Signals and Systems Using MATLAB

Luis F. Chaparro

Chapter 0 --- From the Ground Up!

What is in this chapter?

- Signals and systems and digital technologies
  - Applications
- Analog or discrete?
- Complex numbers and Phasors
- Soft introduction to MATLAB - numerical and symbolic

1948 – Birth of Digital Signal Processing and Communications

- Bell Telephone Laboratories invent transistor
- Manchester University (UK) develop first operational stored-program computer
- Publications
  - Shannon's mathematical theory of digital communications
  - Hamming's theory on error-correcting codes
  - Wiener's *Cybernetics* comparing biological systems with communication and control systems

Moore's law, DSPs and FPGAs

- 1965: Moore (Intel) envisioned number of transistors on a chip would double about every two years
- Digital Signal Processor (DSP): optimized microprocessor for real-time processing. Embedded in larger systems consists of processor, memory and Analog to Digital (ADC) and Digital to Analog (DAC) converters
- Field-Programmable Gate Array (FPGA) device containing programmable logic blocks and interconnects

Examples of Signal Processing Applications

- **Compact-disc (CD) Player (invented in Germany 1982)**

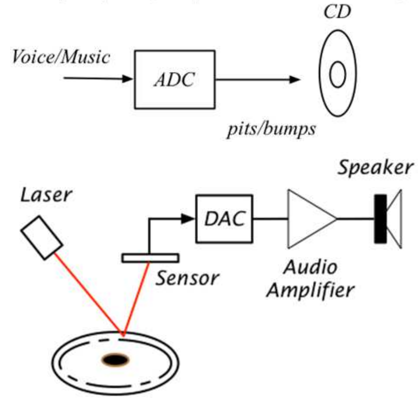

Figure 1: *When playing a CD, the CD player follows the tracks in the disc, focusing a laser on them, as the CD is spun. The laser shines a light which is reflected by the pits and bumps put on the surface of the disc and corresponding to the coded digital signal from an acoustic signal. A sensor detects the reflected light and converts it into a digital signal, which is then converted into an analog signal by the digital to analog converter (DAC). When amplified and fed to the speakers such a signal sounds like the originally recorded acoustic signal.*

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- **Computer-control Systems**
  - Applications: controlling simple systems such as a heater (e.g., keeping a room temperature comfortable while reducing energy consumption) or cars (e.g., controlling their speed), to that of controlling rather sophisticated machines such as airplanes (e.g., providing automatic flight control), or chemical processes in very large systems such as oil refineries.

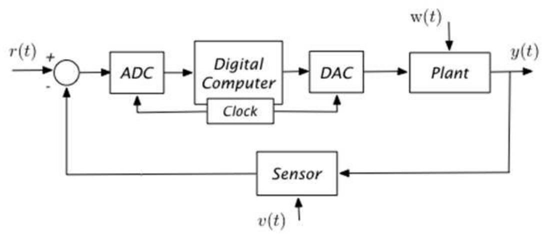

Figure 3. *Computer control system for an analog plant (e.g., cruise control for a car). The reference signal is $r(t)$ (e.g., desired speed) and the output is $y(t)$ (e.g., car speed). The analog signals are converted to digital signals by an ADC, while the digital signal from the computer is converted into an analog signal (an actuator is probably needed to control the car) by a DAC. Thesignals $w(t)$ and $v(t)$ are disturbances or noise in the plant and the sensor (e.g., electronic noise in the sensor and undesirable vibration in the car).*

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Continuous and Discrete Representations

