1. **LP Form**

   Maximize

$$\sum_{all-j}^{n} y_j$$

   Under the constraints

$$y_j \leq \sum_{i|j\in S_j} x_i, \forall j$$

$$\sum_{all-i} x_i = k$$

$$x_i \in [0,1], \forall i$$

$$y_j \geq 0, \forall j$$

   y_j indicates whether element i is covered while x_i whether set i is picked. The first constraint states that every element is counted as most the number of those sets that contains it. The second constraint specifies k as the total number of sets picked.

2. **Randomized Rounding**

   For each x_i, round it to $x_i^* = 1$ with probability of x_i, and to $x_i^* = 0$ otherwise. Then, calculate $y_j^* = \sum_{i|j\in S_j} x_i^*$.

3. The expected value of the rounded solution computed with randomized rounding is $(1-\frac{1}{e})$-approximation.

   The probability of element i being covered is

$$Pr = 1 - \Pi_{i|j\in S_j}(1-x_i) \geq 1 - \Pi_{i|j\in S_j} e^{-x_i^*} \geq 1 - \frac{1}{e}$$

   Because the expected value of the rounded is the same as that of the fractional, the expected value of the overall rounded solution is

$$E[ROUND] = \sum_{all-j}(1-\frac{1}{e}) * y_j = (1-\frac{1}{e})\sum_{all-j} y_j$$

   where $\sum_{all-j} y_j$ is the LP objective. In addition, the optimal solution is no better than the fractional solution $\sum y_j$,

$$OPT \leq \sum_{all-j} y_j.$$

   Thus,

$$E[ROUND] \geq (1-\frac{1}{e})OPT,$$

   the expected value is at least $(1-\frac{1}{e})$ times the optimal.