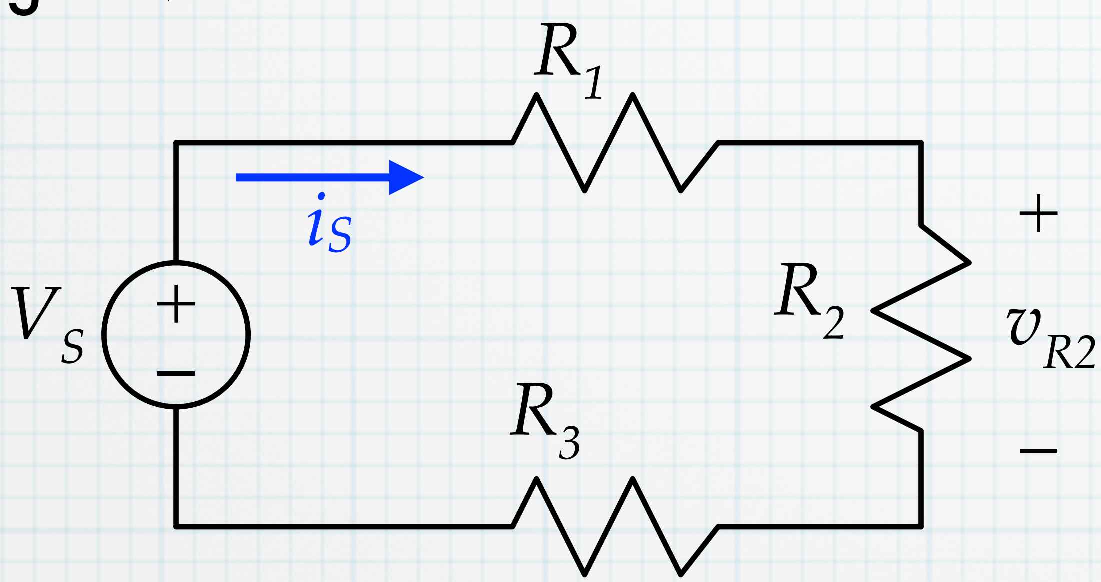
Voltage/current dividers

Trivial to understand, but still very useful.

Use in combination with equivalent resistances to quickly find a particular voltage or current in a circuit.

Voltage divider



Want to know v_{R2}.

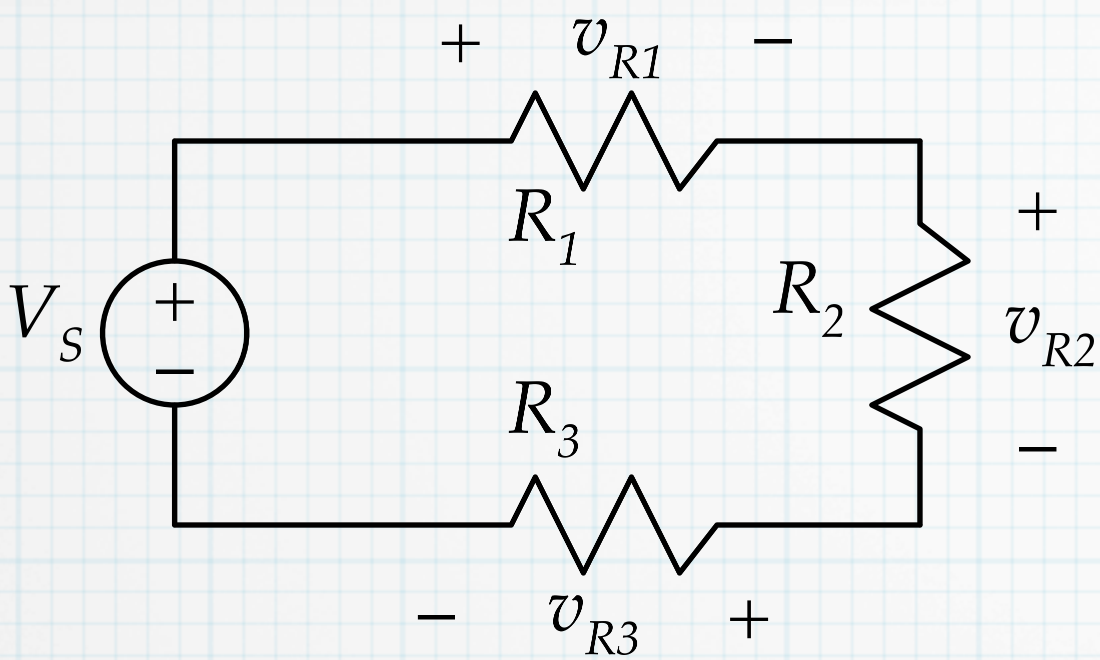
Easily solved with KCL, KVL, & equivalent resistances.

