Homework #4 Solution

1. (20 points)

Magnitude response from poles and zeros—MATLAB

Consider the following filters with the given poles and zeros and dc constant:

$H_1(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zeros $z_1 = 1, z_{2,3} = 1 \pm j\pi$

$H_2(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zeros $z_{1,3} = \pm j\pi$

$H_3(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zero $z_1 = 1$

Use MATLAB to plot the magnitude responses of these filters and indicate the type of filters they are.

See section 5.7.3 in the textbook:

$$H(s) = \frac{\prod_i (s - z_i)}{\prod_k (s - p_k)}$$

$$H_1(s) = \frac{(s-1)(s-1-j\pi)(s-1+j\pi)}{(s+1)(s+1-j\pi)(s+1+j\pi)} = \frac{(s-1)\left((s-1)^2+\pi^2\right)}{(s+1)\left((s+1)^2+\pi^2\right)} = \frac{s^3-3s^2+\left(3+\pi^2\right)s-\left(1+\pi^2\right)}{s^3+3s^2+\left(3+\pi^2\right)s+\left(1+\pi^2\right)}$$

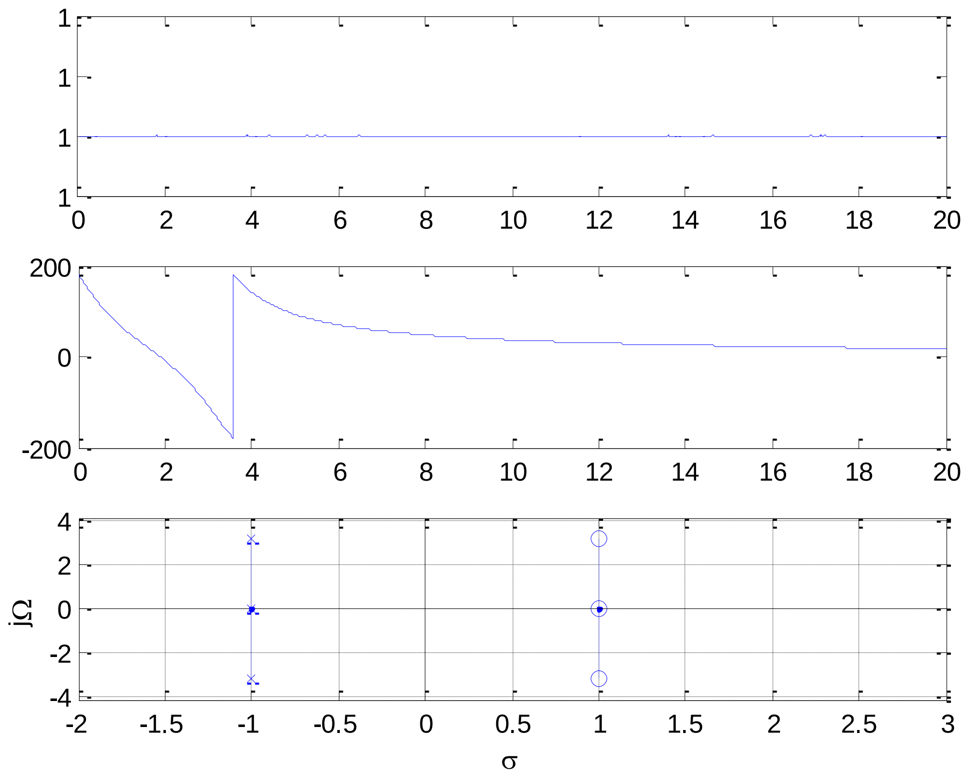
$$H_2(s) = \frac{s^2+\pi^2}{(s+1)\left((s+1)^2+\pi^2\right)}$$