- Discrete-time signal $x[n]$
  analog signal $x(t)$
  sampling process:

$$\boxed{x[n] = x(nT_s) = x(t)_{|t=nT_s} \cdot}$$

or sequence

$$\{\cdots x(-T_s) \;\; x(0) \;\; x(T_s) \;\; x(2T_s) \cdots\}$$

$$\{\cdots x[-1] \;\; x[0] \;\; x[1] \;\; x[2] \cdots\}$$

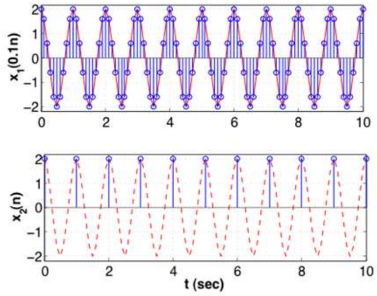

Figure 4. *Sampling an analog sinusoid $x(t) = 2\cos(2\pi t)$, $0 \le t \le 10$, with two different sampling periods $T_{s1} = 0.1$ sec. (top) and $T_{s2} = 1$ sec. (bottom) giving $x_1(0.1n)$ and $x_2(n)$. The sinusoid is shown by dashed lines. Notice the similarity between the discrete-time signal and the analog signal when $T_{s1} = 0.1$ sec., while they are very different when $T_{s2} = 1$ sec. indicating loss of information.*

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Matlab Example

- Inherent discrete-time signals

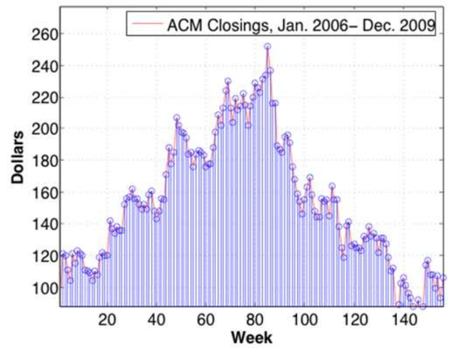


Figure 5. *Weekly closings of ACM stock for 160 weeks in 2006 to 2009. ACM is the trading name of the stock of the imaginary company ACME Inc. makers of everything you can imagine.*

Derivatives and Finite-differences

Derivative operator (measures rate of change analog signal $x(t)$)

$$\boxed{D[x(t)] = \frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}}$$

Finite Calculus forward finite-difference operator

$$\boxed{\Delta[x(nT_s)] = x((n+1)T_s) - x(nT_s)}$$

Integrals and Summations

- Integration is opposite of differentiation

$$I(t) = \int_{t_0}^{t} x(\tau)d\tau$$

(sum of the area under $x(t)$ from t_0 to t)

$$\begin{aligned} \frac{dI(t)}{dt} &= \lim_{h \to 0} \frac{I(t) - I(t-h)}{h} = \lim_{h \to 0} \frac{1}{h} \int_{t-h}^{t} x(\tau)d\tau \\ &\approx \lim_{h \to 0} \frac{x(t) + x(t-h)}{2} = x(t) \end{aligned}$$

- Example: approximation of integrals by sums

$$\int_0^{10} t \ \ dt = \frac{t^2}{2} \Big|_{t=0}^{10} = 50$$

i.e., the area of a triangle of base of 10 and height of 10

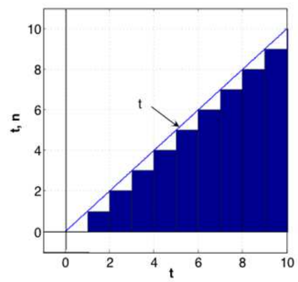


Figure 6. *Approximation of area under $x(t) = t$, $t \geq 0$, 0 otherwise, by pulses of width 1 and height nT_s, where $T_s = 1$ and $n = 0, 1, \cdots$*

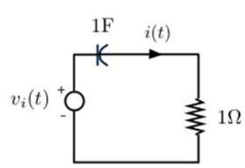
$$\begin{aligned} \sum_{n=0}^{9} p[n] &= \sum_{n=0}^{9} n = 0 + 1 + 2 + \cdots 9 = 0.5 \left[\sum_{n=0}^{9} n + \sum_{k=9}^{0} k \right] \\ &= 0.5 \left[\sum_{n=0}^{9} n + \sum_{n=0}^{9} (9 - n) \right] = \frac{9}{2} \sum_{n=0}^{9} 1 = \frac{10 \times 9}{2} = 45. \end{aligned}$$

