1. The (asymptotically) minimum k at least one head with probability at least $1 - 1/n$

$$Pr = 1 - (\frac{1}{2})^k \geq 1 - \frac{1}{n}$$

$$2^k \geq n$$

$$k \geq logn$$

2. The (asymptotically) minimum k at least $logn$ heads with probability at least $1 - 1/n$

   The Chernoff bound gives that

$$Pr[X \leq (1-\delta)\mu] \leq e^{-\frac{\delta^2\mu}{2}}, 0 < \delta < 1.$$

   Because X follows a binomial distribution, the expected value $\mu = \frac{k}{2}$. If we plug in $\delta = 1 - \frac{logn}{\mu}$, the probability less than $1 - 1/n$ is

$$Pr[X \leq logn] \leq e^{-\frac{(1-\frac{logn}{\frac{k}{2}})^2\frac{k}{2}}{2}} = e^{-\frac{\frac{1og^2n}{k} - 2logn + k}{4}}.$$

   The term $-\frac{\frac{1og^2n}{k} - 2logn + k}{4}$ is dominated by k because $k \geq logn$ as what we have shown. Therefore, asymptotically it is simplied to $-k$ and now we have

$$e^{-k} \leq 1 - (1 - \frac{1}{n}) = \frac{1}{n}.$$

   Solving the inequality, we obtain

$$k \geq logn.$$