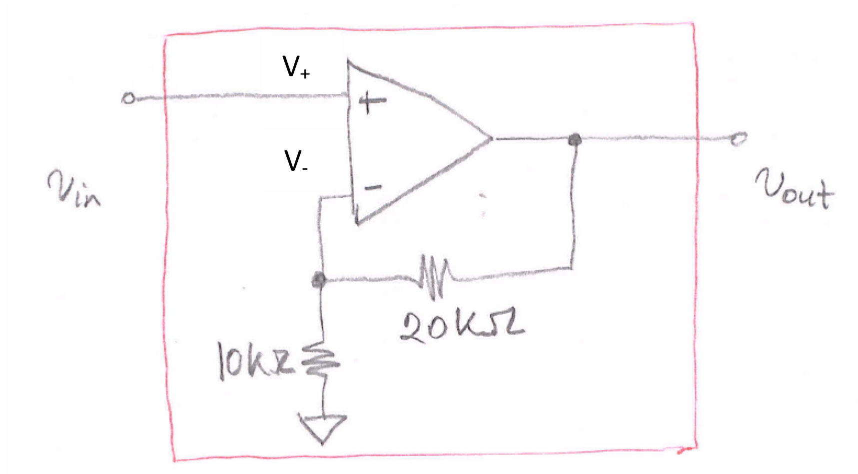
CPE 381: Fundamentals of Signals and Systems
for Computer Engineers

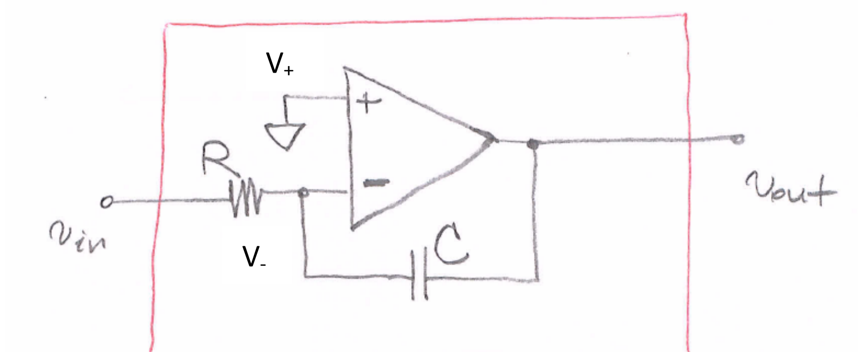
Homework #2 Solution

1. (10 points) What is the transfer function of the following circuits



$$V_{in} = V_+ = V_- = \frac{R_1}{R_1 + R_2} V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} = 3$$



$$V_+ = V_- = 0,\ \ V_{in} - R \cdot i(t) = V_- = 0$$

$$i(t) = \frac{V_{in(t)}}{R}$$

$$V_{out} = V_- - \frac{1}{C}\int_0^t i(\tau)d\tau + V_C^0 = -\frac{1}{C}\int_0^t \frac{V_{in}(t)}{R} d\tau = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$$

For unit-step function V_{in} is constant and

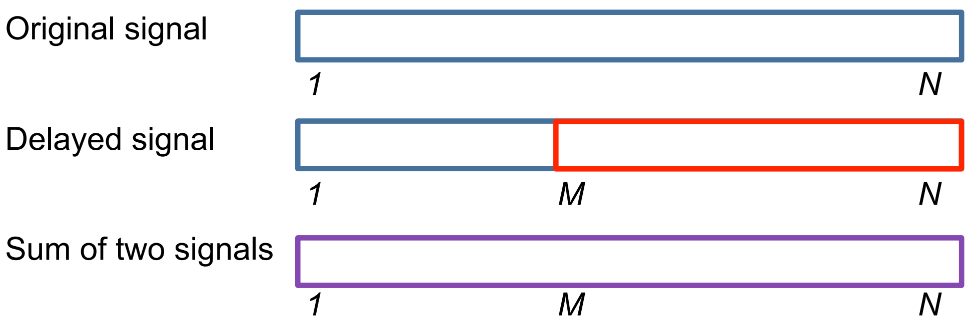
$$\frac{V_{out}}{V_{in}} = -\frac{t}{RC}$$

2. (20 points) Shift in time is equivalent to the shift in array. Therefore, shift of τ seconds is equivalent to shift of M samples.

For sampling frequency F_s, sample interval is $dt = 1/F_s$ and

$$M = \tau / dt = \tau F_s$$

In our case F_s = 8,192 Hz, τ = 0.5 s is equivalent to M = 0.5 * 8,192 = 4,096 samples.

