ (see Fig. 14); $B_0 = 0.5$ Gauss; $\omega_0 = 2\pi \cdot 2129$ Hz; and the coil is circular and is defined by D, d, and N.

The quality factor Q of the coil must not be too high, or the tuning of the resonance frequency in the Earth's field will be difficult. The tuning must be exact, and this is difficult to accomplish because of the varying nature of the Earth's magnetic field. Q should not be larger than 100. To satisfy these conditions we derived the two equations below for G_{coil}. Next to the first equation, which was derived parallel to case A, we derived a second equation where R_1 has been replaced by $(\omega_0 L_1)/Q$.

$$G_{\mathrm{coil}} = \frac{0.3666}{\sqrt{D \left[\frac{D}{d} \right]^2 (1 + 0.05435 \cdot \eta)}} \quad [64]$$

for case A and

$$G_{\mathrm{coil}} = \frac{0.1034}{\frac{D^3}{Q} \left[2.303 \log_{10} \left[\frac{8D}{d} \right] - 1.75 \right] (1 + 0.05435 \cdot \eta)} \quad [65]$$

referring to the case where R_1 is replaced by $\omega_0 L_1/Q$. In Eqs. [64] and [65], $G_{\mathrm{coil}} = [\mathrm{cm}^{-3} s^{-1/2}]$, $D = [\mathrm{cm}]$, η is given by Eq. [38], $N_{\mathrm{opt}} = \sqrt{R_{\mathrm{opt}} R_1/\omega_0^2 L_1^2} = \sqrt{R_{\mathrm{opt}}/Q\omega_0 L_1}$, and $Q = 100$.

If one wishes to calculate the geometry factor, Eq. [64] can be used as long as the values do not exceed those calculated from Eq. [65]. To have $Q \leq 100$, we impose the condition $R_1 \geq \omega_0 L_1/Q$. If R_1 is less than this value, a second resistor R_s must be added in series or a resistor R_p must be added in parallel to the coil. However, Q must be constant at 100.

We can also derive two equations for N_{opt}. The second satisfies the condition $Q = 100$. For both equations, the parameters defined earlier apply.

$$N_{\mathrm{opt}} = \sqrt{R_{\mathrm{opt}} R_1/\omega_0^2 L_1^2} = \sqrt{R_{\mathrm{opt}}/Q\omega_0 L_1} \quad [66]$$

$$N_{\mathrm{opt}} = \frac{923.0}{\sqrt{D^3}} \left[\frac{\frac{D}{d}}{2.303 \cdot \log_{10} \left[\frac{8D}{d} \right] - 1.75} \right] \quad [67]$$

$$N_{opt} = \frac{3273}{\sqrt{QD\left[2.303 \cdot \log_{10}\left(\frac{8D}{d}\right) - 1.75\right]}} \qquad [68]$$

where D = [cm] and Q = 100.

Again, Eq. [67] can be used as long as the values do not exceed the values given by [68], because Q is limited to 100.

Equation [63] shows that T_2^* shows up only in the square root. A fourfold reduction of the linewidth therefore only yields a twofold increase in S/N. The fact that S/N increases only with the square root of T_{me} is known. It is very effective, however, to increase the sample volume and the polarization field B_p. Both quantities contribute proportionally to S/N. Ψ_{max} is a factor depending on the optimal choice for the repetition time T_{re}. Values vary between 0.6 and 1.0 and therefore do not influence S/N much. The field function of the coil F_{coil} reduces S/N when the sample volume is removed from the center of the coil. In the center F_{coil} = 1.

At last we are left with a geometric factor G_{coil}. For the reasons mentioned previously, the maximum Q_1 of the resonance circuit must be limited to 100 for case B. The geometric factor depends mostly on the coil parameters d, D, and N. One can see that G_{coil} is also much more dependent on N/N_{opt} in case A than in case B. Therefore, the optimal number of loops must be adhered to much more precisely in case A than in case B.

CONCLUSION

We have already stated the advantages and disadvantages of method A versus method B, and we have shown that case B will result in a higher S/N than case A. This can clearly be seen by looking at the geometric factor G_{coil}, which is larger in case B. In Fig. 15, we have calculated the ratio of the geometric factors for the two cases, $k_{coil} = (G_{coil})_B/(G_{coil})_A$. The functions show that this factor varies between 1 and 3.3. This limit is due to the imposed restriction that Q_1 shall not exceed 100.

