- **Algorithm**

  We look for e_i, the element not yet covered. We pick all the subsets in S that contain e_i and mark other elements in these subsets as "covered. " We repeat the above greedy process until all elements are covered.

  The above approximation process takes polynomial time of roughly O($\mid S \mid\mid E \mid$) where $\mid S \mid$ is the number of subsets and $\mid E \mid$ the number of elements.

- **Claim 1**: the algorithm is feasible.

  It is assumed that the union of all subsets is universe. Then, if there exists an element that is not covered yet, the algorithm will keep executing. Even in the worse scenario when it has to choose all the given subsets, it will still be the case that all elements are covered by at least one subset.

- **Claim 2**: the algorithm has approximation ratio of f.

  - Suppose we are given a SET-COVER instance x of subsets S and a collection of elements X. Let $C(x)$ denote our approximation solution ranging anywhere from $L(x)$ up to $U(x)$, and $C^*(x)$ the optimal solution to the original NP problem.

    Also, suppose approximation algorithm executes the step of making a greedy choice for k times. That is, we have selected k elements and picked the corresponding subsets by the time every element is covered. For every two of these elements, they do not share any common subset as guaranteed by the algorithm. Thus, $C(x)$ is the sum of occurrences of the k elements.
  - Each of the k elements is covered in at most f subsets.

    The upper bound of the approximation is

    $$C(x) \leq fk$$

    in a worst case when each of them appears in f different subsets.
  - If an element is chosen to be covered, it means that the element has not been covered in any previous subsets. Therefore at least one more subset is required to cover the element.

    The lower bound of optimal solution is

    $$k \leq C^*(x)$$

    in a best case when each of the k elements is covered in exactly one subset.
  - According to the above inequities,

    $$k \leq C^*(x) \leq C(x) \leq fk$$

    Therefore,

    $$1 \leq \frac{C(x)}{C^*(x)} \leq \frac{fk}{k} = f$$

    In other word, the algorithm has an approximation ratio of f.