

Math Homework week 2

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3.1

$$\begin{aligned}\|x + y\|^2 &= \langle x + y, y + x \rangle \\ &= \langle x, y \rangle + \langle y, x \rangle + \langle x, x \rangle + \langle y, y \rangle \\ &= \|x\|^2 + \|y\|^2 + \langle y, x \rangle + \overline{\langle x, y \rangle} \\ &= \|x\|^2 + \|y\|^2 + 2(\langle y, x \rangle)\end{aligned}\tag{1}$$

$$\langle y, x \rangle = \frac{1}{2}(-\|x\|^2 - \|y\|^2 + \|x + y\|^2)$$

$$\begin{aligned}\|x - y\|^2 &= \langle x - y, y - x \rangle \\ &= \langle x, y \rangle + \langle y, x \rangle - \langle x, x \rangle - \langle y, y \rangle \\ &= \|x\|^2 + \|y\|^2 - \langle y, x \rangle - \overline{\langle x, y \rangle} \\ &= \|x\|^2 + \|y\|^2 - 2(\langle y, x \rangle)\end{aligned}\tag{2}$$

$$\langle y, x \rangle = \frac{1}{2}(\|x\|^2 + \|y\|^2 - \|x - y\|^2)$$

combining equations 1 and 2 from above we get

$$\langle y, x \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)$$

3.2

$$\begin{aligned}\|x + iy\|^2 &= \langle x + iy, iy + x \rangle \\ &= \langle x, iy \rangle + \langle iy, x \rangle + \langle x, x \rangle + \langle iy, iy \rangle \\ &= \|x\|^2 + \|iy\|^2 + \langle iy, x \rangle + \overline{\langle x, iy \rangle} \\ &= \|x\|^2 + \|iy\|^2 + 2(\langle iy, x \rangle)\end{aligned}\tag{3}$$

$$\langle y, x \rangle = \frac{1}{2}(-\|x\|^2 - \|iy\|^2 + \|x + iy\|^2)$$

$$\begin{aligned}
\|x - iy\|^2 &= \langle x - iy, iy - x \rangle \\
&= \langle x, iy \rangle + \langle iy, x \rangle - \langle x, x \rangle + \langle iy, iy \rangle \\
&= \|x\|^2 + \|iy\|^2 - \langle iy, x \rangle + \overline{\langle x, iy \rangle} \\
&= \|x\|^2 + \|iy\|^2 - 2\langle iy, x \rangle
\end{aligned} \tag{4}$$

$$\langle y, x \rangle = \frac{1}{2}(\|x\|^2 + \|iy\|^2 - \|x - iy\|^2)$$

combining equations 3 and four give:

$$\langle y, x \rangle = \frac{1}{4}(i\|x + y\|^2 - i\|x - y\|^2)$$

combining with equations 1 and 2 we get:

$$\langle y, x \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2)$$

3.3

$$\cos \theta = \frac{\langle x, y \rangle}{\|x\| \|y\|}$$

$$\begin{aligned}
\langle f, f \rangle &= \sqrt{\langle f * f \rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x * x dx} \\
&= \sqrt{\frac{1}{3} - 0} \\
&= \sqrt{\frac{1}{3}} \\
&= \|f\|
\end{aligned} \tag{5}$$

$$\begin{aligned}
\langle f, f \rangle &= \sqrt{\langle f * f \rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x^5 * x^5 dx} \\
&= \sqrt{\frac{1}{11} - 0} \\
&= \sqrt{\frac{1}{11}} \\
&= \|g\|
\end{aligned} \tag{6}$$

$$\begin{aligned}
\langle f \cdot g \rangle &= \int_0^1 x * x^5 dx \\
&= \int_0^1 x^6 dx \\
&= \frac{1}{7}
\end{aligned} \tag{7}$$

$$\cos \theta = \frac{\frac{1}{7}}{\sqrt{\frac{1}{3}} \sqrt{\frac{1}{11}}} \tag{8}$$

3.3b: x^2, x^4

$$\cos \theta = \frac{\langle x, y \rangle}{\|x\| \|y\|}$$

$$\begin{aligned}
\langle f, f \rangle &= \sqrt{\langle f * f \rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^5 x * x dx} \\
&= \sqrt{\frac{1}{5} - 0} \\
&= \sqrt{\frac{1}{5}} \\
&= \|f\|
\end{aligned} \tag{9}$$

$$\begin{aligned}
\langle f, f \rangle &= \sqrt{\langle f * f \rangle} \\
&= \sqrt{\int_0^1 f(x)f(x)dx} \\
&= \sqrt{\int_0^1 x^4 * x^4 dx} \\
&= \sqrt{\frac{1}{17} - 0} \\
&= \sqrt{\frac{1}{17}} \\
&= \|g\|
\end{aligned} \tag{10}$$

$$\begin{aligned}
\langle f \cdot g \rangle &= \int_0^1 x^2 * x^4 dx \\
&= \int_0^1 x^6 \\
&= \frac{1}{7}
\end{aligned} \tag{11}$$

$$\cos \theta = \frac{\frac{1}{7}}{\sqrt{\frac{1}{5}} \sqrt{\frac{1}{17}}} \tag{12}$$

2 Orthogonality

3 QR Decomposition