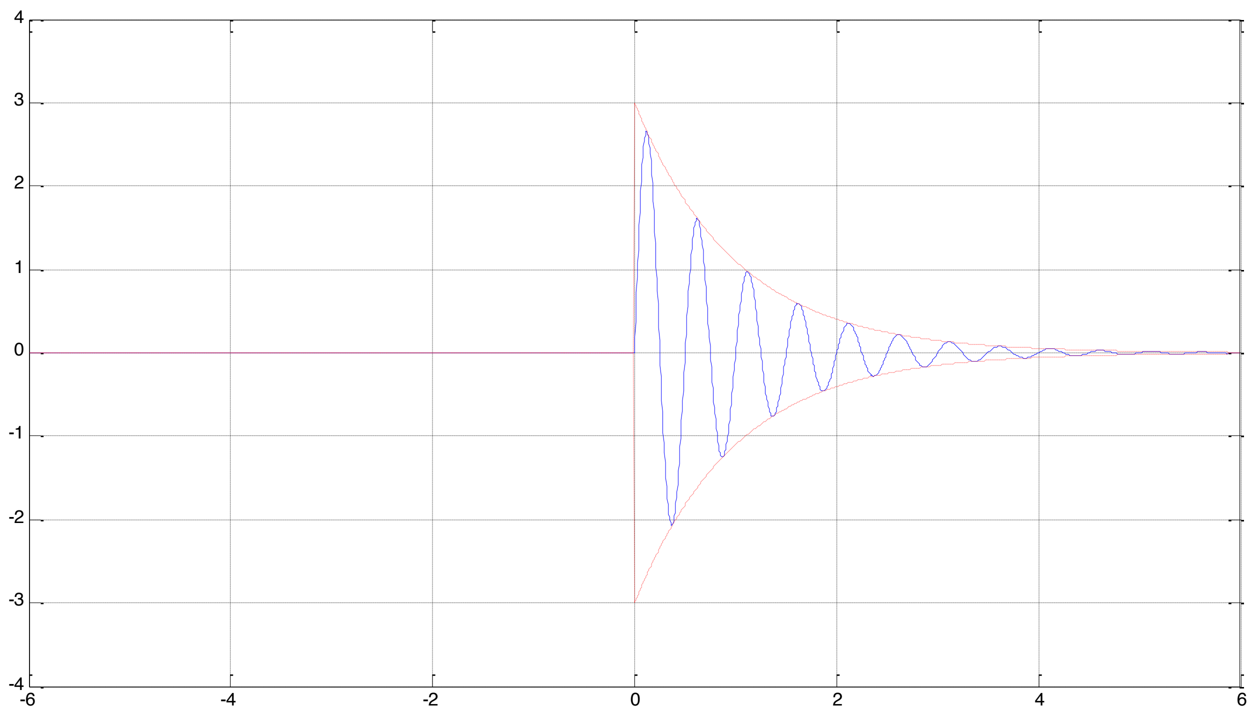
1. A microcontroller system has a 12 bit AD converter, 10,000 bytes of available RAM, and samples stereo audio signal with sampling frequency of 20 KHz. How many seconds of audio signal can be stored in memory?

T = 10000 bytes / (2 channels * 2 bytes/sample * 20000 samples/s) = 0.125 s (125ms)

or optimized:

T = 10000 bytes / (2 channels * 1.5 bytes/sample * 20000 samples/s) = 0.166 s (166ms)

2. Plot function $x(t) = e^{-t} \cdot 3\sin(4\pi t) \cdot u(t)$



3. The output of a causal LTI system with the impulse response $h(t)$ to a causal input $x(t)$ is

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau$$

4. Consider the periodic signal x(t) = cos(0.4π·*t*) + 3·cos(2 π·*t/7*), −∞ < t < ∞..

Is x(t) periodic? If it is, what is the period T_0 of $x(t)$? T_0 = 35 s

$x(t) = x_1(t) + x_2(t)$

$T_1 = 2\pi / (0.2\pi) = 5$ s

$T_2 = 2\pi / (2\pi/7) = 7$ s

$T_0 = N^*T_1 = M^*T_2$ → The least common multiple of 5 and 7 is 35, therefore 7N = 5M → $T_0 = 7^*5 = 35$ s

