HW 11 of Plasma

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1. gas dynamic equation in one dimensional:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho u)}{\partial x} = 0 & (1) \\ \dfrac{\partial(\rho u)}{\partial t} + \dfrac{\partial(\rho u^2 + P)}{\partial x} = 0 & (2) \end{cases}$$

apply $P = C \cdot \rho^{\gamma}$ into (2):

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\gamma P}{\rho} \frac{\partial}{\partial x} \rho = 0 \qquad (3)$$

then we seperate (1) & (3): with $\rho = \rho_0 + \Delta\rho$; $u = u_0 + \Delta u$

$$\begin{cases} \partial_t \Delta\rho + \rho_0 \partial_x \Delta u + u_0 \partial_x \Delta\rho = 0 & (4) \\ \rho_0 \partial_t \Delta u + \rho_0 u_0 \partial_x \Delta u + \gamma P_0/\rho_0 \cdot \partial_x \Delta\rho = 0 & (5) \end{cases}$$

transfer into gas reference: $u_0 = 0$

$$\begin{cases} \partial_t \Delta\rho + \rho_0 \partial_x \Delta u = 0 & (6) \\ \rho_0 \partial_t \Delta u + \gamma P_0/\rho_0 \cdot \partial_x \Delta\rho = 0 & (7) \end{cases}$$

$\partial_t (6) - \partial_x (7)$: $\quad \partial_t^2 \cdot \Delta\rho - \dfrac{\gamma P_0}{\rho_0} \cdot \partial_x^2 \Delta\rho = 0 \qquad (8)$

(8) is a wave function, the phase velocity v_p is:

$$v_p = \sqrt{\gamma P_0/\rho_0}$$

using Einstein relation: $\omega^2 = \omega_0^2 + k^2 \cdot v_p^2$, group velocity is:

$$v_g = \frac{d\omega}{dk} = \frac{k}{\omega} \cdot v_p^2$$

2. E - field in plasma is:

$$E = \frac{ne \cdot \Delta X}{\varepsilon_0} \sqrt{4\pi\varepsilon_0}$$

and electron's velocity $u = \omega_{pe} \cdot \Delta X = \Delta X \sqrt{\frac{ne^2}{\varepsilon_0 m_e}}$

so $$\mathcal{E}_k / \mathcal{E}_e = \frac{\frac{1}{2} n_0 m_e \cdot \Delta X^2 \cdot \frac{ne^2}{\varepsilon_0 m_e}}{\frac{1}{8\pi} 4\pi\varepsilon_0 \cdot \left(\frac{ne \cdot \Delta X}{\varepsilon_0}\right)^2} = 1$$

$\mathcal{E}_k = \mathcal{E}_e$ they are the same.

3. frequency of plasma wave is:

$$(300 \times 10^6 \times 2\pi)^2 \cdot \frac{\varepsilon_0 \cdot m_e}{e^2}$$

$$f_p = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\varepsilon_0 m_e}}$$

requirement of $f_p = 300$ MHz leads to

$$n = 5.923 \times 10^5 \ /m^3$$