Differential and Difference Equations

- A differential equation characterizes the way the system responds to inputs over time
- solution of de obtained using **analog** and **digital computers**
- RC circuit: voltage source $v_i(t)$ (input), $R = 1\ \Omega$, $C = 1$ Farad (with huge plates!)

$$v_i(t) = v_c(t) + \frac{dv_c(t)}{dt} \qquad v_c(t): \text{ output}$$

initial voltage $v_c(0)$ across C.



$$v_c(t) = \int_0^t [v_i(\tau) - v_c(\tau)]d\tau + v_c(0) \qquad t \geq 0$$

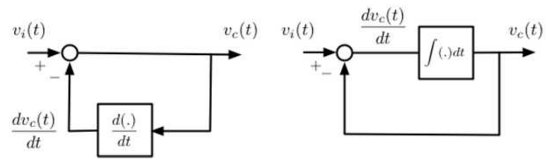


Figure 7. *Realization of first-order differential equation using differentiators (left) and integrators (right)*

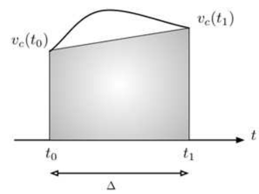


Figure 8. *Approximation of area under the curve by a trapezoid.*

$$v_c(t_1) - v_c(t_0) = \int_{t_0}^{t_1} v_i(\tau)d\tau - \int_{t_0}^{t_1} v_c(\tau)d\tau.$$

First order linear difference equation with constant coefficients approximating the differential equation characterizing the RC circuit

$$\boxed{v_c(nT) = \frac{T}{2+T}[v_i(nT) + v_i((n-1)T)] + \frac{2-T}{2+T}v_c((n-1)T) \qquad n \geq 1}$$

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Complex or Real?

- Time-dependent signals characterized by frequency and damping giving $s = \sigma + j\Omega$ in Laplace transform, or $z = re^{j\omega}$ in the Z-transform
- Complex Numbers and Vectors

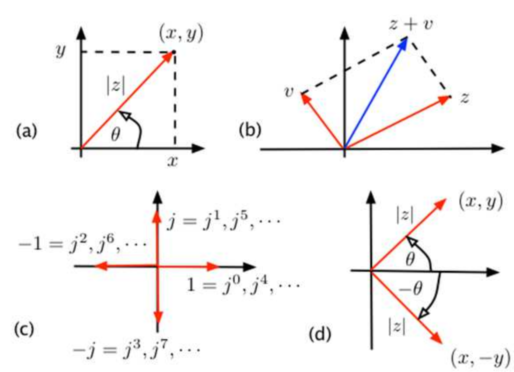

Figure 9. *(a) Representation of a complex number z by a vector ; (b) addition of complex numbers z and v; (c) integer powers of j; (d) complex conjugate.*

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- Euler's identity

$$\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta).}$$

$$\boxed{\cos(\theta) = \mathcal{R}e[e^{j\theta}] = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

while the sine is given by

$$\boxed{\sin(\theta) = \mathcal{I}m[e^{j\theta}] = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$$

- Euler's identity can also be used to find different trigonometric identities. For instance,

$$\cos^2(\theta) = \left[\frac{e^{j\theta} + e^{-j\theta}}{2}\right]^2 = \frac{1}{4}[2 + e^{j2\theta} + e^{-j2\theta}] = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$
$$\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$$
$$\sin(\theta)\cos(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}\,\frac{e^{j\theta} + e^{-j\theta}}{2} = \frac{e^{j2\theta} - e^{-j2\theta}}{4j} = \frac{1}{2}\sin(2\theta)$$

Phasors and Sinusoidal Steady–state

- Sinusoid

$$x(t) = A\cos(\Omega_0 t + \psi) \qquad -\infty < t < \infty$$

where A is the amplitude, $\Omega_0 = 2\pi f_0$ is the frequency in rad/sec and ψ is the phase in radians

- For $v(t) = A\cos(\Omega_0 t + \psi)$ the corresponding **phasor** is

$$\boxed{V = Ae^{j\psi} = A\cos(\psi) + jA\sin(\psi) = A\angle\psi}$$

and such that

$$\boxed{v(t) = \mathcal{R}e[Ve^{j\Omega_0 t}] = \mathcal{R}e[Ae^{j(\Omega_0 t + \psi)}] = A\cos(\Omega_0 t + \psi)}$$

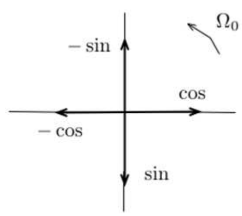


Figure 10. *Generation of sinusoids from phasors of a frequency* Ω_0.

