OSCILLATIONS: SPRING-MASS SYSTEMS

PURPOSE:

The purpose of this experiment is to investigate the motion of a mass oscillating on a spring.

THEORY:

HOOKE'S LAW

Imagine a spring that is hanging vertically from a support. When no mass hangs at the end of the spring, it has a length x_0 . When a mass is added to the spring, its length increases by Δx, $\Delta x = x - x_0$. The spring exerts a restoring force,

$$F = - k \Delta x \tag{1}$$

where Δx is the distance the spring is pulled down and k is called spring constant. The minus sign indicates that the spring's force is always opposite in direction from the displacement of its free end. The equation (1) is known as Hooke's Law.

The spring's force is balanced by the hanging mass, mg. That is $F = - mg$. The Eq. (1) can be written as

$$mg = k \Delta x$$

SIMPLE HARMONIC MOTION

Consider a spring of constant k suspended vertically from above with a mass m attached to its lower end. If the mass is pulled down a small distance (x) from the equilibrium position and released. The spring's restoring force causes the mass to oscillate up and down. The restoring force F is equal to $F = - kx$. When you apply Newton's 2nd law to the equation $F = - kx$, you get $- kx = ma$. Recall that the acceleration, a, can be written as the second derivative of displacement with respect to time. Therefore, $- kx = ma$ can be rewritten as the differential equation,

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x$$

or $$\frac{d^2x}{dt^2} = -\omega^2 x \tag{2}$$

where $\omega^2 = \dfrac{k}{m}$.

A solution of Eq. (2) is $x(t) = A\cos(\omega t + \delta)$, which means that the mass moves in the x direction with simple harmonic motion. The motion repeats itself with angular frequency ω, or equivalently with a period, T, equal to:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad (3)$$

The period of oscillation depends on the mass and the spring constant.

Let's summarize the major assumptions used in arriving at Eqs. (2) & (3).

a. The spring obeys Hooke's law, i.e, is not over-stretched and is repeatable.
b. The force of friction either on the surface or through the air is negligible.

APPARATUS:

Spring, weights, meter stick, lab stand, force sensor, motion sensor, DataStudio and computer.

METHOD:

In this experiment, the force sensor measures the force that stretches the spring as weight is added to one end of the spring. You will measure the amount of distance that the spring stretches, and enter the distances into the computer. The DataStudio program displays the force and distance. The slope of the best fit line of graph of force versus distance is the spring constant "k". The motion sensor measures the oscillation of the mass on the end of the spring. The DataStudio program displays position versus time. It should be the sine wave or cosine wave. You should measure the period, the time it takes for the mass to make complete oscillation.

PROCEDURE:

Part I: Force Sensor Calibration

1. Start the DataStudio by double click the icon "DataStudio". The Experiment Setup window appears. Drag the Force Sensor to analog channel A.

2. Double click icon "Force Sensor" to open the "Sensor Properties" window. Select Tab "Calibration" to do sensor calibration.

   The Calibration window shows the default values (50 Newtons produces 8 Volts, -50 Newtons produces –8 Volts). The force sensor is set up so that a pull away from the sensor is a "negative" force. For example, if a 1 kilogram object is hung vertically from the hook, the force sensor measures –9.8 Newtons (since the force is downward).

a. For the **High Value** calibration point, do not put any mass on the force sensor's hook, and then press the tare button on the side of the force sensor to zero the sensor. Since there is no mass on the sensor's hook, type **0** as the **High Value**. Click "Take Reading" button for High Value.

b. For the **Low Value** calibration point, hang 1 kilogram mass on the sensor's hook. Type **–9.8** as **Low Value**. Click "Take Reading" button for Low Value.

c. Click OK to return to the Experiment Setup window.

Part II: Determining the Spring Constant

In this Part you import the data from keyboard. Double click sensor icon. At the "Sensor Properties" window, select **Fast** and **5 Hz** for Sample Rate, and then click OK to return to the Experiment Setup window. Click "Options" button to open "Sampling Options" window, and then choice "keep data values only when commanded". Type "**Stretch**" for Name and "**m**" for Units.

1. Suspend the spring from the force sensor's hook so that it hangs vertically.

2. Use the meter stick to measure the position of the bottom end of the spring (without any mass added to the spring). Record this measurement as the spring's equilibrium position.

