

# Taylor Series of Arcsine Function

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➡️ *It has been suggested that this article or section be renamed:*

## *Taylor Series of Real Arcsine Function*

*One may discuss this suggestion on the talk page.*

## Theorem

The (real) arcsine function has a Taylor series expansion:

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \frac{x^{2n+1}}{2n+1}$$

which converges for  $-1 \leq x \leq 1$ .

## Proof

From the General Binomial Theorem:

$$\begin{aligned} (1-x^2)^{-1/2} &= 1 + \frac{1}{2}x^2 + \frac{1 \cdot 3}{2 \cdot 4}x^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^6 + \dots \\ (1) : \qquad &= \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} x^{2n} \end{aligned}$$

for  $-1 < x < 1$ .

From Power Series is Termwise Integrable within Radius of Convergence, (1) can be integrated term by term:

$$\begin{aligned} \int_0^x \frac{1}{\sqrt{1-t^2}} dt &= \sum_{n=0}^{\infty} \int_0^x \frac{(2n)!}{2^{2n}(n!)^2} t^{2n} dt \\ \implies \arcsin x &= \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \frac{x^{2n+1}}{2n+1} \qquad \text{Derivative of Arcsine Function} \end{aligned}$$

We will now prove that the series converges for  $-1 \leq x \leq 1$ .

By Stirling's Formula:

$$\begin{aligned} \frac{(2n)!}{2^{2n}(n!)^2} \frac{x^{2n+1}}{2n+1} &\sim \frac{(2n)^{2n} e^{-2n} \sqrt{4\pi n}}{2^{2n} n^{2n} e^{-2n} 2\pi n} \frac{x^{2n+1}}{2n+1} \\ &= \frac{1}{\sqrt{\pi n}} \frac{x^{2n+1}}{2n+1} \end{aligned}$$

Then:

$$\begin{aligned} \left| \frac{1}{\sqrt{\pi n}} \frac{x^{2n+1}}{2n+1} \right| &< \left| \frac{x^{2n+1}}{n^{3/2}} \right| \\ &\leq \frac{1}{n^{3/2}} \end{aligned}$$

By P-Series Converges Absolutely:

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

is convergent.

So by the Comparison Test, the Taylor series is convergent for  $-1 \leq x \leq 1$ .



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