Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #5 Solution

1. (10 points)

a) (4 points) Explain the difference between hard and soft real-time systems.

A system is said to be real-time if the total correctness of an operation depends not only upon its logical correctness, but also upon the processing time for each system state. A system state is created by sampling the system.
Real-time systems perform their operation fast enough to influence the system they control. They are classified according to the consequences of missing a deadline.

Hard real-time systems may generate a total system failure if the deadline is missed.

Soft real-time systems may tolerate missing a processing deadline for a limited period of time that will degrade only the system's quality of service (such as latency)

b) (6 points) Maximum frequency of the input is 600Hz. The microcontroller processes each sample in 1200 clock cyles with clock frequency F_c = 1MHz. Can this system run in real-time?

If the maximum frequency of the signal is 600Hz, the sampling frequency must be at least 1,200Hz (Nyquist criterion). Therefore, sampling time is

$$T_s = 1 / F_s = 1/1200 = 833\ \mu s$$

Processing time is

$$T_p = 1{,}200 \text{ cycles} * T_{cycle} = 1{,}200 * 1\ \mu s = 1.2 \text{ ms}$$

Since Tp > Ts, the system CAN NOT work in real time.

c) (4 points) What is the minimum frequency of the clock that allows real-time operation with 2x oversampling of the input?

Processing time must be less than sampling time

$$T_p = 1{,}200 \text{ cycles} * T_{cycle} = 1200 * 1/F_{clock} < T_s = 1/F_s \rightarrow$$

$$F_{clock} > 1{,}200 * F_s, \qquad \text{Fclock} > 1.44 \text{ MHz}$$

2. (10 points) Consider a sinusoidal signal:

$$x(t) = 5 \cos (14 \pi t + \pi/8)$$

Determine an appropriate sampling period T_s and obtain the discrete-time signal x[n] corresponding to the largest allowed sampling period.

$\omega = 2\pi f = 14\pi \rightarrow f = 7$ Hz $\rightarrow Ts < 1/(2f) = 1/14 = 71.4$ms

$$x[n] = 5\cos\left(14\pi t + \frac{\pi}{8}\right)\Bigg|_{t=n/14} = 5\cos\left(\pi n + \frac{\pi}{8}\right)$$

3. (10 points) Consider a discrete time IIR system represented by the difference equation:

$$y[n] = 0.4\, y[n-1] + x[n]$$

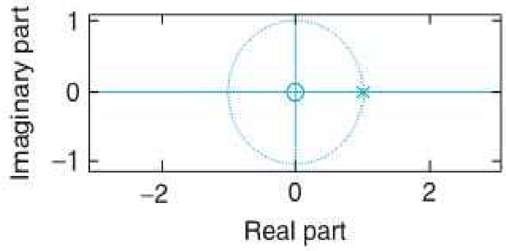
with x[n] as the input and y[n] as the output. Determine the transfer function of the system and find the impulse response. Under what conditions the system is BIBO stable?

See example 9.10 (page 535).

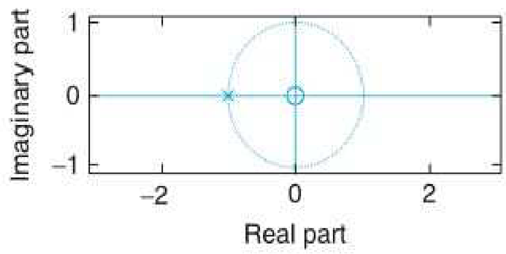
$H(z) = Y(z)/X(z) = 1/(1-0.4z^{-1}) \rightarrow h(n) = 0.4^n u[n]$

The system has a pole at z=0.4 (inside the unit circle) and therefore it is BIBO stable.

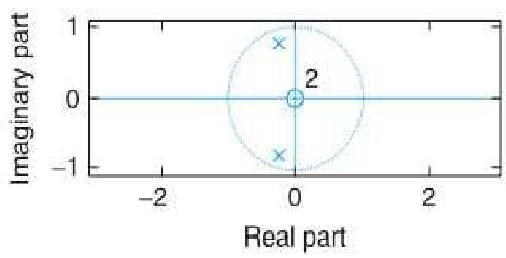
4. (10 points) Describe the effect of pole location on the inverse Z-transform for the following cases.



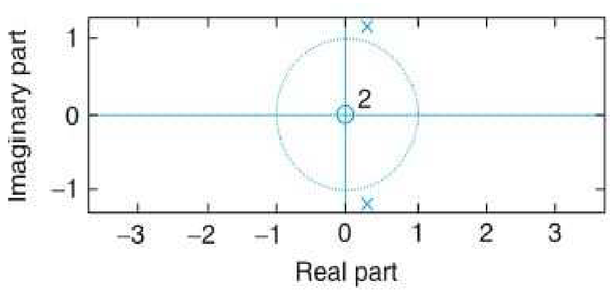
u[n], constant



cosine of frequency π,
constant amplitude



a decaying modulated exponential



a growing modulated exponential

5. (6 points) If X(z) is the Z-transform of a causal signal x[n], then

Initial value is x[0] = $\lim_{Z\to\infty} X(Z)$

Final value is $\lim_{n\to\infty} x[n] = \lim_{Z\to 1} (Z-1)X(Z)$

6. (10 points) Consider an RLC circuit represented by the second-order differential equation

$$\frac{d^2V_c(t)}{dt^2}+\frac{dV_c(t)}{dt}+V_c(t)=V_s(t)$$

where the voltage across the the capacitor V_c(t) is the output and the source V_S(t) = u(t) is the input. Let the initial conditions be zero. Find the voltage across the capacitor using difference equations.

$$\left(\frac{1}{T_s^2}+\frac{1}{T_s}+1\right)v_c(nT_s)-\left(\frac{2}{T_s^2}+\frac{1}{T_s}\right)v_c((n-1)T_s)+\frac{1}{T_s^2}v_c((n-2)T_s)=v_s(nT_s)$$

For T_S=1

$$3v_c[n]-3v_c[n-1]+v_c[n-2]=v_s[n]$$

7. (20 points) Let x[n] = {0, 1, 1, 1, 0} and h[n] = {1.5, 1, 0.5}.
Compute and plot the convolution y[n] = x[n] * h[n].

