Physics 5150

Homework Set # 8

Due 5 pm Thursday 3/22/2018

Problem 1: Diamagnetic Drift

A cylindrical column of hydrogen plasma with uniform $T_e = T_i = \text{const}$ *and with* $n_e = n_i = n(r)$*, where* r *is the cylindrical radius, is immersed in a uniform magnetic field along the z-axis, of strength* B_0*. There is no electric field. The plasma density profile has the form:*

$$n(r) = n_0 \left[1 - (r/a)^2\right], \qquad 0 \le r \le a\,. \tag{1}$$

a) Calculate the ion and electron diamagnetic flows for $0 \le r \le a$.
b) Calculate the associated diamagnetic current in this region.
c) Determine magnetic field resulting from the diamagnetic current calculated in (b).

Problem 2: The Eddington limit

Consider the hydrogen plasma (consisting of electrons and protons only) in the upper atmosphere of a bright massive star [mass M_**, radius* R_**, and luminosity (total radiative power emitted by the star)* L_**]. The radiation pressure of the star exerts an outward radial force on the charged particles in the star's atmosphere:* $f_{\rm rad} = \sigma_T F_{\rm rad}/c$*. Here,* c *is the speed of light,* $F_{\rm rad} = L_*/4\pi R^2$ *is the radiative energy flux per unit area, and* σ_T *is the Thomson scattering cross-section:* $\sigma_T = (8\pi/3) r_j^2$*, where* r_j *is called the classical radius of the particles (index* j *stands for electrons or protons), defined as* $r_j = e^2/(4\pi\epsilon_0 m_j c^2)$ *(equal to* $2.82 \times 10^{-13}\,\text{cm}$ *for electrons and 1836 times smaller for protons). Because the cross-section for protons is by a factor of* $(m_p/m_e)^2$ *times smaller than for electrons, the radiation pressure force on the protons can be completely neglected compared with the radiation pressure on the electrons.*

(a) Calculate the maximum luminosity L_* *that the star can have before the net outward radiation pressure force on the plasma starts to overwhelm the gravitational force on the plasma. Take into account that the radiation pressure acts mostly on the electrons, whereas the gravity acts mostly on the ions. Can the radiation pressure blow the electrons away leaving the heavier ions behind? Why?*

(b) This maximum luminosity is known as the Eddington limit, or the Eddington luminosity, $L_{\rm Edd}$*. It plays a very important role in astrophysics: if the luminosity of a star starts to approach the Eddington limit, the star starts to blow out a powerful radiation-driven wind that causes significant mass loss and thus affects the star's evolution. Estimate this maximum luminosity for the Sun (*$M_* = M_\odot = 2 \times 10^{33}\,\text{g}$*), and compare it with the actual solar luminosity (*$L_* = L_\odot = 4 \times 10^{33}\,\text{erg/s}$*). Do you think the solar wind is driven by the Sun's radiation pressure or by some other force?*

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Problem 3: Quasineutrality

When deriving the (nonrelativistic) MHD equations from the two-fluid theory, one uses the assumption of quasineutrality, $|\rho_e| \equiv e|\Delta n| \equiv e|n_i - n_e| \ll en \simeq en_e$*. Using Poisson's law (*$\nabla \cdot \mathbf{E} = 4\pi\rho_e$*), nonrelativistic Ampere's law (*$\nabla \times \mathbf{B} = 4\pi\mathbf{j}/c$*), and ideal-MHD Ohm's law* $\mathbf{E} = -[\mathbf{u} \times \mathbf{B}]/c$*, show that quasineutrality is indeed a good approximation in the nonrelativistic limit, i.e., when the fluid velocity is much less than the speed of light,* $|\mathbf{u}| \ll c$*.*

Problem 4 (optional): Entropy

Using the variational principle, show that the non-drifting Maxwellian velocity distribution is the result of maximizing the entropy density

$$S = -\int f \ln f d^3\mathbf{v}\,, \tag{2}$$

subject to the constraints of a fixed particle number density, $n = \int f d^3\mathbf{v} = \mathrm{const}$*, and a fixed particle energy density,* $E = \int (mv^2/2)\, f d^3\mathbf{v} = \mathrm{const}$*.*