

For any $\epsilon > 0$, this linear program relaxation has an integrality gap of $2 - \epsilon$ if we can find a case where the upper bound is 2. Consider the case where we have n items, each of which takes $(0.5 + a)$ space where a is a super small nonnegative number. Suppose the LP after relaxation generates a fractional solution k equal to the total space of all items. Then, an optimal solution needs at least $2k$ bins because every pair of items that takes slightly larger than one bin now requires two whole bins. The integrality gap for this case is thus 2.