Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #1 Solution

1. How much memory do you need to store audio or audio and video recording of one lecture (80 minutes)?

   a) sampling at 8,000 Hz and 8 bits/sample.

   M_a = 80 min * 60 sec * 8000 samples/s * 1 byte/sample = 38,400,000 B = 36.6 MB

   Remember 1MB = 2^{10} bytes = 1,048,576 bytes

   b) CD quality recording (44.1 KHz, 16 bits/sample, stereo recording)

   M_b = 80 min * 60 sec * 2 channels *44,100 samples/s * 2 bytes/sample = 807.5 MB

   c) CD quality with 20 times compression (MP3 format)

   $M_c = M_b$ / 20 = 40.4 MB

   d) audio and video

   M_v = 80 min * 60 sec * 30 fps * 640 * 480 * 3 Bytes (24 bits) + M_a

   = 126,562 MB + 36.6 MB ≈ 124 GB

2. Write a script in Matlab to plot cosine signal with frequency 1Hz and amplitude 2 for three seconds. Generate a discrete-time signal x[n] = x(nTs) = x(t) | t=nTs for the sampling interval Ts=0.1 sec, Ts = 0.5 sec, and Ts = 1 sec.

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% CPE381 HW1-2
Tsa=0.0001;             % Ts=10^-4
t=0:Tsa:3;              % t=0..3s
A=2;                    % Amplitude = 2
f=1;                    % frequency = 1 Hz
x=A*cos(2*pi*f*t);
t1=0:0.1:3;             % other sampling intervals
x1=A*cos(2*pi*f*t1);
t2=0:0.5:3;
x2=A*cos(2*pi*f*t2);
t3=0:1:3;
x3=A*cos(2*pi*t3);
figure
subplot(221)
plot(t,x),xlabel('t[s]'),title(‘Analog signal’)
subplot(222)
plot(t,x),hold,stem(t1,x1,'r'),xlabel('t[s]'),title(‘Ts = 0.1 s’)
subplot(223)
plot(t,x),hold,stem(t2,x2,'r'),xlabel('t[s]'),title(‘Ts = 0.5 s’)
subplot(224)
plot(t,x),hold,stem(t3,x3,'r'),xlabel('t[s]'),title(‘Ts = 1 s’)
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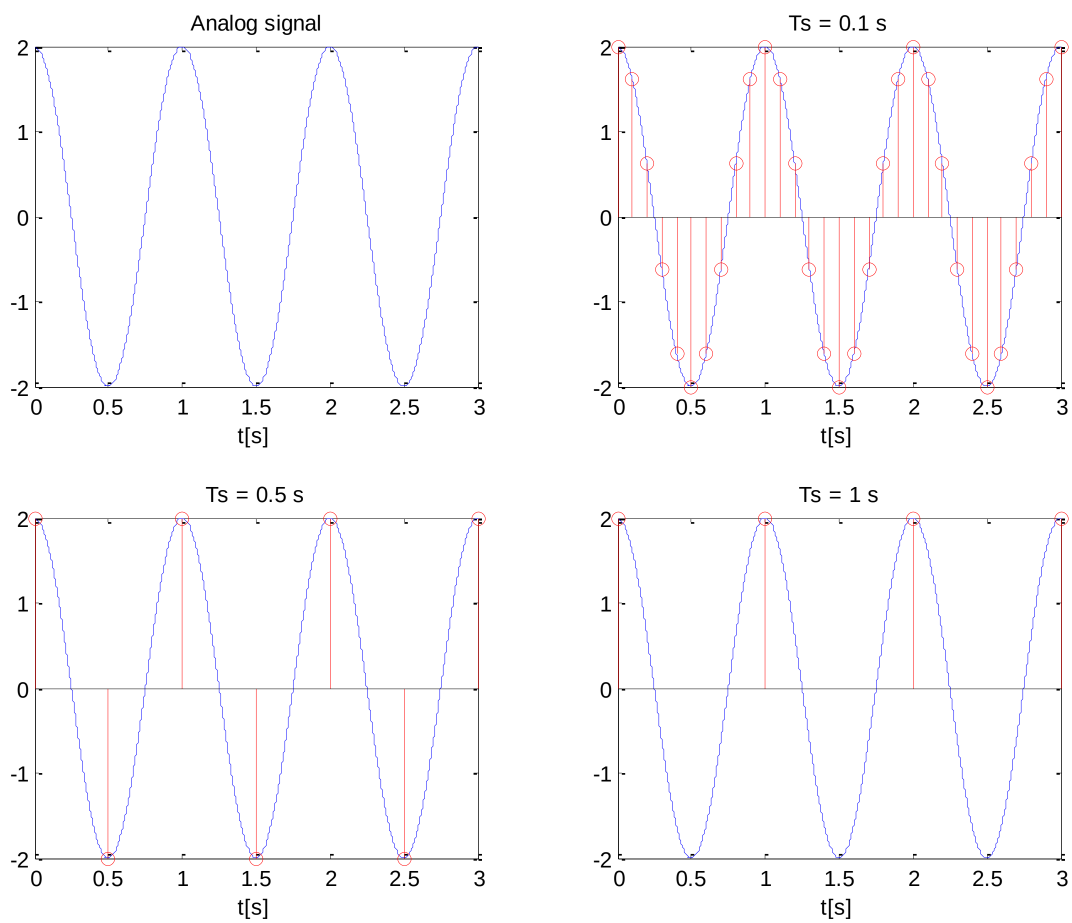
Minimum sampling frequency is Ts = 0.5 s (T/2 or 2*max_f)!



