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Hw 3

CPE 381

1. $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$

Initial Conditions $= \emptyset$. $Y(s) = \mathcal{L}[y(t)]$, $X(s) = \mathcal{L}[x(t)]$

$$Y(s)(s^2 + 3s + 2) = X(s)$$

Impulse response $y(t) = h(t)$

$$H(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$A = \left.\frac{1}{s+2}\right|_{s=-1}$, $A = 1$ $\quad B = \left.\frac{1}{s+1}\right|_{s=-2}$, $B = -1$

9 $h(t) = \mathcal{L}^{-1}[H(s)] = (e^{-t} - e^{-2t})\overset{?}{(u(t))}$

$x(t) = u(t)$ $\quad X(s) = 1/s$

Unit-Step Resp: $S(t) = \mathcal{L}^{-1}[S(s)]$

$$S(s) = \frac{H(s)}{s} = \frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$A = \left.\frac{1}{(s+1)(s+2)}\right|_{s=0} = 1/2$ $\quad B = \left.\frac{1}{s(s+2)}\right|_{s=-1} = -1$ $\quad C = \left.\frac{1}{s(s+1)}\right|_{s=-2} = 1/2$

9 $S(t) = 0.5u(t) - e^{-t}u(t) + 0.5e^{-2t}u(t)$

HW3

2. $H(s) = \dfrac{s}{s^2+s+1}$

Find unit step response, $s(t)$.

Step Response in Laplace domain: $U(s) = 1/s$

From Notes: $X(s) \; \boxed{H(s)} \; Y(s)$ $\quad Y(s) = X(s) \cdot H(s)$

Response = input • transfer funct.

So, $\mathcal{L}\left[s(t)\right] = \dfrac{s}{s^2+s+1}\left[\dfrac{1}{s}\right] = \dfrac{1}{s^2+s+1}$

then $s(t) = \mathcal{L}^{-1}\left[\dfrac{1}{s^2+s+1}\right]$

Find roots for $s^2+s+1=0$, $s = 0.5 \pm 0.866i$

Complex roots, so complete the square for $s^2+s+1=0$,
Follow format $s^2+2as+a^2 = C$,

$s^2 + 2(1/2)s + (1/4) = -3/4$, $\quad (s+1/2)^2 + 3/4 = 0$

we know $\mathcal{L}\left[e^{-at} \cdot \sin(\Omega_0 t)\, u(t)\right] = \dfrac{\Omega_0}{(s+a)^2 + \Omega_0^2}$

So $\mathcal{L}\left[s(t)\right] = \dfrac{\sqrt{3/4}}{(s+1/2)^2 + 3/4}$

And $s(t) = \left[e^{-1/2 t} \cdot \sin\left(\left(\sqrt{3}/2\right)t\right) u(t)\right]$

Response from $\left(u(t) - u(t-1)\right)$: $\quad s(t) - s(t-1)$

3.

Signal transfer function: $H(s) = \frac{2}{s-1}$

tentatively, $h(t) = 2e^{t}$ which grows rather than damps.

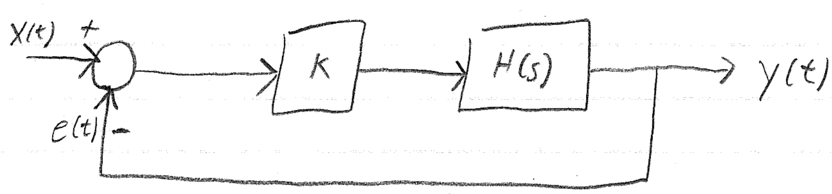
Transfer function for the negative feedback system:

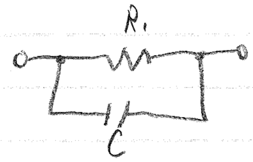
$$G(s) = \frac{Y(s)}{X(s)} = \frac{KH(s)}{1+KH(s)} \qquad X(t) \equiv u(t)$$

$$E(s) = X(s)\big(1 - G(s)\big) = \frac{1}{s}\big(1 - G(s)\big) = \frac{1}{s}\left(\frac{1+KH(s) - KH(s)}{1+KH(s)}\right)$$

