Homework Set #1

Due 5 pm Thursday, Jan. 25, 2018

Problem 1:
Compute the pressure, in atmospheres, exerted by a thermonuclear plasma on its container. Assume $k_B T_e = k_B T_i = 20$ keV, $n=10^{14}$ cm^{-3}*, and* $p = n k_B(T_i + T_e)$.

Solution:
$P = n k_B (T_i+T_e) = 6.4 \times 10^5$ N/m$^2 \approx 6.3$ atm.

Problem 2:
Consider two infinite, parallel plates at $x= \pm d$*, set at potential* $\varphi=0$*. The space between them is uniformly filled by a 1-species gas of density* n *of particles with charge* q*. Find the potential distribution* $\varphi(x)$ *between the plates.*

Solution:
Poisson's equation in 1D reads: $\varphi''(x) = - 4\pi n q$. Since the density is uniform, the right-hand side is constant and this equation can be easily integrated:

$$\varphi(x) = - 2 \pi nq x^2 + Bx + C.$$

Applying the boundary conditions $\varphi(-d) = 0 = \varphi(d)$, we immediately get

$$\varphi = 2\pi nq (d^2-x^2).$$

Problem 3:
In laser fusion, the core of a small pellet of DT plasma is compressed to a density of $n=10^{27}$ cm^{-3} *at a temperature of* 50,000,000 K*. Estimate the number of particles in a Debye sphere in this plasma.*

Solution:
(Important: don't forget to convert density in cm^{-3} into m^{-3}!)
Debye length: $\lambda_D = 69 (T/n)^{1/2}$ m $\approx 1.54 \times 10^{-11}$ m ;
Volume of the Debye sphere: $V_D=(4/3) \pi \lambda_D^3 \approx 1.5 \times 10^{-32}$ m^3 ;
Number of particles in the Debye sphere: $N_D = n V_D \approx 15$.

Problem 4:

Assuming an isothermal atmosphere at $k_B T$ = 20 C, *estimate by what factor the atmospheric pressure in Boulder is lower than in New York.*

Solution:

From the vertical hydrostatic pressure balance, $dP/dz = -\rho(z)\, g$, and the relationship $P = nk_B T = \rho k_B\, T/m$, where k_B is Boltzmann's constant and $m \approx 29\, m_p$, where m_p is the proton mass, is the average mass of "air molecules" (this is because air mostly consists of N_2 with mass 28 and O_2 with mass 32), we find that the air pressure and density decrease exponentially with height as

$$P(z) = P(0)\, exp(-z/H),$$

where $H = k_B\, T/mg \approx 8.5\ km$ is the pressure scale-height. Thus, the atmospheric pressure in Boulder, which is about 1.65 km above sea level, is approximately 82% of that in New York, which is at sea level.

Problem 5:

Show that, in a non-relativistic plasma ($kT_e << m_e\, c^2$)*, the mutual Coulomb (electrostatic) force between two typical nearby electrons is much stronger than their mutual Lorentz (magnetic) force.*

Solution:

Here we are interested in a simple order-of-magnitude, back-of-the-envelop estimate, so we'll ignore factors of order unity. We will do this in cgs units.

The magnitude of the Coulomb force between two electrons separated by a distance l is $F_E = e^2/l^2$. The magnitude of the Lorentz force between these two charges can be estimated as $F_M \sim e\, B\, v/c$, where v is the velocity of one of the particles (particle 1), B is the characteristic strength of the magnetic field created by the other particle (particle 2) at the position of particle 1, and where we have ignored numerical factors of order unity related to the relative orientation angle between the vectors $\boldsymbol{B}$ and $\boldsymbol{v}$. And, of course, e here is the electron charge and c is the speed of light. Again ignoring a factor of order unity due to the angle between the velocity of particle 2 and the separation vector between the two particles, we can estimate the magnetic field B as $B \sim ev/cl^2$.

Thus we get: $F_M \sim e^2 v^2/l^2 c^2$ --- which is by a factor $v^2/c^2 \sim kT_e\,/m_e\, c^2$ smaller than the typical electric force F_E.