Physics 5150

Homework Set # 12

Due 5 pm Thursday 4/26/2018

<u>Problem 1:</u> Energy Partitioning in Alfvén Waves

Consider an incompressible Alfvén wave propagating parallel to a background magnetic field $\mathbf{B_0} = B_0\,\hat{z}$. *What is the greater: the plasma kinetic energy density due to the perturbed velocity,* $E_{\rm kin} = \rho_0|u_1|^2/2$, *or the magnetic energy density of the perturbed magnetic field,* $E_{\rm magn} = |B_1|^2/(8\pi)$, *and by what factor?*

<u>Problem 2:</u> Alfvén Speed

Calculate the Alfvén speed in the following environments, assuming a fully ionized pure hydrogen plasma, $m_i = m_p$, $n_i = n_e$ *(please pay attention to units):*
(a) Tokamak: $B_0 = 4$ *Tesla;* $n_e = 10^{14}\,{\rm cm}^{-3}$;
(b) Earth Magnetosphere: $B_0 = 20$ nT *(nano-Tesla);* $n_e = 0.1\,{\rm cm}^{-3}$;
(c) Solar Corona: $B_0 = 100$ *G;* $n_e = 10^9\,{\rm cm}^{-3}$.

<u>Problem 3:</u> Relativistic Alfvén Wave

Derive the dispersion relation, and also the phase and group velocities, of an Alfvén wave propagating parallel to the background magnetic field ($[\mathbf{k} \times \mathbf{B}_0] = 0$*) in the case when the formal expression for the Alfvén velocity that we derived in class,* $V_A = B_0/\sqrt{4\pi\rho_0}$, *exceeds the speed of light (this is equivalent to saying that the magnetic field is so strong that the magnetic energy density,* $B_0^2/8\pi$, *is greater than one half of the plasma rest-mass energy density,* $\rho_0 c^2$*).*

(Hint: Since we are considering linear waves, the wave's amplitude is treated as being as infinitesimally small. In particular, this means that the perturbed velocity, $\mathbf{u}_1$, *is much smaller than the speed of light, i.e., the actual plasma motions are non-relativistic. Then, only one equation in our derivation needs to be modified: namely, the displacement current needs to be kept in in Ampere's law.)*

<u>Problem 4:</u> Fast Magnetosonic Wave

Using compressible ideal MHD equations with an adiabatic equation of state $P \propto \rho^\gamma$, *derive the dispersion relation for a fast magnetosonic wave propagating perpendicular to a uniform background magnetic field* $\mathbf{B_0} = B_0\,\hat{z}$. *Find the phase and group velocities of the wave.*