- **Algorithm**

  We denote M_1 to be all the items such that $p_i > w_i$, and M_2 the rest of the items.

  If the total weight of all items is no greater than one, accept all items if the total price is greater than one and reject if not greater.

  If the total weight is greater than one, then for each item, if the item $u_i \in M_1$, then pack it using the First Fit Algorithm; otherwise, reject it. Repeat through all items until the remaining items has a total weight of less than or equal to one. Then, pack all remaining items into one extra bin.

- **The approximation algorithm has a ratio of 2.**

  We use $FF(M)$ to denote the objective value of packing M using the First Fit Algorithm. In the first two cases, it is trivially true that the case that the approximation produces the same result as that in an optimal solution.

  Otherwise, we denote $P(x)$ as the total space of all items in x and $W(x)$ the total weight. The approximated value consists of $FF(M_1)$ and $P(M_2)$. Assume an optimal solution packs k bins, rejects the item set S_1 and accepts the item set S_2. Then we have

  $$OPT = k + P(S_1) \leq W(S_2) + P(S_1) \leq W(M_1) + P(M_2),$$

  which simplifies to

  $$OPT \leq W(M_1) + P(M_2).$$

  We consider a feasible solution as to pack the whole M_1 optimally and to reject the whole M_2, which is not greater than OPT. So it follows that

  $$W(M_1) + P(M_2) \geq \frac{OPT - P(M_2) - 1}{2} + P(M_2) \geq \frac{OPT - 1}{2}.$$

  According to the First Fit Algorithm, the objective value $FF(M')$ is bounded by $2 * W(M')$ given $W(M') > \frac{1}{2}$. Combining the above two inequalities, we see that

  $$FF(M_1) + P(M_2) \leq 2(\sum_{u_i \in M_1} w_i + P(M_2)) = 2(W(M_1) + P(M_2)) \leq 2OPT.$$

  Which gives a 2-approximation.

(Reference: Bin packing problems with rejection penalties and their dual problems by Gyrgy Dsa and Yong He; discussed with Diqiu Zhou)