

We assume that the graph is connected and our goal is to partition the graph with minimal edge cost. The following LP is given.

$$\min \sum_{(i,j) \in E} c_{ij} d_{ij}$$

$$d_{ij} - p_i + p_j \geq 0, \forall (i,j) \in E$$

$$p_s - p_t \geq 1$$

$$d_{ij} \in [0, 1], \forall (i,j) \in E$$

$$p_i \in [0, 1], \forall i \in V$$

The constraints $p_i \in [0, 1]$ and $d_{ij} \leq 1$ are not necessary because, when the objective value is minimal, it must be the case that they both are satisfied.

d_{ij} represents whether the edge is cut; p_i denotes which partition of the graph an edge belongs to.

We can make p_i and p_j differ at most one by constructing a new solution. We multiply each p_i by a factor of $\frac{1}{p_a - p_b}$, where p_a is the maximum and p_b the minimum among all p_i .

Then we consider d_{ij} . The smaller d_{ij} is, the smaller the objective function will be. When the graph is optimally partitioned into two parts, there must be a corresponding LP solution where

$$d_{ij} = p_i - p_j \leq 1.$$

Suppose that is not the case, then we set $d'_{ij} = 1$. The solution is still feasible since the above inequalities still hold true. This contradicts the fact that the graph is optimally solved. Now we have d_{ij} all between k and $k + 1$ where k is some real number. If $k = 0$, we are good; we can set

$$d'_{ij} = d_{ij} - k$$

to obtain a new LP solution that satisfies $d_{ij} \in [0, 1]$. Thus, we get rid of the two constraints.