

Physics 5150

Homework Set # 2

Due 5 pm Thursday 2/1/2018

Problem 1: Debye Shielding:

Consider a positive point test charge immersed in a plasma. Calculate the electrostatic potential and charge density distributions around the test charge and then find the total plasma charge of the Debye shielding cloud. How big is it compared with the test charge? For simplicity, assume that the ions are fixed and that $e\phi \ll k_B T_e$.

Problem 2: Heliospheric Termination Shock:

The Solar Wind is a supersonic flow of hydrogen plasma streaming outward from the Sun with a constant radial velocity of 400 km/sec, and a density that is equal to about 8 cm^{-3} at 1 AU (Astronomical Unit, the distance between the Sun and the Earth) and that falls off with distance as r^{-2} . The solar wind is stopped by the pressure of the interstellar medium (ISM) at the so-called Heliospheric Termination Shock, where the ram pressure of the wind (ρv^2) equals to the total pressure of the ISM, $P_{\text{ISM}} = 4 \times 10^{-13} \text{ Pa}$. Approximately, how far from the Sun is the termination shock?

Problem 3: Pulsars:

A pulsar (a rotating magnetized neutron star) has a radius $R = 10 \text{ km}$ and a mass $M = 1.4 M_\odot$, where $M_\odot = 2 \times 10^{30} \text{ kg}$ is the mass of the Sun. The pulsar spins around its rotation axis with a period $P = 1 \text{ sec}$. Multi-year observations show that the pulsar is slowly spinning down, so that its rotation period increases at a rate of $\dot{P} = 10^{-8} \text{ sec/year}$. It is believed that the corresponding decay of the pulsar's rotational energy ($E_{\text{rot}} = I\Omega^2/2$, where I is the pulsar moment of inertia and $\Omega = 2\pi/P$ is its angular rotation rate) powers the pulsar wind by creating and ejecting highly relativistic electrons and positrons. These **relativistic particles** then fill the pulsar wind nebula (like the Crab Nebula) and gradually radiate away all their energy via synchrotron radiation. In the following, assume that the system is in a steady state and take the pulsar moment of inertia to be $I = (2/5)MR^2$. Also assume that each electron and positron has initial energy $\epsilon = \gamma m_e c^2$, with a Lorentz factor $\gamma = 10^4$.

- (a) What is the total luminosity (i.e., power) of the nebula?
- (b) How many electrons and positrons are created by the pulsar per second?

Problem 4: Quasar Accretion Power:

A galaxy consisting of 10^{12} Sun-like stars, each star shining at $L_\odot = 4 \times 10^{26} \text{ W}$, is located 300 million light years away from us. Right at the center of the galaxy there is a supermassive black hole with a mass $M_{\text{BH}} = 10^8 M_\odot$. The black hole has an accretion disk around it and accretes gas at a rate of $\dot{M} = 1 M_\odot/\text{year}$. A fraction $\eta = 10\%$ of the rest-mass energy ($E = mc^2$) of the accreted material is converted to radiation that escapes the system. What is the total radiative luminosity of the accreting black hole? How does it compare with the total star light coming from the galaxy?