

1. Algorithm

Guess the most expensive pump i^* used by the optimal solution and discard all items of cost more than w_{i^*} . Then, sort the rest of the pumps by $\frac{w_i}{s_i}$. Start from the most economic item, we pick until the total number of gallons exceeds D . The algorithm is guaranteed to yield some feasible solution because pumps we discard are not in an optimal solution.

2. The algorithm is a 2-approximation.

Suppose that, adding i th pump in the sorted sequence of pumps will result in the total gallons exceeding D . Suppose that it has s_i and w_i . Because the greedy algorithm ensures that each solution to a subproblem is most "dense," by the time the greedy algorithm is about to add the i th pump, the total gallons is less than or equal to that in an optimal solution. As for the i th pump, w_i is less than the optimal because it is a part of that solution. The approximation result is the sum of the total gallons before the i th pump and w_i , each of which is less than an optimal. Therefore, the approximation generates a result within twice the optimal.