Physics 5150

Homework Set # 9

Due 5 pm Thursday 4/5/2018

Problem 1:

In our derivation of the MHD equations, we used Ampere's law in the form $\nabla \times \mathbf{B} = 4\pi\mathbf{j}/c$*, thereby neglecting the displacement current* $\partial_t \mathbf{E}$*. Within the non-relativistic ideal MHD framework (i.e.,* $\mathbf{E} = -[\mathbf{u} \times \mathbf{B}]/c$ *with* $|\mathbf{u}| \ll c$*) is this a good approximation? Does it remain to be valid in relativistic MHD (when* $|\mathbf{u}| \sim c$*)?*

Problem 2:

A star with a radius $R_* = 1$ million km *and a uniform magnetic field* $B_* = 100\,\mathrm{Gauss}$ *on its surface ends its life by suddenly collapsing into a very compact and dense neutron star of radius* $R_{ns} = 10\,\mathrm{km}$*. The collapse is so rapid that the magnetic field doesn't have time to diffuse away and hence the perfect flux-freezing assumption holds.*

(a) *What is the strength of the resulting magnetic field on the surface of the neutron star?*

(b) *What is the electron cyclotron frequency in this magnetic field, and to which part of the electromagnetic spectrum does the frequency correspond?*

Problem 3:

Consider an infinitely-long straight wire carrying a current I*, surrounded by vacuum. Calculate the magnetic tension and magnetic pressure gradient forces at a distance* R *from the wire. Which force is stronger? Where is the net force (the sum of the two forces) pointing?*

Problem 4:

Our planet's magnetosphere protects life on Earth from the harmful flow of charged particles streaming from the Sun, known as the solar wind. Estimate roughly how far from the Earth (in centimeters and in Earth's radii) is the pressure of the magnetosphere's dipole magnetic field able to stop the incoming solar wind? Assume that the magnetic field is $0.3\,\mathrm{G}$ *at the Earth's equator and drops off inversely proportionally to* R^3 *and that the solar wind is a fully ionized hydrogen plasma with a velocity of* $400\,\mathrm{km/sec}$ *and a density of* $10\,\mathrm{cm}^{-3}$*. Neglect the thermal and magnetic pressure of the solar wind itself and only take into account its ram pressure.*