$$i_S = \frac{V_S}{R_{eq}} = \frac{V_S}{R_1 + R_2 + R_3}$$

$$v_{R2} = i_S R_2 = \frac{R_2}{R_1 + R_2 + R_3} V_S$$

That's it.

The total voltage across a series string is divided among the resistors according to a simple ratio.

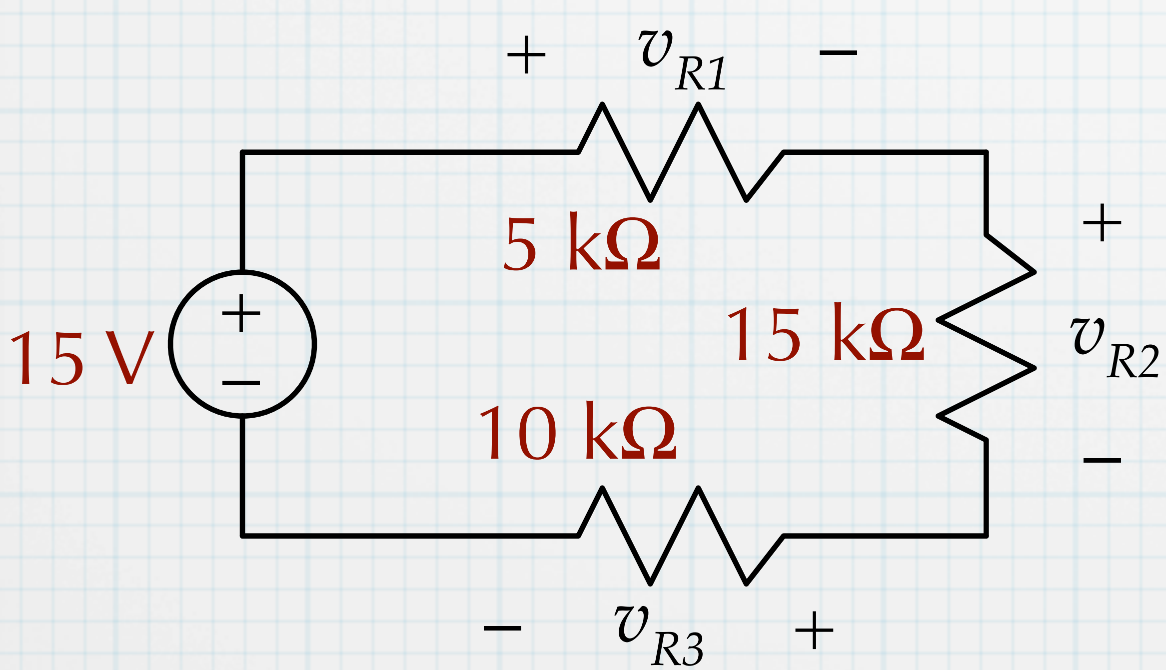


Easy to find the other voltages, too.

$$v_{R1} = \frac{R_1}{R_1 + R_2 + R_3} V_S$$

$$v_{R3} = \frac{R_3}{R_1 + R_2 + R_3} V_S$$

Put in some values.



