

HW 5

1. a) Hard Real Time: System Must be able to process input as it appears.

Soft Real Time: System may occasionally be overloaded by input and create a backlog, delaying processing for some time.

b) $f_{\text{input}} = 600\text{Hz}$; 1200cc/sam ; $F_c = 1\text{MHz}$

$$f_{\text{sample}} = \text{at least } 2 \times 600\text{Hz} = 1200\text{Hz}$$

$$1200\text{Hz} \times 1200\text{cc/sam} = 1,440,000 > 1,000,000\text{Hz}$$

→ System is not real time.

c) $f_{\text{max}} = 600\text{Hz}$, 1200cc/sam ,

$$f_{\text{nyquist}} = 2 \times 600\text{Hz} = 1200\text{Hz}$$

$$\text{Minimum Clock: } 1200\text{Hz} \times 1200\text{cc} = \boxed{1,440,000\text{Hz}}$$

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2.

$$x(t) = 5 \cos(14\pi t + \pi/8)$$

$5 \rightarrow$ amplitude, $\pi/8 \rightarrow$ phase, $14 \rightarrow$ frequency

$$\cos(2\pi t) \rightarrow 1 \text{ Hz}$$

$$\cos(14\pi t) \rightarrow 7 \text{ Hz}$$

$$\text{Largest } F_s = 2 \times 7 \text{ Hz} = 14 \text{ Hz}, \quad T_s = 0.07 \text{ s}$$

$$x[n] = 5 \cos(14\pi t + \pi/8) \Big|_{t=0.07n} = \boxed{5 \cos(n\pi + \pi/8)}$$

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3.

$$y[n] = 0.4 y[n-1] + x[n]$$

$$\text{Transfer Function } H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.4z^{-1}}$$

$$\text{Impulse Response } h[n] = \mathcal{Z}^{-1}[H(z)] = 0.4^n u[n]$$

For the system to be BIBO stable, the poles of $H(z)$ must fall within the unit circle of the z -plane.

$H(z) = \frac{1}{1 - 0.4z^{-1}} = \frac{z}{z - 0.4}$, there is a pole at $z = 0.4$, which is inside the unit circle: the system is BIBO stable.

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4. a) Signal is of minimum frequency $f=0\text{Hz}$.

b) Signal is of maximum Nyquist frequency $f=\frac{1}{2}F_s$

c) Zeros within unit circle, signal is damped.

d) Zeros outside unit circle, signal increases by some magnitude e^n .

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5.6/

$X(z)$ is Z-transform of causal signal $x[n]$.

$$x[0] = \frac{X(z)}{u(z)z^{-0}} = \frac{X(z)}{u(0)}$$

$$\lim_{n \rightarrow \infty} x[n] = 0$$

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7.

$$x[n] = \{0, 1, 1, 1, 0\} \quad \text{and} \quad h[n] = \{1.5, 1, 0.5\}$$

Find $y[n] = x[n] * h[n]$.

$$y[n] = \sum_{k=0}^n x[k]h[n-k]$$

$$y[0] = x[0]h[0] = 0.0$$

$$y[1] = x[0]h[1] + x[1]h[0] = 1.5$$

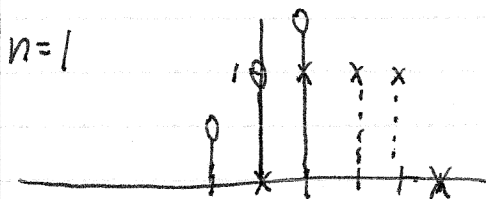
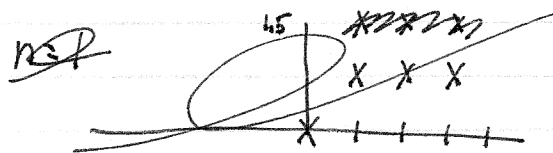
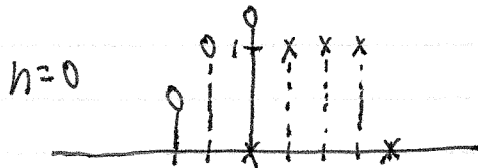
$$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] = 2.5$$

$$y[3] = x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0] = 3.0$$

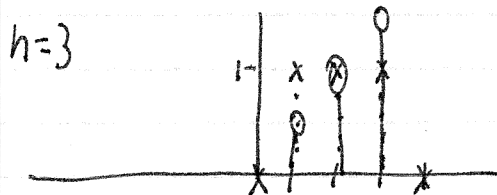
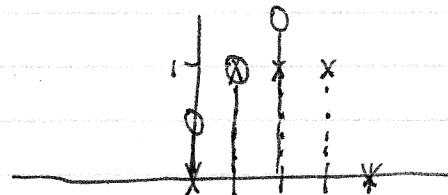
$$y[4] = x[0]h[4] + x[1]h[3] + x[2]h[2] + x[3]h[1] + x[4]h[0] = 1.5$$

$$y[5] = x[0]h[5] + x[1]h[4] + x[2]h[3] + x[3]h[2] + x[4]h[1] + x[5]h[0] = 0.5$$

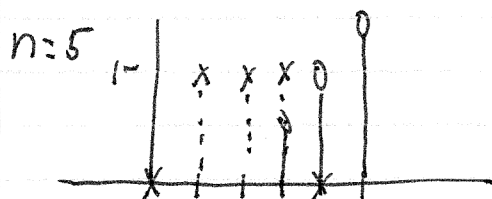
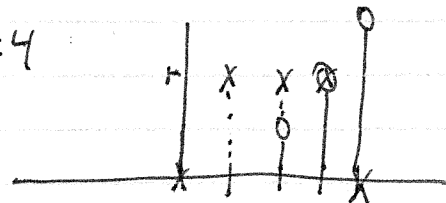
$$y[6] = x[0]h[6] + x[1]h[5] + x[2]h[4] + x[3]h[3] + x[4]h[2] + x[5]h[1] + x[6]h[0] = 0.0$$



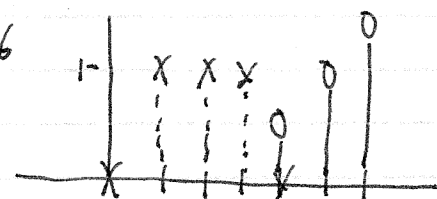
$n=2$



$n=4$



$n=6$



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8.

$$X[n] = u[n] - u[n-4] \quad h[n] = 0.5^n u[n]$$

$$Y(z) = H(z) X(z)$$

$$= \frac{1}{(1-0.5z^{-1})(1-z^{-1})(1+(z-4)^{-1})}$$

$$\frac{1}{2} - 4 = -3.5$$

$$= \frac{A}{(1-0.5z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{C}{(1+(z-4)^{-1})}$$

$$z = \frac{1}{2}, \quad A = -1 \cdot \frac{5}{7} = -\frac{5}{7}$$

$$z = 1, \quad B = 0.5 \cdot \frac{2}{3} = \frac{1}{3}$$

$$z = 3, \quad C = \frac{5}{6} \cdot \frac{2}{3} = \frac{5}{9}$$

$$Y(z) = \frac{-5/7}{(1-0.5z^{-1})} + \frac{1/3}{(1-z^{-1})} + \frac{5/9}{(1+(z-4)^{-1})}$$

$$y[n] = -\frac{5}{7}(0.5^n)u[n] + \left(\frac{1}{3}\right)u[n] + \left(\frac{5}{9}\right)u[n-4]$$