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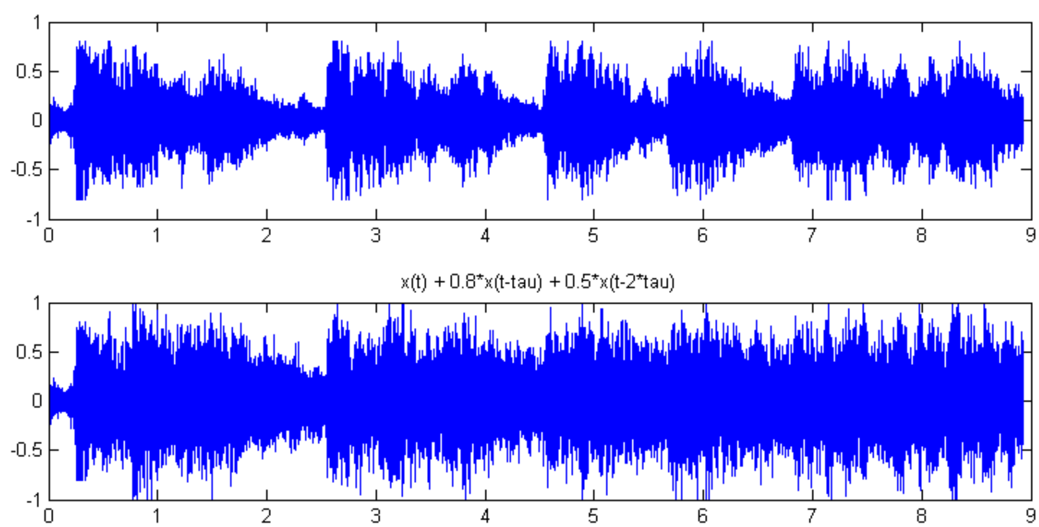
% CPE381 HW2_1
load handel
N=length(y);              % length of sound array
% time
dt=1./Fs;
t=(1:N)*dt;
tau=0.5;                  % delay in seconds [s]
Ndelay=round(Fs*tau);     % shift in samples
y1=[zeros(Ndelay,1); y(1:(N-Ndelay))];     % signal delayed for tau
y2=[zeros(2*Ndelay,1); y(1:(N-2*Ndelay))]; % signal delayed for 2*tau
% final output
yy=y+0.8*y1+0.5*y2;
figure
subplot(211)
plot(t,y)
subplot(212)
plot(t,yy),axis([0 9 -1 1])
title('x(t) + 0.8*x(t-tau) + 0.5*x(t-2*tau)')
% you can even hear the result!
sound(yy)
```



3. (10 points)

Example Find (i) impulse response of capacitor and (ii) its unit step response. $C = 1$ F.

C: $v_c(0) = 0$,

$$v_c(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

Impulse response:

$$i(t) = \delta(t) \quad \Rightarrow \quad v_c(t) = h(t) = \frac{1}{C}\int_0^t \delta(\tau)d\tau = \frac{1}{C}u(t)$$

$C = 1$F, unit-step response

$$v_c(t) = \int_{-\infty}^{\infty} h(t-\tau)i(\tau)d\tau = \int_{-\infty}^{\infty} \frac{1}{C}u(t-\tau)u(\tau)d\tau$$

$$v_c(t) = \frac{1}{C}\int_0^t d\tau = \frac{1}{C}r(t)$$

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4. (15 points) Find transfer function of the circuit in problem 1.b.

If we use complex impedance Z_c, output will be

$$V_{out} = V_- - Z_c I = -\frac{1}{Cs}\cdot\frac{V_{in}}{R}$$

$$\frac{V_{out}}{V_{in}} = H(s) = -\frac{1}{RCs}$$

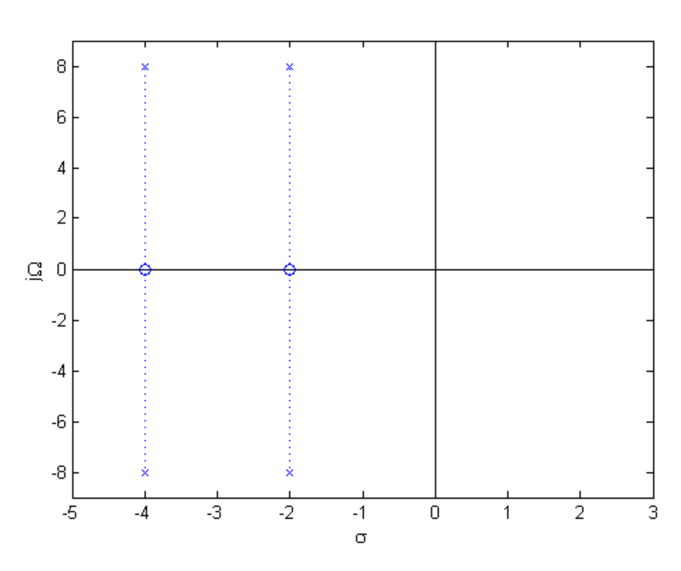
5. (20 points)