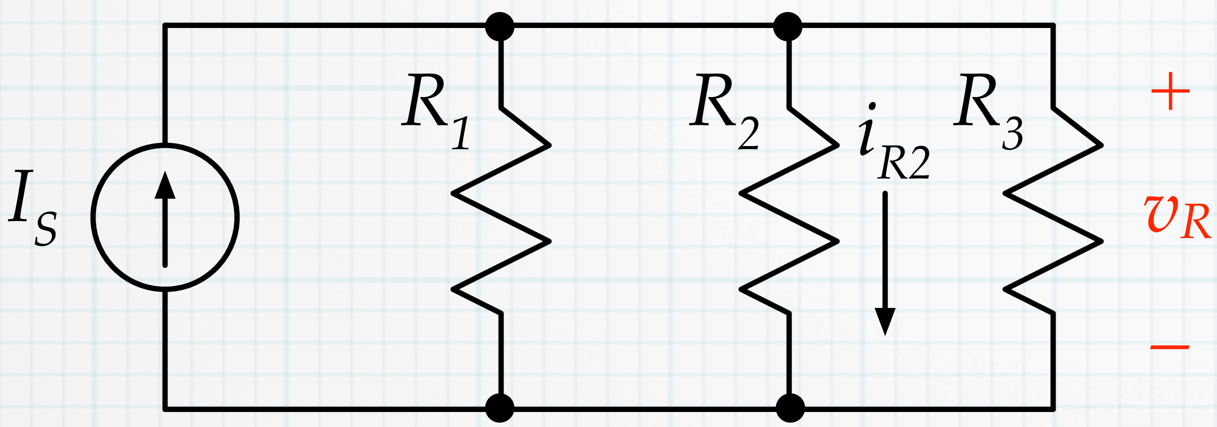
$$v_{R1} = \frac{5\text{k}\Omega}{5\text{k}\Omega + 15\text{k}\Omega + 10\text{k}\Omega}(15\text{V}) = 2.5\text{V}$$

$$v_{R2} = \frac{15\text{k}\Omega}{5\text{k}\Omega + 15\text{k}\Omega + 10\text{k}\Omega}(15\text{V}) = 7.5\text{V}$$

$$v_{R3} = \frac{10\text{k}\Omega}{5\text{k}\Omega + 15\text{k}\Omega + 10\text{k}\Omega}(15\text{V}) = 5\text{V}$$

Current divider

Same idea, but with current.



Want to know i_{R2}.

Easily solved with KCL, KVL, & equivalent resistances.

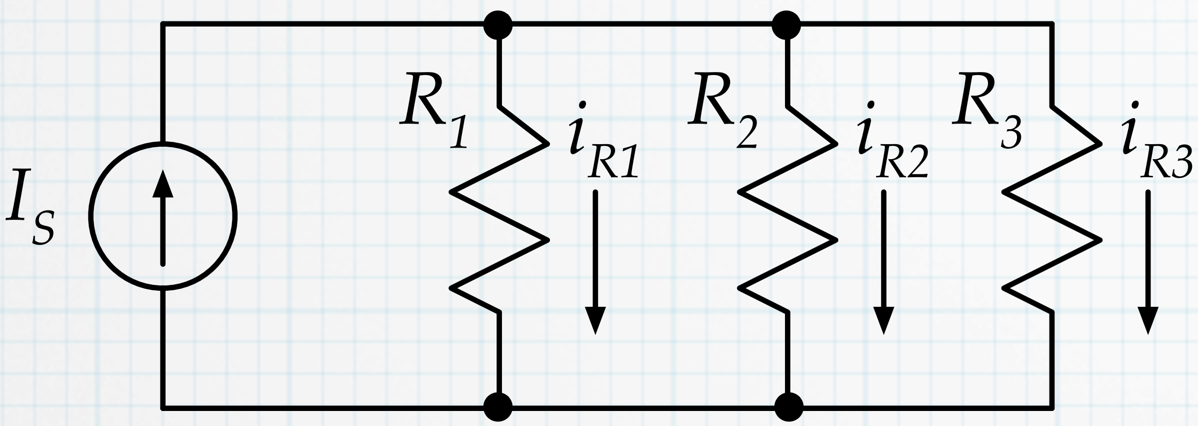
$$v_R = I_S R_{eq}$$

$$= \frac{I_S}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$i_{R2} = \frac{v_R}{R_2}$$

$$= \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S$$

The total is divided according a simple ratio determined by the resistors.

