Physics 5150
Homework Set # 7
Due 4 pm Friday 3/16/2018!

Problem 1: Pressure tensor
Show that if the distribution function is isotropic (i.e., spherically symmetric), then the pressure tensor is diagonal.

Problem 2: Energy Distribution Function
Consider a gas with a given isotropic velocity distribution function, $f(\mathbf{v}) = f(|v|)$. What is the energy *distribution function, $F_\epsilon(\epsilon)$ of such a gas? Here, $\epsilon = mv^2/2$ is the kinetic energy of a particle and F_ϵ is normalized according to*

$$\int_0^\infty F_\epsilon(\epsilon)\, d\epsilon = n\,. \tag{1}$$

Problem 3: Maxwellian distribution function
The general form of a drifting Maxwellian distribution is

$$f(\mathbf{v}) = n_0 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp[-m(\mathbf{v} - \mathbf{U}_0)^2/2kT]\,. \tag{2}$$

Show by explicit calculation that:

(a) *the plasma density corresponding to this distribution is equal to n_0;*

(b) *the average particle velocity corresponding to this distribution is equal to $\mathbf{U}_0$;*

(c) *the average particle kinetic energy is $\bar{\mathcal{E}} = (3/2)kT + mU_0^2/2$;*

(d) *the pressure tensor $\mathbf{\Pi}$ for this distribution is diagonal and isotropic — i.e., that $\mathbf{\Pi} = \mathrm{diag}\{p,p,p\}$ or, equivalently, $\Pi_{ij} = p\delta_{ij}$ — and that the scalar pressure is given by $p = nkT$.*

Problem 4:
Consider air (80% N_2 and 20% O_2) at normal pressure and at a room pressure (20 C). Assuming (non-drifting) Maxwellian distribution function for the air molecules, how many nitrogen molecules in 1 m^3 have velocities in the range between 1000 m/sec and 1001 m/sec? How many have velocities between 2000 m/sec and 2001 m/sec?

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Problem 5:

A particle species s has a distribution function of the form

$$f_s(\mathbf{x}, \mathbf{v}, t) = n_s \left(\frac{m_s}{2\pi k T_s}\right)^{3/2} \exp\left[-\frac{m_s v^2/2 + q_s\phi}{kT_s}\right], \tag{3}$$

where $\phi = \phi(\mathbf{x})$ is an electrostatic potential (constant in time), $\mathbf{E} = -\nabla\phi$, and the particle number density $n_s = n_{0s}$ and temperature $T_s = T_{0s}$ are both uniform in space and constant in time. Show that this distribution function satisfies the Vlasov equation with $\mathbf{B} = 0$,

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{q_s\mathbf{E}}{m_s}\cdot\nabla_{\mathbf{v}} f = 0\,. \tag{4}$$