The problem can be viewed as a special kind of cases for Metric-TSP with "terminals" when all vertexes in a graph are "terminals. "The result should accord with greedy Steiner algorithm that gives O(log $n$).

Let $i_1$, $i_2$, ..., $i_{|V|}$ be the order in which the solution produced by approximation visits all vertexes in $G$. Let $p_i$ be the $i$th edge chosen that connects to a nearest vertex and $c(i)$ the cost of $p_i$. Then it must be the case that, for all $1 \leq j \leq\mid V \mid$,

$$c(i_j) \leq \frac{2 * OPT}{j}$$

where $OPT$ denotes the minimal possible cost. We prove this by contradiction as shown below.

Suppose $c(i_j) > \frac{2*OPT}{j}$ for some $j$. Let $i'_1$, $i'_2$, ..., $i'_{|V|}$ be same vertexes in the new order. We pick any two vertexes $i'_a$ and $i'_b$ such that each is either $i_{|V|}$ or in $S'$. Assume $1 \leq a < b \leq\mid V \mid$ and then $c(i'_b) \leq \frac{2*OPT}{j}$ according to ???, which leads to contradiction.

Adding up costs of individual vertexes, we have a total cost

$$\sum_{j=1}^{|V|} \frac{2 * OPT}{j} = 2 * OPT * \sum_{j=1}^{|V|} \frac{1}{j}.$$

In this case we view $2 * OPT$ as constant and notice that $\sum_{j=1}^{|V|} \frac{1}{j}$ is bounded by O(log $\mid V \mid$). Therefore, this algorithm gives a O(log $n$)-approximation for the Metric-TSP problem where $n =\mid V \mid$.