What is the average power of $x(t)$?

$$\int_0^x \cos^2(x)dx = \int_0^x \frac{1}{2}(1+\cos(2x)) = \frac{1}{2}\int_0^x dx + \frac{1}{4}\int_0^x \cos(y)dy = \left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^x$$

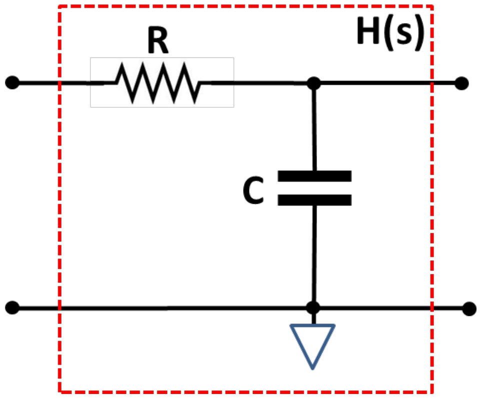
$$\int_0^t \cos^2(x)dx = \left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^t = \frac{t}{2}+\frac{1}{4}\sin(2t) \Rightarrow for\ t = T\ \int_0^T \cos^2(x)dx = \frac{1}{2}$$

$$P_{x1} = \frac{1}{T_1}\int_0^{T_1} x_1^{\,2}(t)dt = \frac{1}{0.5}\cdot\left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^{T_1} = \frac{1}{T_1}\left(\frac{T_1}{2}+\frac{1}{4}\sin\left(12\pi\cdot\frac{1}{6}\right)\right) = 1\cdot\frac{1}{2} = 0.5$$

$$P_{x2} = \frac{1}{T_2}\int_0^{T_2} x_2^{\,2}(t)dt = \frac{1}{T_2}\int_0^{T_2}(3\cos(16\pi t))^2 dt = 9\cdot\frac{1}{T_2}\int_0^{T_2}\cos^2(16\pi t)dt = 9\cdot\frac{1}{2} = 4.5$$

$P = P_{x1} + P_{x2} = 0.5 + 4.5 = 5$

5. (4 points)



$$H(s)=\frac{Y(s)}{X(s)}=\frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}=\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

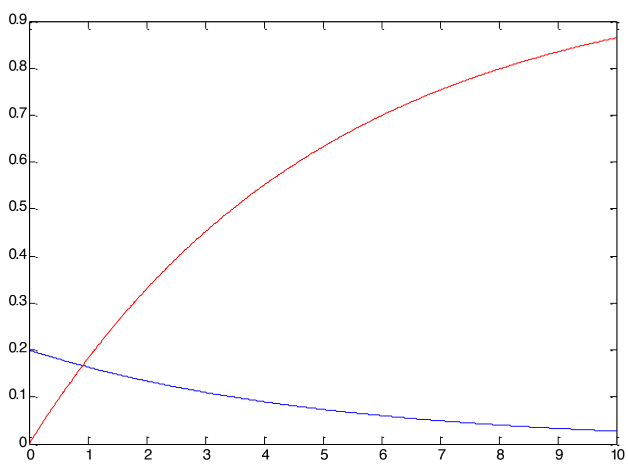
$$h(t)=\frac{1}{RC}\cdot e^{-\frac{t}{RC}}$$

b) (6 points) Step response:

$$\frac{1}{RC}=\frac{1}{5M\Omega\cdot 1\mu F}=0.2$$

$$S(s)=\frac{1}{s}H(s)=\frac{1}{s}\frac{0.2}{s+0.2}=\frac{A}{s}+\frac{B}{s+0.2}=\frac{1}{s}-\frac{1}{s+0.2}$$

$$s(t)=\left(1-e^{-0.2\cdot t}\right)\cdot u(t)$$



6. A system with input x(t) and output y(t) is defined by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Find the impulse response *h(t)* and the unit-step response *s(t)*.