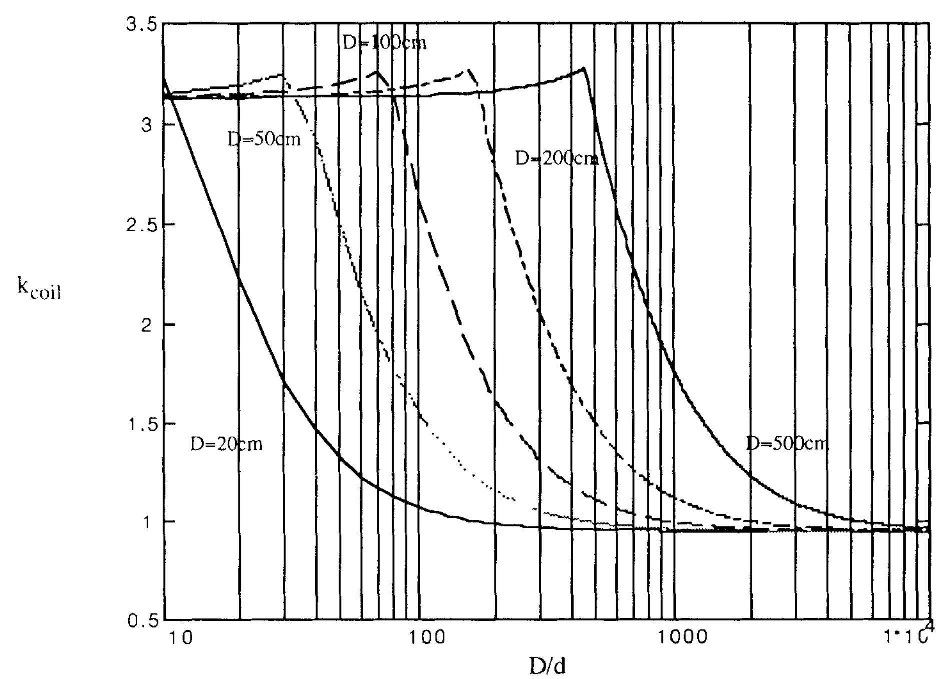

Figure 15. Ratio of geometric factors for the cases A and B. $k_{coil} = (G_{coil})_B/(G_{coil})_A$.

We now give three example calculations. Example 1 relates to case A and to a coil with a relatively small diameter (D = 20 cm). The enclosed sample has a volume of V = 1,000 cm^3. These dimensions are close to those in our laboratory setup. We measured the spin-lattice relaxation time T_2^* as $5.7 \cdot 10^{-3}$. Examples 2 and 3 use the same setup except that in example 2 the coil is directly connected (case A) and in example 3 the coil is part of a resonance transformation (case B) at the amplifier. The coil itself is much larger; its diameter is D = 200 cm. The sample volume has been kept the same as in example 1 to allow for a comparison. A more realistic volume of 10 L would have increased S/N by a factor of 10. Because we expect to perform these kinds of experiments outdoors — removed from interference — we consider the homogeneity of the Earth's field to be very good, and we use a T_2^* of 2 s.

In all three examples parameters equal D/d = 100, V = 1,000 cm^3, T_1 = 2 s, T_{me} = 100 s, and F_{coil} = 1.

Example 1 (case A)

Given parameters:

$$D = 20 \text{ cm}$$
$$B_p/B_0 = 20/0.5 = 40$$
$$T_2^* = 5.7 \cdot 10^{-3} \text{ s}$$

Derived parameters:

$$T = 3 \cdot T_2^* = 17.1 \cdot 10^{-3} \text{ s}$$
$$T/T_1 = 0.00855$$
$$(T_{re}/T_1)_{opt} = 1.25$$
$$\Psi_{max} = 1 \text{ (from Fig. 14)}$$
$$N = N_{opt} = 330 \text{ (Eq. [60])}$$
$$G_{coil} = 7.3 \cdot 10^{-4} \text{ cm}^{-3} \text{ s}^{-1/2} \text{ (Eq. [59])}$$