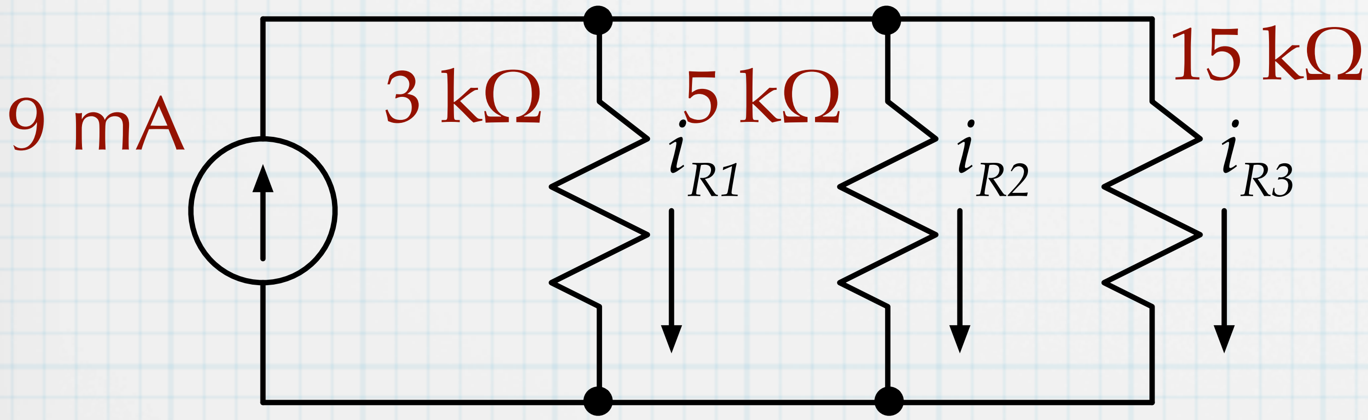


The other currents are just as easy.

$$i_{R1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S$$

$$i_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S$$

Plug in some numbers.