The Laplace transform of the differential equation gives

$$[s^2Y(s) - sy(0) - \frac{dy(t)}{dt}|_{t=0}] + 3[sY(s) - y(0)] + 2Y(s) = X(s)$$
$$Y(s)(s^2 + 3s + 2) - (s + 3) = X(s)$$

so we have that

$$Y(s) = \frac{X(s)}{(s+1)(s+2)} + \frac{s+3}{(s+1)(s+2)}$$
$$= \frac{1 + 3s + s^2}{s(s+1)(s+2)} = \frac{B_1}{s} + \frac{B_2}{s+1} + \frac{B_3}{s+2}$$

We find B_1 = 0.5, B_2 = 1, and B_3 = - 0.5.

therefore:

$$y(t) = \left[0.5 + e^{-t} - 0.5 \cdot e^{-2t}\right] \cdot u(t)$$

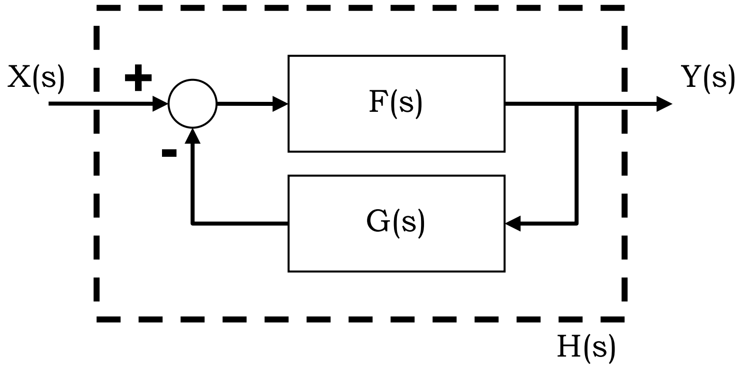
steady state response is

$$0.5 \cdot u(t)$$

and transient response is

$$\left[e^{-t} - 0.5 \cdot e^{-2t}\right] \cdot u(t)$$

7. (5 points) What is the transfer function H(s) of the system represented below?



Y(s) = F(s)*(X(s) - G(s)*Y(s)) = F(s)*X(s) - F(s)*G(s)*Y(s)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

b) (10 points) Transfer function of the feedback block is

$$G(s) = \frac{Z_{R||C}}{Ls + Z_{R||C}}$$

$$Z_{R||C} = \frac{R\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{RCs + 1}$$

$$G(s) = \frac{R}{R + Ls(RCs + 1)} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Transfer function of the system is:

$$H(s) = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

$F(s) = A$ and $G(s) = \frac{\frac{1}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$

$$H(s) = \frac{A}{1+A\left(\frac{\frac{1}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}\right)} for\ A \to \infty\ H(s) = LCs^2 + \frac{L}{R}s + 1$$

8. Determine the fundamental frequency ω_0 of

$$x(t) = 2 + 8 \cdot \sin\left(\frac{2\pi}{6} t\right)$$

and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\omega_0 = 2\pi/6$

$$\begin{aligned} x(t) &= 2 + 8 \cdot \frac{1}{2}\left(e^{j\frac{2\pi}{6}t} - e^{-j\frac{2\pi}{6}t}\right) = a_{-1}e^{-j\omega_0 t} + a_0 e^{j0t} + a_1 e^{j\omega_0 t} \\ &= a_{-1}e^{-j\frac{2\pi}{6}t} + a_0 + a_1 e^{j\frac{2\pi}{6}t} \\ &= -4e^{-j\frac{2\pi}{6}t} + 2 + 4e^{j\frac{2\pi}{6}t} \end{aligned}$$

and $a_{-1} = -4$, $a_0 = 2$, $a_1 = 4$.

9. Represent magnitude and phase line spectra of raised sine signal:

$$x(t) = 2 + 8 \cdot \sin(100t) + 6 \cdot \cos(200t)$$