

Natural Language Processing

Jim Martin -- Lecture 3

CSCI 5832

Today

- Language Modeling
 - Probabilistic language models
 - N -gram approach
 - Independence assumptions
 - Practical Issues
 - Dealing with zeroes

Word Prediction

- Guess the next word...
 - *So I notice three guys standing on the ____*

What kinds of knowledge did you use to come up with those predictions?

Word Prediction

- We can formalize this task as a problem in discrete probability
 - Given a vocabulary, compute a probability distribution over that vocabulary given the preceding words. $P(w_n | w_{1:n-1})$
 - Or assign a probability to a sequence. $P(w_{1:n})$
 - We'll call a model that can do this a Probabilistic Language Model

Applications

- It turns out that the ability to assess the probability of a sequence is extremely useful. It is at the core of many applications
 - Automatic speech recognition
 - Handwriting and character recognition
 - Spam detection
 - Sentiment analysis
 - Spelling correction
 - Machine translation
 - Summarization

Speech Recognition

- Initial acoustic/signal system proposes two hypotheses for an input sentence
 - *Its hard to wreck a nice beach*
 - *Its hard to recognize speech*
- Job of the language model is to say which of those is more likely

Discrete Probability Review

We're concerned with the *probability of the outcome of discrete events*.

- I flip a coin, what's the probability of it coming up heads?
- What's the probability that it will snow tomorrow?
- What's the probability that school will be closed the following day, given that its snowing when you went to bed?

Discrete Probability Review

- Probabilities are beliefs about an event outcome expressed as a number between 0 and 1.
- The sample space is the set of all possible outcomes.
- An event is some particular outcome.
- A prior is a probability we hold in the absence of any other evidence.
- A conditional is a probability we hold given some set of evidence.

Snow Days

- Probability that school will be closed the following day (C), given that it was snowing when you went to bed (S)
 - $P(C \mid S)$ “Probability of C given S”

Snow Days

- Probability that school will be closed the following day (C), give that it was snowing when you went to bed (S)
 - $P(C \mid S) = P(C \wedge S) / P(S)$

Snow Days

- Probability that school will be closed the following day (C), give that it was snowing when you went to bed (S)
 - $P(C \mid S) = P(C \wedge S) / P(S)$
 - How would we go about assessing this fraction and what would it mean?

Snow Days

- $P(C \mid S) = P(C \wedge S) / P(S)$
- Let's look at the parts:
 - $P(C \wedge S)$ this is a prior probability made up of two events. We want to assess the probability of the intersection of these events. Let's use frequencies. Out of some school records:
 - $\text{Count}(\text{days that it snowed and school was subsequently closed}) /$
 - $\text{Count}(\text{days it snowed in the record})$

Snow Days

- $P(C \mid S) = P(C \wedge S) / P(S)$
- Let's look at the parts:
 - $P(S)$ this is a prior. Let's use frequencies. Out of some school records:
 - $\text{Count}(\text{days that it snowed}) /$
 - $\text{Count}(\text{days in the record})$

Snow Days

- $P(C \mid S) = P(C \wedge S) / P(S)$
 $= \text{Count}(\text{closed} \wedge \text{snowed}) /$
 $\text{Count}(\text{snowed})$

Out of all the days it snowed, what was the fraction of the days that the schools subsequently closed.

Probability and Language

- With respect to “language models” we’ll be mainly concerned with the probability of sentences (or sequences of linguistic units)
 - The sentence is the event
 - The sample space is the space of all possible sentences
 - (wait what?)
 - We’d like to assign a probability to that event
 - (this is a strange notion)

Chomsky

“... it must be recognized that the notion of *"probability of a sentence"* is an entirely useless one, under any known interpretation of this term.”

“Entirely useless” is a pretty strong claim.
One that turns out to be incorrect.



Language Modeling

$$P(w_n | w_{1:n-1})$$

- How might we go about calculating a conditional probability over word sequences?
 - One way is to use the definition of conditional probabilities and look for counts. So to get
 - $P(\textit{the} \mid \textit{its water is so transparent that})$
- By definition that's
 - $\frac{P(\textit{its water is so transparent that the})}{P(\textit{its water is so transparent that})}$
- Let's try to get each of those from counts in a large corpus.

Easy Estimate

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{\text{Count}(\text{its water is so transparent that the})}{\text{Count}(\text{its water is so transparent that})}$$

Crude Estimate

- According to Google those counts are 1320 and 1420 so the conditional probability we want is...
 - $P(\text{the } | \text{ its water is so transparent that}) = 0.93$

Crude Estimate

- How about “matrix”
 - That gives you a 0. $0/1420 = 0$
- How about “you”
 - ◆ That gives you a 1. $1/1420 = 0.0007$
- How about “she”
 - ◆ That gives you a 0. $0/1420 = 0$
 - This seems wrong. “she” should not be the same as “matrix”

Language Modeling

- Unfortunately, for most sequences, and for most text collections, we won't get good estimates using counting alone.
 - We're likely to get a lot of 0 counts, leading to 0 probabilities for sequences that are entirely plausible.
- Clearly, we'll have to be a more clever to make counting work.
 - First, we'll use the chain rule for probability
 - And then apply a particularly useful independence assumption

The Chain Rule

- Recall the definition of conditional probabilities

$$P(A | B) = \frac{P(A \wedge B)}{P(B)}$$

- Rewriting:

$$P(A \wedge B) = P(A | B)P(B)$$

- For sequences...

