HW 10 of Plasma

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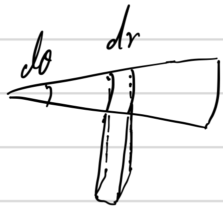
1. according to the Ampre's law, with $\vec{B} = B_\varphi \hat{e}_\varphi$ (cylindrical coordinate)

$$\mu_0 j_z(r) = \frac{1}{r}\frac{d}{dr}(r B_\varphi),$$

considering $j_z(r) = j_z \sim$ Const. $B_\varphi(r) = \frac{1}{2}\mu_0 j_z r$

then choose derivative volume current of $dV = r d\theta \cdot dr \cdot dz$



Now it's Lorentz force is

$$df_B = dV \cdot j_z B_\varphi(r) \quad (-\hat{e}_r)$$

and balanced by gas pressure $P(r)$.

$$df_p = r d\theta \cdot dz \cdot \frac{dP}{dr} \cdot dr \quad (-\hat{e}_r).$$

Equilibrum requires $df_B + df_p = 0$, and boundary condition of $P(a) = 0$

$$\frac{1}{2}\mu_0 j_z^2 \cdot r = -dP$$

$$P(r) = \frac{1}{4}\mu_0 j_z^2 (a^2 - r^2)$$

2. $$\beta = \frac{2n k_B T}{B^2/8\pi} = \frac{16\pi\, n\, k_B T}{B^2}$$, apply the parameters

	1)	2)	3)
β	3.22×10^5	1.39×10^5	4.16×10^9

neither of these enviroment is magnetically - dominated.

3. starting from $\nabla \times B = \alpha_0 \cdot B$, $B = B_\theta \hat{e}_\theta + B_z \hat{e}_z$

so $\alpha (B_\theta \hat{e}_\theta + B_z \cdot \hat{e}_z) = \frac{1}{r} \frac{d}{dr} (r B_\theta) \hat{e}_z - \frac{d}{dr} B_z \cdot \hat{e}_\theta$

$$\Rightarrow \begin{cases} \alpha B_\theta = - \frac{d}{dr} B_z \\ \alpha B_z = \frac{1}{r} \frac{d}{dr} (r B_\theta) \end{cases}$$

then we get:

$$\begin{cases} \alpha^2 B_\theta + \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} (r B_\theta) \right) = 0 \\ \alpha^2 B_z + \frac{1}{r} \frac{d}{dr} \left(r \cdot \frac{d}{dr} B_z \right) = 0 \end{cases}$$

$\left(\frac{1}{x} (y + x y') \right)' + \alpha^2 y = 0.$

$\left(\frac{y}{x} + y' \right)'$

$y'' + \frac{y'}{x} - \frac{y}{x^2} + \alpha^2 y = 0$

the B_z should be root of Bessel equation (zero order):

$$r (r B_z')' + \alpha^2 r^2 B_z = 0$$

with Neumann boundary condition $B_z' (r=0) = 0$

the solution should be: $B_z = B_0 \cdot J_0 (\alpha r)$

the J_0 is zeroth order of first kind Bessel function

similarly, $B_\theta = B_1 \cdot J_1 (\alpha r)$

J_1 is first order of first kind Bessel function

given $\alpha B_\theta = - \frac{d}{dr} B_z$

$J_0'(\alpha r) = - \alpha \cdot J_1 (\alpha r)$, so $B_0 = B_1 = B$

$B_z = B \cdot J_0(\alpha r)$; $B_\theta = B J_1 (\alpha r)$