- **Linear Program**

  Let $[n]$=[1, 2, ..., n].

  The LP minimizes

$$\sum_{b=1}^{n} x_b + \sum_{i=1}^{n}[w_i * (1 - \sum_{b=1}^{n} x_{b,i})]$$

  under constraints

$$\sum_{i=1}^{n}(s_i * x_{b,i}) \leq x_b, \forall b$$

$$\sum_{b=1}^{n} x_{b,i} \leq 1, \forall i$$

$$0 \leq x_b \leq 1, \forall b$$

$$0 \leq x_{b,i} \leq 1, \forall b \forall i$$

  Variable $x_b = 1$ if and only if the bth bin is opened. $x_{b,i}$ is an indicator variable that is 1 if and only of item i is assigned to the bth bin, otherwise 0. The first constraint limits the space each bin packs and the second constraint specifies that an item can only be packed once.

- **LP Rounding**

  If $1 - \sum_{b=1}^{n} x_{b,i} \geq \frac{1}{2}$ for item i, we pack it; otherwise, discard it.

- **LP and Rounding gives an O(1)-approximation.**

  During the rounding, $x_{b,i}$ increases at most twice the fractional value. Because the optimal solution is no better than the LP solution over real numbers, rounding leads to a two approximation factor. Then, consider the items that are packed rather than discarded. Greedy packing has a two approximation ratio, as discussed in the last homework. The cost resulting from packing is no greater than the total cost. Therefore, the overall solution is within four times of that in an optimal solution.