Substituted into Eq. [58], we have $(S/N)_{ac}$ = 15.59

Example 2 (case A)

Given parameters:

$$D = 200 \text{ cm}$$
$$B_p/B_0 = 1$$
$$T_2^* = 2 \text{ s}$$

Derived parameters:

$$T = 3 \cdot T_2^* = 6 \text{ s}$$
$$T/T_1 = 3$$
$$(T_{re}/T_1)_{opt} = 5.54$$
$$\Psi_{max} = 0.613 \text{ (from Fig. 14)}$$
$$N = N_{opt} = 105 \text{ (Eq. [60])}$$
$$G_{coil} = 5 \cdot 10^{-5} \text{ cm}^{-3} \text{ s}^{-1/2} \text{ (Eq. [59])}$$

Substituted into Eq. [58], we have $(S/N)_{ac}$ = 0.3065

Example 3 (case B)

Same parameters as in example 2 except for:

$$N = N_{opt} = 10.5 \text{ (Eqs. [67] and [68]}$$
$$G_{coil} = 1.6 {*} 10^{-4} \text{ cm}^{-3} \text{ s}^{-1/2} \text{ (Eq. [65])}$$

Substituted into Eq. [63], we have $(S/N)_{ac}$ = 0.9808

Example 2 (case A) provides a S/N factor of 51(!), smaller than in example 1 (laboratory setup; see data in Fig. 16). This difference is reduced in example 3 to a factor of 16. Note, however, that in examples 2 and 3 the Earth's field was used exclusively; in example 1 we used a very strong polarization field of 20 Gauss.

Experimental Data

Figure 16 shows two spectra obtained with a setup based on the theoretical considerations for low-field NMR shown in this paper. The geometry of the laboratory setup is shown in Fig. 1.