```
%CPE381: HW2_3

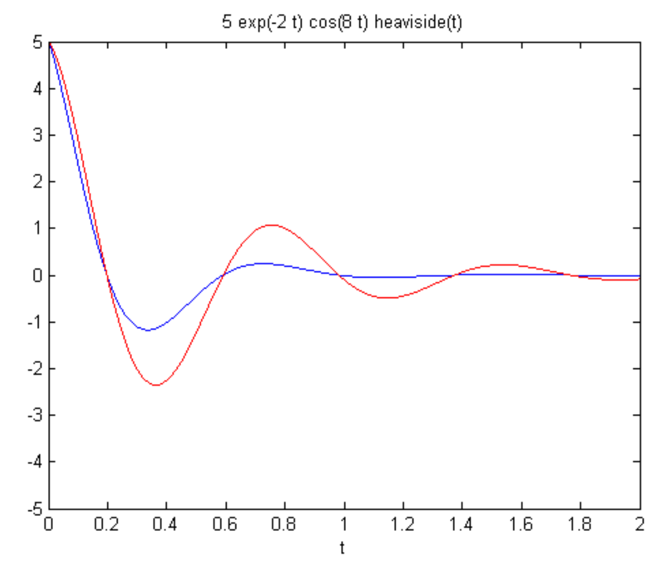
syms t x1 x2
x1=5*exp(-2*t)*cos(8*t)*heaviside(t);
x2=5*exp(-4*t)*cos(8*t)*heaviside(t);

X1=laplace(x1)
% X1 = 5*(s+2)/(s^2+4*s+68)
% plot
splane([5 10],[1 4 68])

X2=laplace(x2)
% X2 = 5*(s+4)/(s^2+8*s+80)
figure
% plot
splane([5 20],[1 8 80])
```



Zeros and poles of X1 and X2



Output y1 and y2

Discuss the changes in the s plane and describe their effect on function in time domain

Zeros and poles shifted to the left (larger absolute values of σ); consequently, signal in time domain is more attenuated (dumped).

6. (10 points)

Causal functions and constants:	αf(t)	⇔	αF(s)
Linearity:	αf(t) + βg(t)	⇔	αF(s) + βG(s)
Time shifting:	f(t – α)	⇔	$e^{-\alpha s}$F(s)
Frequency shifting:	$e^{\alpha t}$f(t)	⇔	F(s – α)
Multiplication by t:	tf(t)	⇔	$-\frac{dF(s)}{ds}$
Derivative:	$\frac{df(t)}{dt}$	⇔	sF(s) – f(0-)
Second derivative:	$\frac{d^2f(t)}{dt^2}$	⇔	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral:	$\int_{0-}^{t} f(t')dt$	⇔	$\frac{F(s)}{s}$
Expansion/Contraction:	f(αt)α ≠ 0	⇔	$\frac{1}{\lvert a \rvert}F(\frac{s}{\alpha})$

7. (15 points) Example 3.13

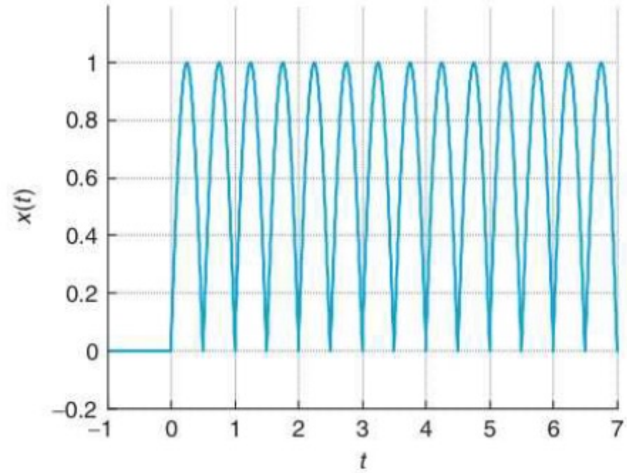


FIGURE 3.13
Full-wave rectified causal signal.

Solution

The first period of the full-wave rectified signal can be expressed as

$$x_1(t) = \sin(2\pi t)u(t) + \sin(2\pi(t-0.5))u(t-0.5)$$

and its Laplace transform is

$$X_1(s) = \frac{2\pi(1+e^{-0.5s})}{s^2+(2\pi)^2}$$

And the train of these sinusoidal pulses

$$x(t) = \sum_{k=0}^{\infty} x_1(t-0.5k)$$

will then have the following Laplace transform:

$$X(s) = X_1(s)[1+e^{-s/2}+e^{-s}+\cdots] = X_1(s)\frac{1}{1-e^{-s/2}} = \frac{2\pi(1+e^{-s/2})}{(1-e^{-s/2})(s^2+4\pi^2)}$$