Physics 5150

Homework Set # 4

Due 5 pm Thursday 02/15/2018

Problem 1: **Larmor radius.**
Compute the Larmor radius r_L for the following cases (neglecting $v_\|$):
(a) 10-keV electron in the Earth's magnetic field of 0.5 G.
(b) A solar wind proton with velocity 300 km/s, $B = 5$ nT (nano-Tesla).
(c) A 1-keV He^+ ion in the solar atmosphere near a sunspot, where $B = 0.5$ kG.
(d) A 3.5 MeV He^{++} ash alpha-particle in an 8 Tesla DT fusion reactor.

Problem 2: **Larmor radius.**
In the TFTR (Tokamak Fusion Test Reactor in Princeton, NJ), the plasma was heated by injection of 200-keV neutral deuterium (D) atoms, which, after entering the magnetic field, are converted into 200-keV D ions (with atomic weight $A = 2$) by charge exchange. These ions are confined within the tokamak only if their Larmor radius r_L is less than the minor radius $a = 0.6$ m of the toroidal plasma inside the tokamak. Compute the maximum (i.e., corresponding to particles with $v_\| = 0$) Larmor radius of these D ions in a 5 Tesla magnetic field to see whether this confinement condition is satisfied.

Problem 3: $E \times B$ **drift.**
An unneutralized electron beam has a density $n_e = 10^{14}\,\mathrm{m}^{-3}$ and radius $a = 1$ cm and flows along a $B = 2$ T uniform and straight magnetic field in the z-direction. Calculate the magnitude and direction of the $\mathbf{E} \times \mathbf{B}$ drift at $r = a$ caused by the electric field due to the beam's charge.

Problem 4: **Cyclotron and Plasma Frequency.**
There two important fundamental frequency scales in plasma physics (it is more convenient to work in the cgs units in this problem): (i) the (electron) cyclotron frequency, $\Omega_{ce} \equiv eB/m_e c$ (which we have covered in class), and (ii) the electron plasma frequency, $\omega_{pe} \equiv (4\pi n_e e^2/m_e)^{1/2}$ (which we will study later). Let us say we want to determine which one of these two frequencies is higher under given conditions. Express the ratio of the two frequencies in terms of the ratio of two energy densities: the magnetic field energy density, $B^2/8\pi$, and the electron rest-mass energy density, $n_e m_e c^2$.

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Problem 5: Ultra-relativistic gyro-motion.
Calculate the gyro-radius and the gyro-frequency of an ultra-relativistic particle of energy $\epsilon \equiv \gamma mc^2$, $\gamma \gg 1$, in a uniform constant magnetic field $\mathbf{B}$, for an arbitrary pitch angle α (angle between the particle's velocity and $\mathbf{B}$).

Problem 6: Rotating Plasma Column.
A cylindrical column of plasma rotates around its central axis (as though it were a rigid solid) at an angular velocity ω_0. A constant uniform magnetic field $\mathbf{B}_0$ is present parallel to the axis of rotation.

(a) Assuming that the rigid rotation can be described by $v_E = \boldsymbol{\omega_0} \times \mathbf{r}$, where $v_E = c\,\mathbf{E} \times \mathbf{B}/B^2$ (using cgs units), compute the electric field $\mathbf{E}$ in the plasma column in cylindrical (r, ϕ, z) coordinates.

(b) Is there a polarization charge $\rho_q = (4\pi)^{-1}\nabla \cdot \mathbf{E}$ associated with this electric field? If so, how does ρ_q depend on the distance from the central axis?

(c) Find the electrostatic potential φ such that $\mathbf{E} = -\nabla\varphi$.