We can use contradiction to prove that there is no additive 100-approximation algorithm A for TSP unless P = NP.

Suppose for any instance I, every solution produced by the algorithm costs at most $OPT(I) + \alpha$ where $\alpha = 100$. Also, suppose that we know the guaranteed optimal solution, $OPT(I)$, for I.

Unless P = NP, we can always construct another instance I' that has the same vertexes and edges but larger edge weights by multiplying the original ones by a fixed constant x. We express the cost of the new approximated solution as $OPT(I') + \alpha'$.

Because ratios between weights are reserved, the new cost for I' is simply x times the original cost for I. The new approximated cost for I' is $x(OPT(I) + \alpha)$ while the new optimal cost $x * OPT(I)$. Rewritten the expression, we find that $\alpha' = x * \alpha$.

According to math rules, we can always find a sufficiently large constant x such that α' is larger than 100. This contradicts our assumption that any solution produced by the algorithm costs no larger than $OPT(I) + 100$. Therefore, the original statement is true.

(Discussed with Diqiu Zhou)