

Signals and Systems Using MATLAB

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Chapter 1 --- Continuous- time Signals

What is in this chapter?

- Classification of time-dependent signals
- Continuous-time signals
- Basic operations -- even and odd signals
- Periodic signals
- Finite-energy, finite-power signals
- Signal representation using basic signals

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Classification of Time-dependent Signals

- *Predictability* : random or deterministic
- *Variation of time and amplitude*: continuous-time, discrete-time, digital
- *Energy*: finite or infinite energy
- *Repetitive behavior*: periodic or aperiodic
- *Symmetry with respect to the time origin*: even or odd
- *Support*: finite or infinite l of the signal outside of which the signal is always zero.

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Analog to Digital and Digital to Analog Conversion

- **Analog to digital converter (ADC):** converts analog signal into a digital signal
- **digital to analog converter (DAC):** convert digital signal into an analog signal

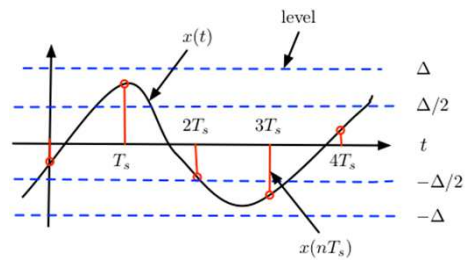


Figure 1. Discretization in time and in amplitude of an analog signal. The parameters are the sampling period T_s and the quantization level Δ . In time, samples are taken at uniform times $\{nT_s\}$, and in amplitude the range of amplitudes is divided into finite number of levels so that each sample value is approximated by them.

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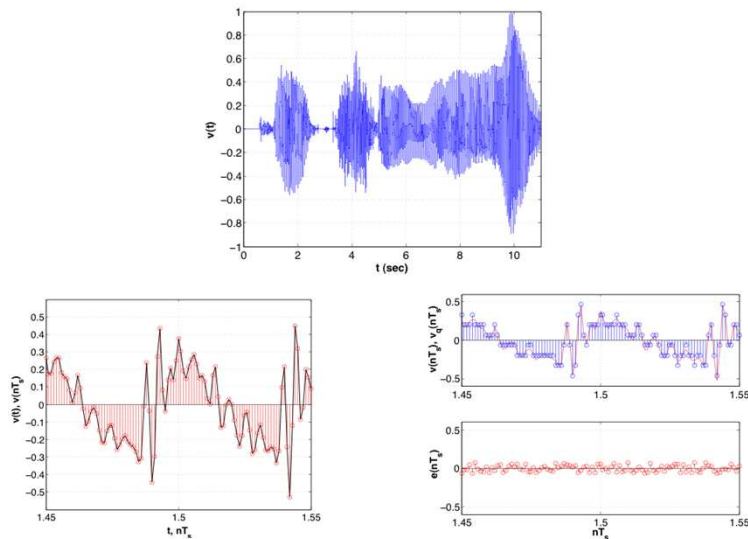


Figure 2. A segment of the speech signal shown on top is sampled and quantized. The bottom left figure displays the speech segment (continuous line) and the sampled signal (vertical samples) using a sampling period $T_s = 10^{-3}$ sec. The bottom-right figure shows the sampled and the quantized signal. The quantization error, difference between the sampled and the quantized signals, is displayed in the bottom-right figure.

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Continuous-time Signals

A continuous-time signal:

$$x(\cdot) : \mathcal{R} \rightarrow \mathcal{R} \text{ (C)}$$

$$t \quad x(t)$$

Independent variable is time t
 $x(t_0)$, is a real (or a complex) value
 t and $x(t)$ vary continuously, if needed, from $-\infty$ to ∞ .

