- **Decision Problem**

  The corresponding decision problem to the optimization problem is, if given an undirected graph, whether there exists a subset S of at most n vertices such that every vertex has some edge connected with a vertex in S.

- **MIN-DOM is NP.**

  A certificate of this problem is a subset S of selected vertices. For every vertex v in S we iterate through its edges and check whether the vertex on the other side of the edge is in S. The process takes polynomial time of roughly O($\mid V \mid\mid E \mid$).

- **The problem is NP-hard.**

  We find VERTEX-COVER known to be NP-complete.

  - **Reduction**

    Given a VERTEX-COVER instance of an undirected graph G, we are able to construct a corresponding MIN-DOM of an undirected graph G' in the way described below.

    For every vertex in G, we add a vertex to G' respectively. For every edge in G, we add an edge between corresponding vertexes. G and G' have exactly same E and V so far.

    Then, again for every edge between v_i and v_j in G, we add an extra vertex v_k this time in G', as well as adding two edges (v_i,v_k) and (v_j,v_k).

    The reduction function runs in polynomial time O($\mid E \mid + \mid V \mid$).

  - **Claim 1: If $G \in$ VERTEX-COVER, then $G' \in$ MIN-DOM.**

    If vertex selection for G is satisfactory, then for every edge between v_i and v_j in G', either v_i or v_j is selected. Because there exists an edge between every pair of v_i, v_j and v_k, any one of the three being selected will result in the other two being connected to a selected vertex. Therefore, all vertexes are covered in G'.

  - **Claim 2: If $G' \in$ MIN-DOM, then $G \in$ VERTEX-COVER.**

    Notice that G' is composed of a pile of "triangles, " each contains three vertexes that has three edges between them. By reduction, at least one of the three vertexes has only two edges. For each "triangle, " we mark one such vertex as a "new" vertex; for every edge between two "old" vertexes, we mark it as an "old" edge.

    If G' after reduction is satisfactory, then for every vertex, at least one of its adjacent vertexes is selected. Because every vertex must belong to at lease one "triangle," for every "triangle" in G', we can categorize it into two cases based on what type of vertexes is selected.

    * **Case 1: the selected vertex is "new. "**

      The vertex is connected to exactly two vertexes that have corresponding vertexes in G. We select one of the two in G.

    * **Case 2: the selected vertex is "old. "**

      We select its corresponding vertex in G. The vertex may have already been selected, but repeatedly selecting them do not lead to any contradiction.

    For every triangle in G', the edge between the two vertexes is the only edge that has a counterpart in G, and the edge will be covered in both cases. Therefore all the edges in original G are covered.

  Thus MIN-DOM is NP-hard.

- **The problem is NP-Complete.**

  Because the decision problem is both NP and NP-hard, it is NP-complete. The original optimization problem is no easier than the decision problem, so it is NP-complete as well.