3. Pr0.12 $w = e^z \quad z = 1+j1$

(a) If $w = e^z$

$$\log(w) = z$$

(b) The real and imaginary of w are

$$w = e^z = e^1 e^{j1} = e^{1+j1} = \underbrace{e\cdot\cos(1)}_{\textit{real part}} + \underbrace{j\cdot e\cdot\sin(1)}_{\textit{imaginary part}}$$

(c) The imaginary parts are cancelled and the real parts added twice

$$w + w^* = 2 * \mathcal{R}e(w) = 2\cdot e\cdot\cos(1)$$

(d) $|w| = e$ and $\angle w = 1$.

(e) Using the result in (a)

$$|\log(w)|^2 = |z|^2 = (2^{1/2})^2 = 2$$

(f) According to Euler's equation and solution b)

$$w = e\cdot\cos(1) + j\cdot e\cdot\sin(1)$$
$$w^* = e\cdot\cos(1) - j\cdot e\cdot\sin(1)$$

$$\rightarrow \cos(1) = 0.5\,(e^{j} + e^{-j}) = 0.5\,(w/e + w^*/e)$$

4. Pr 0.14

Use Euler's identity to find an expression for *cos*(α) *cos*(β), and from the relation between cosines and sines obtain an expression for *sin*(α) *sin*(β).

$$\begin{aligned}
\cos(\alpha)\cos(\beta) &= 0.5\,(e^{j\alpha} + e^{-j\alpha})\,0.5(e^{j\beta} + e^{-j\beta}) \\
&= 0.25\,(e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}) + 0.25(e^{j(\alpha-\beta)} + e^{-j(\alpha-\beta)}) \\
&= 0.5\cos(\alpha+\beta) + 0.5\cos(\alpha-\beta)
\end{aligned}$$

$$\begin{aligned}
\sin(\alpha)\sin(\beta) &= \cos(\alpha - \pi/2)\cos(\beta - \pi/2) \\
&= 0.5\cos(\alpha - \pi/2 + \beta - \pi/2) + 0.5\cos(\alpha - \pi/2 + \beta + \pi/2) \\
&= 0.5\cos(\alpha + \beta - \pi) + 0.5\cos(\alpha - \beta) \\
&= -0.5\cos(\alpha+\beta) + 0.5\cos(\alpha-\beta)
\end{aligned}$$

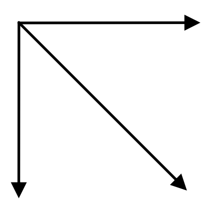
5. (modified Pr 0.23)

Plot signals $y(t) = A\sin(\Omega_0 t)$ and $x(t) = A\cos(\Omega_0 t)$.

(a) $\sin(\Omega_0 t) = \cos(\Omega_0(t - T_0/4)) = \cos(\Omega_0 t - \pi/2)$, since $\Omega_0 = 2\pi/T_0$.

(d) Suppose then you have the sum of two sinusoids, for instance z(t) = x(t) + y(t), adding the corresponding phasors for x(t) and y(t) at some time (e.g., t = 0), which is just a sum of two vectors, you should get a vector and the corresponding phasor. Get the phasor for z(t) and the expression for it in terms of a cosine.

$$z(t) = \Re\left[\sqrt{2}Ae^{-j\pi/4}e^{j\Omega_0 t}\right] = \sqrt{2}A\cos(\Omega_0 t - \pi/4)$$



6. Consider the analog signal

$$x(t) = \sin(2\pi t + \theta) \qquad -\infty < t < \infty$$

Determine the value of θ for which x(t) is even and odd.

