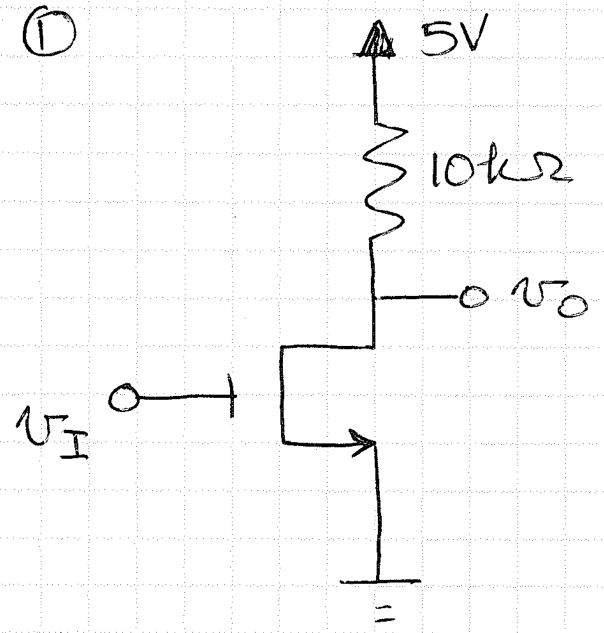
Problem Set # 7 - Solutions

①



$V_t = 1.5V$

$k_n' \frac{W}{L} = 0.2 \frac{mA}{V^2}$

at $v_I < 1.5V$ $v_O = 5V$

point A => $v_I = 1.5V$
$v_O = 5V$

or $V_{OA} = 5V$
$V_{IA} = 1.5V$

at edge of saturation (point B)

$v_O = v_I - V_t$ $\quad i_D = \frac{1}{2} k_n' \frac{W}{L} (v_I - V_t)^2$

$V_{OB} = V_{IB} - 1.5$ or $V_{IB} = V_{OB} + 1.5$

$I_{DB} = 0.1 (V_{IB} - 1.5)$

$I_{DB} = 0.1 V_{OB}^2$

by ohm's law $\quad V_{OB} = 5 - 10 I_{DB} = 5 - 10(0.1 V_{OB}^2)$

$V_{OB} = 5 - V_{OB}^2$

or

$V_{OB}^2 + V_{OB} - 5 = 0$

$$V_{OB} = \frac{-1 \pm \sqrt{1^2 - 4(-5)}}{2}$$

$\boxed{V_{OB} = 1.79 V}$ or $-2.79 V$

① cont

$$V_{IB} = V_{OB} + 1.5 \qquad (V_{OB}, V_{IQ}) = (1.79, 3.29)$$

$$\boxed{V_{IB} = 3.29\,V}$$

at point C => triode region

$$v_I = 5V \qquad V_{IC} = 5V$$

$$i_D = k'_n \frac{W}{L}\left[(v_{GS} - V_t)v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

