LABORATORY # 05

Basic Filters and Frequency Response

Purpose

The goal of this laboratory is to study the frequency characteristics of Low and High Pass Filters. Specifically, signal amplitude, signal phase, and how they are related to input frequency will be explored. In addition, the concept of cutoff frequency will be introduced.

Theoretical Background

A filter is a device or configuration that allows a specific range of input frequencies to pass and rejects signals whose frequency is outside of the specified range. Among the many types of filters, in this laboratory you will be introduced to basic Low and High Pass Filters. These filters consist of nothing more than a configuration made up of a single resistor and capacitor.

Low Pass Filter

Low Pass Filters allow low frequencies to pass and block high frequencies. The ideal Low Pass Filter should allow all frequencies below some specified frequency, call it f_c, to pass and allow no frequencies above f_c to pass. It is not possible to design an ideal Low Pass Filter. Therefore the design characteristics of the filters used in this laboratory will be non-ideal. It is important to be aware there are more sophisticated filtering devices on the market that approach the ideal case.

Figure 5.1, on the next page, shows a typical configuration for a Low Pass Filter. Through the mathematical development to follow it will be shown that the circuit is acting as a Low Pass Filter at the output terminal A-B. The input frequency and amplitude of E, will directly influence the magnitude and phase of Vc. In other words, at certain input frequencies the Low Pass Filter will operate on the input **E** to produce $\mathbf{V_c}$. The frequency where this action occurs is determined by values chosen for the resistor (R) and the capacitor (C).

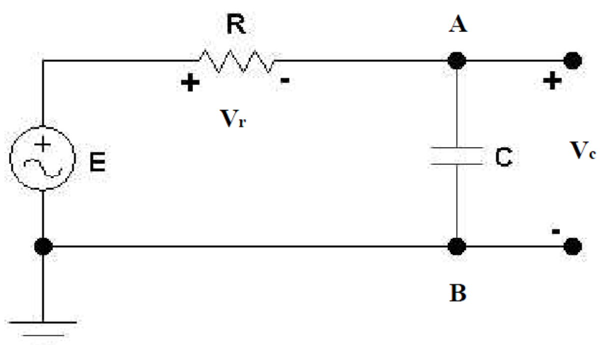


Figure 5.1

The voltage across the capacitor in Figure 5.1 can be written as

$$V_c = I(-jX_c)$$

Where

$$I = \frac{E}{Z}$$

And

$$Z = R - jX_c$$

After substitution V_c can be rewritten as

$$V_c = \frac{-jEX_c}{R - jX_c}$$

Recalling that

$$X_c = \frac{1}{2\pi fC}$$

And substitution leads to

$$\frac{V_c}{E} = \frac{1}{1 + j2\pi fRC}$$

If we use the definition of gain, and take the magnitude, the final form can be written as

$$\left|\frac{V_c}{E}\right| = \frac{1}{\sqrt{(1 + (2\pi fRC)^2)}}$$

From inspection of Equation 11.1 it is apparent that as the frequency (f) increases, the magnitude of the gain decreases. In short, the circuit in Figure 11.1 works as a Low Pass Filter. Also take note that the product RC has an effect on the overall gain. The phase angle for the Low Pass Filter can be expressed as

$$\tan(\phi) = -2\pi fRC$$

Note that the phase is initially at zero radians, where ideal gain is at its maximum. This shows that the phase of the output will approach -π/2 radians as the frequency increases.

High Pass Filter

High Pass Filters allow high frequencies to pass and block low frequencies. The ideal High Pass Filter should allow all frequencies above some specified frequency, call it f_c, to pass and allow no frequencies below f_c to pass. It will be shown mathematically that the circuit is acting as a High Pass Filter at the output terminal A-B, in Figure 5.2.

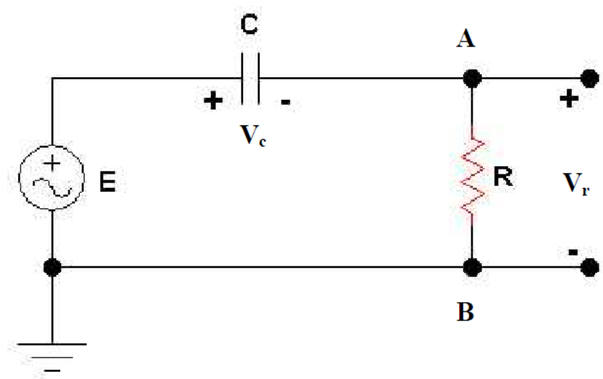


Figure 5.2

The voltage across the resistor, in Figure 5.2, can be written as

$$V_r = IR$$

$$I = \frac{E}{Z}$$

$$Z = R - jX_c$$

Using the definition of Xc, and making substitutions, allows the equation to be rewritten as

$$\frac{V_r}{E} = \frac{j2\pi f RC}{1 + j2\pi f RC}$$

Using the definition of gain and taking the magnitude gives the final form of

$$\left|\frac{V_r}{E}\right| = \frac{2\pi f RC}{\sqrt{(1 + (2\pi f RC)^2)}}$$

Noticing that 2RC in above Equation is a constant for any given circuit leads to

$$\left|\frac{V_r}{E}\right| \propto \frac{f}{\sqrt{1 + f^2}}$$

This equation is recognizable as an increasing function of frequency that approaches unity as the frequency increases. Therefore, the circuit in Figure 5.2 works as a High Pass Filter. The phase angle for the High Pass Filter can be expressed as

$$\tan(\phi) = \frac{1}{2\pi f RC}$$

Note the phase is initially at π/2 radians, where ideal gain is zero. It shows that the phase of the output will approach zero radians as the frequency increases.

Cutoff Frequency

The cutoff frequency, denoted by fc, is defined as the frequency at which the magnitude of gain is equal to

$$\frac{1}{\sqrt{2}}$$

Considering the Low Pass Filter configuration

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1+(2\pi f RC)^2)}}$$

Replacing f with f_c, and then solving for the cutoff frequency yields

$$f_c = \frac{1}{2\pi RC}$$

This is true for both Low and High Pass Filters. You determine the cutoff frequency by setting the product value of (RC). Keeping in mind that a low pass configuration will pass frequencies below f_c and the high pass configuration will pass frequencies above f_c.

Theoretical Analysis

Let the input signal be E(t) = 42 cos(t), with = 2f, R = 1 K, and C = 1uF for both the Low and High Pass Filter sections. <u>Note E = 4. You will need this below.</u>

Low Pass Filter

Use the circuit in Figure 5.1 for steps 1 through 4.

- Fill up a table similar to the one given below. (Table 5.1).
- Plot Gain Magnitude vs. frequency. Use data from Table 5.1.
- Plot Phase vs. frequency. Use data from Table 5.1.

Table 5.1

f (HZ)	Gain Magnitude	Phase Angle (Degree)
25	986.923	-8.918
50	953.159	-17.424
75	903.847	-25.209
100	846.126	-32.116
150	727.351	-43.274
200	622.443	-51.459
250	536.893	-57.491
300	468.573	-62.028
400	369.669	-68.282
500	303.298	-72.326
600	256.391	-75.129
800	195.132	-78.736
1000	157.173	-80.948

High Pass Filter

Use the circuit in Figure 5.2 for steps 1 through 4.

- Fill up a table similar to the one given below. (Table 5.2).
- Plot Gain Magnitude vs. frequency. Use data from Table 5.2.
- Plot Phase vs. frequency. Use data from Table 5.2.

Table 5.2

f (HZ)	Gain Magnitude	Phase Angle (Degree)
25	155.025	81.082
50	299.442	72.576
75	425.918	64.791
100	531.637	57.884
150	685.471	46.726
200	782.162	38.541
250	843.298	32.509
300	883.165	27.972
400	929.010	21.718
500	952.797	17.674
600	966.502	14.871
800	980.736	11.264
1000	987.546	9.052

Simulation

Low Pass Filter

Draw the circuit similar to Figure 5.3 on the next page. Note the connection of a new instrument. This instrument is called a Bode Plotter. The Bode Plotter produces a graph of a circuit's frequency response. It is capable of measuring a signal's voltage gain and phase shift. Both gain and phase are plotted against frequency on the screen. The vertical and horizontal scales, F and I, should be preset to display the area of interest on the Bode screen.
For the simulation section let your input signal be defined as $e(t) = 4\sqrt{2}\cos(wt)$,
where = 2f, f = 100 Hz, R = 1 K, and C = 1uF.

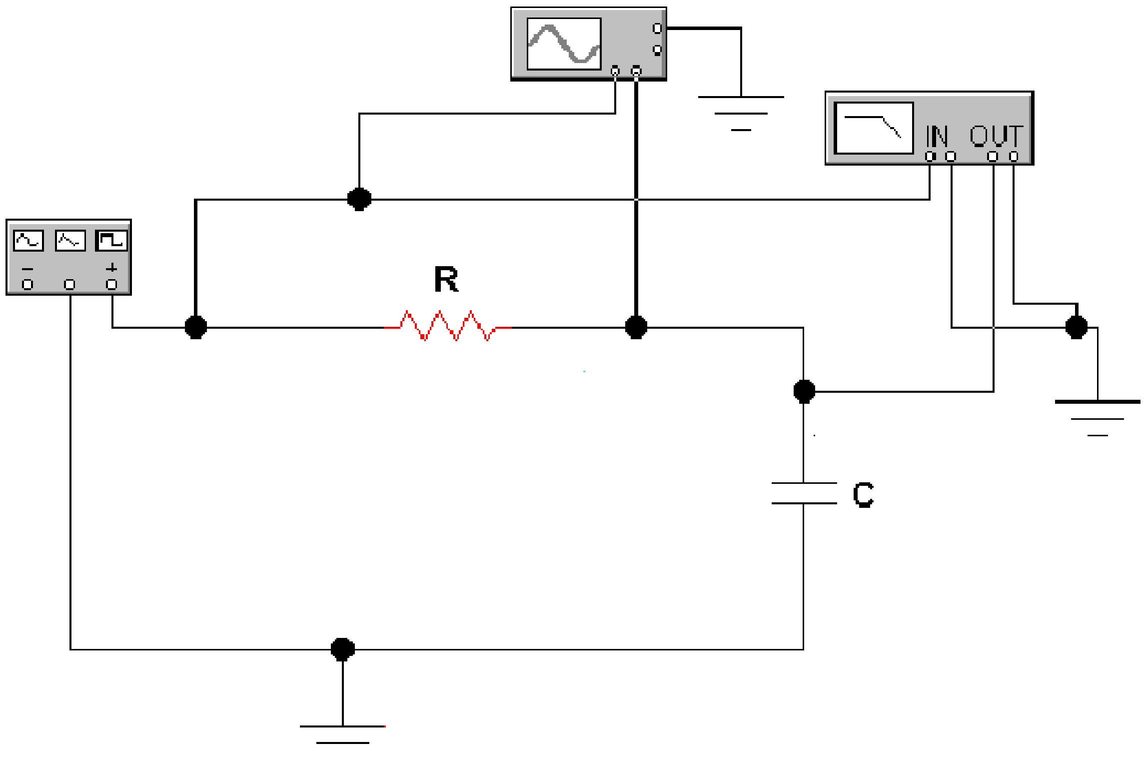


Figure 5.3

- Open the Bode Plotter. Set the plotter for a magnitude reading by depressing the Magnitude button. Then depress Linear (Lin) buttons under both the Vertical and Horizontal headings. Next Set I = 0 and F = 1 under the Vertical heading, and I = 1 Hz and F = 2 kHz under the Horizontal heading. See Figure 5.4 below.
- Set up your function generator with the defined inputs and simulate the circuit. Make a printout of the Bode Plotter output. Interpret the output of the plot. Does it show Low Pass or High Pass Filter characteristics?
- Determine the cutoff frequency by moving the cursor line. Position the cursor line by clicking on the left and right arrows. The cursor position is displayed in the windows next to the arrows. Recall the cutoff frequency occurs when the output magnitude is 0.707, the magnitude of the input. Make a print of the output with the cursor line positioned at the cutoff frequency on the screen.
- Make the necessary changes in settings on the Bode Plotter in order to get Phase characteristic. Depress the Phase button. Then simulate the circuit and make a printout of the output.