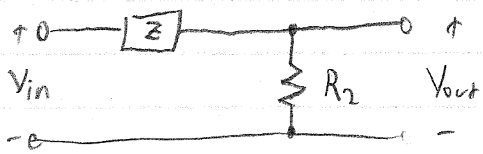
$$= \left(\frac{1}{s + sH(s)K}\right) = \frac{1}{\frac{s^2 - s}{s-1} + \left(\frac{2s}{s-1}\right)K} = \frac{1}{s} + \left(\frac{s-1}{2s}\right)\frac{1}{K} = \frac{2K+s-1}{2sK}$$

poles of $E(s)$: $\cancel{s^2(1+KH(s))}$ for $K \neq 0$, real roots



4. a) $Z =$ 

$$Z = \frac{R_1 \cdot \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}$$

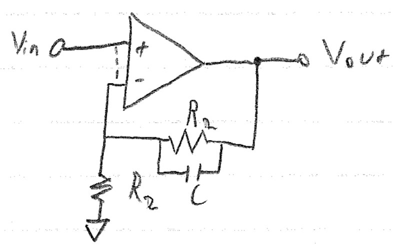


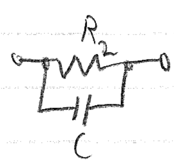
$$V_{out} = \frac{R_2}{Z + R_2} = \frac{R_2}{\left(\frac{R_1 \cdot \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}\right) + R_2}$$

Simplify Z: $\frac{R_1}{Cs} \cdot \left(\frac{1}{R_1} + \frac{Cs}{1}\right) = \frac{R_1}{Cs}\left(\frac{1 + R_1 Cs}{R_1}\right) = \frac{R_1(1 + R_1 Cs)}{R_1 Cs}$

Simplify V_{out}: $\left(\frac{R_2}{1}\right)\left(\frac{R_1 Cs}{R_1 + R_1^2 Cs} + \frac{1}{R_2}\right) = \left(\frac{R_2}{1}\right)\left(\frac{R_2 R_1 Cs + 1}{R_2(R_1 + R_1^2 Cs)}\right)$

$= \left(\frac{R_2 R_1 Cs}{R_1 + R_1^2 Cs}\right) = \frac{R_2}{Z + R_2} = H(s)$

b) 



$= R_F = Z$

Noninverting, $V_{out} = 1 + R_F/R_2 = 1 + \frac{Z}{R_2} = \frac{R_2}{R_2} + \frac{Z}{R_2} = \frac{Z + R_2}{R_2} = \left(\frac{R_2}{Z + R_2}\right)^{-1}$

$V_{out} = H(s) = \left(\frac{R_1 + R_1^2 Cs}{R_2 R_1 Cs}\right)$

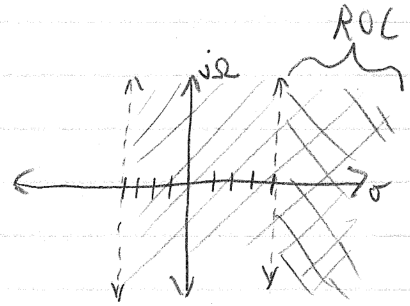
5. $X(s) = \frac{1}{(s+4)(s-4)} = \frac{1}{s^2-16}$ roots = 4, -4

$x(t) = \mathcal{L}^{-1}[X(s)]$

$X(s) = \frac{A}{(s+4)} + \frac{B}{(s-4)}$ $A = \frac{1}{(s-4)}\Big|_{s=-4} = -1/8$ $B = \frac{1}{(s+4)}\Big|_{s=4} = 1/8$

$x(t) = \left(-\frac{1}{8}e^{-4t} + \frac{1}{8}e^{4t}\right)$

$\frac{1}{(s+4)(s-4)}$



6. Second Order DE. $y''(t) + 5y'(t) + 4y(t) = x(t)$

Reformat: $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = x(t)$

7 Impulse Resp: Initial Conditions $= \emptyset$, $Y(s) = H(s) \cdot X(s)$

$$H(s) = \frac{1}{s^2 + 5s + 4} = \frac{1}{(s+4)(s+1)} = \frac{A}{(s+4)} + \frac{B}{(s+1)}$$

$$A = \frac{1}{(s+1)}\Big|_{s=-4} = -\tfrac{1}{3} \qquad B = \frac{1}{(s+4)}\Big|_{s=-1} = \tfrac{1}{3}$$

$$h(t) = \left(-\tfrac{1}{3} e^{-t} + \tfrac{1}{3} e^{-4t}\right)(u(t))$$

7 Unit Step Resp: $X(t) = u(t)$, $X(s) = 1/s$

$$S(s) = Y(s) = \frac{H(s)}{s} = \frac{1}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+1)}$$

$$A = \frac{1}{(s+4)(s+1)}\Big|_{s=0} = 1/4 \quad B = \frac{1}{s(s+1)}\Big|_{s=-4} = \frac{1}{12} \quad C = \frac{1}{s(s+4)}\Big|_{s=-1} = -1/3$$

$$s(t) = \left(1/4 + \tfrac{1}{12} e^{-4t} - 1/3\, e^{-t}\right) u(t)$$