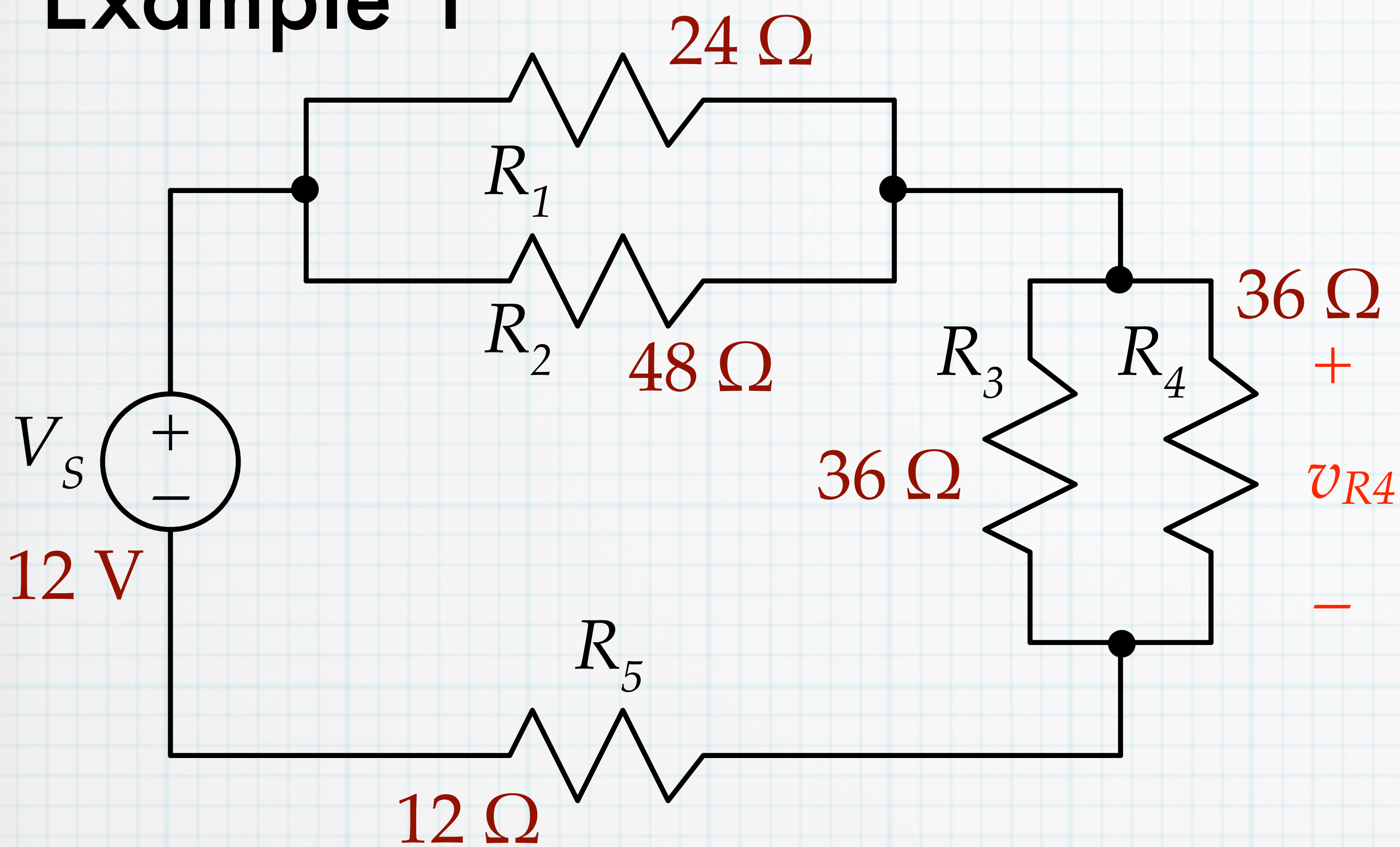


$$i_{R1} = \frac{\frac{1}{3\text{k}\Omega}}{\frac{1}{3\text{k}\Omega} + \frac{1}{5\text{k}\Omega} + \frac{1}{15\text{k}\Omega}} (9\text{mA}) = 5\text{mA}$$

$$i_{R2} = \frac{\frac{1}{5\text{k}\Omega}}{\frac{1}{3\text{k}\Omega} + \frac{1}{5\text{k}\Omega} + \frac{1}{15\text{k}\Omega}} (9\text{mA}) = 3\text{mA}$$

$$i_{R3} = \frac{\frac{1}{15\text{k}\Omega}}{\frac{1}{3\text{k}\Omega} + \frac{1}{5\text{k}\Omega} + \frac{1}{15\text{k}\Omega}} (9\text{mA}) = 1\text{mA}$$