3. Click the **Start** button to begin data recording.

4. Press the tare button on the side of the force sensor to zero the force sensor. Click **Keep** button and type in 0. Click OK to record your typed in value.

5. Add 20 grams of mass to the end of the spring. Measure the new position of the end of the spring. Record the difference between the new position and equilibrium position as **Δx** or "stretch" (in meters).

6. Click **Keep** button and type in the value of **Δx** (in meters). Click OK.

7. Add 10 grams to the spring and repeat the measurement of the new position of the end of the spring.

8. Click **Keep** button. Type the new **Δx** and then click OK to record your typed in value.

9. Continue to add mass in 10 gram increments until you have added 70 grams. Measure the new stretched position of the end of the spring each time you add mass. Click Keep button and type in each new Δx. Click OK each time to record your value.

10. Click the Stop button (the red color button, side of Keep button) to end data recording. The recorded data, **Run #1**, will appear in the Data list.

11. Drag the Force Data from "Data List" to Graph in "Display List". The graph shows Force vs. Time. Then drag the Stretch Data from "Data List" to Horizontal Axis of the graph. The horizontal axis will be changed from Time to Stretch. The plot shows **Force vs. Stretch.**

12. Click "Fit" button on menu bar, and then select "Linear Fit". The statistics area will show the slope for the line of 'best fit' for your data. The slope is the spring constant "k". Record the value of k.

    To get the best result, click and drag the cursor to draw a rectangle over just that region of the graph that shows the best straight line through the most data points.

Part III: Measuring the period of Spring-Mass system

1. Delete the graph, the recorded data and Force Sensor Icon by clicking it and then press the **Delete** key from keyboard.

2. Click "Options" button to open "Sampling Options" window, and then cancel Keyboard input.

3. Drag the Motion sensor to the Digital Channel 1. Double click sensor icon. At the "Sensor Properties" window, click Tab "Motion Sensor" and then select **30** for Trigger Rate. Click OK to return to the Experiment Setup window.

4. Put 50 grams weight on the end of the spring.

   To avoid damage to the motion sensor, tape (tie) the weight to the spring.

5. Place the motion sensor on the table directly beneath the weight.

6. Adjust the position of the spring so that the minimum distance from the weight to the motion sensor is greater than 40 cm at the bottom of the weight's movement.

7. Pull the mass down to stretch the spring about 5 cm. Release the mass. Let it oscillate a few times so that the weight will move up-and-down without much side-to-side motion.

8. Click the "Start" button to begin recording data. Continue recording for approximately 10 complete periods and then click "Stop" button to end data recording.

9. Drag the position data “Run # 1” to **Graph** on Displays list. The graph shows position vs. time.

10. Click the **Smart Tool** button. Use the cross hair to find the average period of oscillation of the weight. Move the cross hair to the first peak in the plot of position versus time and read the value of time. Record the value of time in the Data section.

11. Move the cross hair to each consecutive peak in the plot and record the value of time.

12. Find the period of each oscillation by calculating the difference between the times for each successive peak. Find the average of the period. Record you result.

13. Calculate the theoretical value for the period of oscillation based on the measured value of the spring constant of the spring and the weight.

Part IV: Questions

1. When the position of the weight is farthest from the equilibrium position, what is the velocity of the weight? (Hint: Drag the Velocity Data from Data List to the graph of Position vs. Time to display the Velocity vs. Time. Move the cross hair to a peak on the position plot, and move the cross hair on the velocity plot. Compare the data of Velocity with the data of Position at the same time.)

2. When the absolute value of the velocity of the weight is greatest, where is the weight relative to the equilibrium position?

DATA SHEET
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Part II: Determining the spring constant

Equilibrium Position = ______ m

Mass (g)	20	30	40	50	60	70
Δx, "Stretch" (m)						

Spring Constant, k = ________ N/m

Part III: Measuring the period of Spring-Mass system

Peak #	1	2	3	4	5	6	7
Time (sec)							
Period (sec)							

Average period of oscillation, T = ________ sec

The theoretical value for the period of oscillation:

m = _____ kg $\quad\quad$ $T = 2\pi\sqrt{\frac{m}{k}}$ = _______ sec

The percent difference between your calculated value and the measured value:

$$\text{percent difference} = \left|\frac{calculated - measured}{calculated}\right| \times 100\% = \_\_\_\_\_\_\_$$