- **Claim 1: The general's algorithm indeed works.**

  Suppose we are given a BLOCKING instance x of an undirected graph G that might contain k "garrison-vertexes" and none or some "empty vertexes."

  The algorithm will separate all the vertexes into at least k subgraphs G_i, each of which contains exactly one garrison. It is obvious that, starting from any garrison, a soldier can only travel to the vertexes in the same subgraph and thus cannot travel to another garrison. Therefore all garrisons are isolated from one another.

- **Claim 2: The algorithm has an approximation ratio of 2.**

  An optimal solution partitions the graph G into k subgraphs to ensure that none of the garrisons can connect to each other.

  We use E_{ij} to denote the set of all edges that connect subgraphs G_i to G_j in an optimal solution. That is, every edge in the set has one vertex in G_i and one in G_j, and is counted only once in one of the subsets. For convenience purpose we set $E_{ii} = 0$. Then we can represent the optimal solution as

  $$C^*(x) = \mid E_{12} \mid + \mid E_{13} \mid + \mid E_{23} \mid + \mid E_{14} \mid + ...$$

  We use E_A to denote the set of all edges starting from G_A in an potimal solution. That is, every edge in the set has one vertex in G_A and one in some other subgraph. By definition, we have

  $$\mid E_i \mid = \mid E_{i1} \mid + \mid E_{i2} \mid + \mid E_{i3} \mid + ... + \mid E_{ik} \mid$$

  For every pair of the subgraphs G_i and G_j, every cut edge between them will be counted in both E_{ij} and E_{ji}. In addition, for any $i \neq j$ we have $\mid E_{ij} \mid = \mid E_{ji} \mid$ because they contain the same edges. Therefore,

  $$\mid E_1 \mid + \mid E_2 \mid + \mid E_3 \mid + ... \mid E_k \mid = 2(\mid E_{12} \mid + \mid E_{13} \mid + \mid E_{23} \mid + \mid E_{14} \mid + ...) = 2C^*(x).$$

  By the general's algorithm, we are ensured that B_i is the optimal for every single garrison,

  $$\mid B_i \mid \leq \mid E_i \mid .$$

  The general's solution counts the union of B_i for every garrison i where overlapping edges are counted only once,

  $$C(x) = \mid B_1 \cup B_2 \cup B_3 \cup ... \cup B_{k-1} \mid \leq \mid B_1 \mid + \mid B_2 \mid + ... + \mid B_{k-1} \mid .$$

  Taking into account what has been analyzed above,

  $$C(x) \leq \mid B_1 \mid + \mid B_2 \mid + ... + \mid B_{k-1} \mid \leq \mid E_1 \mid + \mid E_2 \mid + ... + \mid E_{k-1} \mid = 2C^*(x) - \mid E_k \mid < 2C^*(x).$$

  Since $2C^*(x)$ is the upper bound of $C(x)$,

  $$1 \leq \frac{C(x)}{C^*(x)} < 2.$$

  Therefore, the algorithm has an approximation ratio of 2.