# PROJECTILE MOTION

## OBJECTIVE:

This experiment provides you the opportunity to study the laws of projectile motion. This experiment will allow you to calculate the velocity of projectile from the range and height it travels. Also, you will be able to estimate the effects on the calculated results that are produced by the measurement errors.

## METHOD:

In summary you will use a simple inclined plane constructed of a section of PVC pipe to launch a small steel ball. By adjusting the starting position of the ball within the pipe, you can control the velocity of the ball as it leaves the end of the pipe. Once the ball leaves the pipe it becomes a free projectile. You will measure the range and height that the ball travels as a free projectile. From this free-projectile data, you can calculate the velocity. These measurements and calculations permit you to verify some of the laws of projectile motion.

You will make a series of measurements that will:

1. experimentally verify that the initial velocity of the ball can be controlled by adjusting the height through which it rolls. Moreover, you will show that this velocity is proportional to the square root of the height traveled down the inclined plane.
2. experimentally show that the horizontal range of the free projectile is proportional to the vertical height of free-fall. This range is also proportional to the square root of the vertical distance of free fall.

## APPARATUS:

PVC pipe with an approximate 120 degree bend, Steel ball, Lab stand, Blocks and Meter stick.

## THEORY:

### Range of a Projectile in Free-Fall

When a freely falling body has an initial velocity, v, in the x direction only, it will travel a distance, x, as

$$x = v\,t \qquad \text{Eq.1}$$

where x = the horizontal distance traveled, v = the initial velocity, and t the time to impact.

The vertical distance traveled in the same time is

$$y = \tfrac{1}{2}\, g\, t^2 \qquad \text{Eq.2}$$

Equations 1 and 2 may be combined to yield

$$v = x\sqrt{\frac{g}{2y}} \qquad \text{Eq.3}$$

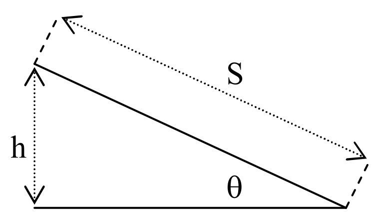
Equation 3 can be used to calculate the initial velocity of the projectile used in this experiment by using horizontal range and vertical height measurements data. The velocity of the projectile can calculated without time measurements provided that acceleration of gravity is known. The acceleration due to gravity is 979.6 cm/sec in Huntsville Alabama.

**Ball Rolling Down an Incline**

The concept of the moment of inertia of rigid bodies predicts the following result: that a solid sphere rolling down an incline without slipping will be accelerated at a uniform rate independent of the mass of the sphere. Moreover, the theory predicts that the acceleration will be

$$a = 5/7\, g \sin\theta \qquad \text{Eq.4}$$

Where θ = the angle at which the plane is inclined from the horizontal.