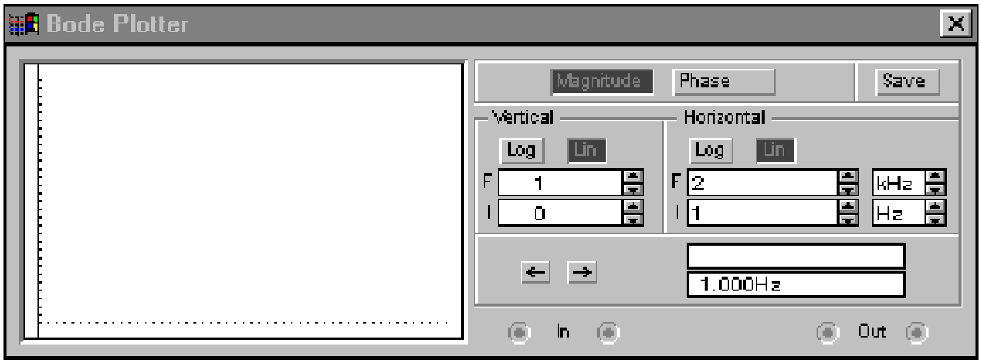


Figure 5.4

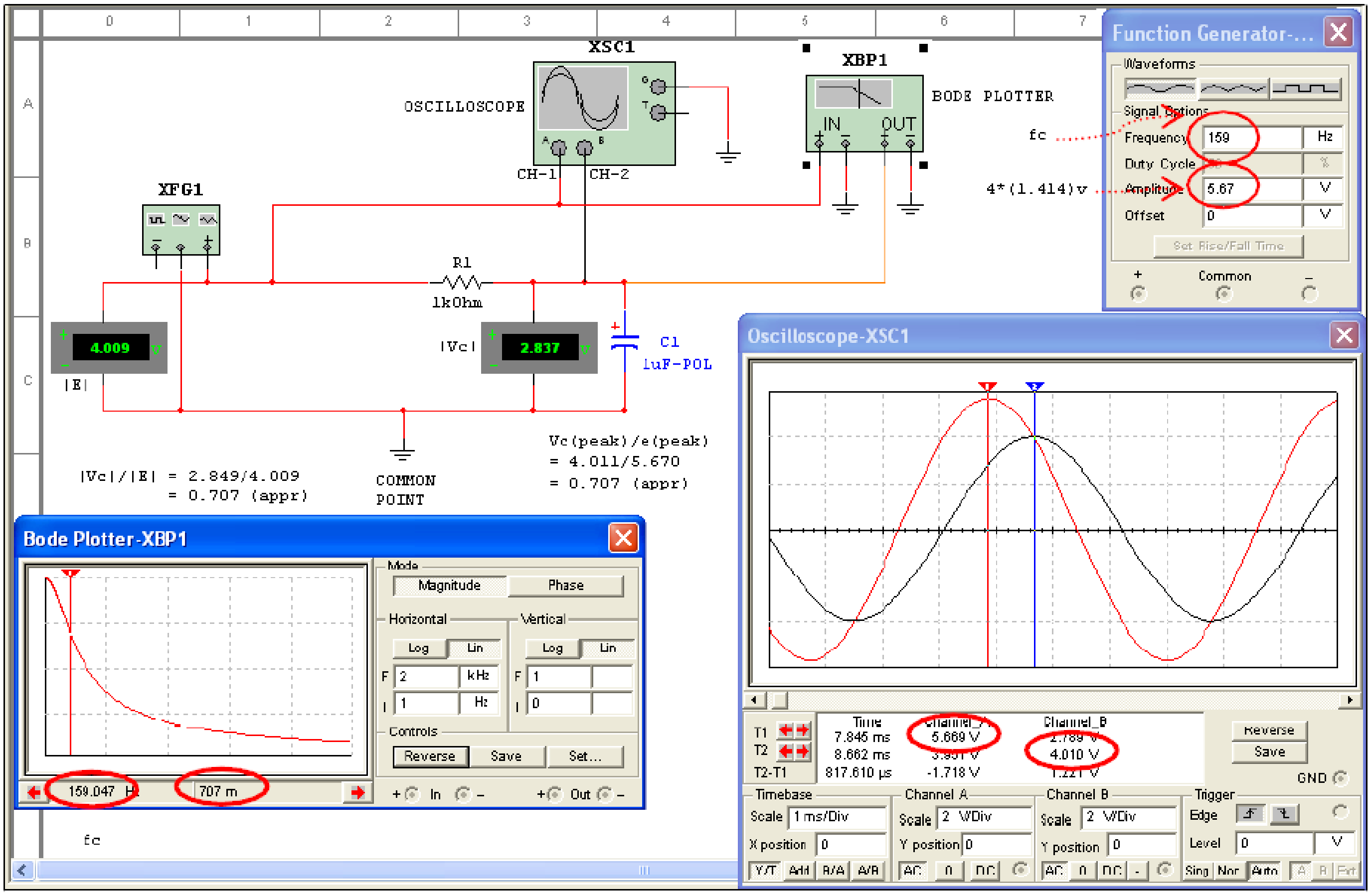
High Pass Filter

Use the same circuit as in Figure 5.3 but switch the positions of R and C to convert the circuit into a High Pass Filter configuration. Use the same values from the Low Pass Filter simulation for your input and circuit parameters. Repeat steps 1 through 3 from Low Pass Filter simulation. However, you will need to reset the Bode Plotter settings to get the magnitude and phase characteristics for the High Pass Filter. Turn the circuit on and adjust F and I for both the Vertical and Horizontal settings. Once you have a viewable output, stop the simulation. <u>Now restart the simulation and take your data</u>.

