HW2 of Plasma

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1. according to (2.2.9), potential around the positive charge is.

$$\phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} \cdot e^{-r/\lambda_D} \qquad \left(\lambda_D = \sqrt{\frac{\varepsilon_0 \cdot k_B T}{n_0 \cdot e^2}} \right)$$

and given poission equation, charge density ρ satisfies: $-\frac{\rho}{\varepsilon_0} = \nabla^2 \phi$

because $e\phi / k_B T \ll 1$, apply (2.2.5)

$$\rho = -\frac{n_0 \cdot e^2}{k_B \cdot T} \phi \quad ; \quad \rho = -\frac{n_0 e^2 Q}{4\pi \varepsilon_0 \cdot k_B T} \cdot \frac{e^{-r/\lambda_D}}{r}$$

then the total charge of the Debye shielding cloud is:

$$q_D = \int_0^{+\infty} 4\pi r^2 \, dr \cdot \rho$$

$$= -\frac{n_0 e^2 Q}{\varepsilon_0 \cdot k_B \cdot T} \cdot \lambda_D^2 \cdot \int_0^{+\infty} x \cdot e^{-x} \, dx = -Q$$

$q_D = -Q$ prefectly sheid the charge.

2. according to $\frac{1}{2} m_H v^2 = \frac{3}{2} kT$ & $P = nkT$

$$P = \frac{1}{3} n \cdot m_H \cdot v^2 \qquad \text{apply into distance dependent } n.$$

$$P = \frac{1}{3} n_0 \cdot m_H v^2 \cdot \left(\frac{R_0}{R}\right)^2$$, apply $v = 400$ km/s, $m_H = 1.6737 \times 10^{-27}$ kg.

$n_0 = 8 \times 10^{-6}$ /m³. $P = 4 \times 10^{-13}$ Pa.

$$\frac{R}{R_0} = \sqrt{\frac{n_0 \, m_H \cdot v^2}{3 \cdot P}} = 4.23 \times 10^{-5} \quad , \quad R = 4.23 \times 10^{-5} \text{ AU}$$

3. nebula power P is derivative of pulsar energy, $-\frac{dE}{dt}$ with

$$E = \frac{1}{2} I \cdot \omega^2 \quad , \text{apply } I = \frac{2}{5} M \cdot R^2 , \quad \omega = \frac{2\pi}{P}$$

$$\frac{dE}{dt} = -\frac{4\pi^2}{5} M \cdot R^2 \cdot \frac{2}{P^3} \cdot \frac{dP}{dt} \quad , \text{so} \quad \boxed{\text{Power} = \frac{8\pi^2}{5} M \cdot R^2 \cdot \frac{\dot{P}}{P^3}}$$

, with $M = 1.4 \times 2 \times 10^{30}$ kg ; $R = 10^4$ m , $P = 1.$s. $\dot{P} = 10^{-8} / (365 \times 3600 \times 24)$

(a) $\text{Power} = 1.402 \times 10^{24}$ W.

(b) nebula power is consist of relativistic electron & positron, whose number n satisfies: $2n \cdot \gamma \cdot m_e \cdot c^2 = \text{Power}$

so $n = 8.5627 \times 10^{32}$ /s

4. accreting BH has power of :

$$\text{Power} = \eta \cdot \dot{M} c^2 \quad , \text{apply } \eta = 0.1 \quad \dot{M} = 2 \times 10^{30} / \text{year}$$

$$\text{Power} = 5.7 \times 10^{38} \text{ W} ,$$

and $L = 4 \times 10^{26}$ W , so accreting BH power is much bigger than L.