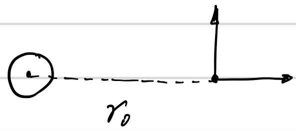
HW5 of Plasma

Chen Tang.

1. consider both gradient & curvature drift,

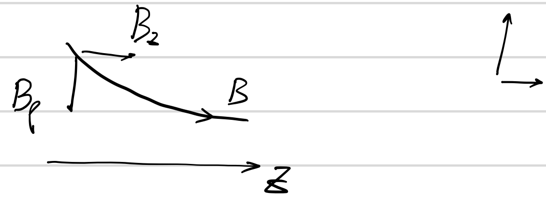


guiding center drift velocity is $\vec{v}_B$:

$$\vec{v}_B = \vec{v}_{\nabla B} + \vec{v}_R = \frac{2K_\parallel + K_\perp}{qB} \cdot \frac{\vec{R}_c \times \vec{B}}{R_c^2 \cdot B}$$

now $\vec{B} = \frac{\mu_0 I}{2\pi r} \cdot \hat{e}_\theta$, and $v_\parallel = v_\perp = v_0$, $r = r_0$ $\left(\vec{R}_c = r_0 \cdot \hat{e}_r\right)$

$$\vec{v}_B = \frac{3m v_0^2}{q} \cdot \frac{2\pi}{\mu_0 I} \hat{e}_z$$



2. in reference of partice $v_\parallel \hat{e}_z$, extra electric field generated as $\vec{E}' = -\frac{\vec{v}_\parallel}{c} \times \vec{B} = \frac{v_\parallel}{c} B_\rho \left(\hat{e}_z \times \hat{e}_\rho\right) = \frac{v_\parallel B_\rho}{c} \cdot \hat{e}_\theta$

and this angular E field accelerates the $v_\perp$ and resulting in perpendicular energy increase.

3. magnetic momentum μ keeps constant during motion:

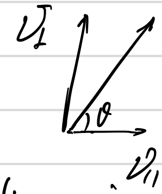
$$\mu = \frac{m \cdot v_\perp^2}{2B} = \text{Const.}$$

and magnetic flux enclosed is $\varphi_m = B \cdot \pi r_c^2$, with $r_c = \frac{m v_\perp}{qB}$

so $\varphi_m = \frac{\pi m^2 v_\perp^2}{q^2 B} = \frac{2\pi m}{q^2} \mu$, φ_m is also a constant.

4. $\mu = K_\perp / B$ is constant, meaning that $K_{\perp max} = K_{\perp min} \cdot \frac{B_{max}}{B_{min}}$.

$K_{\perp max} = 4 K_{\perp min}$, so when $K_{\perp max} > (K_{\perp min} + K_{\parallel})$, particle escapes.



criteria equation is $3 K_{\perp min} = K_{\parallel}$, or $\sqrt{3}\, v_\perp = v_\parallel$

and launch angle θ satisfies $\tan\theta = \frac{1}{\sqrt{3}}$, $\theta = \frac{\pi}{6}$. the escaped ions fractions is:

$$P_e = \frac{1}{4\pi} \cdot 2 \times \int_0^{\pi/6} 2\pi r \cdot \sin\theta \, d\theta = 1 - \frac{\sqrt{3}}{2}$$

and the trapped particels fraction $P_t = 1 - P_e = \sqrt{3}/2$.

5 (1) initial $v_\parallel = v_\perp = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2E_k}{m_p}} = 978.67$ km/s

because the moving mirror will recoil $v_\parallel$ with extra speed $2v_m$, $v_\parallel$ increase after every reflection, until $v_\parallel / v_\perp \geqslant \sqrt{R_m - 1} = 2$. It requires

$$[v_\parallel / 2v_m] + 1 = 49 \text{ times reflection to meet this criteria.}$$

and final energy would be: $E_k = \frac{1}{2} m \left(v_\perp^2 + (v_\parallel + 4v_m)^2 \right) = 25.027$ keV

(2) assuming motion in mirror is harmonic, we calculate the single period by last recoil, where length of mirror is $A = \frac{1}{2} L - v_m t$, t is the time of total process, $T = \frac{2\pi A}{2 v_\parallel}$ and $24.5\, T = t$, so

$$\pi \left(\frac{1}{2} L - v_m t \right) \cdot 24.5 = v_\parallel \cdot t$$

$$t = \frac{24.5\, \pi L}{2 v_\parallel + 2\pi \cdot 24.5 \cdot v_m} = 6.85 \times 10^{10}\ s$$