$$H_3(s) = \frac{s-1}{(s+1)\left((s+1)^2+\pi^2\right)}$$

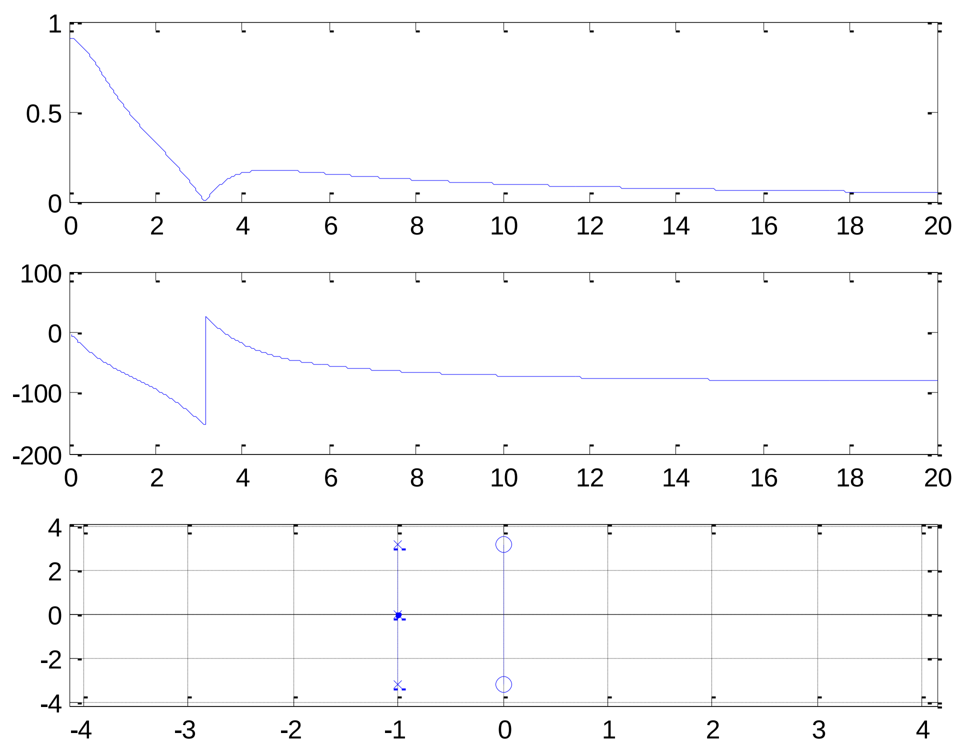
```
% CPE381 HW4_3
clear all; clf
n=[1 -3 3+pi^2 -(1+pi^2)];
d=[1 3 3+pi^2 (1+pi^2)];
figure(1)
wmax=20;
freqresp_s(n,d,wmax)
n1=[0 1 0 pi^2]; d1=d;
figure(2)
freqresp_s(n1,d1,wmax)
n2=[0 0 1 -1]; d2=d;
figure(3)
freqresp_s(n2,d2,wmax)

function
    [w,Hm,Ha]=freqresp_s(b,a,wmax)
w=0:0.01:wmax;
H=freqs(b,a,w);
Hm=abs(H);
Ha=angle(H)*180/pi;
figure
subplot(311)
plot(w,Hm)
subplot(312)
plot(w,Ha)
subplot(313)
splane(b,a)
```