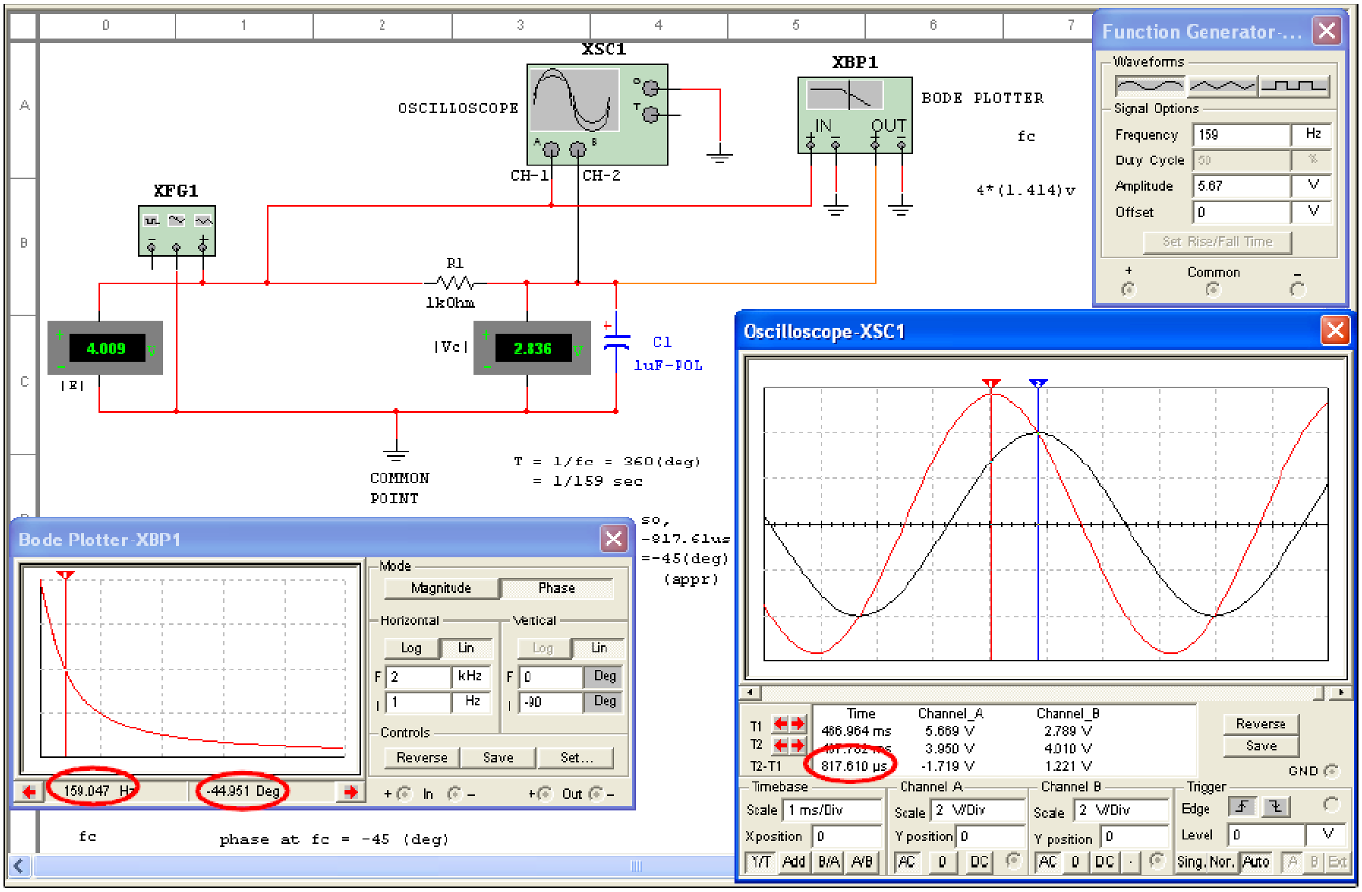
Laboratory Procedure

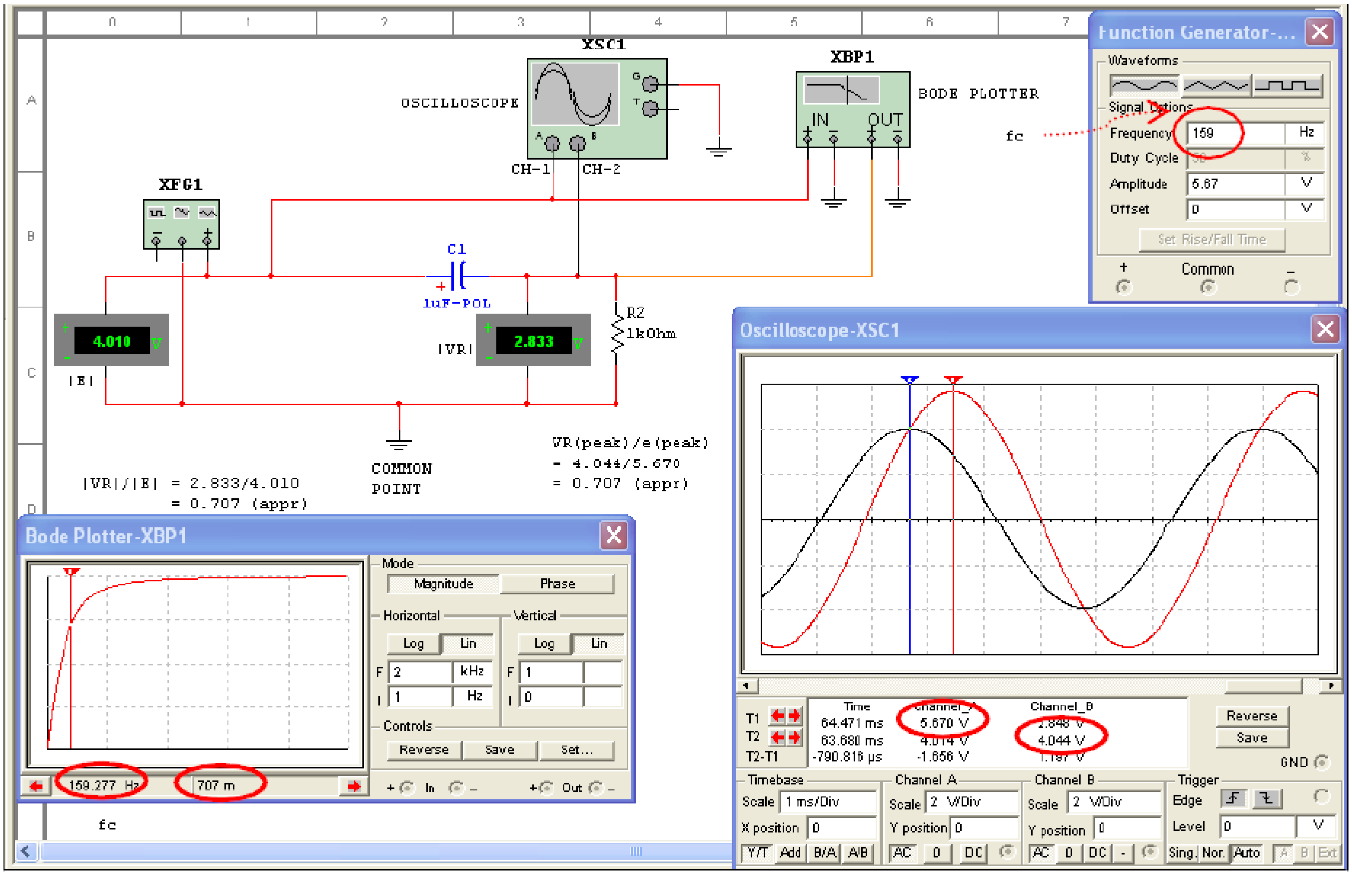
Build both a Low Pass Filter (Figure 5.1) and a High Pass Filter (Figure 5.2) in the laboratory. You should use the Oscilloscope and Multimeter to make your measurements. Follow all the steps from the Theoretical Analysis.