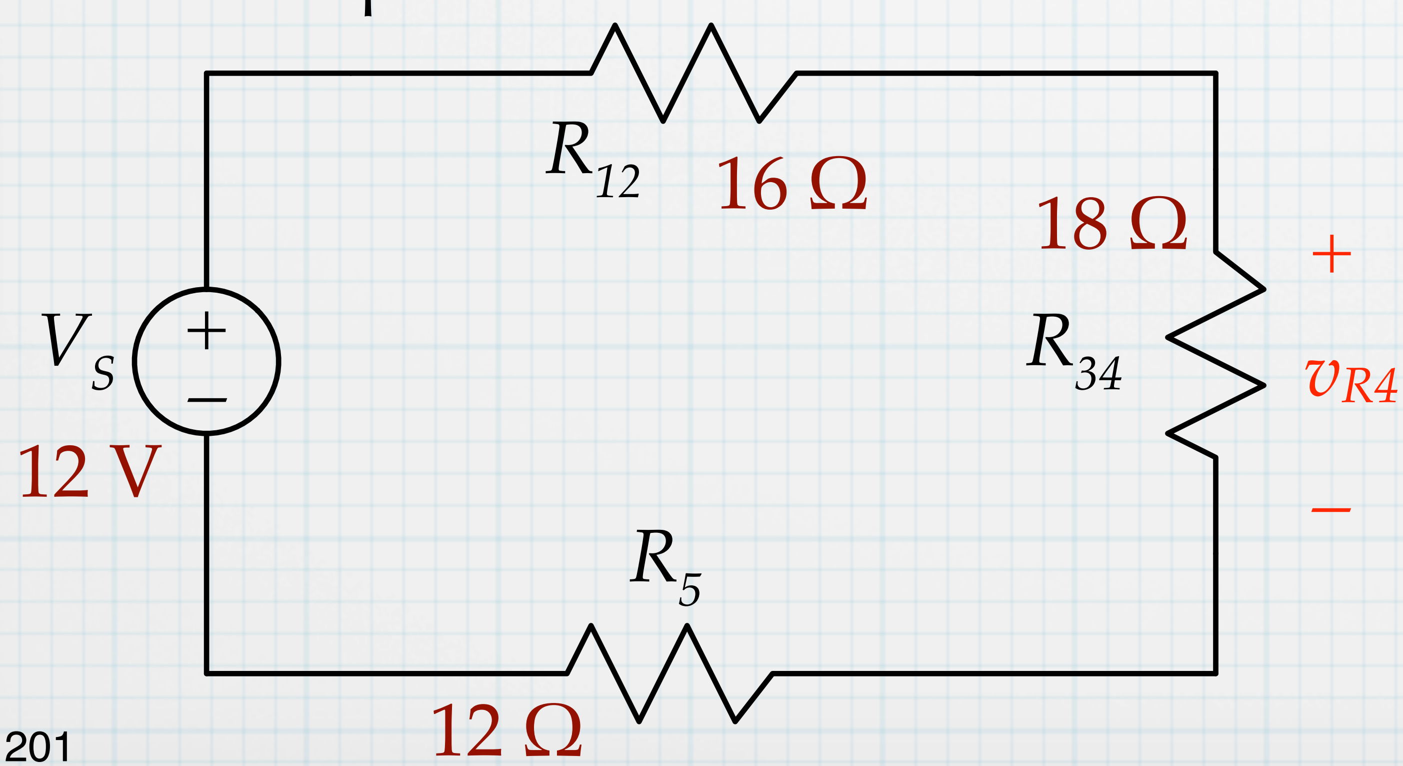
Example 1



In the circuit, find v_{R4}.

$v_{R4} = v_{R3}$

Combine parallel combinations.



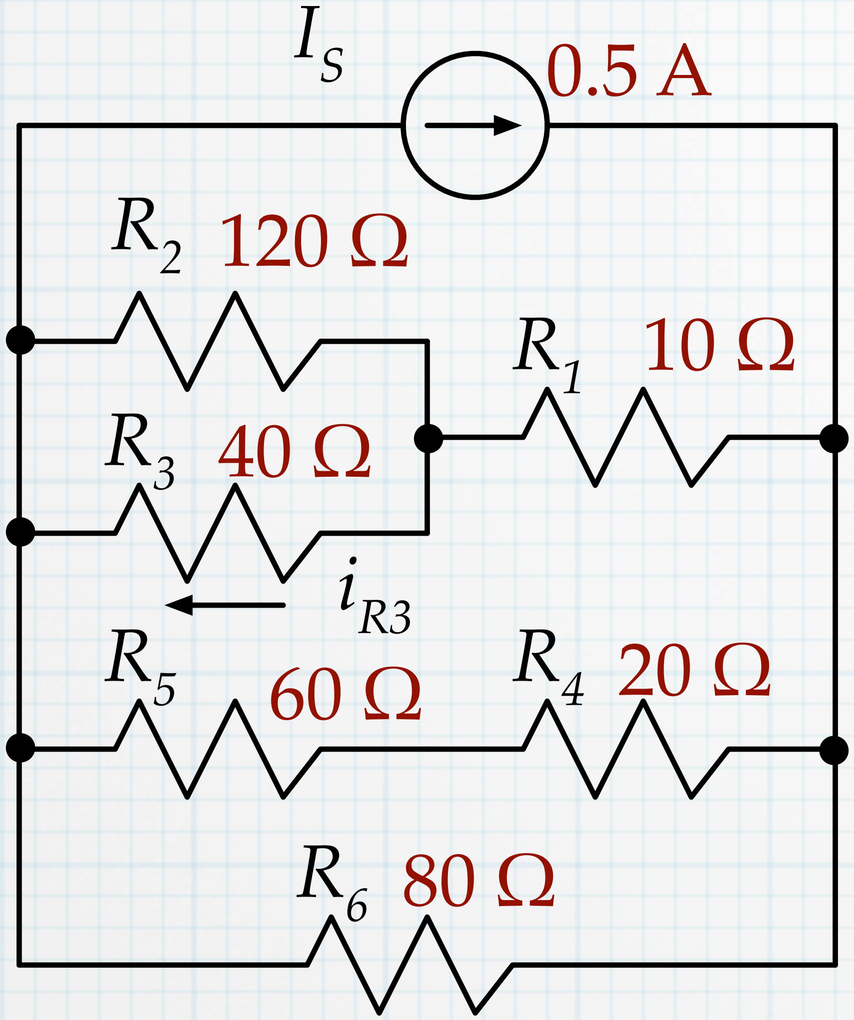
Use voltage divider.

