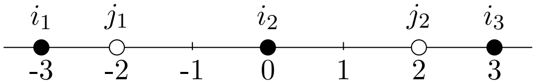
We show an instance in which the given algorithm yields an arbitrarily large value. Suppose there are three facilities, i_1, i_2, and i_3, and the cost of opening each facility is 1, 1, and a large constant x, respectively. There are also two clients j_1 and j_2, the connection cost for each j to i is simply their distance in the following graph, which form a metric.



In an optimal solution, j_1 goes to i_1 and j_2 goes to i_2. In this case, the opening cost is $1 + 1 = 2$ and the connection cost is $1 + 2 = 3$, the optimal is thus 5.

If we run the given algorithm, however, the objective value is not bounded when x, or the opening cost of i_3, is a large constant. For client j_1, the LP gives $\Delta_1 \geq 1$. Therefore, the algorithm assigns j_1 to i_1 and then remove i_1, j_1, and i_2 that are within the radius $2\Delta_1 \geq 2$. Now, i_3 is the only left facility for j_2. Because x is arbitrarily large, the solution given by the approximation is not bounded.

In fact, the given algorithm sometimes fails to generate a feasible solution. Consider such case that the setup is the same as the above case except that i_3 is absent. j_2 is assigned to nowhere because all the facilities have been removed.