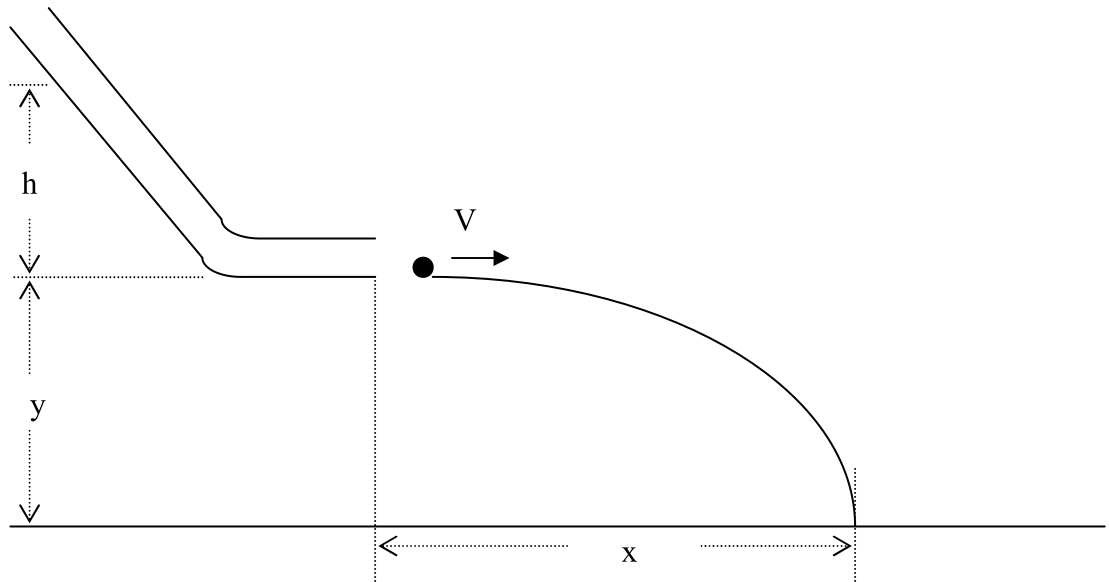

Recalling that $v^2 = 2as$ and $\sin\theta = h/s$ , it follows that Eq.4 can be rewritten as

$$v^2 = \frac{10}{7} gh \qquad \text{Eq.5}$$

or that

$$v = \sqrt{\frac{10}{7} gh} \qquad \text{Eq.6}$$

Equation 6 then suggests a means for us to control the velocity of the projectile. By adjusting the height, h, the velocity at the end of the inclined plane may be varied. We must emphasize that equation 6 is accurate only when the sphere rolls without slipping, a result that may not entirely valid inside the smooth PVC pipe used in this experiment. The motion of ball in the PVC pipe should be combined rolling with sliding. However, the velocity should be proportional to $\sqrt{gh}$ and the constant should lie between $\sqrt{\frac{10}{7}}$ and $\sqrt{2}$ .




**DETAILED PROCEDURE:**

**Part 1: Velocity as a Function of Height**

During this part of experiment you will keep the free-fall distance of the projectile constant. You will vary the height that ball rolls in the pipe. There are 5 holes in the side of the pipe that allow you to set the starting height and get 5 different velocities. Measure the projectile range resulting from each position. Use the carbon paper to make a permanent record of the impact point of each trail as demonstrated by your instructor. You can measure the range distance after allowing 5 to 10 balls to roll through the tube.

Use equation 3 to calculate a velocity from this range-vertical free-fall data. Then plot these computed velocities as a function of the square roots of the starting height in the pipe. (Is this result a straight line?). Alternately, you could plot the velocity as function of the height in the tube. (In this case your result will not be linear plot.)

**Part 2. Range as Function of Free-Fall Height**

During this part of the experiment you will keep the height that the ball rolls in the pipe a constant. Vary the free-fall height of the projectile. Measure the range as function of the free-fall height. Even if you do not know the precise value of the velocity in equation 3, it predicts the relationship between the horizontal range and vertical free-fall distance. Plot this data to determine if “x” and “y” are related as predicted by Eq.3.

**Additional Questions:**

1. If this same projectile motion experiment were performed on the moon where the gravity is 1/6th that of the earth would the ranges be longer, sorter, or the same as the values you measured here on earth? Justify your answer.

2. If the end of the pipe were angled up slightly (instead of being level) what would happen to the values of "x".

3. Explore the 5/7 factor from equation 4. Do this by using your data from table I.

**Table I. Velocity as a Function of Height Rolled**

y = ________

| | h | $\sqrt{h}$ | $x_1$ $x_2$ $x_3$ $x_4$ $x_5$ | $\bar{x} \pm \sigma_x$ | $v = x\sqrt{g/2y}$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

**Table II. Range as a Function of Free-Fall Height**

h = ________

| | y | $x_1$ $x_2$ $x_3$ $x_4$ $x_5$ | $\bar{x} \pm \sigma_x$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |