

MST-TSP is a way of approximating travelling salesman problem (TSP). It first finds a minimum spanning tree (MST) using some sort of algorithm, then doubles all the edges in the tree, and finally adopts the loop to form a solution.

A MST connects all vertexes in a graph with minimum cost. Any feasible solution to TSP should also connect all vertexes. In fact, after cutting an edge in an optimal TSP solution, it still connects all vertexes. A whole path involves at least n edges. By contradiction, at least one of the edges in a feasible TSP solution should weight at least $\frac{1}{n}$ times the total weight. Therefore, the heaviest edge is at least $\frac{1}{n}$ times the total weight. $MST \leq (1 - \frac{1}{n}) TSP[OPT]$ where $TSP[OPT]$ denotes the weight in an optimal solution to TSP.

When edges in MST are doubled, edge weights are doubled too. If the graph follows triangle inequality, we can get rid of more weights by further replacing certain pairs of edges. The result should be less than two times the MST, written as $MST-TSP < 2MST$ where MST-TSP denotes the solution produced by approximation algorithm.

Combining both inequalities above, we get $MST-TSP < 2MST \leq 2(1 - \frac{1}{n}) TSP[OPT]$. The left hand side is an approximation and the right hand side is an optimal. Therefore MST-TSP is a $2(1 - \frac{1}{n})$ -approximation algorithm.

(Discussed with Diqu Zhou)