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Basic Signal Operations

- *Signal addition:* $z(t) = x(t) + y(t)$ using adder
- *Constant multiplication* — $z(t) = \alpha x(t)$ using constant multiplier
- *Time/ frequency shifted:*
 - $x(t)$ delayed by τ : $x(t - \tau)$
 - $x(t)$ advanced by τ : $x(t + \tau)$
 - $x(t)$ shifted in frequency or frequency modulated: $x(t)e^{j\Omega_0 t}$
- *Time scaled:* $x(\alpha t)$, constant α
 - $\alpha = -1$, $x(-t)$, reversed in time or *reflected*
 - $\alpha \neq 1$, signal compressed/expanded
- *Time windowed:* $z(t) = x(t)w(t)$, window signal $w(t)$

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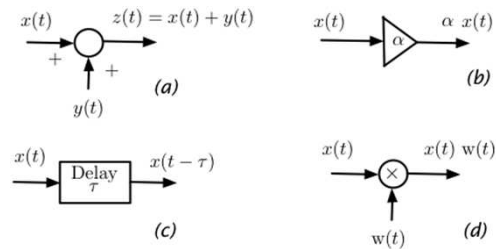
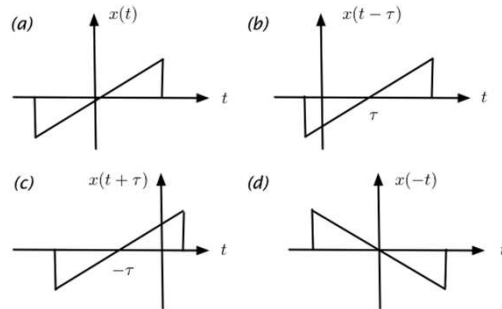


Figure 3. Diagrams of basic signal operations: (a) adder, (b) constant multiplier, (c) delay, and (d) time-windowing or modulation.



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Example Consider an analog pulse

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $x(t - 2)$, $x(t + 2)$, and $x(-t)$.

Solution

$$x(t - 2) = \begin{cases} 1 & 0 \leq t - 2 \leq 1 \text{ or } 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$x(0)$ (in $x(t)$ occurs at $t = 0$) in $x(t - 2)$ occurs when $t = 2$, so signal has been shifted to the right 2, $x(t - 2)$ is “delayed” by 2 with respect to $x(t)$

$$x(t + 2) = \begin{cases} 1 & 0 \leq t + 2 \leq 1 \text{ or } -2 \leq t \leq -1 \\ 0 & \text{otherwise} \end{cases}$$

$x(0)$ for $x(t + 2)$ occurs at $t = -2$ which is ahead of $t = 0$

$$x(-t) = \begin{cases} 1 & 0 \leq -t \leq 1 \text{ or } -1 \leq t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

or mirror image of the original, e.g., $x(1)$ occurs when $t = -1$.

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Example Compare $x(-t+2)$ to $x(t)$ above

Solution

$x(-t+2)$ is reflected, but advanced or delayed by 2?

t	$x(-t+2)$
2	$x(0) = 1$
1.5	$x(0.5) = 1$
1	$x(1) = 1$
0	$x(2) = 0$
-1	$x(3) = 0$

then $x(-t+2)$ reflected and “delayed” by 2

Even and Odd Signals

Even and odd signals are defined as follows:

$$x(t) \text{ even : } x(t) = x(-t)$$

$$x(t) \text{ odd : } x(t) = -x(-t)$$

Even and odd decomposition: Any signal $y(t)$ is representable as a sum of an even component $y_e(t)$ and an odd component $y_o(t)$

$$y(t) = y_e(t) + y_o(t)$$

where

$$y_e(t) = 0.5 [y(t) + y(-t)]$$

$$y_o(t) = 0.5 [y(t) - y(-t)]$$

Example Consider

$$x(t) = \cos(2\pi t + \theta) \quad -\infty < t < \infty$$

- What θ makes $x(t)$ even, odd?
- For $\theta = \pi/4$ is $x(t)$ even or odd?