$$v_{R4} = \frac{R_{34}}{R_{12} + R_{34} + R_5} V_S$$

$$= \frac{18\Omega}{16\Omega + 18\Omega + 12\Omega} (12\text{V})$$

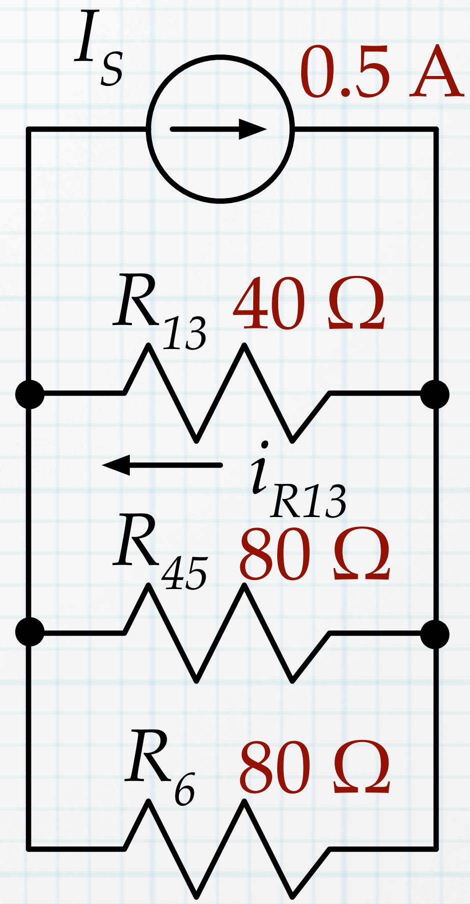
$$= 4.69\text{V}$$

Example 2



In the circuit, find i_{R3}.

Find the equivalent resistance in each branch.



Use a current divider to find i_{R13}.

$$i_{R13} = \frac{\frac{1}{R_{13}}}{\frac{1}{R_{13}} + \frac{1}{R_{45}} + \frac{1}{R_6}} I_S$$

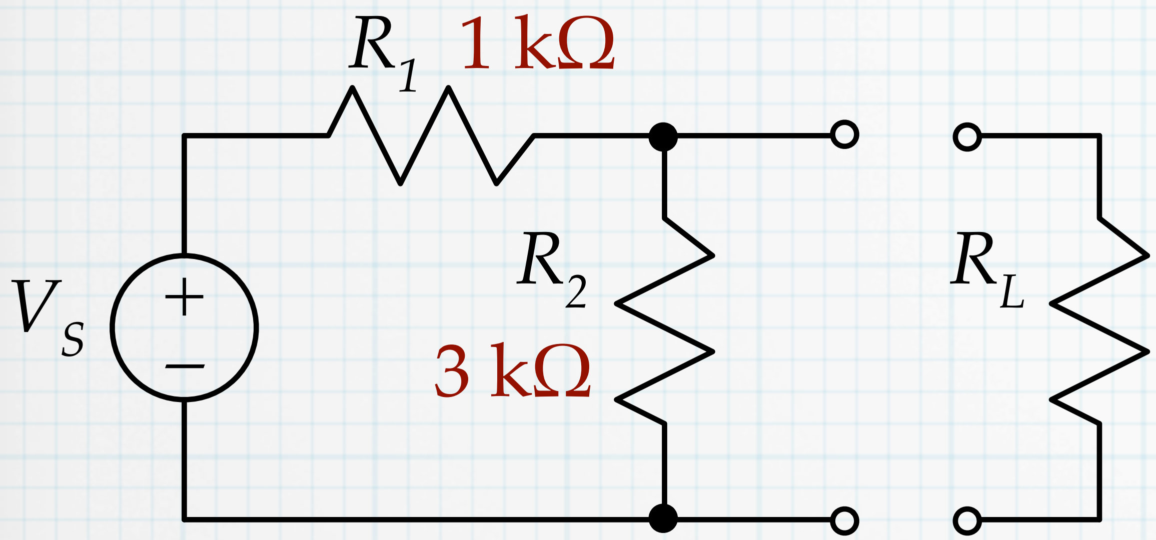
$$= \frac{\frac{1}{40\Omega}}{\frac{1}{40\Omega} + \frac{1}{80\Omega} + \frac{1}{80\Omega}} (0.5\text{A}) = 0.25\text{A}$$

Referring back to the original circuit, i_{R13} divides between R_2 and R_3.

$$i_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_2} + \frac{1}{R_3}} i_{R13}$$

$$= \frac{\frac{1}{40\Omega}}{\frac{1}{120\Omega} + \frac{1}{40\Omega}} (0.25\text{A}) = 0.1875\text{A}$$

Example 3



In the simple divider circuit at right, if R_L is attached in parallel with R_2, the voltage across R_1 doubles. What is the value of R_L?

