PHYS/ASTR-5150 Spring 2018

Physics 5150

Homework Set # 3

Due 5 pm Thursday 2/8/2018

SOLUTIONS

Problem 1: Thermonuclear fusion in stars

A 5 billion year old main-sequence star called Sol, belonging to the spectral type G2, has a mass $M_\odot = 2 \times 10^{33}\,\mathrm{g}$ and a total luminosity $L_\odot = 4 \times 10^{33}\,\mathrm{erg/s}$. The star is powered by the pp chain thermonuclear reaction, the net result of which is that 4 protons are turned into a He^4 nucleus (an α-particle) releasing 26.7 MeV of energy in the process.

(a) Assuming that the star's parameters have not changed appreciably since it was born, how much hydrogen (by mass) has the star consumed over its lifetime?

(b) In addition to producing helium, each pp chain reaction also produces 2 neutrinos that fly out of the star freely, without interacting with anything (they carry only 2% of the total fusion energy, so the corresponding energy losses can be neglected). How many neutrinos fly each second through a $1\,\mathrm{cm}^2$ thumbnail of John Bahcall standing on a seashore on a small rocky planet called Terra, 150 mln km from the star?

Solution:

The number of pp-chain reactions happening in the Sun each second is

$$\dot{N} = \frac{L_\odot}{\epsilon} = 0.93 \times 10^{38}\,\mathrm{sec}^{-1}\,. \tag{1}$$

(a) In each reaction 4 hydrogen nuclei (protons) are consumed. Therefore, the total hydrogen consumption rate is

$$\dot{M}_H = 4 m_p \dot{N} \simeq 6.3 \times 10^{11}\,\mathrm{kg/sec}\,. \tag{2}$$

Then, the amount of hydrogen consumed over the star's lifetime is

$$\Delta M_H = \dot{M}_H \times 5 \cdot 10^9 \times 3.15 \times 10^7\,\mathrm{sec} \simeq 10^{29}\,\mathrm{kg}\,. \tag{3}$$

(b) The total neutrino production rate is

$$\dot{N}_\nu = 2\dot{N} \simeq 1.9 \times 10^{38}\,\mathrm{sec}^{-1}\,. \tag{4}$$

The neutrino flux at $R = 1$ AU then is:

$$\Gamma_\nu = \frac{\dot{N}_\nu}{4\pi R^2} \simeq 6.6 \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}\,. \tag{5}$$

Problem 2: Fusion power plant

What is the annual tritium mass consumption of a continuously (24×7) operating D-T thermonuclear fusion reactor with the total (α-particles plus neutrons) fusion power output of 1 GW?

Solution:

Fusion energy released in 1 reaction is $\epsilon \simeq 17\,\mathrm{MeV} \simeq 2.7 \times 10^{-12}\,\mathrm{J}$. The number of reactions per unit time is then:

$$\dot{N} = \frac{P}{\epsilon} \simeq 3.7 \times 10^{20}\,\mathrm{sec}^{-1}\,. \tag{6}$$

The number of tritium nuclei consumed each year is

$$N_T = \dot{N} \times 3.16 \cdot 10^7\,\mathrm{sec/yr} \simeq 1.16 \times 10^{28}\,, \tag{7}$$

and hence the annual tritium mass consumption is

$$M_T = N_T \times 3m_p \simeq 58\,\mathrm{kg}\,. \tag{8}$$

Problem 3: Tokamak plasma density:

Equilibrium and stability consideration limit the total (electrons + ions) plasma pressure inside a tokamak to be less than about 10% of the pressure of the confining magnetic field. What is the maximum plasma electron density in a tokamak with a magnetic field of 5 Tesla and temperature $T_i = T_e = 10$ keV?

Solution:

The maximum plasma pressure is $P_{\max} = 0.1\,P_{\mathrm{mag}} = 0.1\,B^2/8\pi$. The corresponding plasma density, related to the plasma pressure via $P_{\max} = n_{\max}(T_e + T_i)$, is

$$n_{\max} = 0.1\,\frac{B^2}{8\pi(T_e + T_i)} \simeq 3 \times 10^{14}\,\mathrm{cm}^{-3}\,.$$