Solution

Reflection $x(-t) = \cos(-2\pi t + \theta)$, then:

(i) $x(t)$ even if $x(t) = x(-t)$ or

$$\begin{aligned} \cos(2\pi t + \theta) &= \cos(-2\pi t + \theta) \\ &= \cos(2\pi t - \theta) \end{aligned}$$

or $\theta = -\theta$ or $\theta = 0, \pi$

$x_1(t) = \cos(2\pi t)$ and $x_2(t) = \cos(2\pi t + \pi) = -\cos(2\pi t)$ are even

(ii) $x(t)$ odd if $x(t) = -x(-t)$ or

$$\cos(2\pi t + \theta) = -\cos(-2\pi t + \theta) = \cos(-2\pi t + \theta \pm \pi) = \cos(2\pi t - \theta \mp \pi)$$

or $\theta = -\theta \mp \pi$ or $\theta = \mp \pi/2$

$\cos(2\pi t - \pi/2) = \sin(2\pi t)$ and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$ are odd

$\theta = \pi/4$, $x(t) = \cos(2\pi t + \pi/4)$ is neither even nor odd

Example Consider

$$x(t) = \begin{cases} 2\cos(4t) & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Find its even and odd decomposition
- What would happen if $x(0) = 2$ instead of 0, i.e., when we define the sinusoid at $t = 0$? Explain.

Solution

$x(t)$ is neither even nor odd because $x(t) = 0$ for $t \leq 0$

$$x_e(t) = 0.5[x(t) + x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ \cos(4t) & t < 0 \\ 0 & t = 0 \end{cases}$$

$$x_o(t) = 0.5[x(t) - x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ -\cos(4t) & t < 0 \\ 0 & t = 0 \end{cases}$$

adding them gives $x(t)$

For $x(0) = 2$,

$$x_e(t) = 0.5[x(t) + x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ \cos(4t) & t < 0 \\ 2 & t = 0 \end{cases}$$

odd component is the same

even component has a discontinuity at $t = 0$.

Periodic and Aperiodic Signals

An analog signal $x(t)$ is **periodic** if

- it is defined for all possible values of t , $-\infty < t < \infty$, and
- there is a positive real value T_0 , the **period** of $x(t)$, such that

$$x(t + kT_0) = x(t)$$

for any integer k .

The period of $x(t)$ is the smallest possible value of $T_0 > 0$ that makes the periodicity possible. Thus, although NT_0 for an integer $N > 1$ is also a period of $x(t)$ it should not be considered the period.

Example Periodic signal $x(t)$ of period T_0 , are following signals periodic? if so their periods

1. $y(t) = A + x(t)$,
2. $z(t) = x(t) + v(t)$, $v(t)$ periodic of period $T_1 = NT_0$, where $N > 0$ integer,
3. $w(t) = x(t) + u(t)$, $u(t)$ periodic of period T_1 , not multiple of T_0 , conditions for $w(t)$ periodic

Solution

(a) $y(t)$ periodic of period T_0 , i.e., integer k , $y(t + kT_0) = A + x(t + kT_0) = A + x(t)$ since $x(t)$ is periodic of period T_0 .

(b) $T_1 = NT_0$ period of $x(t)$
 $z(t)$ periodic of period T_1 , integer k

$$z(t + kT_1) = x(t + kT_1) + v(t + kT_1) = x(t + kNT_0) + v(t) = x(t) + v(t)$$

(d) $w(t)$ periodic if

$$\frac{T_1}{T_0} = \frac{N}{M}$$

where $N > 0$ and $M > 0$ integers not divisible by each other

$$w(t + MT_1) = x(t + MT_1) + u(t + MT_1) = x(t + NT_0) + u(t + MT_1) = x(t) + u(t)$$

Example $x(t) = e^{j2t}$, $y(t) = e^{j\pi t}$

- $z(t) = x(t) + y(t)$, periodic?
- $w(t) = x(t)y(t)$, periodic?
- $p(t) = (1 + x(t))(1 + y(t))$ periodic?

