- **The problem is NP.**

  A certificate of this problem is a satisfying assignment A to ϕ's variables. We can check in polynomial time whether every clause contains at least one True (T) and one False (F).

- **The problem is NP-hard.**

  We find 3-SAT known to be NP-complete.

  - **Reduction**

    Given an instance of 3-SAT containing k clauses, we are able to construct an instance of NAE-3-SAT with $2k$ clauses.

    For every clause of ϕ in the following form

    $$(x \vee y \vee z)$$

    we append two clauses to ϕ'

    $$(x, y, a) \wedge (z, \neg a, f).$$

    where a has same value for both clauses. The only reason we add f to the clause is that we want a 3-CNF.

    The reduction runs in polynomial time proportional to the number of clauses.

  - **Claim 1: If $\phi \in$ 3-SAT, then $\phi' \in$ NAE-3-SAT.**

    There are two possible cases when $\phi \in$ 3-SAT.

    Case one is when x or y (or both) is true. We assign a in both clauses to be false, f to be false, and rewrite ϕ as

    $$(x, y, F) \wedge (z, T, F).$$

    The first clause is true because we assume that at least one of x and y is true; the second clause is also true as we have a true and a false no matter what z is.

    Case two is when both x and y are false. We assign a in both clauses to be true and f still to be false. Because we know that ϕ is satisfactory, z must be true. ϕ is then rewritten as

    $$(F, F, T) \wedge (T, F, F).$$

    Both clauses above are true.

    To sum up, no matter what values x, y and z have, ϕ' is always satisfactory.

  - **Claim 2: If $\phi' \in$ NAE-3-SAT, then $\phi \in$ 3-SAT.**

    * When f is assigned a value of false, there are following two possible cases with different x, y, and z.

      Case one is when a is true, we rewrite the formula as

      $$(x, y, T) \wedge (z, F, F).$$

      Because ϕ' is true, we can infer that z is true. Therefore we have at least z to be true.

      Case two is when a is false, we rewrite the formula as

      $$(x, y, F) \wedge (z, T, F).$$

      The second clause is always true with at least one T and one F, so we do not get extra information from it. The first clause is known to be true, so we can conclude that at least one of x and y is true.

  * When f is assigned a value of true, we flip the values of x, y, z, f, and a. Because each satisfying NAE-3-SAT must have at least one literal true and one literal false, it must also be satisfactory after being flipped. Then we can assign the truth assignments to the new NAE-3-SAT the way we have discussed above because now f is false.

  Therefore there always exists a truth assignment to the original 3-SAT that is satisfactory.

  Thus this problem is NP-hard.

- **The problem is NP-Complete.**

  Because the problem is both NP and NP-hard, it is also NP-Complete.