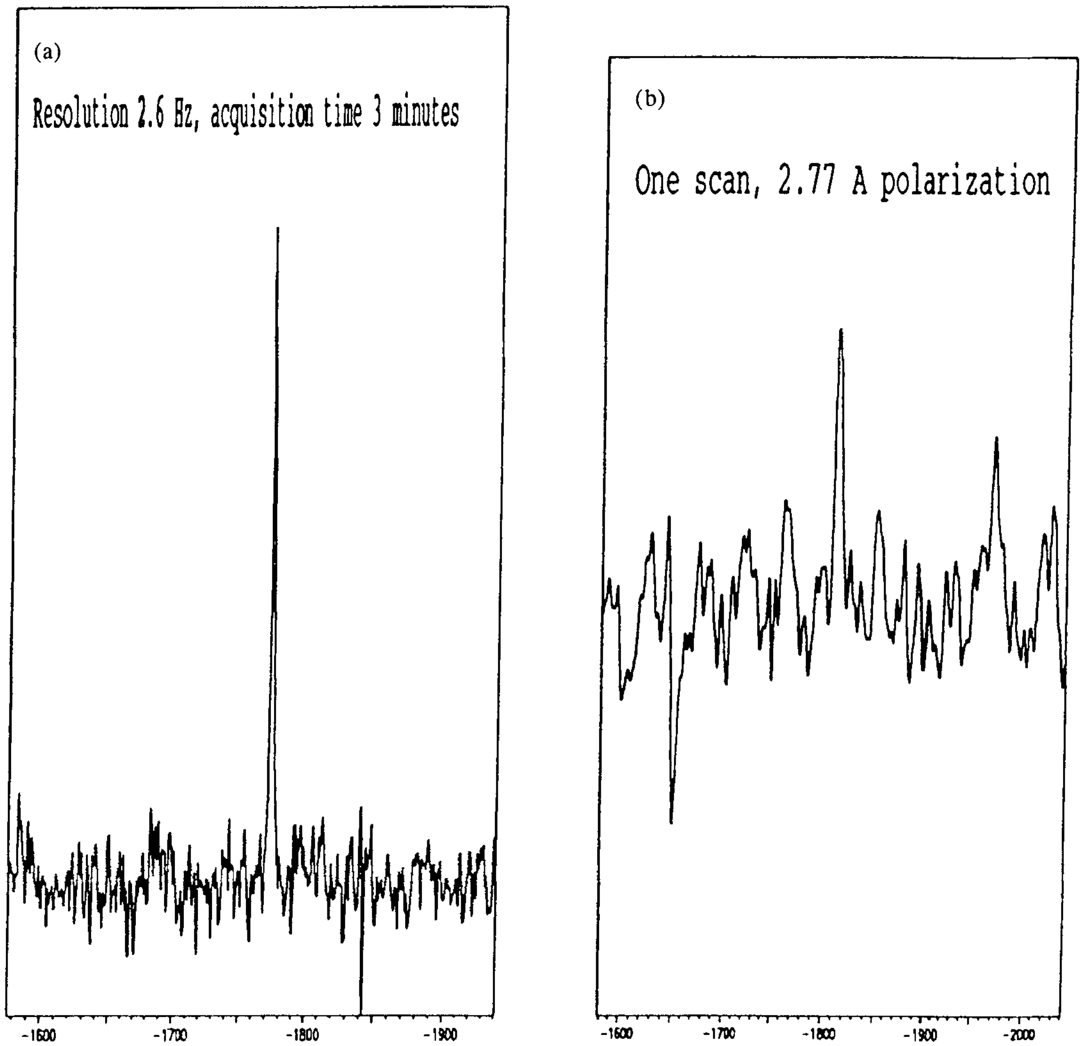

Figure 16. Experimental data obtained in the Earth's field. (a) Signal was acquired in 3 min (n = 32 scans). The resolution is 2.6 Hz. (b) One scan experiment with a polarization current of 2.77A.

SYMBOLS

Constants

μ_0	= permeability constant	= $4\pi \cdot 10^{-9}$ Vs/Acm	[Vs/Acm]
k_B	= Bolzmann constant	= $1.380 \cdot 10^{-23}$ Ws/K	[VAs/K]

h	=	Planck's constant = $6.625 \cdot 10^{-34}$ Ws^2	[VAs^2]
N_A	=	Avogadro's number = $6.023 \cdot 10^{23}$	[mol^{-1}]
I	=	quantum number	
γ	=	gyromagnetic ratio	[(Gauss $\cdot$ s)$^{-1}$; cm^2/Vs^2]
	=	$2.675 \times 10^4 \frac{1}{\text{Gauss} \cdot \text{s}} = 2.675 \times 10^{12}$ cm^{2}Vs^2	
k_3	=	$-8.6 \cdot 10^{-13}$ VA/(Gauss$\cdot$cm^3)	[VA/(Gauss$\cdot$cm^3)]
k_4	=	$-2.702 \cdot 10^{-13}$ V/cm^2	[V/cm^2]
k_5	=	0.638173	[1]

Electrical Dimensions

ΔU_i	induced potential due to spins precessing in ΔV	[V]
U_i	induced potential due to spins precessing in V	[V]
u_1	induced potential in coil with 1 loop	[V]
u_N	induced potential in coil with N loops	[V]
u_3, a_0	signal voltage at amplifier	[V]
$\sqrt{\overline{u_r^2}}$	potential noise at amplifier (effective value)	[V]
$\sqrt{\overline{u_{r3}^2}}, Y_{eff}$	total noise voltage at input to amplifier (effective value)	[V]
$\sqrt{\overline{u_{R_1}^2}}$	noise of resistance [ohm noise] of receiver coil (effective value)	[V]
I_1	current in coil with only one loop	[A]
I_p	current in the polarizing coil	[A]
$\sqrt{\overline{i_r^2}}$	current noise of amplifier (effective value)	[A]
R_1	ohm resistance of coil with only one loop	[Ω]
X_1	reaction of coil with one loop	[Ω]
R_N	ohm resistance of coil with N loops	[Ω]
R_{in}	input resistance of amplifier	[Ω]
R_{opt}	optimal source resistance of amplifier	[Ω]
L_1	induction of coil with one loop	[H]
L_N	induction of coil with N loops	[H]

