

- **MP-SCHED is NP.**

A certificate of instance (X, m, d) is m sequences of jobs. We can check in polynomial time $O(|X|)$ whether the sum of job lengths in each sequence is not greater than d .

- **MP-SCHED is NP-hard.**

We find 2-PARTITION known to be NP-Complete.

- **Reduction**

We only consider 2-PARTITION instances where non-negative integers are involved. If N contains some negative integers, we can transform it to N' by finding the smallest integer n_{min} and adding the absolute value of it to every number in N so that the smallest integer in N' is now 0 with $t' = t + n_{min} |N|$.

We construct a MP-SCHED instance with two processors with a deadline $d = \frac{L}{2}$ where L denotes the total length of all jobs. For each $n \in N$, we can add the same x to X . The reduction function runs in polynomial time $O(|N|)$.

- **Claim 1: If $N \in 2\text{-PARTITION}$, then $(X, m, d) \in \text{MP-SCHED}$.**

If $N \in 2\text{-PARTITION}$, then d must be an integer and that there exists two partitions of N where each partition sums up to $\frac{L}{2}$. We can arbitrarily permute jobs in each partition since the order does not change their sum. By construction $d = \frac{L}{2}$, we conclude that all sequences of jobs are finished exactly at d in the corresponding MP-SCHED instance.

- **Claim 2: If $(X, m, d) \in \text{MP-SCHED}$, then $N \in 2\text{-PARTITION}$.**

If the reduced instance $(X, m, d) \in \text{MP-SCHED}$, then there exists two sequences in X , each sequence containing numbers that sum up to no larger than d . We also know that the total length of jobs is L , so it must be the case that both sequences have a duration of exactly $\frac{L}{2}$. For each job in a sequence we have the same number to the corresponding subset. Then there exists a way of partitioning N .

Thus MP-SCHED is NP-hard.

- **MP-SCHED is NP-Complete.**

Because MP-SCHED is both NP and NP-hard, it is also NP-Complete.