$$v_O = 5 - 10 i_D$$

$$I_{DC} = 0.2\left[(5-1.5)V_{OC} - \frac{1}{2}V_{OC}^2\right]$$

$$I_{DC} = 0.7V_{OC} - 0.1V_{OC}^2$$

$$V_{OC} = 5 - 10 I_{DC} = 5 - 7V_{OC} - V_{OC}^2$$

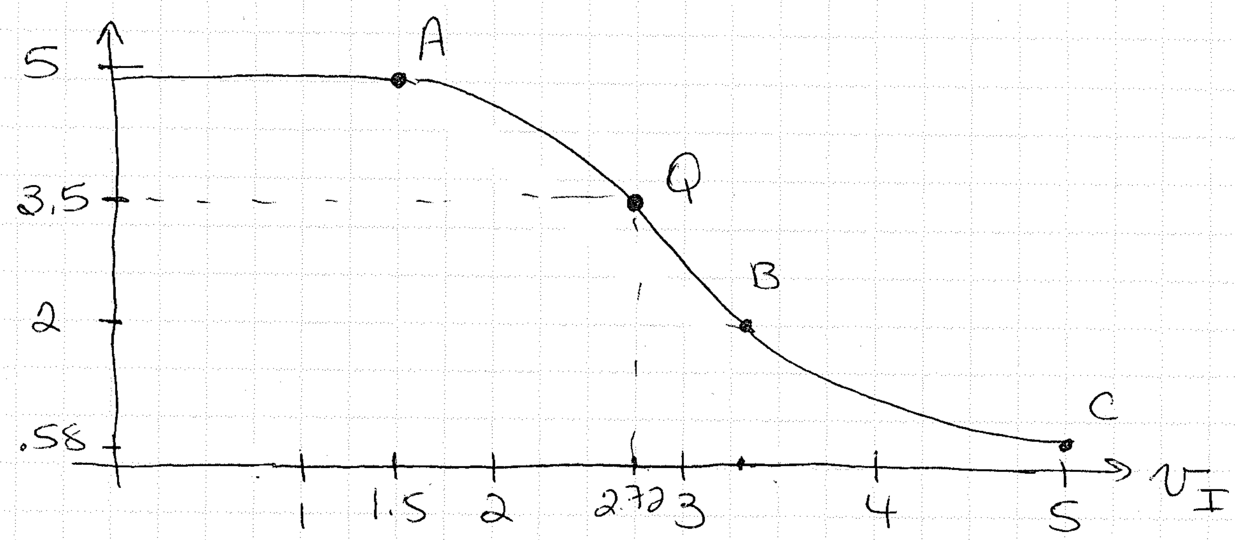
so $\quad V_{OC}^2 + 8V_{OC} - 5 = 0$

$$V_{OC} = \frac{-8 \pm \sqrt{64 - 4(-5)}}{2}$$

$$\boxed{V_{OC} = 0.58V} \text{ or } -8.58V$$

$$(V_{OC}, V_{IC}) = (0.58V, 5V)$$

1.



b)

$$I_{DQ} = 0.15\,mA = \frac{1}{2} k'_n \frac{W}{L} (V_{IQ} - V_t)^2$$

$$0.15 = \frac{1}{2}(.2)(V_{IQ} - 1.5)^2$$

$$(V_{IQ} - 1.5)^2 = 1.5$$

$$\boxed{V_{IQ} = 2.72\,V}$$

$$V_{OQ} = 5 - 10 I_{DQ} = 3.5\,V \qquad \boxed{V_{OQ} = 3.5\,V}$$

c)