Solution

$$x(t) = \cos(2t) + j \sin(2t) \text{ periodic, } T_0 = \pi$$

$$y(t) = \cos(\pi t) + j \sin(\pi t) \text{ periodic, } T_1 = 2$$

- (1) $z(t)$ periodic if $T_1/T_0 = 2/\pi$ is rational, which is not
 (2) $w(t) = x(t)y(t) = e^{j(2+\pi)t} = \cos(\Omega_2 t) + j \sin(\Omega_2 t)$ periodic
 $\Omega_2 = 2 + \pi = 2\pi/T_2$ so $T_2 = 2\pi/(2 + \pi)$
 (3) $1 + x(t)$, periodic, $T_0 = \pi$
 $1 + y(t)$, periodic, $T_1 = 2$

$$p(t) = 1 + x(t) + y(t) + x(t)y(t)$$

$x(t) + y(t)$ not periodic, then $p(t)$ is not periodic

1. Analog sinusoids of frequency $\Omega_0 > 0$ are periodic of period $T_0 = 2\pi/\Omega_0$. If $\Omega_0 = 0$ the period is not well defined.
2. The sum of two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 , is periodic if the ratio of the periods T_1/T_2 is a rational number N/M , with N and M non-divisible. The period of the sum is $MT_1 = NT_2$.
3. The product of two sinusoids is periodic. The product of two periodic signals is not necessarily periodic.

Finite Energy and Finite Power Signals

The **energy** and the **power** of an analog signal $x(t)$ are defined for either finite or infinite support signals as:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

The signal $x(t)$ is then said to be **finite energy**, or **square integrable**, whenever

$$E_x < \infty$$

The signal is said to have **finite power** if

$$P_x < \infty$$

Example Energy and power of

- (a) $x(t) = \cos(\pi t/2 + \pi/4)$, $-\infty < t < \infty$,
 (b) $y(t) = (1+j)e^{j\pi t/2}$, $0 \leq t \leq 10$, zero otherwise,
 (c) $z(t) = 1$, for $0 \leq t \leq 10$ and zero otherwise.

Finite energy, finite power or both?

Solution

Energy

$$\begin{aligned} E_x &= \int_{-\infty}^{\infty} \cos^2(\pi t/2 + \pi/4) dt \rightarrow \infty \text{ infinite energy} \\ E_y &= \int_0^{10} |(1+j)e^{j\pi t/2}|^2 dt = 2 \int_0^{10} dt = 20 \text{ finite energy} \\ E_z &= \int_0^{10} dt = 10 \text{ finite energy} \end{aligned}$$

Power:

$$P_y, P_z = 0, \quad y(t), z(t) \text{ finite energy}$$

For $x(t)$, let $T = NT_0$:

$$\begin{aligned} P_x &= \lim_{T \rightarrow \infty} \frac{2}{2T} \int_0^T \cos^2(\pi t/2 + \pi/4) dt = \lim_{N \rightarrow \infty} \frac{1}{NT_0} \int_0^{NT_0} \cos^2(\pi t/2 + \pi/4) dt \\ &= \lim_{N \rightarrow \infty} \frac{1}{NT_0} \left[N \int_0^{T_0} \cos^2(\pi t/2 + \pi/4) dt \right] = \frac{1}{T_0} \int_0^{T_0} \cos^2(\pi t/2 + \pi/4) dt \\ &= \frac{1}{8} \int_0^4 \cos(\pi t + \pi/2) dt + \frac{1}{8} \int_0^4 dt = 0 + 0.5 = 0.5 \\ &\quad \uparrow \\ &\text{red } \cos^2(\theta) = 0.5 + 0.5 \cos(2\theta) \end{aligned}$$

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Example Consider

$$x(t) = \cos(2\pi t) + \cos(4\pi t), \quad -\infty < t < \infty$$

$$y(t) = \cos(2\pi t) + \cos(2t), \quad -\infty < t < \infty$$

Periodic? Power?