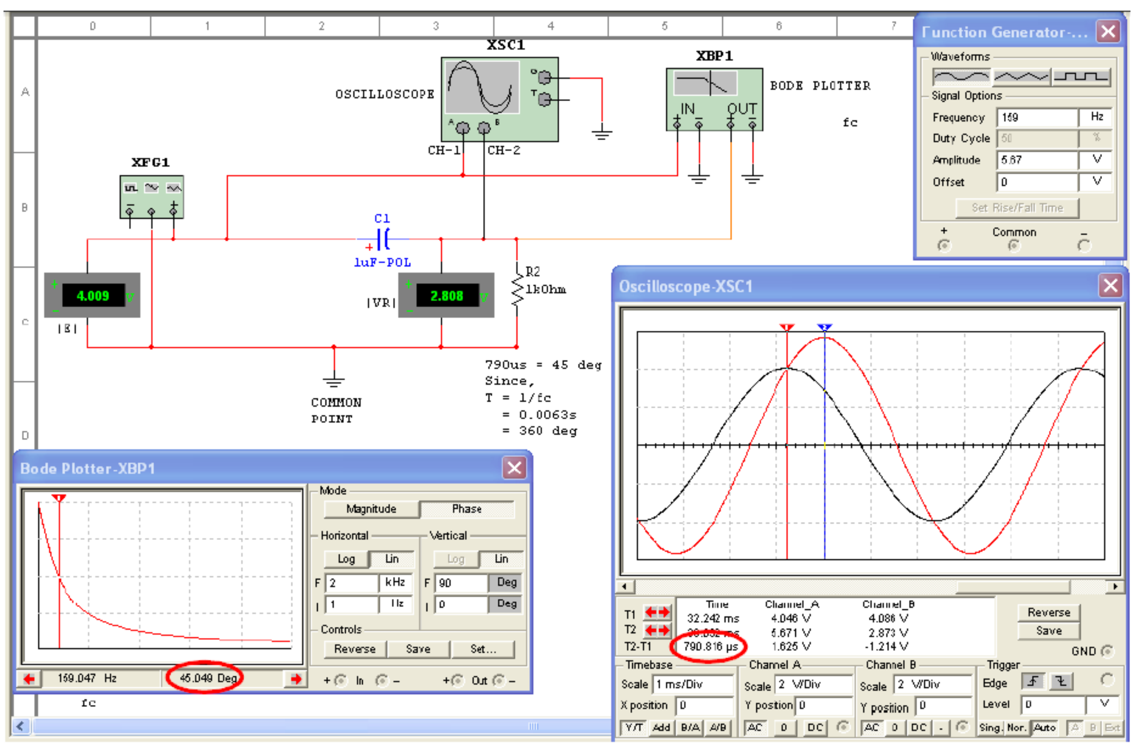
Simulation Results



XSC1
OSCILLOSCOPE
CH-1
CH-2
XBP1
BODE PLOTTER
IN
OUT
XFG1
R1
1kOhm
4.009
|E|
|Vc|
2.837
C1
1uF-POL
|Vc|/|E| = 2.849/4.009
= 0.707 (appr)
COMMON
POINT
Vc(peak)/e(peak)
= 4.011/5.670
= 0.707 (appr)
fc
4*(1.414)v
Function Generator-...
Waveforms
Signal Options
Frequency 159 Hz
Duty Cycle
Amplitude 5.67 V
Offset 0 V
Set Rise/Fall Time
Common
Oscilloscope-XSC1
Time
7.845 ms
8.662 ms
817.610 µs
Channel_A
5.669 V
-1.718 V
Channel_B
4.010 V
Reverse
Save
GND
Timebase
Scale 1 ms/Div
X position 0
Channel A
Scale 2 V/Div
Y position 0
Channel B
Scale 2 V/Div
Y position 0
Trigger
Edge
Level 0 V
Bode Plotter-XBP1
Mode
Magnitude
Phase
Horizontal
Vertical
Log
Lin
F 2 kHz
I 1 Hz
F 1
I 0
Controls
Reverse
Save
Set...
159.047 Hz
707 m
In
Out
fc


XSC1
OSCILLOSCOPE
CH-1
CH-2
XBP1
BODE PLOTTER
fc
4*(1.414)v
XFG1
R1
1kOhm
4.009
|E|
|Vc|
2.836
C1
1uF-POL
COMMON
POINT
T = 1/fc = 360(deg)
= 1/159 sec
so,
-817.61us
=-45(deg)
(appr)
fc
phase at fc = -45 (deg)
Function Generator
Frequency 159 Hz
Amplitude 5.67 V
Offset 0 V
Bode Plotter-XBP1
159.047 Hz
-44.951 Deg
Oscilloscope-XSC1
T1 466.964 ms 5.669 V 2.789 V
T2-T1 817.610 µs -1.719 V 1.221 V
Timebase Scale 1 ms/Div
Channel A Scale 2 V/Div
Channel B Scale 2 V/Div


XSC1
OSCILLOSCOPE
XBP1
BODE PLOTTER
fc
XFG1
C1
1uF-POL
R2
1kOhm
4.010
|E|
2.833
|VR|
CH-1
CH-2
COMMON
