

$$\sum_{i \in [n]} \min[S_i, \max[D - s(A), 0]]x_i \geq D - s(A)$$

Intuitively, the left hand side is the total gallons of the remaining pumps and the right hand side is the goal for these pumps. It should hold true for any combination of pumps because the addition of gallons is linear.

Mathematically, we break down the constraints by cases.

For any subset of pumps, if  $D - s(A) > 0$ , we construct a new case with those in  $[n]$ . In the previous problem, we have proved that a feasible integral solution satisfies

$$\sum_{i \in [n]} \min[s_i, D]x_i \geq D.$$

Therefore, the new instance satisfies

$$\sum_{i \in [n]} \min[S_i, D - s(A)]x_i \geq D - s(A),$$

which will be exactly the same expression above given the condition  $D - s(A) > 0$ .

When  $D - s(A) \leq 0$ , according to what has been proved, the constraint is satisfied as well.