Math Homeyork yeek 2

Tom Curran

July 2, 2018

1

<u>3.1</u>

$$\begin{aligned} \|x+y\|^2 &= \langle x+y, y+x\rangle \\ &= \langle x,y\rangle + \langle y,x\rangle + \langle x,x\rangle + \langle y,y\rangle \\ &= \|x\|^2 + \|y\|^2 + \langle y,x\rangle + \overline{\langle x,y\rangle} \\ &= \|x\|^2 + \|y\|^2 + 2(\langle y,x\rangle) \end{aligned} \tag{1}$$

$$\langle y,x\rangle = \frac{1}{2}(-\|x\|^2 - \|y\|^2\|x+y\|^2)$$

$$\begin{aligned} \|x-y\|^2 &= \langle x-y, y-x\rangle \\ &= \langle x,y\rangle + \langle y,x\rangle - \langle x,x\rangle + \langle y,y\rangle \\ &= \|x\|^2 + \|y\|^2 - \langle y,x\rangle + \overline{\langle x,y\rangle} \\ &= \|x\|^2 + \|y\|^2 - 2(\langle y,x\rangle) \end{aligned} \tag{2}$$

$$\langle y,x\rangle = \frac{1}{2}(\|x\|^2 + \|y\|^2 - \|x-y\|^2)$$

combining equations 1 and 2 from above we get

$$\langle y,x\rangle = \frac{1}{4}(\|x+y\|^2 - \|x-y\|^2)$$

<u>3.2</u>

$$\begin{aligned} \|x+iy\|^2 &= \langle x+iy, iy+x\rangle \\ &= \langle x,iy\rangle + \langle iy,x\rangle + \langle x,x\rangle + \langle iy,iy\rangle \\ &= \|x\|^2 + \|iy\|^2 + \langle iy,x\rangle + \overline{\langle x,iy\rangle} \\ &= \|x\|^2 + \|iy\|^2 + 2(\langle iy,x\rangle) \end{aligned} \tag{3}$$

$$\langle y,x\rangle = \frac{1}{2}(-\|x\|^2 - \|iy\|^2\|x+iy\|^2)$$

$$
\begin{aligned}
\|x-iy\|^2 &= \langle x-iy, iy-x\rangle \\
&= \langle x, iy\rangle + \langle iy, x\rangle - \langle x, x\rangle + \langle iy, iy\rangle \\
&= \|x\|^2 + \|iy\|^2 - \langle iy, x\rangle + \overline{\langle x, iy\rangle} \\
&= \|x\|^2 + \|iy\|^2 - 2(\langle iy, x\rangle)
\end{aligned} \tag{4}
$$

$$\langle y, x\rangle = \frac{1}{2}(\|x\|^2 + \|iy\|^2 - \|x-iy\|^2)$$

combining equations 3 and four give:

$$\langle y, x\rangle = \frac{1}{4}(i\|x+y\|^2 - i\|x-y\|^2)$$

combining with equations 1 and 2 we get:

$$\langle y, x\rangle = \frac{1}{4}(\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2)$$

<u>3.3</u>

$\cos\theta = \frac{\langle x,y\rangle}{\|x\|\|y\|}$

$$
\begin{aligned}
\langle f, f\rangle &= \sqrt{\langle f * f\rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x * x dx} \\
&= \sqrt{\frac{1}{3} - 0} \\
&= \sqrt{\frac{1}{3}} \\
&= \|f\|
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\langle f, f\rangle &= \sqrt{\langle f * f\rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x^5 * x^5 dx} \\
&= \sqrt{\frac{1}{11} - 0} \\
&= \sqrt{\frac{1}{11}} \\
&= \|g\|
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\langle f \cdot g\rangle &= x * x^5 \\
&= x^6
\end{aligned} \tag{7}
$$

$$
\cos\theta = \frac{x^6}{\sqrt{\frac{1}{3}}\sqrt{\frac{1}{11}}} \tag{8}
$$

<u>3.3b</u>: x^2, x^4
$\cos\theta = \frac{\langle x,y\rangle}{\|x\|\|y\|}$

$$
\begin{aligned}
\langle f, f\rangle &= \sqrt{\langle f * f\rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^5 x * x dx} \\
&= \sqrt{\frac{1}{5} - 0} \\
&= \sqrt{\frac{1}{5}} \\
&= \|f\|
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\langle f, f\rangle &= \sqrt{\langle f * f\rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x^4 * x^4 dx} \\
&= \sqrt{\frac{1}{17} - 0} \\
&= \sqrt{\frac{1}{17}} \\
&= \|g\|
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\langle f \cdot g\rangle &= x^2 * x^4 \\
&= x^6
\end{aligned} \tag{11}
$$

$$
\cos\theta = \frac{x^6}{\sqrt{\frac{1}{5}}\sqrt{\frac{1}{17}}} \tag{12}
$$