HW 8 of Plasma

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1. (1) According to the momentum equation.

$$n_i m_i (\partial_t V_i + V_i \cdot \nabla V_i) = - \nabla P_i + n_i q_i (E + V_i \times B)$$

under steady state and no electric field.

$$- \nabla P_i + n_i \cdot q_i \cdot V_i \times B = 0$$, so diamagnetic drift flow is

$$\vec{V_i} = - \frac{\nabla P_i \times B}{n_i q B^2}$$

as thermal pressure $P_i = P_e = n(r) \cdot kT$,

$$\nabla P_i = n_0 kT \cdot \nabla [1 - (r/a)^2] = - n_0 kT \cdot \frac{2r}{a^2} \hat{e}_r$$ (Cylindrical Coordinate)

so $$\vec{V_i} = - \frac{n_0 kT}{n(r) e B_0} \cdot \frac{2r}{a^2} \hat{e}_\varphi = - \frac{kT}{eB_0} \cdot \frac{2r}{a^2 - r^2} \hat{e}_\varphi$$

and $\vec{v_e} = - \vec{v_i}$

(2) current $\vec{J} = n q \vec{v}$, so $J_i = J_e = - \frac{kT}{B_0} \cdot n_0 \cdot \frac{2r}{a^2} \hat{e}_\varphi$

(3). choose any circle current at $r_0 \sim r_0 + dr$, current areal denisty σ is

$$\sigma = 2 J(r) dr \hat{e}_\varphi$$, given Ampere's circuital Law.

$$d\vec{B}_\sigma = \begin{cases} \mu_0 \sigma \hat{e}_z & r < r_0 \\ 0 & r > r_0 \end{cases}$$

so diamagnetic magnetic field $\vec{B}_\sigma(r) =$

$$\vec{B}_\sigma(r) = \int_r^a 2\mu_0 J(r) dr \cdot \hat{e}_z = - \frac{2kT}{B_0} \cdot \frac{n_0}{a^2} (a^2 - r^2) \hat{e}_z$$

2. (a) radiation pressure force f_{rad} = :

$$f_{rad} = \frac{8\pi}{3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 \cdot \frac{L_*}{4\pi R^2 \cdot c} \quad \text{(on electrons)}$$

gravitational force $f_G = \frac{GM_*}{R^2} m_p$, when f_{rad} overwhelem f_G.

$$L_* = \frac{GM_*}{\cancel{R^2}} \cdot m_p \cdot \cancel{4\pi}^{2} \cancel{R^2} \cdot c \cdot \frac{3}{{}_2\cancel{8\pi}} \cdot \left(\frac{4\pi\varepsilon_0 \cdot m_e \cdot c^2}{e^2}\right)^2$$

$$= 24\, G M_* m_p \cdot \pi^2 \varepsilon_0^2 m_e^2 \cdot c^5 / e^4$$

if the radiation blow all electrons away, ions left will reject with each other due to the coulomb force (way bigger than gravity force), there is no other force to bound ions together. So the left part of star will also blow itself away, it's impossible just leave ions to form a star.

(b) $L_{*\,sun} = G M_* m_p \cdot 3c/2 \cdot r_e^{-2}$, apply all the numbers

$L_{*\,sun} = 1.2625 \times 10^{31}\ J/s = 1.2625 \times 10^{38}\ erg/s$

$L_\odot = 4 \times 10^{33}\ erg/s \ll L_{*\,sun}$

Sun's radiation pressure due to current luminosity is much smaller than the benchmark for balancing gravity, so solar wind is definitely driven by other force.

3. $\nabla \times B = 4\pi \vec{j}/c$, so $B \sim 4\pi \vec{j}/c$

and electric field due to plasma current is

$$E = -\vec{u}\times\vec{B}/c \sim 4\pi\, \vec{j}\cdot\vec{u}/c^2$$

and non-neutrality charge density according to Poisson's law is

$$\rho_e = E/4\pi \sim \vec{j}\cdot\vec{u}/c^2 \quad , \quad \text{apply } \vec{j} = en\cdot\vec{u}$$

so $|\rho_e| = e|\Delta n| \sim en\cdot\frac{u^2}{c^2} \ll en$, so quasineutrality approximation is good!

4. apply lagrange multiplies into variation, we have

$$\delta \int S + \alpha n + \beta E \; d^3\vec{v} = 0$$

$\Rightarrow \delta \int -f\ln f + \alpha f + \beta\cdot\frac{mv^2}{2}\cdot f \; d^3\vec{v} = 0$, variation of f satisfies maximizing

so $\frac{\delta}{\delta f}\int \cdots = -\ln f - 1 + \alpha + \beta\cdot\frac{mv^2}{2} = 0$

$\Rightarrow \ln f = \alpha - 1 + \beta\cdot\frac{mv^2}{2}$; $f = e^{\alpha-1}\cdot e^{\beta\cdot\frac{mv^2}{2}} \sim A\cdot e^{\beta\cdot\frac{mv^2}{2}}$

which is in the maxiwellian form

now normalize the f, fixed particle number n requires $\int f \, d\vec{v}^3 = n$, so:

$$\int A \cdot e^{\beta \cdot \frac{mv^2}{2}} d^3_v = n \text{ , } \quad \beta \text{ has to be negative, } \beta \to -\beta$$

and $A \cdot \left(\frac{2\pi}{\beta m}\right)^{\frac{3}{2}} = n$, also energy conservation requires:

$$\frac{1}{n} \int A \cdot \frac{mv^2}{2} \cdot e^{-\beta \cdot \frac{mv^2}{2}} d^3_{\vec{v}} = E, \quad \text{so } \frac{mA}{2n} \cdot \sqrt{\pi} \cdot 3 \left(\frac{2}{\beta m}\right)^{\frac{3}{2}} = E$$

so $$A = n \left(\frac{3m}{4\pi E}\right)^{\frac{3}{2}} , \quad \beta = \frac{3}{2E}$$

$$f_{(\vec{v})} = n \left(\frac{3m}{4\pi E}\right)^{\frac{3}{2}} \cdot \exp\left(-\frac{3mv^2}{4E}\right) \quad ; \quad v^2 = |\vec{v}|^2$$