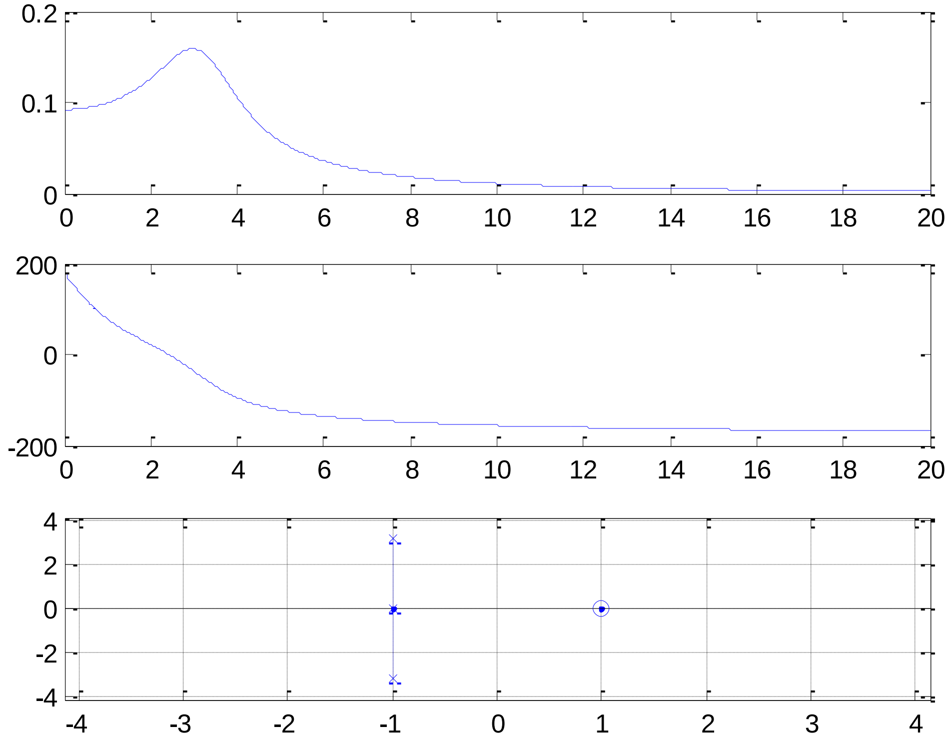
```
function splane(num,den)
% function splane
% input: coefficients of numerator (num) and
denominator (den) in
% decreasing order
% output: pole/zero plot
% use: splane(num,den)
%
z=roots(num);
p=roots(den);
A1=[min(imag(z)) min(imag(p))];A1=min(A1)-1;
B1=[max(imag(z)) max(imag(p))];B1=max(B1)+1;
N=20;
D=(abs(A1)+abs(B1))/N;
im=A1:D:B1;
Nq=length(im);
re=zeros(1,Nq);
A=[min(real(z)) min(real(p))];A=min(A)-1;
B=[max(real(z)) max(real(p))];B=max(B)+1;
stem(real(z),imag(z),'o:')
hold on
stem(real(p),imag(p),'x:')
hold on
%plot(re,im,'k');xlabel('\sigma');ylabel('j\Om
ega')
grid
% axis([A -A min(im) max(im)])
axis([min(im) max(im) min(im) max(im)]);
hold off
```



H1 is an all-pass filter.



H2 is a notch filter; it behaves like low-pass filter at low frequencies.



H3 is a low-pass filter.

2. (20 points) An ideal low pass filter *H(s)* with zero phase and magnitude response:

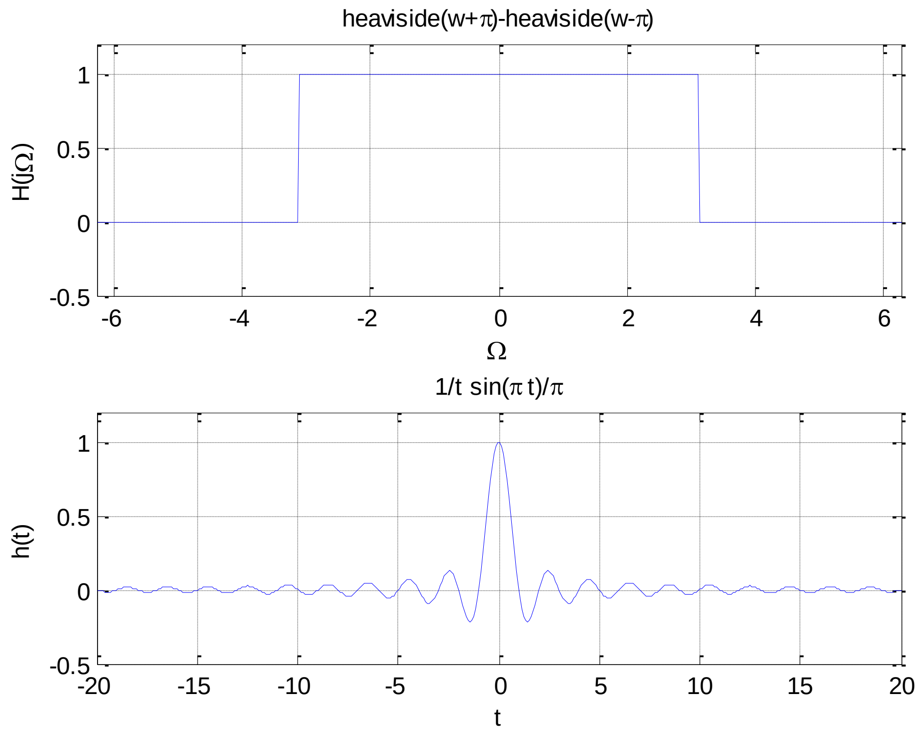
$$|H(j\Omega)| = \begin{cases} 1 & -\pi \le \Omega \le \pi \\ 0 & otherwise \end{cases}$$