Even: $\theta = -\pi/2$
Odd: $\theta = 0$ or π

7. $x(t) = e^{-|t|}$

a) $$E_x = \int_{-\infty}^{\infty} x^2(t)dt = 2\int_{0}^{\infty} \left(e^{-t}\right)^2 dt = 2\frac{e^{-2t}}{-2}\Big|_0^{\infty} = 1$$

b) the signal is absolutely integrable (section 1.3.4) as

$$\int_{-\infty}^{\infty} |x(t)|dt = 2\int_{0}^{\infty} e^{-t}dt = 2\frac{e^{-t}}{-1}\Big|_0^{\infty} = 2 < \infty$$

c) $y(t) = e^{-t}\cos(2\pi t)u(t)$

$$E_y = \int_{0}^{\infty} y^2(t)dt = \int_{0}^{\infty} e^{-2t}\cos^2(2\pi t)dt < \int_{0}^{\infty} e^{-2t}dt = E_x/2 = 1/2$$

d)

$$v_R(t) + \frac{1}{C}\int_{0}^{t} i(\tau)d\tau = 1$$

$$e^{-t} + \frac{1}{RC}\left(1 - e^{-t}\right) = 1$$

for $t \to \infty$ $1/RC = 1 \rightarrow R = 1M\Omega$.

8.

Pr. 1.3(a) Let $x(t) = x_1(t) + x_2(t) = \cos(2\pi t) + 2\cos(\pi t)$, so that $x_1(t)$ is a cosine of frequency $\Omega_1 = 2\pi$ or period $T_1 = 1$, and $x_2(t)$ is a cosine of frequency $\Omega_2 = \pi$ or period $T_2 = 2$. The ratio of these periods $T_2/T_1 = 2/1$ is a rational number so $x(t)$ is periodic of period $T_0 = 2T_1 = T_2 = 2$.
(b)(c) The average power of $x(t)$ is given by

$$P_x = \frac{1}{T_0}\int_0^{T_0} x^2(t)dt = \frac{1}{2}\int_0^2 [x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t)]dt$$

Using the trigonometric identity $\cos(\alpha)\cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta)$ we have that the integral

$$\begin{aligned}\frac{1}{2}\int_0^2 2x_1(t)x_2(t)dt &= \frac{1}{2}\int_0^2 4\cos(2\pi t)\cos(\pi t)dt \\ &= \int_0^2 [\cos(\pi t) + \cos(3\pi t)]dt = 0\end{aligned}$$

since $\cos(\pi t) + \cos(3\pi t)$ is periodic of period 2 and so its area under a period is zero. Thus,

$$\begin{aligned}P_x &= \frac{1}{2}\int_0^2 [x_1^2(t) + x_2^2(t)]dt \\ &= \frac{1}{2}\int_0^2 x_1^2(t)dt + \frac{1}{2}2\int_0^1 x_2^2(t)]dt \\ &= P_{x_1} + P_{x_2}\end{aligned}$$

so that the power of $x(t)$ equals the sum of the powers of $x_1(t)$ and $x_2(t)$ which are sinusoids of different frequencies, and thus orthogonal as we will see later.
Finally,

$$\begin{aligned}P_x &= \frac{1}{2}\int_0^2 \cos^2(2\pi t)dt + \int_0^1 4\cos^2(\pi t)dt \\ &= \frac{1}{2}\int_0^2 [0.5 + 0.5\cos(4\pi t)]dt + \int_0^1 4[0.5 + 0.5\cos(2\pi t)]dt \\ &= 0.5 + 2 = 2.5\end{aligned}$$

remembering that the integrals of the cosines are zero (they are periodic of period 0.5 and 1 and the integrals compute their areas under one or more periods, so they are zero).
(d) The components of $y(t)$ have as periods $T_1 = 2\pi$ and $T_2 = 2$ so that $T_1/T_2 = \pi$ which is not rational so $y(t)$ is not periodic. In this case we need to find the power of $y(t)$ by finding the integral over an infinite support of $y^2(t)$ which will as before give

$$P_y = P_{y_1} + P_{y_2}$$

In the case of harmonically related signals we can use the periodicity and compute one integral. However, in either case the power superposition holds.

9.

Pr. 1.4 (a) The signal $x_1(t) = 4\cos(\pi t)$ has frequency $\Omega_1 = 2\pi/2$ so that the period of $x_1(t)$ is $T_1 = 2$. Likewise the signal $x_2(t) = -\sin(3\pi t + \pi/2)$ has frequency $\Omega_2 = 3\pi = 2\pi/(2/3)$ so that it is periodic of period $T_2 = 2/3$

(b) Yes, $x(t)$ is periodic. The ratio of the two periods is

$$\frac{T_1}{T_2} = \frac{2}{2/3} = 3$$

so that

$$T_0 = T_1 = 3T_2 = 2$$

is the period of $x(t) = x_1(t) + x_2(t)$.

(c) In general, if the ratio of the periods of two periodic signals is

$$\frac{T_1}{T_2} = \frac{M}{K}$$

for integers M and K, not divisible by each other, then $T_0 = KT_1 = MT_2$ is the period of the sum of the periodic signals. If the ratio is not rational (i.e., M and/or K are not integers) then the sum of the two periodic signals is not periodic.