$$A_v = -R_D k'_n \frac{W}{L} (V_{IQ} - V_t)$$

$$= -10(.2)(2.72 - 1.5)$$

$$= 2.45\,V/V$$

① cont $|V_{IQ} - V_{IB}| = |2.72 - 3.29|$

$= 0.57\,V$

$|V_{IQ} - V_{IA}| = |2.72 - 1.5|$

$= 1.22\,V$

maximum amplitude, swing for input voltage

$v_{i\,max} = 0.57\,V$

swing $v_{I\,max} = 2.72 \pm 0.57\,V$

$i_D = 0.1(v_I - 1.5)^2$

$v_O = 5 - i_D(10)$

v_I	i_D	v_O
* 3.29	0.32mA	1.79V
2.72V	0.15mA	3.5V
2.15	0.042mA	4.58V

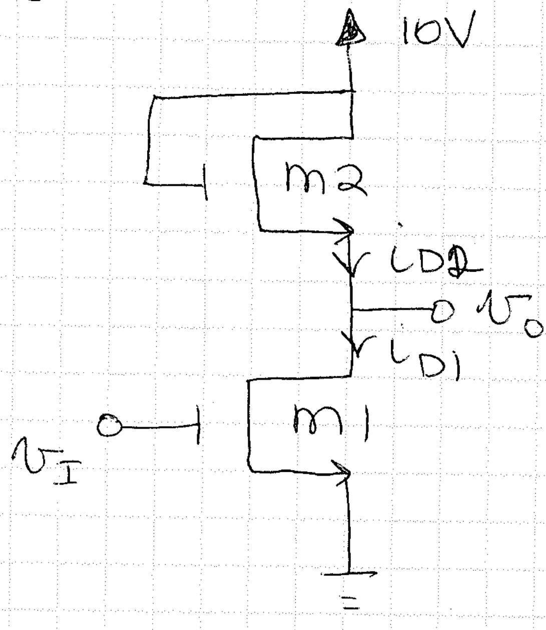
* edge of sat to triode

$\Delta v_O = 4.58 - 3.5$ (high swing)
$= 1.08\,V$

$\Delta v_O = 3.5 - 1.79$ (low swing)
$= 1.71$

max output swing $= 1.71\,V$

② PS#7 - Solutions



$k'_{n1} \frac{W}{L} = 100 \frac{mA}{V^2}$

$k'_{n2} W/L = 10 mA/V^2$

device m2 in saturation

device m1 in saturation

for $v_I \geq V_{t1}$

$v_I \leq v_O + V_{t1}$

$v_{GS2} = 10 - v_O$

$v_{GS1} = v_I$

$$i_{D2} = \frac{1}{2} k'_{n2} (v_{GS2} - V_{t2})^2$$

$$i_{D1} = \frac{1}{2} k'_{n1} (v_{GS1} - V_{t1})^2$$

$$i_{D2} = 5(10 - v_O - 1)^2 \qquad i_{D1} = i_{D2}$$

$$i_{D1} = 50 (v_I - 1)^2$$

$$\sqrt{5 (9 - v_O)^2} = \sqrt{50 (v_I - 1)^2}$$

$$\sqrt{5} (9 - v_O) = \sqrt{50} (v_I - 1)$$

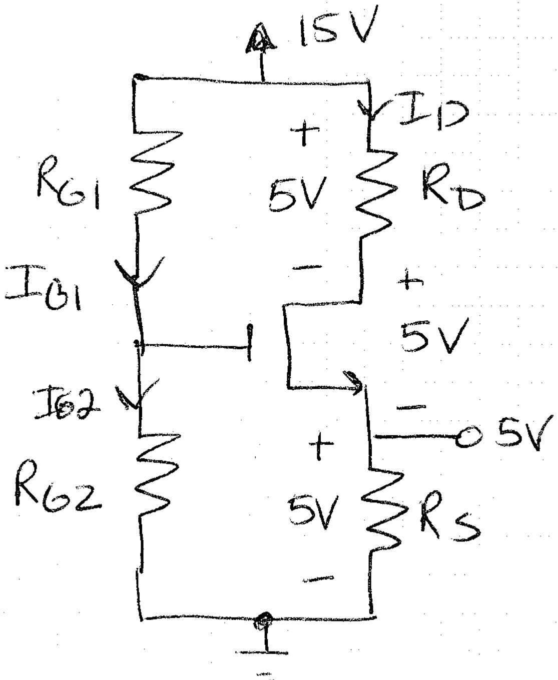
$$9 - v_O = \frac{\sqrt{50}}{\sqrt{5}} (v_I - 1)$$

$$v_O = 9 - \frac{\sqrt{50}}{\sqrt{5}} (v_I - 1)$$

$$A_v = \frac{dv_O}{dv_I} = \frac{-\sqrt{50}}{\sqrt{5}} = -3.16 V/V$$

PS #7 - Solutions

(3)