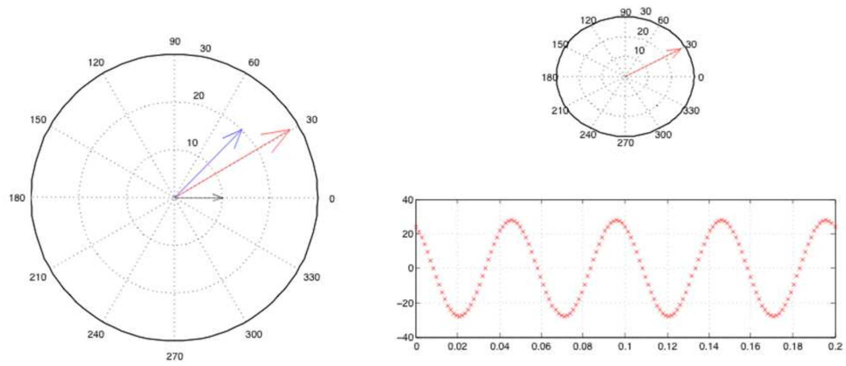
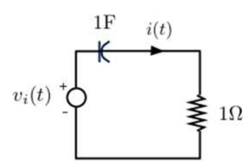


Figure 11. *Sum of phasors $I_1 = 10e^{j0}$ (black) and $I_2 = 20e^{j\pi/4}$ (blue) with the result in red (left), sinusoid (right bottom) generated by phasor $I = I_1 + I_2$ (right top).*

- Sinusoidal steady–state using phasors: RC circuit ($R = 1\ \Omega$, $C = 1$ F, input $v_i(t) = A\cos(\Omega_0 t)$, and $v_c(t)$ output)

$$
\begin{aligned}
v_c(t) &= \mathcal{R}e\left[V_c e^{j\Omega_0 t}\right] \qquad V_c = Ce^{j\psi} \text{ phasor for } v_c(t)\\
\frac{dv_c(t)}{dt} &= \frac{d\mathcal{R}e[V_c e^{j\Omega_0 t}]}{dt} = \mathcal{R}e\left[V_c \frac{de^{j\Omega_0 t}}{dt}\right] = \mathcal{R}e\left[j\Omega_0 V_c e^{j\Omega_0 t}\right]\\
v_i(t) &= \mathcal{R}e\left[V_i e^{j\Omega_0 t}\right] \quad \text{where} \quad V_i = Ae^{j0}
\end{aligned}
$$



in the differential equation

$$
\begin{aligned}
v_i(t) &= \frac{dv_c(t)}{dt} + v_c(t)\\
\mathcal{R}e\left[V_c(1 + j\Omega_0)e^{j\Omega_0 t}\right] &= \mathcal{R}e\left[Ae^{j\Omega_0 t}\right]
\end{aligned}
$$

so that

$$V_c = \frac{A}{1 + j\Omega_0} = \frac{A}{\sqrt{1 + \Omega_0^2}} e^{-j\tan^{-1}(\Omega_0)} = Ce^{j\psi}$$

Sinusoidal steady–state response

$$v_c(t) = \mathcal{R}e\left[V_c e^{j\Omega_0 t}\right] = \frac{A}{\sqrt{1 + \Omega_0^2}} \cos(\Omega_0 t - \tan^{-1}(\Omega_0))$$

Ratio

$$\frac{V_c}{V_i} = \frac{1}{1 + j\Omega_0}$$