Department of Electrical and Computer Engineering
The University of Alabama in Huntsville
Spring 2015

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Due: Monday, February 2 at 2:15 pm
Please bring hardcopy to the class and upload softcopy to Canvas

Student name:

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1 5	2 17	3 10	4 10	5 10	6 5	7 15	8 10	9 18	Total

Homework #1

1. (5 points)

How much memory do you need to store audio or audio and video recording of one lecture (80 minutes)? Consider the following conditions:

a) sampling at 8,000 Hz and 8 bits/sample.
b) CD quality recording (44.1 KHz, 16 bits/sample, stereo recording).
c) CD quality with 20 times compression (MP3 format).
d) video 640x480 pixels at 30 frames/second; RGB components (one byte each); audio as described in a).

2. (17 points) (Pr 0.2)

Write a script in Matlab to plot cosine signal with frequency 1Hz and amplitude 2 for three seconds. Generate a discrete-time signal $x[n] = x(nTs) = x(t) \mid t=nTs$ for the sampling interval Ts=0.1 sec, Ts = 0.5 sec, and Ts = 1 sec.

Determine for which values of Ts the discrete-time signal has lost the information in the analog signal. Use MATLAB to plot the analog signal (use the plot function) and the resulting discrete-time signals (use the stem function). Superimpose the analog and the discrete-time signals for $0 \le t \le 3$; use subplot to plot the four figures as one figure. For plotting the analog signal use Ts = 10^{-4}. You also need to figure out how to label the different axes and have the same scales and units.

3. (10 points) (Pr 0.12)

Consider a function of $z = 1 + j1$,

$$w = e^z$$

(a) Find $\log(w)$.
(b) Find the real and the imaginary parts of w.
(c) What is $w+w^*$, where w^* is the complex conjugate of w?
(d) Determine $|w|$, $\angle w$.
(e) What is $|\log(w)|^2$?
(f) Express $\cos(1)$ in terms of w using Euler's equation.

4. (10 points) (Pr 0.14)

Use Euler's identity to find an expression for *cos*(α) *cos*(β), and from the relation between cosines and sines obtain an expression for *sin*(α) *sin*(β).

5. (10 points) Phasors (modified Pr 0.23)

Plot signals y(t)= A sin($\Omega_0 t$) and x(t)= A cos($\Omega_0 t$).

(a) Do you see now how these functions are connected? How many radians do you need to shift in a positive or negative direction to get a sine from a cosine, etc.

(d) Suppose then you have the sum of two sinusoids, for instance z(t) = x(t) + y(t), adding the corresponding phasors for x(t) and y(t) at some time (e.g., t = 0), which is just a sum of two vectors, you should get a vector and the corresponding phasor. Get the phasor for z(t) and the expression for it in terms of a cosine.

6. (5 points)

Consider the analog signal

$$x(t) = \sin(2\pi t + \theta) \qquad -\infty < t < \infty$$

Determine the value of θ for which x(t) is even and odd.

7. (15 points) (Pr 1.1)

1.1. Signal energy and RC circuit—MATLAB

The signal $x(t) = e^{-|t|}$ is defined for all values of t.

(a) Plot the signal $x(t)$ and determine if this signal is finite energy. That is, compute the integral

$$\int_{-\infty}^{\infty} |x(t)|^2 dt$$

and determine if it is finite.

(b) If you determine that $x(t)$ is absolutely integrable, or that the integral

$$\int_{-\infty}^{\infty} |x(t)| dt$$

is finite, could you say that $x(t)$ has finite energy? Explain why or why not. *Hint:* Plot $|x(t)|$ and $|x(t)|^2$ as functions of time.

(c) From your results above, is it true the energy of the signal

$$y(t) = e^{-t}\cos(2\pi t)u(t)$$

is less than half the energy of $x(t)$? Explain. To verify your result, use symbolic MATLAB to plot $y(t)$ and to compute its energy.

(d) To discharge a capacitor of 1 mF charged with a voltage of 1 volt we connect it, at time $t = 0$, with a resistor of $R\ \Omega$. When we measure the voltage in the resistor we find it to be $v_R(t) = e^{-t}u(t)$. Determine the resistance R. If the capacitor has a capacitance of 1 μF, what would be R? In general, how are R and C related?

8. (10 points) (Pr 1.3)

Consider the periodic signal $x(t) = \cos(2\Omega_0 t) + 2\cos(\Omega_0 t)$, $-\infty < t < \infty$, and $\Omega_0 = \pi$. The frequencies of the two sinusoids are said to be harmonically related (one is a multiple of the other).

(a) Determine the period T_0 of $x(t)$.

(b) Compute the power P_x of $x(t)$.

(c) Verify that the power P_x is the sum of the power P_1 of $x_1(t) = \cos(2\pi t)$ and the power P_2 of $x_2(t) = 2\cos(\pi t)$.

(d) In the above case you are able to show that there is superposition of the powers because the frequencies are harmonically related. Suppose that $y(t) = \cos(t) + \cos(\pi t)$ where the frequencies are not harmonically related. Find out whether $y(t)$ is periodic or not. Indicate how you would find the power P_y of $y(t)$. Would $P_y = P_1 + P_2$ where P_1 is the power of $\cos(t)$ and P_2 is the power of $\cos(\pi t)$? Explain what is the difference with respect to the case of harmonic frequencies.

9. (18 points) (Pr 1.4)

1.4. Periodicity of sum of sinusoids—MATLAB

Consider the periodic signals $x_1(t) = 4\cos(\pi t)$ and $x_2(t) = -\sin(3\pi t + \pi/2)$.

(a) Find the periods of $x_1(t)$ and $x_2(t)$.

(b) Is the sum $x(t) = x_1(t) + x_2(t)$ periodic? If so, what is its period?

(c) In general, two periodic signals $x_1(t)$ and $x_2(t)$ having periods T_1 and T_2 such that their ratio $T_1/T_2 = M/K$ is a rational number (i.e., M and K are positive integers), then the sum $x(t) = x_1(t) + x_2(t)$ is periodic. Suppose the rationality condition is satisfied and $M = 3$ and $K = 12$. Determine the period of $x(t)$.

(d) Determine whether $x(t) = x_1(t) + x_2(t)$ is periodic when

- $x_1(t) = 4\cos(2\pi t)$ and $x_2(t) = -\sin(3\pi t + \pi/2)$
- $x_1(t) = 4\cos(2t)$ and $x_2(t) = -\sin(3\pi t + \pi/2)$

Use symbolic MATLAB to plot $x(t)$ in the above two cases and confirm your analytic results about the periodicity or lack of periodicity of $x(t)$.