Solution

$\cos(2\pi t)$, $\cos(4\pi t)$ periods $T_1 = 1$ and $T_2 = 1/2 \Rightarrow x(t)$ periodic ($T_1/T_2 = 2$)
 with period $T_1 = 2T_2 = 1$, and harmonically related frequencies
 $\cos(2t)$, period $T_3 = \pi \Rightarrow y(t)$ not periodic ($T_1/T_3 = 1/\pi$ not rational), frequencies 2π and 2 not harmonically related

$$\begin{aligned} x^2(t) &= \cos^2(2\pi t) + \cos^2(4\pi t) + 2\cos(2\pi t)\cos(4\pi t) \\ &= 1 + \frac{1}{2}\cos(4\pi t) + \frac{1}{2}\cos(8\pi t) + \cos(6\pi t) + \cos(2\pi t) \\ P_x &= \frac{1}{T_0} \int_0^{T_0} x^2(t) dt = 1 \end{aligned}$$

$$\begin{aligned} y^2(t) &= \cos^2(2\pi t) + \cos^2(2t) + 2\cos(2\pi t)\cos(2t) \\ &= 1 + \frac{1}{2}\cos(4\pi t) + \frac{1}{2}\cos(4t) + \cos(2(\pi+1)t) + \cos(2(\pi-1)t) \\ P_y &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T y^2(t) dt \\ &= 1 + \frac{1}{2T_4} \int_0^{T_4} \cos(4\pi t) dt + \frac{1}{2T_5} \int_0^{T_5} \cos(4t) dt \\ &\quad + \frac{1}{T_6} \int_0^{T_6} \cos(2(\pi+1)t) dt + \frac{1}{T_7} \int_0^{T_7} \cos(2(\pi-1)t) dt = 1 \end{aligned}$$

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If

$$\begin{aligned}x(t) &= \cos(2\pi t) + \cos(4\pi t) = x_1(t) + x_2(t) \\y(t) &= \cos(2\pi t) + \cos(2t) = y_1(t) + y_2(t)\end{aligned}$$

then $P_{x_1} = P_{x_2} = P_{y_1} = P_{y_2} = 0.5$ so that

$$\begin{aligned}P_x &= P_{x_1} + P_{x_2} = 1 \\P_y &= P_{y_1} + P_{y_2} = 1\end{aligned}$$

The power of a sum of sinusoids,

$$x(t) = \sum_k A_k \cos(\Omega_k t) = \sum_k x_k(t)$$

with harmonically or non harmonically related frequencies $\{\Omega_k\}$, is the sum of the power of each of the sinusoidal components,

$$P_x = \sum_k P_{x_k}$$

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Representation using Basic Signals

Complex Exponentials

A complex exponential is a signal of the form

$$\begin{aligned}x(t) &= Ae^{at} \\&= |A|e^{r t} [\cos(\Omega_0 t + \theta) + j \sin(\Omega_0 t + \theta)] \quad -\infty < t < \infty\end{aligned}$$

where $A = |A|e^{j\theta}$, and $a = r + j\Omega_0$ are complex numbers.

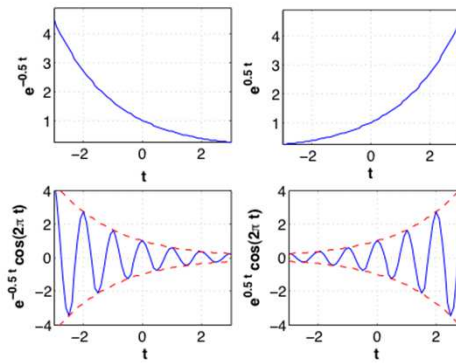
- A and a are real,

$$x(t) = Ae^{at} \quad -\infty < t < \infty, \text{ decaying exponential } (a < 0), \text{ growing exponential } (a > 0)$$

- A is real, $a = j\Omega_0$,

$$x(t) = Ae^{j\Omega_0 t} = \underbrace{A \cos(\Omega_0 t)}_{\mathcal{Re}[x(t)]} + j \underbrace{A \sin(\Omega_0 t)}_{\mathcal{Im}[x(t)]}$$

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Analog exponentials: decaying exponential (top left), growing exponential (top right), modulated exponential decaying and growing (bottom left and right).