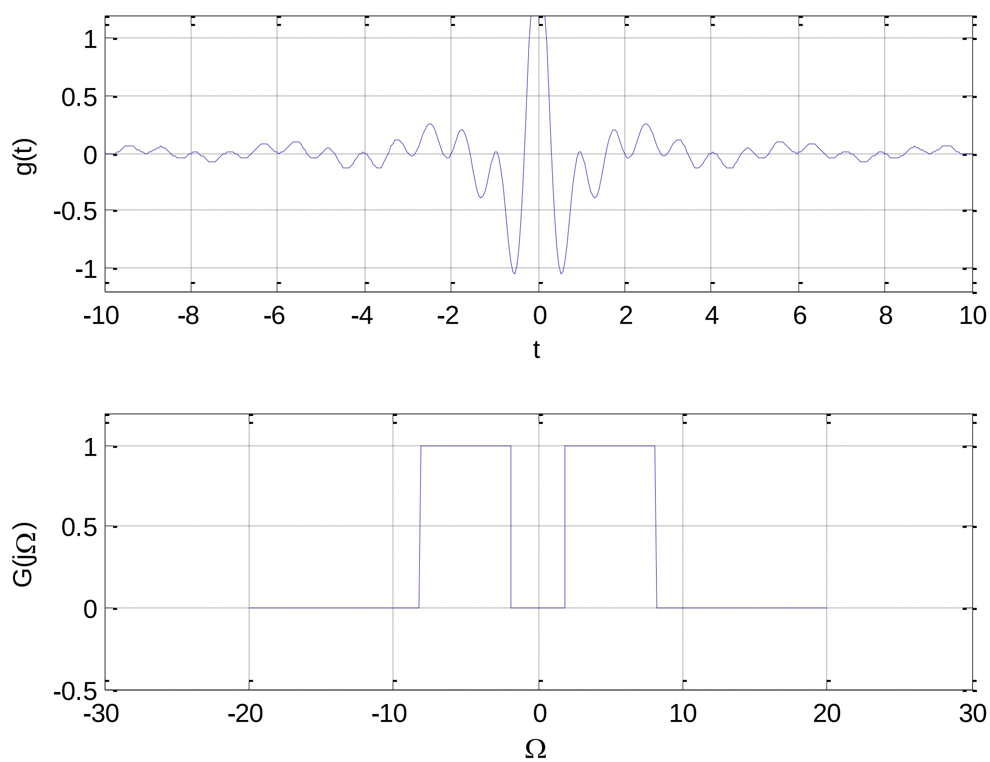
a) The impulse response is h(t) = sin(πt)/ πt , which is non-causal since h(t) $\neq$ 0 for t<0. (textbook 5.7.2. and page 365)

b) What is the effect of shifting the central frequency of the ideal filter for 5π ?

The bandpass filter. It can be implemented using ideal low-pass filter by shifting the central of the ideal low-pass filter

g(t) = 2*h(t)*cos(5*π*t)
and G(jΩ) = H(j(Ω - 5π)) + H(j(Ω + 5π))


heaviside(w+π)-heaviside(w-π)
H(jΩ)
Ω
1/t sin(π t)/π
h(t)
t


g(t)
t
G(jΩ)
Ω

3. (10 points)
A 12-bit AD converter is used to digitize signal with negative reference $V_{R-} = 0.5V$ and positive reference $V_{R+} = 2.5V$.

a) (3 points) What is the quantization step?
b) (3 points) What is the output of the AD converter for $V_{in} = 2.2$ V ?
c) (2 points) What is the output of the AD converter for $V_{in} = 0.4$ V ?
d) (2 points) What is the output of the AD converter for $V_{in} = 3$ V ?

