Physics 5150

Homework Set # 6

Due 5 pm Thursday 3/1/2018

Problem 1: Gradient and Curvature drifts.
The equation for a dipole magnetic field in spherical coordinates (r, θ, ϕ) is given by (in SI units)

$$\vec{B} = \frac{\mu_0 M}{4\pi} \frac{1}{r^3} (2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}),$$

where M is the magnetic moment, and $\hat{r}$ and $\hat{\theta}$ are the unit vectors in the r and θ directions, respectively.

(a) Show that the equation for a magnetic field line is $r = R\sin^2\theta$, where R is the radius of the magnetic field line at the equator ($\theta = \pi/2$).

(b) Show that the radius of curvature of a magnetic field line at the equator is $R_c = R/3$. [Hint: in general, the radius of curvature is given by $R_c = |(\hat{b}\cdot\nabla)\hat{b}|^{-1}$, where $\hat{b} \equiv \vec{B}/B$ is a unit vector in the direction of the magnetic field.]

(c) Compute the curvature drift of a particle with a positive charge q and parallel velocity $v_\parallel$ at a radial distance R at the equator.

(d) Compute the ∇B drift of a particle with a positive charge q and perpendicular velocity $v_\perp$ at a radial distance R at the equator.

(e) Compare the directions and magnitudes of the curvature and ∇B drifts at the equator.

Problem 2:
A particle is trapped in a magnetic mirror field given by

$$B_z = B_0\left[1 + \left(\frac{z}{L}\right)^2\right]$$

and has a total kinetic energy $E = mv^2/2$ and pitch angle α_0 at $z = 0$. Find the oscillation (bounce) frequency in terms of L, E, and α_0.

Problem 3:
Consider a one-dimensional gas of particles with a velocity distribution function that has a triangular shape:

$$f(v_x) = A\,(1 - \frac{|v_x|}{v_0}), \quad |v_x| \le v_0\,, \tag{1}$$

and

$$f(v_x) = 0 \quad |v_x| > v_0\,. \tag{2}$$

(a) Express the constant parameter A in terms of the particle density n and v_0.

(b) Calculate the following quantities in terms of v_0:
(i) the average particle velocity, $< v_x >$;
(ii) the average magnitude of particle velocity, $< |v_x| >$;
(iii) the average particle kinetic energy, $< mv_x^2/2 >$.