HW9 of Plasma

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1. standard Ampere law in GS unit is:

$$\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c} \partial_t E$$

where the displacement part comes from $E = -\frac{1}{c} u \times B$ given MHD framework, so approximately the extra $(\nabla \times B)_\Delta$ has order of:

$$(\nabla \times B)_\Delta \sim \frac{1}{c^2} \partial_t u \times B$$

When $|u| \ll c$, $\partial_t u$ is always very small compared to c, the approximation is very good. On the other hand, even $|u| \sim c$ sometimes $\partial_t u$ is still very small, so in relativistic MHD, the approximation is not bad sometimes.

2. (a) Invariant flux has form of:

$$\Phi_m = B \cdot 4\pi R^2$$

since $\Phi_m = \Phi_m'$, new magnetic field B' is:

$$B' = B_* \cdot \frac{R_*^2}{R_{ns}^2}$$, calculated result is $B' = 10^{12}$ Gauss

(b) cyclotron frequency of new B' is

$$\omega = \frac{eB'}{m} = 1.759 \times 10^{19} \text{ rad/s}$$

with $f = \omega / 2\pi = 2.8 \times 10^{18}$ Hz, it correspond to X-rays (Hard or soft)

3. first of all, the magnetic field B in cylindrical coordinate is

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{e}_\varphi$$

the magnetic pressure force $\vec{f}_p = -\nabla\left(\frac{B^2}{2\mu_0}\right)$

so $\vec{f}_p = \frac{\mu_0 I^2}{4\pi^2} \cdot \frac{1}{R^3} \hat{e}_\rho$

and magnetic tensim force $\vec{f}_T = \frac{1}{\mu_0}(B\cdot\nabla)B$

$$\vec{f}_T = \frac{B^2}{\mu_0}(\hat{e}_B\cdot\nabla)\hat{e}_B = -\frac{\mu_0 I^2}{4\pi^2 R^2}\cdot\frac{1}{R}\hat{e}_\rho \ ; \ \left((\hat{e}_\varphi\cdot\nabla)\hat{e}_\varphi = -\frac{\hat{e}_\rho}{R}\right)$$

two forces have same strength, and the net force is zero.

4. outside the surface of earth, magnetic force has form of

$B = \frac{B_0 \cdot R_m^3}{r^3}$, and it's pressure is:

$$P_B = \frac{1}{2\mu_0} B^2 = \frac{B_0^2 R_m^6}{2\mu_0} \cdot \frac{1}{r^6}$$

on the othe hand, pressure force from solar wind is

$P_s = \rho u^2$, when $P_B \geq P_s$, the magnetic field could stop solar wind, so

$$\frac{B_0^2 R_m^6}{2\mu_0} \cdot \frac{1}{r^6} = \rho \cdot u^2$$

$r = 4.578 \times 10^9$ cm $\sim$ $7.15\ R_m$