Greedy Cover is to pick a set that covers at least $(1-\epsilon)$ times the maximum number of uncovered elements at each greedy step. Its approximation ratio is $1-\frac{1}{e^{1-\epsilon}}$, proved as follows.

- **Claim 1:** $x_{i+1} \geq \frac{z_i*(1-\epsilon)}{k}$.

  Pf. Suppose x_i is the number of new elements covered by ith set chosen, y_i the total elements covered after ith set chosen, and $z_i = OPT - y_i$ where OPT is the maximum number of covered elements. k is the number of sets picked when algorithm terminates. The first set covers at least $\frac{1}{k}$ of the uncovered elements and for the same reason we have

  $$x_{i+1} \geq \frac{z_i*(1-\epsilon)}{k}.$$

- **Claim 2:** $z_k \leq (1-\frac{1-\epsilon}{k})^k * OPT$.

  Pf. We use mathematical induction. The base case is when $i=1$ and $z_1 \leq (1-\frac{1-\epsilon}{k})*OPT$ is trivially true.

  Suppose $z_i \leq (1-\frac{1-\epsilon}{k})^i * OPT$ is true, then using Claim 1 we get

  $$z_{i+1} \leq z_i - x_{i+1} \leq z_i * (1-\frac{1-\epsilon}{k}) \leq (1-\frac{1-\epsilon}{k})^{i+1} * OPT.$$

  After we plug in $i = k-1$, it becomes

  $$z_k \leq (1-\frac{1-\epsilon}{k})^k * OPT.$$

Thus the algorithm has an approximation ratio of $(1-\frac{1-\epsilon}{k})^k = 1-\frac{1}{e^{1-\epsilon}}$.