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Sinusoids

Sinusoids are of the general form

$$A \cos(\Omega_0 t + \theta) = A \sin(\Omega_0 t + \theta + \pi/2) \quad -\infty < t < \infty$$

where A is the amplitude of the sinusoid, $\Omega_0 = 2\pi f_0$ (rad/sec) is the frequency, and θ is a phase shift. The frequency and time variables are inversely related,

$$\Omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$\begin{aligned} \cos(\Omega_0 t) &= \frac{1}{2}(e^{j\Omega_0 t} + e^{-j\Omega_0 t}) \\ \sin(\Omega_0 t) &= \frac{1}{2j}(e^{j\Omega_0 t} - e^{-j\Omega_0 t}) \end{aligned}$$

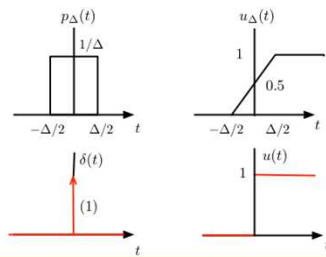
Modulation systems in communications

$$A(t) \cos(\Omega(t)t + \theta(t))$$

- **Amplitude modulation or AM:** $A(t)$ changes according to the message, frequency and phase constant,
- **Frequency/Phase modulation or FM:** $\Omega(t)/\theta(t)$ changes according to the message, amplitude and phase constant,

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Unit-step, Unit-impulse and Ramp Signals



The impulse signal $\delta(t)$ is:

- zero everywhere except at the origin where its value is not well defined, i.e., $\delta(t) = 0$, $t \neq 0$, undefined at $t = 0$,
- its area is unity, i.e.,

$$\int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 1 & t > 0 \\ 0 & t < 0. \end{cases}$$

The unit-step signal is

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

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The $\delta(t)$ and $u(t)$ are related as follows:

$$\begin{aligned} u(t) &= \int_{-\infty}^t \delta(\tau) d\tau \\ \delta(t) &= \frac{du(t)}{dt} \end{aligned}$$

Calculus: $\Delta \rightarrow 0$, relation between $u(t)$ and $\delta(t)$

$$\begin{aligned} u_{\Delta}(t) &= \int_{-\infty}^t p_{\Delta}(\tau) d\tau \\ p_{\Delta}(t) &= \frac{du_{\Delta}(t)}{dt} \end{aligned}$$

The ramp signal is defined as

$$r(t) = t u(t)$$

Its relation to the unit-step and the unit-impulse signals is

$$\begin{aligned} \frac{dr(t)}{dt} &= u(t) \\ \frac{d^2r(t)}{dt^2} &= \delta(t) \end{aligned}$$

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Example For

$$\begin{aligned}x_1(t) &= \cos(2\pi t)[u(t) - u(t-1)] \\x_2(t) &= u(t) - 2u(t-1) + u(t-2)\end{aligned}$$

represent them as the sum of a continuous signal and unit step signals, and find their derivatives.