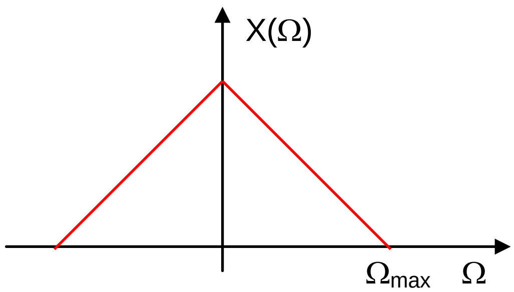
a) The quantization step is
$\Delta=(V_{r+} - V_{r-})/(2^{12}-1) = (2.5-0.5)/4095 \approx 0.49$ mV

b) The output of the AD converter is
$ADout=(V_{in} - V_{r-})/ \Delta = (2.2 - 0.5) / 0.49 \text{ mV} = 3481$

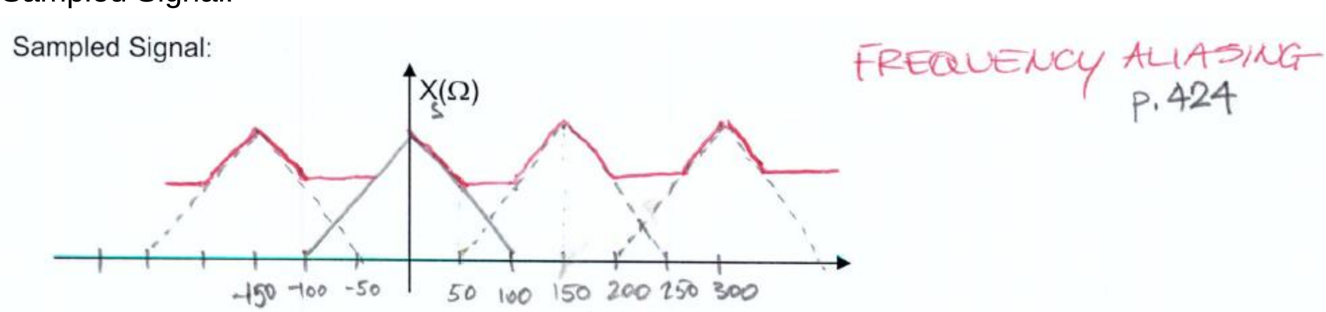
c) The output of the AD converter is 0

d) The output of the AD converter is 4095 (all ones)

4. (10 points) Figure below represents spectrum of band limited signal with maximum frequency $\Omega_{max} = 100$ Hz. Represent spectrum of the same signal sampled at Fs = 150 Hz. Describe the effect.



Sampled Signal:



5. (10 points) We are trying to decide between a 12-bit and 16 bit ADC. The signals in this application are known to have frequencies that do not exceed 5KHz. The dynamic range of the signal is 2.5V. Determine an appropriate sampling period and compare the percentage of error the two ADCs of interest.

   Take a look at example 8.5, page 519.

   Fs > 2Fmax → Fs>10KHz, for Fs=20,000 samples/sec → Ts=50μs.

   ε [%] = 100·Δ / range [%] = 100*(range/(2^n - 1)/range = 100 / (2^n – 1)

   ε_1 = 100/4095 = 0.024 %

   ε_2 = 100/65535 = 0.0015 %

6. A discrete time IIR system with input x[n] and output y[n] is represented by the equation:

   y[n] = 0.2·y[n-2] + x[n] n ≥ 0

a) find the impulse response *h(n)* of the system, by assuming that initial conditions are zero (y[n]=h[n]=0, n<0) and x[n]=δ[n].

   h[0] = 0.2*h[-2] + 1 = 1
   h[1] = 0.2*h[-1] + 0 = 0
   h[2] = 0.2*h[0] + 0 = 0.2
   h[3] = 0.2*h[1] + 0 = 0
   h[4] = 0.2*h[2] + 0 = 0.2^2

