- **The problem is NP.**

  A certificate of this problem is an assignment of all the logs in each case. We can check in polynomial time proportional to the number of logs whether each log fits into a case without overlapping with other logs.

- **The problem is NP-hard.**

  We find 2-PARTITION known to be NP-complete.

  - **Reduction**

    Given a 2-PARTITION instance of a set of numbers N, we are able to construct an instance with two cases. Each of the cases has a size 1 by $k = \frac{L}{2}$ where L denotes the total length of all the logs in X. For each log in X, we add the value of its length to N.

    The reduction function runs in polynomial time O($\mid N \mid$).

  - **Claim 1: If $N \in$ 2-PARTITION, then $X \in$ the problem.**

    If N can be partitioned into two subsets with equal sum, then there exists two partitions of N, each of which sums up to $\frac{L}{2}$. We have the corresponding logs in the two cases respectively. Because the cases have a width of one, all the logs in a same case are parallel to the longer side of the case. The logs are placed one after one, in an arbitrary order. Each case has some logs with a total length of $\frac{L}{2}$. Therefore they fit "perfectly, " without extra room in the cases.

  - **Claim 2: If $X \in$ the problem, then $N \in$ 2-PARTITION.**

    If the logs fit into the two cases, then there exists two partitions in X, each containing numbers that sum up to no larger than $\frac{L}{2}$. We also know that the total sum of numbers in N is L, so it must be the case that each group of numbers sum up to exactly $\frac{L}{2}$. Then there exists a way of partitioning N.

  Thus this problem is NP-hard.

- **The problem is NP-Complete.**

  Because the problem is both NP and NP-hard, it is also NP-complete.