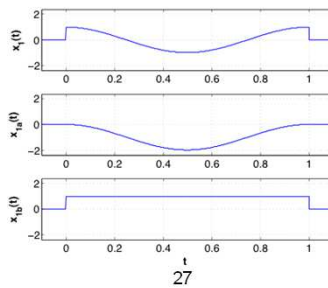
Solution

$$x_1(t) = \underbrace{(\cos(2\pi t) - 1)[u(t) - u(t-1)]}_{\text{continuous}} + \underbrace{[u(t) - u(t-1)]}_{\text{discontinuous}}$$

$$\begin{aligned}\frac{dx_1(t)}{dt} &= -2\pi \sin(2\pi t)[u(t) - u(t-1)] + (\cos(2\pi t) - 1)[\delta(t) - \delta(t-1)] + \delta(t) - \delta(t-1) \\&= -2\pi \sin(2\pi t)[u(t) - u(t-1)] + \delta(t) - \delta(t-1)\end{aligned}$$

$x_2(t)$, jump discontinuities at $t = 0$, $t = 1$ and $t = 2$ so discontinuous, continuous component 0

$$\frac{dx_2(t)}{dt} = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

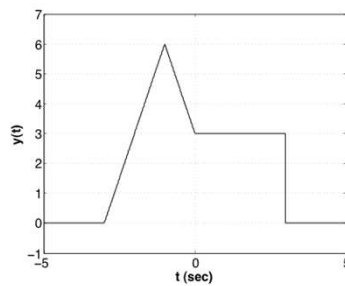


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Example-MATLAB

$$y(t) = 3r(t+3) - 6r(t+1) + 3r(t) - 3u(t-3)$$

plot it and verify analytically that the obtained figure is correct.

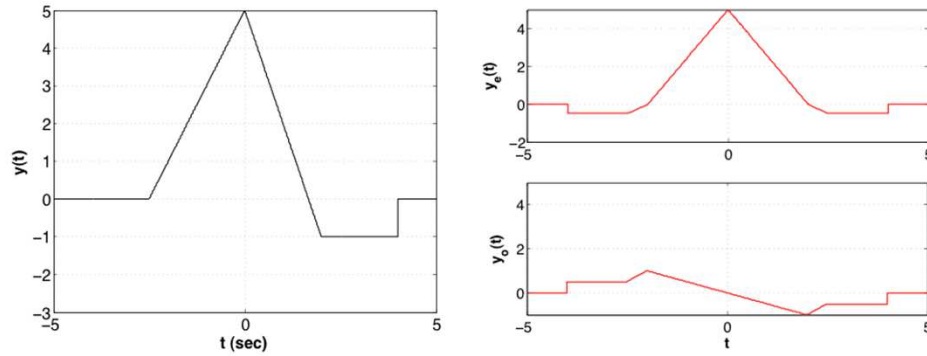


Analytically,

- $y(t) = 0$ for $t < -3$ and for $t > 3$, signal in support $-5 \leq t \leq 5$
- $-3 \leq t \leq -1$, $y(t)$ is $3r(t+3) = 3(t+3)$ which is $y(-3) = 0$, $y(-1) = 6$
- $-1 \leq t \leq 0$, $y(t)$ is $3r(t+3) - 6r(t+1) = 3(t+3) - 6(t+1) = -3t + 3$, $y(-1) = 6$, $y(0) = 3$,
- $0 \leq t \leq 3$, $y(t)$ is $3r(t+3) - 6r(t+1) + 3r(t) = -3t + 3 + 3t = 3$,
- $t \geq 3$, $y(t)$ is $3r(t+3) - 6r(t+1) + 3r(t) - 3u(t-3) = 3 - 3 = 0$.

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Example From plot of signal $y(t)$, obtain its formula, and its even and odd components.



Signal $y(t) = 2r(t + 2.5) - 5r(t) + 3r(t - 2) + u(t - 4) - u(t - 5)$ (left), even component $y_e(t)$ (right-top), odd component $y_o(t)$ (right-bottom).