Magnetic Dimensions

$\vec{H}_p$	field strength vector of polarizing field	[A/cm^2]
B_0	induction of Earth's field (0.5 Gauss)	[Gauss]
$B_p(t)$	time dependence of induction of polarizing field	[Gauss]
B_p	induction of polarizing field	[Gauss]
B_1	induction due to current I_1 in coil with one loop	[Gauss]
$\vec{b}_1$	unit vector of $\vec{B}_1$	
(B_1/I_1)	characteristic expression for induction dependence for coil with only one loop	[Gauss/A]
$M_p(t)$	time dependence of magnetic vector component in direction of polarizing field B_p	[Gauss]
M_p	magnetic vector component in direction of polarizing field B_p	[Gauss]
$\vec{M}$	magnetization vector of the spins	[A/cm]
M_0	magnetization of the spins $= \lvert\vec{M}\rvert$	[A/cm]
$\vec{m}$	unit vector in the direction of $\vec{M}$	
χ_0	magnetic susceptibility of the spins	
χ_0'	magnetic susceptibility in magnetic units $\chi_0' = \dfrac{\chi_0}{4\pi}$	
$\vec{\mu}_k$	magnetic dipole vector for a single spin	[A cm^2]
μ_{tot}	magnetic dipole vector for all spins in a volume element ΔV	[A cm^2]

Geometric Dimensions

z	symmetric distance from center of coil	[cm]
D	diameter of circular coil	[cm]
d	diameter of coil loop package	[cm]
q	area of single loop	[cm^2]
q_{tot}	area of total loop package	[cm^2]
V	volume of sample to be measured	[cm^3]
ΔV	volume element of sample	[cm^3]

Time Dimensions

T_1	spin-lattice relaxation time	[s]
T_2^*	spin-spin relaxation time in inhomogeneous field (time constant of FID)	[s]
T	acquisition time of FID	[s]
T_{EM}	time constant of EM	[s]
T_{re}	repetition time of FIDs	[s]
T_{me}	total measurement time available	[s]

Frequencies

ω	$= 2\pi f =$ circular frequency	[Radian·Hz]
ω_0	precession frequency of protons	[Radian·Hz]
$\Delta f, F$	frequency bandwidth of noise, given by frequency response of amplifier	[Hz]

Other Dimensions

T_{R_1}	temperature of noise resistance R_1	[K]
i	$= \sqrt{-1}$	[1]
d	density of sample	[g/cm^3]
T_A	absolute temperature	[K]
N_S	number of spins in a molecule	
N	total number of spins per unit volume	[cm^{-3}]
N_α	number of spins antiparallel to polarization field	[cm^{-3}]
N_β	number of spins parallel to polarization field	[cm^{-3}]
N_p	number of surplus spins parallel to polarization field $N_p = N_\beta - N_\alpha$	[cm^{-3}]

Dimensionless Quantities

(S/N)	S/N before FT
$(S/N)_{max}$	S/N before FT when $G = G_{opt}$
$(S/N)_\omega$	S/N after FT
$(S/N)_{ac}$	S/N after N_{ac} acquisitions (i.e., at end of T_{me})
G_{coil}	geometric factor of coil

F_{coil}		field function of coil
$F(D/d)$	=	$[2.303 \cdot \log_{10}(8D/d)] - 1.75$ = form factor of coil
Ψ		factor dependent on length of experiment parts
Ψ_{max}		maximum value for Ψ when $T_{re}/T_1 = (T_{re}/T_1)_{opt}$
$(T_{re}/T_1)_{opt}$		optimal value of (T_{re}/T_1), leading to $\Psi = \Psi_{max}$
N		number of loops of coil
N_{opt}		optimal number of loops to maximize (S/N)
n_{ac}		number of accumulated FIDs within T_{me}
G		ratio of ideal transformer at input to preamplifier
G_{opt}		optimal G, for which (S/N) = $(S/N)_{max}$
η	=	$(G_{opt}/G)^2 + (G/G_{opt})^2$
β	=	ratio of actual to maximum component M_p at the beginning of FID
$Q_{(\omega)}$	=	$\omega L_1/R_1$, quality factor of coil as a function of ω
Q_1	=	$\omega_0 L_1/R_1$, quality factor for $\omega = \omega_0$

Conversion Table

1 Gauss	=	$1 \cdot 10^{-8}$ Vs/cm^2
1 Tesla	=	$1 \cdot 10^4$ Gauss
	=	$1 \cdot 10^{-4}$ Vs/cm^2
1 Oersted	=	$10/4\pi$ A/cm

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