Without R_L,

$$v_{R1} = \frac{R_1}{R_1 + R_2} V_S$$

With R_L,

$$v'_{R1} = \frac{R_1}{R_1 + R_{eq}} V_S$$

$$= 2\frac{R_1}{R_1 + R_2} V_S$$

From the two expressions for v'_{R1}

$$\frac{R_1}{2} + \frac{R_2}{2} = R_1 + R_{eq}$$

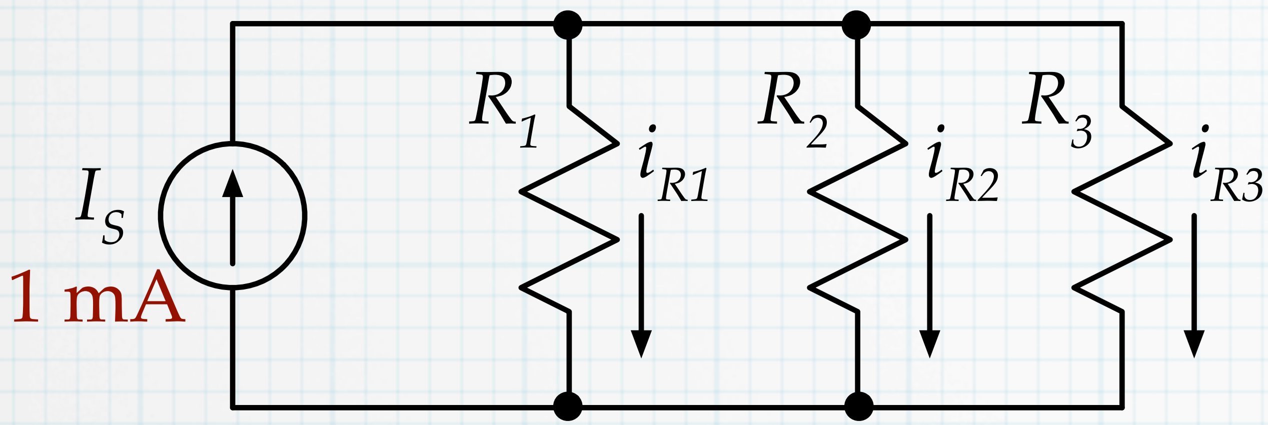
$$R_{eq} = \frac{R_2 - R_1}{2} = \frac{3\text{k}\Omega - 1\text{k}\Omega}{2} = 1\text{k}\Omega$$

Then

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_L}$$

$$R_L = \frac{1}{\frac{1}{R_{eq}} - \frac{1}{R_2}} = 1.5\text{k}\Omega$$

Example 4



For the simple current divider at right, design it (i.e. choose resistor values) so that the currents are in the ratio i_{R1}:i_{R2} :i_{R3} = 1:2:4 and the total power dissipated in the resistors is 10 mW.

From the current divider equation, we know that the currents are proportional to the inverse of the resistance in each branch.

$$\frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} = 1 : 2 : 4$$

Therefore,

$$R_1 : R_2 : R_3 = 4 : 2 : 1$$

$R_2 = 2R_3$ and $R_1 = 2R_2$ ($R_1 = 4R_3$).

The equivalent resistance of the three in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{4R_3} + \frac{1}{2R_3} + \frac{1}{R_3} = \frac{1.75}{R_3}$$

$$R_{eq} = \frac{R_3}{1.75}$$

Find R_{eq},

$$P = I_S^2 R_{eq} \longrightarrow R_{eq} = \frac{P}{I_S^2} = 10\text{k}\Omega$$

Finally:

$R_3 = 1.75R_{eq} = 17.5$ kΩ; $R_2 = 2R_3 = 35$ kΩ; $R_1 = 2R_2 = 70$ kΩ