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Example Use $r(t)$ and $u(t)$ to represent the triangular signal $\Lambda(t)$ and its derivative.

$$\Lambda(t) = \begin{cases} t & 0 \leq t \leq 1 \\ -t + 2 & 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution

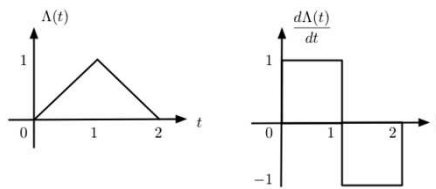
$$\Lambda(t) = r(t) - 2r(t - 1) + r(t - 2)$$

In fact,

$$\begin{aligned} \Lambda(t) &= r(t) = t \quad \text{for } 0 \leq t \leq 1 \\ &= r(t) - 2r(t - 1) = t - 2(t - 1) = -t + 2 \quad \text{for } 1 < t \leq 2 \\ &= r(t) - 2r(t - 1) + r(t - 2) = t - 2(t - 1) + (t - 2) = 0 \quad t > 2 \end{aligned}$$

Derivative:

$$\frac{d\Lambda(t)}{dt} = u(t) - 2u(t - 1) + u(t - 2) = \begin{cases} 1 & 0 \leq t \leq 1 \\ -1 & 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

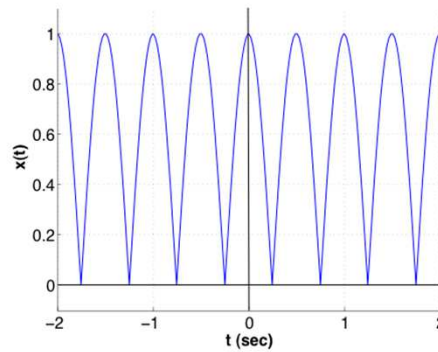


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Example Full-wave rectified signal

$$x(t) = |\cos(2\pi t)| \quad -\infty < t < \infty$$

Representation for a period, and represent $x(t)$ in terms of shifted versions of it.



Solution

Period, $0 \leq t \leq T_0 = 0.5$:

$$p(t) = x(t)[u(t) - u(t - 0.5)] = |\cos(2\pi t)|[u(t) - u(t - 0.5)]$$

$$x(t) = \sum_{k=-\infty}^{\infty} p(t - kT_0)$$

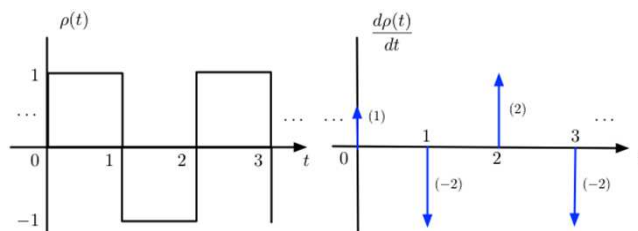
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Example Generate causal train of pulses, repeating every 2 units of time using first period. Find its derivative.

$$s(t) = u(t) - 2u(t - 1) + u(t - 2)$$

Solution

$$\rho(t) = \sum_{k=0}^{\infty} s(t - 2k)$$



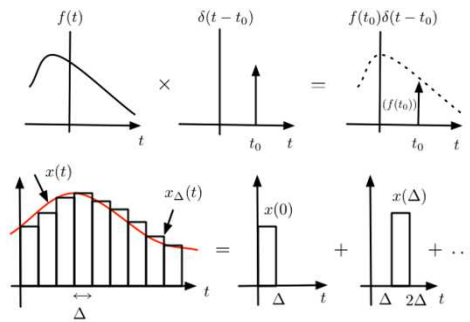
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Generic Representation of Signals

$$\begin{aligned}\int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt &= \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)dt = f(\tau) \int_{-\infty}^{\infty} \delta(t-\tau)dt \\ &= f(\tau) \quad \text{for any } \tau\end{aligned}$$

By the sifting property of the impulse function $\delta(t)$ any signal $x(t)$ can be represented by the following **generic representation**:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau.$$



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What have we accomplished?

- Signal classification
- Symmetry, periodicity, energy/power for continuous-time signals
- Signal representation using basic signals (unit-step, impulse, ramp, exponential)

Where do we go from here?

- Connect signals and systems
- Develop theory that approximates behavior of most systems
- Time and frequency analysis

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