$I_D = 2mA$

$V_t = 1.2V$

$k'_n = 80 \frac{\mu A}{V^2}$

$W/L = \frac{240}{6} = 40$

$I_{G1} = I_{G2} = 1\mu A$

$R_D = \frac{5V}{2mA} = 2.5k\Omega$

$R_S = \frac{5V}{2mA} = 2.5k\Omega$

Assume saturation

$$I_D = \frac{1}{2}(80)(40)(V_{GS} - 1.2)^2$$

$$(\mu A)\ 2000 = \frac{1}{2}(80)(40)(V_{GS} - 1.2)^2$$

$$(V_{GS} - 1.2)^2 = 1.25$$

$V_{GS} = 2.32V$

$V_S = 5V$

$V_G = 7.32V$

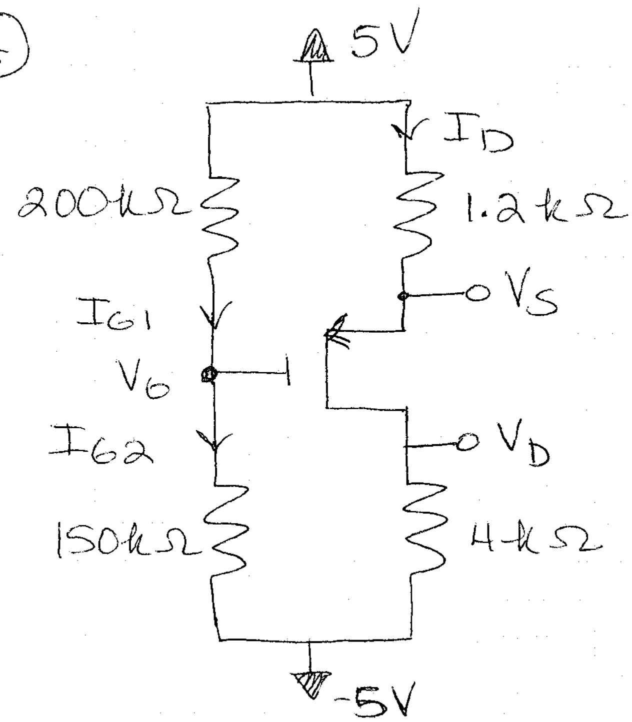
$$R_{G1} = \frac{15 - V_G}{1\mu A}$$

$$= 7.68\,M\Omega$$

$$R_{G2} = \frac{V_G}{1\mu A} = 7.32\,M\Omega$$

PS#7

(4)



$k'_p \frac{W}{L} = 0.25 \frac{mA}{V^2}$

$V_t = -1V$

$I_{G1} = \frac{5 - V_G}{200}$ (mA)

$I_{G2} = \frac{V_G + 5}{150}$ (mA)

$I_{G1} = I_{G2}$

$\frac{5 - V_G}{200} = \frac{V_G + 5}{150}$

$5 - V_G = \frac{200}{150}(V_G + 5)$

$5 - V_G = 1.33 V_G + 6.67$

$-2.33 V_G = 1.67$

$\boxed{V_G = -0.714 V}$

$V_{SG} = V_S - V_G = V_S + 0.714$

Assume saturation

$I_D = \frac{1}{2} k'_p \frac{W}{L} (V_{SG} - |V_t|)^2$

$I_D = \frac{1}{2}(.25)(V_{SG} - 1)^2$

$I_D = 0.125(V_S + .714 - 1)^2$

$I_D = 0.125(V_S - 0.286)^2$

also

$I_D = \frac{5 - V_S}{1.2}$ (mA)

$\frac{5 - V_S}{1.2} = 0.125(V_S - 0.286)^2$

$5 - V_S = 0.15(V_S^2 - 0.572 V_S + .0818)$

$5 - V_S = 0.15 V_S^2 - .0858 V_S + 0.1227)$

$0 = 0.15 V_S^2 + 0.9142 V_S - 4.988$

or $V_S^2 + 6.095 V_S - 33.25 = 0$

solve

$V_S = \frac{-6.095 \pm \sqrt{(6.095) - 4(-33.2}}{2}$

$V_S = \frac{-6.095 \pm 13.044}{2}$

$\boxed{V_S = 3.47 V}$ or $-9.57V$

PS#7 - Solution

④ cont

$V_{SG} = 3.54\ V$ so $\boxed{V_{GS} = -3.54V}$

for $V_S = 3.47V$ $I_D = 1.27mA$

$$\frac{V_D - (-5)}{4} = I_D \qquad V_D = 4I_D - 5 = 0.069V$$

so $\boxed{V_{DS} = -3.40V}$ $V_{SD} = 3.40V$

in saturation $V_{SD} \geq V_{SG} - |V_t|$

$3.40 \geq 3.54 - 1$ ✓

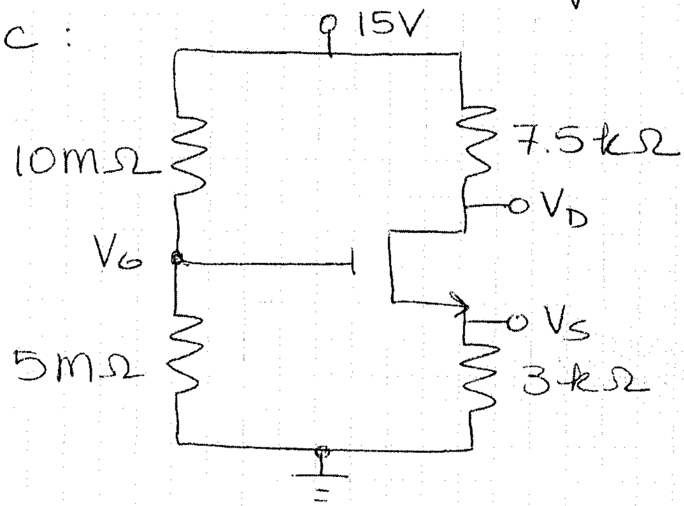
assumption holds.

