Notes on the Doppler laser cooling limit

The Doppler laser cooling limit will in general depend on details of the laser cooling set-up. Some cases are discussed in Itano/Wineland, PRA 25, 35 (1982). A few of these cases are summarized below and calculated for the case of Be^+. Calculated natural linewidth of the Doppler laser cooling transition in Be+ is $\Gamma = 2\pi \times (17.98\ MHz)$. The limits below assume low laser intensity (no saturation effects) and uniform illumination of the ions. Following the Itano/Wineland manuscript, formulas are given for the minimum kinetic energy in the axial direction $\langle E_{Kz}\rangle_{min}$. The Doppler temperature is then obtained from $\frac{1}{2}k_B T_D = \langle E_{Kz}\rangle_{min}$.

3-D laser cooling; i.e. laser cooling with lasers along three axes with equal intensity

This seems difficult to implement in a Penning trap because the rotation makes it difficult to get equal scatter rates along three orthoginal axes. In any case, this is what is usually meant by the Doppler laser cooling limit. For isotropic scattering,

$$\langle E_{Kz}\rangle_{min} = \frac{1}{4}\hbar\Gamma$$

$$T_D = \frac{1}{2}\frac{\hbar\Gamma}{k_B} = 0.43\,\mathrm{mK}$$

cooling along one axis; minimal recoil heating from cooling with orthogonal laser beams; isotropic scattering

This is the Doppler cooling temperature that should be obtained by the dynamics code in the limit that scattering from the in-plane cooling beam is weak compared to scattering from the parallel (to the magneteic field) cooling laser beam.

$$\langle E_{Kz}\rangle_{min} = \frac{1}{6}\hbar\Gamma$$

$$T_D = \frac{1}{3}\frac{\hbar\Gamma}{k_B} = 0.29\,\mathrm{mK}$$

cooling along one axis; minimal recoil heating from cooling with orthogonal laser beams; cooling on an $\Delta M = \pm 1$ transition

This is the Doppler cooling limit assuming the angular distribution of scattered photons appropriate for the $\Delta M = \pm 1$ cooling transition employed in the experiments.

$$\langle E_{Kz} \rangle_{min} = \frac{1}{5} \hbar \Gamma$$

$$T_D = \frac{2}{5} \frac{\hbar \Gamma}{k_B} = 0.35\,\mathrm{mK}$$

For a harmonic oscllator in thermal equilibrium, the average potential and thermal energies should be equal. The approximate mean occupation number $\langle n \rangle$ is $\langle n \rangle \approx 2 \langle E_{Kz} \rangle_{min} / (\hbar \omega_z)$. However, it probably makes sense to quote the different mode energies in terms of temperatures. The formulas above are derived for laser cooling of single particles. We anticipate that they should characterize the Doppler laser cooling limit for the different drumhead modes. Do the simulations calculate the kinetic energy of a particle? kinetic + potential?