POINT
|VR|/|E| = 2.833/4.010
= 0.707 (appr)
VR(peak)/e(peak)
= 4.044/5.670
= 0.707 (appr)
Function Generator-...
Waveforms
Signal Options
Frequency 159 Hz
Duty Cycle
Amplitude 5.67 V
Offset 0 V
Set Rise/Fall Time
Common
Oscilloscope-XSC1
Time
Channel_A
Channel_B
T1 64.471 ms 5.670 V 2.848 V
T2 63.680 ms 4.014 V 4.044 V
T2-T1 -790.816 µs -1.656 V 1.197 V
Reverse
Save
GND
Timebase
Scale 1 ms/Div
X position 0
Channel A
Scale 2 V/Div
Y position 0
Channel B
Scale 2 V/Div
Y position 0
Trigger
Edge
Level 0 V
Bode Plotter-XBP1
Mode
Magnitude
Phase
Horizontal
Vertical
Log
Lin
F 2 kHz
I 1 Hz
F 1
I 0
Controls
Reverse
Save
Set...
159.277 Hz
707 m
fc


XSC1
OSCILLOSCOPE
XBP1
BODE PLOTTER
fc
XFG1
C1
1uF-POL
R2
1kOhm
4.009
|E|
2.808
|VR|
CH-1
CH-2
COMMON
POINT
790us = 45 deg
Since,
T = 1/fc
= 0.0063s
= 360 deg
Function Generator-...
Waveforms
Signal Options
Frequency 159 Hz
Duty Cycle
Amplitude 5.67 V
Offset 0 V
Set Rise/Fall Time
Common
Oscilloscope-XSC1
Time
Channel_A
Channel_B
T1 32.242 ms 4.046 V 4.086 V
T2 5.671 V 2.873 V
T2-T1 790.816 µs 1.625 V -1.214 V
Reverse
Save
GND
Timebase
Scale 1 ms/Div
X position 0
Channel A
Scale 2 V/Div
Y position 0
Channel B
Scale 2 V/Div
Y position 0
Trigger
Edge
Level 0 V
Bode Plotter-XBP1
Mode
Magnitude
Phase
Horizontal
Vertical
Log
Lin
F 2 kHz
I 1 Hz
F 90 Deg
I 0 Deg
Controls
Reverse
Save
Set...
159.047 Hz
45.049 Deg
fc