Physics 5150

Homework Set # 13 (OPTIONAL)

Due: N/A

Problem 1: **Electrostatic Electron Waves**

Consider a one-dimensional electron-ion plasma with ions providing a uniform stationary neutralizing background and with the electron unperturbed velocity distribution function that has a triangular shape:

$$f(v_x) = A\,(1 - \frac{|v_x|}{v_0}), \quad |v_x| \le v_0\,, \tag{1}$$

and

$$f(v_x) = 0 \quad |v_x| > v_0\,. \tag{2}$$

Consider electrostatic electron plasma waves in this plasma. Use the Vlasov and Poisson equations to derive the real part of the dispersion relation $\omega(k)$ for electrostatic electron plasma assuming that the phase velocity is large: $v_\phi \gg v_0$ [remember to keep the first two terms when Taylor-expanding $(\omega - kv)^{-2}$ where needed]. Calculate the phase and group velocities of the wave. Ignore Landau damping.

Problem 2: **Landau Damping and the "bump-on-tail" instability**

An infinite, uniform plasma with fixed ions has an electron distribution function composed of (1) a Maxwellian distribution of "plasma" electrons with density n_p and temperature T_p at rest in the laboratory frame, and (2) a drifting Maxwellian distribution of "beam" electrons with density n_b and temperature $T_b \ll T_p$ centered at $\mathbf{v} = V\hat{x}$. If n_b is infinitesimally small, plasma oscillations in the x direction are Landau-damped. If n_b is relatively large, there will be an electrostatic two-stream instability. Find the critical value of the density ratio n_b/n_p at which the instability sets in, assuming that the beam velocity V is much greater than the thermal velocity of the beam electrons, $V \gg v_{\rm th,b} \equiv (2k_BT_b/m_e)^{1/2}$, but, at the same time, $V v_{\rm th,b} \ll v_{\rm th,p}^2 \equiv 2k_BT_p/m_e$.
Hint: the condition for the instability onset can be found by setting the slope of the combined distribution function to zero.