Supplemental Problems Mar. 5, 2013

Find the power series represention and interval of convergence.

(1.) $f(x) = \frac{1}{1+x}$

(2.) $f(x) = \frac{1}{1-x^3}$

(3.) $f(x) = \frac{1}{5-x}$

(4.) $f(x) = \frac{-1}{(1+x)^2}$ Hint: $\frac{d}{dx}\left(\frac{1}{1+x}\right) = \frac{-1}{(1+x)^2}$

(5.) $f(x) = \ln(5-x)$ Hint: $\int \frac{1}{5-x}\,dx = -\ln(5-x)$ for $|x|<5$

Use a power series to approximate these definite integrals to five decimal places.

(6.) $\int_0^{0.2} \frac{1}{1-x^3}\,dx$

(7.) $\int_0^{\frac{1}{3}} \frac{\tan^{-1}x}{x}\,dx$

Answers

1. $\sum_{n=0}^{\infty} (-1)^n x^n \qquad -1 < x < 1$

2. $\sum_{n=0}^{\infty} x^{3n} \qquad -1 < x < 1$

3. $\frac{1}{5} \sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^n \qquad -5 < x < 5$

4. $\sum_{n=1}^{\infty} (-1)^n n x^{n-1} = \sum_{n=0}^{\infty} (-1)^{n+1} (n+1) x^n \qquad -1 < x < 1$

5. $\ln 5 - \sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^{n+1} \cdot \frac{1}{(n+1)} \qquad -5 < x < 5$

6. .20040

7. .32937