PS#7 Solutions

(5)

a) $V_t = 1V$ $\quad k'_n \frac{W}{L} = 2 \frac{mA}{V^2}$

at dc:
assume
$\lambda = 0$



$V_G = 15 \cdot \frac{5}{10+5} = 5V$

$V_S = 3(1) = 3V$

$V_{GS} = 2V$

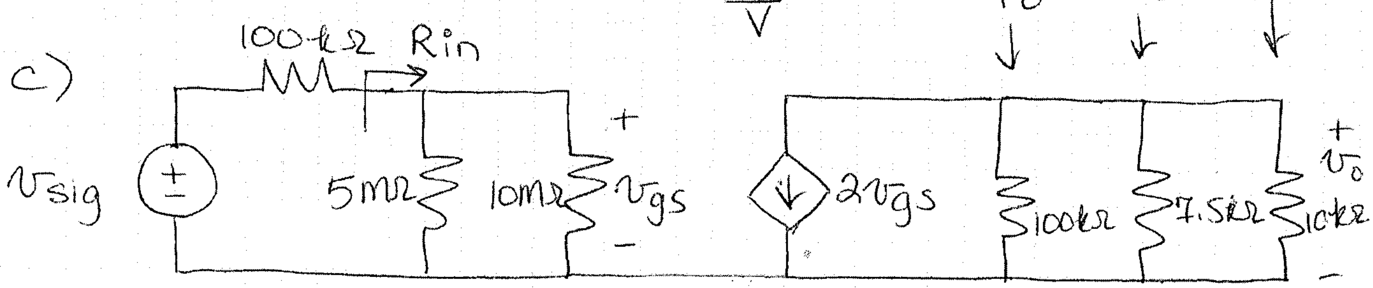
$I_D = 1mA$

$V_D = 15 - 7.5(1)$
$= 7.5V$

b) $r_o = \frac{V_A}{I_D} = \frac{100}{1 \times 10^{-3}} = 100k\Omega$

$g_m = k'_n \frac{W}{L}(V_{GS} - V_t)$
$= 2(2-1) = 2 \frac{mA}{V}$

c)



$v_{gs} = v_{sig} \frac{(5M \| 10M)}{100k + (5M \| 10M)} \Rightarrow \frac{v_{gs}}{v_{sig}} = 0.971 \, V/V$

$R_{in} = 5M \| 10M\Omega = 3.33 M\Omega$

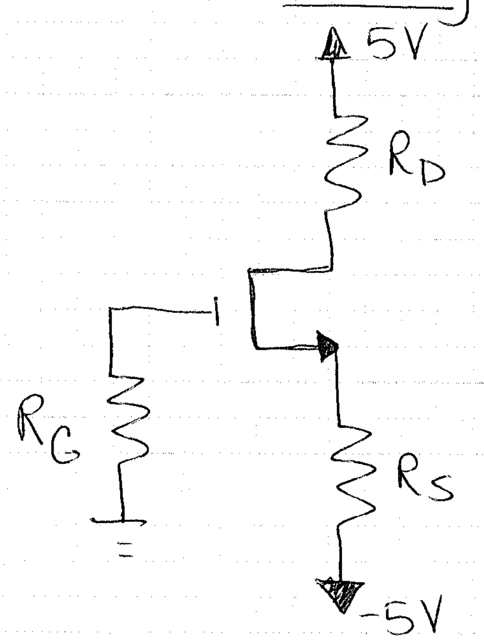
(5) cont

$$\frac{v_o}{v_{gs}} = -2\,(100 \| 7.5 \| 10)$$

$$= -8.22\ V/V$$

$$\frac{v_o}{v_{sig}} = \frac{v_o}{v_{gs}} \cdot \frac{v_{gs}}{v_{sig}} = -8.22\,(.971) = -7.98\ V/V$$