Please take a look at animation on Angel.

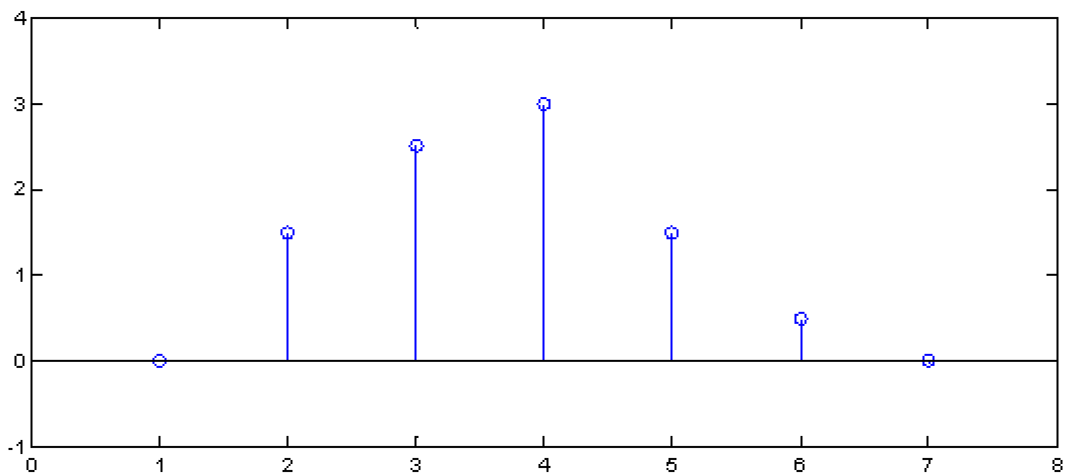
Convolution for causal LTI system:

$$y(t) = \int_{0}^{t} x(\tau)h(t-\tau)d\tau$$

		0	1	1	1	0	
0.5	1	1.5					→ **0**
	0.5	1	1.5				→ **1.5**
		0.5	1	1.5			→ **2.5**

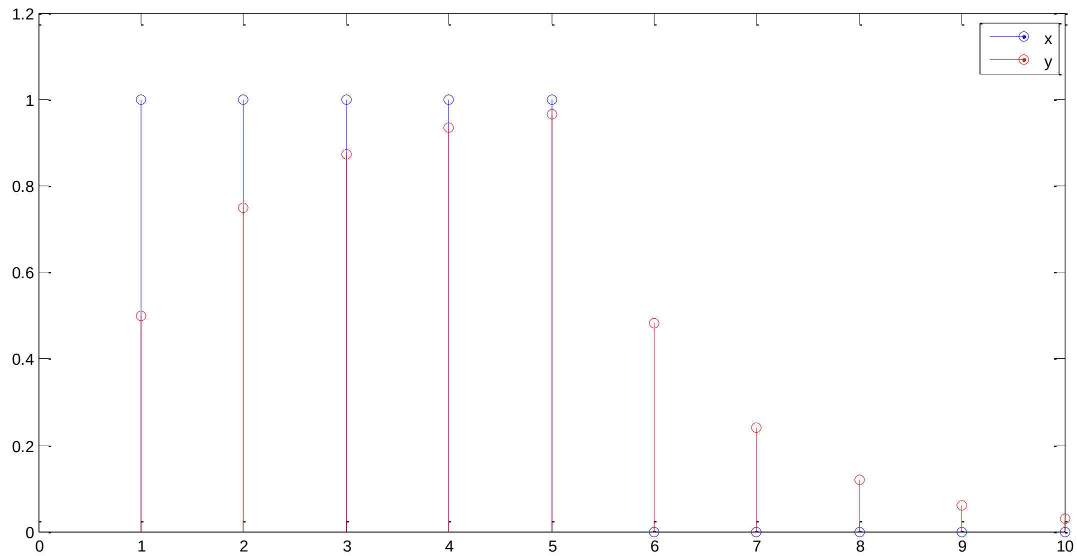
...

Result: { 0 1.5 2.5 3.0 1.5 0.5 0 }



8. (10 points) Let $h[n]=0.5^n u[n]$. What would be the response of the system y[n] to x[n] = u[n] – u[n-4] ? Plot the output y[n].

y[n] = conv(x[n],h[n])



9. (2 points) The frequency support of the DTFT of a discrete-time signal is *INVERSELY* proportional to the time support of the signal.

10. (4 points) The DTFT of a discrete-time signal x[n] is represented as:

$$X\left(e^{j\omega}\right)=\sum_{n} x[n]e^{-j\omega n},\ -\pi \le \omega \le \pi$$

while the inverse transform can be represented as:

$$x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}d\omega$$

11. (4 points) Represent DTFT of the following signals:

a) δ[n]

$1, \quad -\pi \le \omega \le \pi$

b) cos(ω_0n) u[n]

$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right], \quad -\pi \le \omega \le \pi$