or

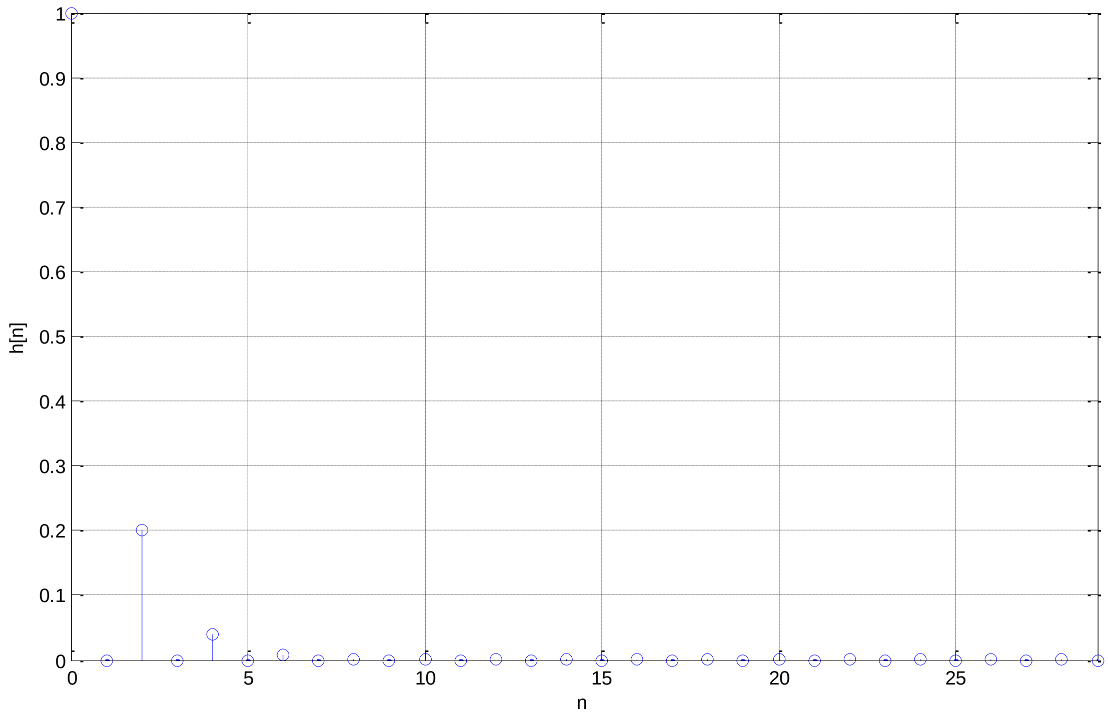
$$h[n]=\begin{cases}0.2^{n/2} & \text{for } n\geq 0 \text{ and even}\\ 0 & \text{otherwise}\end{cases}$$

b) find the impulse response alternatively by using recursive relation between x[n] and y[n].

   h[n] = 0.2*h[n-2] + δ[n]
   h[n-2] = 0.2*h[n-4] + δ[n-2]
   …
   h[n] = δ[n] + 0.2*δ[n-2] + 0.2*δ[n-4] + … (the same result as above)

c) plot h[n] using MATLAB function filter.

```
clear all;clf
a=[1 0 -0.2];
b=1;
x=[1 zeros(1,29)];
h=filter(b,a,x);
n=0:29;
figure(1)
stem(n,h); axis([0 29 0 1]);
grid;ylabel('h[n]'); xlabel('n')
```



7. (15 points) An FIR filter is represented as:

$$y[n]=\sum_{k=0}^{5} k\cdot x[n-k]$$

a) find and plot the impulse response of this filter.

y[n] = 0*δ[n] + 1*δ[n-1] +2*δ[n-2] +3*δ[n-3] +4*δ[n-4] +5*δ[n-5]

b) is this a causal and stable filter? Explain.

The filter is causal since the output depends only on previous values of the input and h[n]=0 for n < 0.

c) find and plot the unit-step response s[n] for this filter.

for x[n] = u[n]

$$s[n]=\sum_{k=1}^{5} k\cdot u[n-k]=u[n-1]+2u[n-2]+3u[n-3]+4u[n-4]+5u[n-5]$$

d) what is the maximum value of the output if the maximum input is 5?

if |x[n]| <5 → |y[n]| < 5 * $\sum_{k=0}^{5} k\cdot |x[k]|$ = 75   the bound is 75.

e) plot h[n] and s[n] using MATLAB function filter.

```
clear all; clf
b=[0 1 2 3 4 5];
a=1;
x=[1 zeros(1,100)];
h=filter(b,a,x);
n=0:19;
figure(1)
subplot(211)
stem(n,h(1:20)); ylabel('h[n]')

% unit step response
x=ones(1,100);
s=filter(b,a,x);
n=0:19;
subplot(212)
stem(n,s(1:20));ylabel('s[n]'); xlabel('n')
```