(6) dc biasing



$V_t = 1V$

$k'_n \frac{W}{L} = 0.8 \frac{mA}{V^2}$

$R_{in} = R_G = 10 M\Omega$

$I_D = 0.1 = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$

$0.1 = 0.4 (V_{GS} - 1)^2$

$V_{GS} = 1 \pm 0.5$

$\boxed{V_{GS} = 1.5V}$ or $+0.5V$

$V_G = 0 \quad V_S = -1.5V$

$$R_S = \frac{-1.5 - (-5)}{0.1} = 35\,k\Omega$$

$V_D = 5 - R_D(0.1)$ $\quad (R_D \sim k\Omega)$

or $R_D = (5 - V_D)/0.1$

largest R_D occurs when V_D is small

$V_{Dmin} \Rightarrow V_{DSmin} = V_{GS} - V_t$

(6) cont

for ± 1V swing

$$V_{DS} - 1 = V_{GS} - V_t$$
$$V_{DS} - 1 = 0.5$$
$$V_{DS} = 1.5V$$

$$V_D - V_S = 1.5$$
$$V_D = 1.5 + \overset{-1.5V}{\cancel{V_S}} = 0$$
$$\boxed{V_D = 0}$$

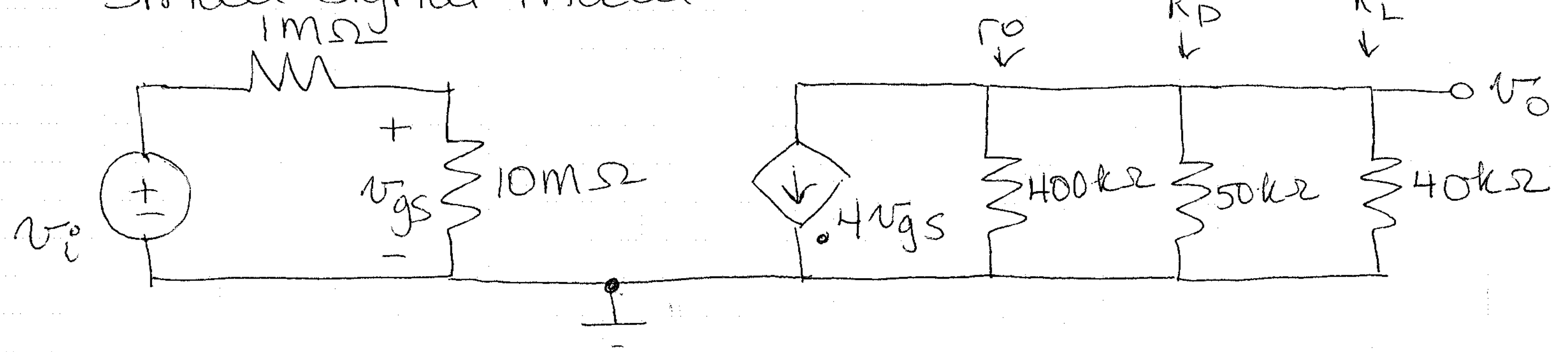
$$R_D = \frac{5-0}{.1} = 50k\Omega$$

by $V_{DSQ} = 1.5V$ $\quad I_{DQ} = 0.1mA$
$V_{GSQ} = 1.5V$

$$g_m = 4k'_n \frac{W}{L}(V_{GS} - V_t) = 0.4mA/V^2$$

$$r_o = \frac{V_A}{I_D} = \frac{40}{.1} = 400k\Omega$$

small signal model



$$v_o = -0.4 v_{gs}(400 \| 50 \| 40)$$
$$= -8.42 v_{gs}$$

$$v_{gs} = v_i \frac{10}{10+1}$$
$$= 0.909 v_i$$

(6) cont

$$\frac{v_o}{v_{gs}} = -8.42 \text{ V/V} \qquad \frac{v_{gs}}{v_i} = 0.909$$

$$\frac{v_o}{v_i} = \frac{v_o}{v_{gs}} \cdot \frac{v_{gs}}{v_i} = -7.65 \text{ V/V}$$