KATER'S PENDULUM

OBJECTIVE:

The objective of this experiment is to allow you to study the motion of a physical pendulum in the shape of a rectangular bar known as Kater's pendulum. By performing this experiment you will get new insight into rigid body motion and the concept of moment of inertia. You will also make a series of measurements to high degree of precision and accuracy in order to calculate a value of the acceleration due to gravity, g.

METHOD:

In summary, you will

a. measure the length and width of the rectangular bar,
b. find position of the center of mass of the bar,
c. suspend the bar in each hole on one side of the bar and measure the period of the pendulum to +/- 0.002 seconds,
d. measure the point of suspension relative to the center of mass,
e. compute the value of "g" and "k", the radius of gyration.

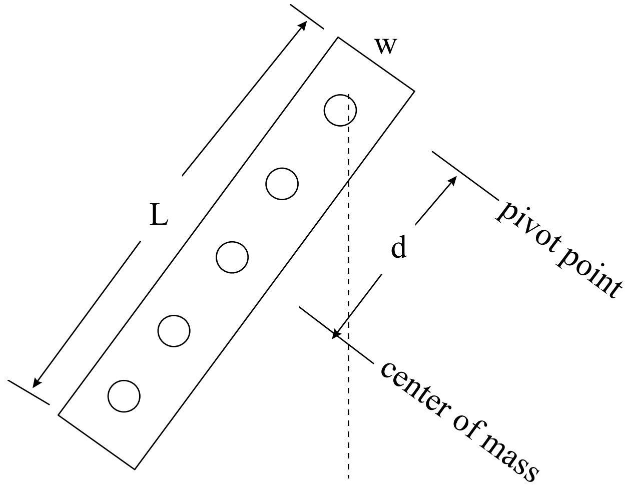
APPARATUS:

Kater's pendulum, meter stick, vernier caliper, photogate timer.

THEORY:

Kater's pendulum is a long bar of length L, width w, and thickness t, with holes drilled through it spaced at regular intervals. The pendulum can be mounted on a knife-edge at any of the holes and its period measured. The theory predicts that the period of oscillation will be a function of the distance, d, of the center of mass from the pivot point. The period of oscillation will also be a function of the mass m, and the moment of inertia about the pivot, I. That is,

$$T = 2\pi\sqrt{\frac{I}{mgd}} \tag{1}$$



There is an alternate way of expressing the period that eliminates the explicit use of the mass of the bar. This requires a definition of a new term, "the radius of gyration", denoted as k. The moment of inertia of a body about an axis through the center of mass depends upon both the mass and the geometric distribution of mass. We can define a new constant, k, so that

$$I = mk^2 \tag{2}$$

k is known as the radius of gyration of the body. Use of this constant eliminates the need to know the mass of the body. For example, the moment of inertia of a long rod of mass m and length L is

$$I = m\frac{L^2}{12} \quad \text{and} \quad k = \frac{L}{\sqrt{12}}$$

When we use the radius of gyration about the center of mass as a substitute for I in Eq. (1), we get

$$T = 2\pi\sqrt{\frac{k^2 + d^2}{gd}} \tag{3}$$

where d = distance from the CM to the pivot point
k = radius of gyration about the center of mass
g = acceleration due to gravity

We can rearrange Eq. (3) to get a useful way of expressing things, as follows,

$$d^2 = \frac{g}{4\pi^2}T^2 d - k^2 \tag{4}$$

This may not look particularly useful until you recognize that if y = d^2 and x = T^2d, then we can rewrite Eq. (4) as

$$y = \frac{g}{4\pi^2} x - k^2 \tag{5}$$

A plot of y as function of x will give a straight line whose slope is $g/4\pi^2$ and whose intercept is k^2 .

PROCEDURE:

1. First locate and mark the center of mass of the bar by balancing it in the center hole.

2. Measure and record the length and width of the bar for your later calculations.

3. Measure the distance, d, from the CM to each hole on one side of the bar.

4. Suspend the bar and measure the period, T, for each hole on one side of the center of mass of the pendulum to about +/- 0.0002 second. Use the photogate timer in the pendulum mode and with a resolution of 0.1 millisecond. To be sure of the timing measurement, repeat each period measurement three or more times and average your results.

5. Record the average period for each hole on one side of the center of mass of the pendulum.

Comment:

The reason for requiring this high degree of resolution, i.e., 0.0001 seconds, can be made clear as follows:

If you use Eq. (3) to predict what the period would be for a pendulum 100 cm long and using $k = \frac{L}{\sqrt{12}}$, you would get

d (cm)	T (sec)
20	1.576
25	1.533
30	1.526
35	1.539
40	1.656

An interesting feature of this pendulum is that the period decreases to a minimum value and then increases again. Most important, however, is that a quick look at this table shows that the period is changing in the third decimal place near the minimum period. Therefore, it's necessary to measure the period with resolution exceeding 0.001 seconds to clearly observe that is occurring.

CALCULATIONS:

1. For every pair of data, T and d, calculate x and y as defined by Eq. (5). Tabulate and estimate the effect of your errors in measuring d and T on the accuracy of x and y.

2. Plot a graph of y vs. x. If you have done everything right the data should fall in a straight line with the errors too small to show on the paper.

3. Compute the slope and intercept of the line using either least squares regression or using the rise over run of the data. Use your estimates of the errors in x and y to estimate the error in the slope and the intercept.

4. Compute the value of "g" from your slope and compare to the accepted value of g.

5. Compute the radius of gyration, k, from your intercept data. Compare this result to a calculated value of k from the geometric relation

$$k^2 = \frac{L^2 + w^2}{12}$$

6. We have neglected the effect of the holes in this experiment. It can be estimated that the radius of gyration would be increased by about 0.2%. Could your experiment, in principle, see this effect?

DATA ORGANIZATION SHEET
EXP KATER'S PENDULUM

L = ________ cm W = __________ cm

d (cm)	T (sec.)	$\overline{T}$ (sec.)	Y = d^2	X = $\overline{T}^2 d$

y = bx + a $\overline{X}$ = ______ $\overline{Y}$ = ______

$$\sigma_x = \sqrt{\frac{\sum\left(x-\overline{X}\right)^2}{N}} =$$

$$b = \frac{\sum\left(x-\overline{X}\right)\left(y-\overline{Y}\right)}{N\sigma_x^2} =$$

a = $\overline{Y}$ - b$\overline{X}$ =

$g = 4\pi^2 b$ (cm/s^2)	% diff. of g with 9.8 (cm/s^2)	$K = \sqrt{-a}$ (cm)	$K = \sqrt{\frac{L^2+W^2}{12}}$ (cm)	% diff. of K