- **Double-Hamiltonian is NP.**

  A certificate of this problem is a sequence of vertexes in graph G. We check if it starts and ends at one specific vertex. For every vertex in the sequence we check whether there is an edge to the last item and the next one, and increment occurrence by one. Then we check if every vertex in the graph is visited exactly twice.

  The process takes polynomial time of roughly O($\mid V \mid$).

- **Double-Hamiltonian is NP-hard.**

  We find Hamiltonian-Circle known to be NP-complete.

  - **Reduction**

    Given a Hamiltonian-Circle instance x of an undirected graph G, we are able to construct a corresponding Double-Hamiltonian instance x' of an undirected graph G' in the way described below. We duplicate the whole graph G to G' in instance x' with same E and V. Ina addition, for each vertex v' in G' we append a "triangle," or a group of three vertexes in such way that an edge exists between any two of them and exactly one vertex also connects to v'.

    For convenience purpose, in a triangle we label the original vertex as v'_0, the one connecting to it v'_a, and the other two v'_b and v'_c respectively.

    The reduction function runs in polynomial time O($\mid E \mid + \mid V \mid$).

  - **Claim 1: If instance $x \in$ Hamiltonian-Circle, then $x' \in$ Double-Hamiltonian.**

    If instance x' for G is satisfactory, then there exists a path that starts and ends at a specific vertex v_s, and visits every vertex exactly once. To get a new path to Double-Hamiltonian instance x', we replace vs in the original path with a new path,

    $$v'_0 \to v'_a \to v'_b \to v'_c \to v'_b \to v'_c \to v'_a \to v'_0.$$

    This piece of path starts and ends at v', which maintains same connections to other vertex counterparts in original G. It is guaranteed that the new path forms a circle that starts at v'_s and ends at v'_s. Also, every vertex in G' is visited exactly twice since the original vertex v is visited exactly once. Therefore, G' is satisfactory for x'.

  - **Claim 2: If $x' \in$ Double-Hamiltonian, then $x \in$ Hamiltonian-Circle.**

    By reduction, G' is composed of some "ordinary" vertexes and some "triangles." According to our observations, vertexes that are a part of a triangle can only be achieved by visiting a bridging vertex v'_0. If there is a path to visit every vertex exactly twice, it must be the case that they are visited in either one of the following two ways:

    $$v'_0 \to v'_a \to v'_b \to v'_c \to v'_b \to v'_c \to v'_a \to v'_0$$

    or

    $$v'_0 \to v'_a \to v'_c \to v'_b \to v'_c \to v'_b \to v'_a \to v'_0.$$

    They are essentially the same because v'_b and v'_c can be swapped.

    For every triangle in G', Therefore all the edges in original G are covered. We can then replace each of the paths with v, the corresponding vertex of v'. The new path is a Hamiltonian circle in G that visits every vertex exactly once. So G is satisfactory for x.

  Thus the problem is NP-hard.

- **Double-Hamiltonian is NP-Complete.**

  Because Double-Hamiltonian is both NP and NP-hard, it is NP-complete.