- $P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

- In general

- $P(x_1, x_2, x_3, \dots, x_n) = P(x_1)P(x_2 | x_1)P(x_3 | x_1, x_2) \dots P(x_n | x_1 \dots x_{n-1})$

The Chain Rule

$$\begin{aligned}P(w_{1:n}) &= P(w_1)P(w_2|w_1)P(w_3|w_{1:2})\dots P(w_n|w_{1:n-1}) \\&= \prod_{k=1}^n P(w_k|w_{1:k-1})\end{aligned}$$

P(its water was so transparent)=

P(its)*

P(water|its)*

P(was|its water)*

P(so|its water was)*

P(transparent|its water was so)

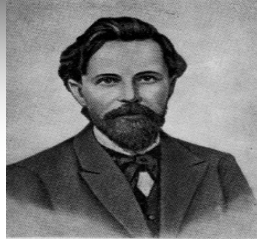
Unfortunately

- There are still a lot of problematically long sequences in this version.
- In general, we'll never be able to get enough data to compute the statistics for those longer prefixes
 - Same problem we had for the original sequence.

Independence Assumption

- Make the simplifying assumption
 - $P(\text{lizard} | \text{the, other, day, I, was, walking, along, and, saw, a}) = P(\text{lizard} | a)$
- Or maybe
 - $P(\text{lizard} | \text{the, other, day, I, was, walking, along, and, saw, a}) = P(\text{lizard} | \text{saw, a})$
- That is, the probability in question is to some degree *independent* of its earlier history

Markov Assumption



Replace each component in the product with an approximation (assuming a prefix of size $N - 1$)

$$P(w_n | w_{1:n-1}) \approx P(w_n | w_{n-N+1:n-1})$$

Bigram version

$$P(w_n | w_{1:n-1}) \approx P(w_n | w_{n-1})$$

Estimating Bigram Probabilities

- The Maximum Likelihood Estimate (MLE)

$$P(w_i \mid w_{i-1}) = \frac{\textit{count}(w_{i-1}, w_i)}{\textit{count}(w_{i-1})}$$

Example

- $\langle s \rangle$ I am Sam $\langle /s \rangle$
- $\langle s \rangle$ Sam I am $\langle /s \rangle$
- $\langle s \rangle$ I do not like green eggs and ham $\langle /s \rangle$

$$\begin{array}{lll} P(I | \langle s \rangle) = \frac{2}{3} = .67 & P(\text{Sam} | \langle s \rangle) = \frac{1}{3} = .33 & P(\text{am} | I) = \frac{2}{3} = .67 \\ P(\langle /s \rangle | \text{Sam}) = \frac{1}{2} = 0.5 & P(\text{Sam} | \text{am}) = \frac{1}{2} = .5 & P(\text{do} | I) = \frac{1}{3} = .33 \end{array}$$

$$P(w_n | w_{n-N+1:n-1}) = \frac{C(w_{n-N+1:n-1} w_n)}{C(w_{n-N+1:n-1})}$$

Berkeley Restaurant Project

- *can you tell me about any good cantonese restaurants close by*
- *mid priced thai food is what I'm looking for*
- *tell me about chez panisse*
- *can you give me a listing of the kinds of food that are available*
- *I'm looking for a good place to eat breakfast*
- *when is caffe venezia open during the day*

Bigram Counts

- Vocabulary size is 1446 $|V|$
- Out of 9222 sentences
 - “I want” occurred 827 times

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|----|------|-----|-----|---------|------|-------|-------|
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

Bigram Probabilities

- Divide bigram counts by the prefix unigram counts to get bigram probabilities.

| i | want | to | eat | chinese | food | lunch | spend |
|------|------|------|-----|---------|------|-------|-------|
| 2533 | 927 | 2417 | 746 | 158 | 1093 | 341 | 278 |

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|----|------|-----|-----|---------|------|-------|-------|
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 0 | 2 | 0 | 6 | 2 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

$$P(\text{want} | i) = 827/2533 = .336$$

Bigram Probabilities

- Divide bigram counts by the prefix unigram counts to get bigram probabilities.

| i | want | to | eat | chinese | food | lunch | spend |
|------|------|------|-----|---------|------|-------|-------|
| 2533 | 927 | 2417 | 746 | 158 | 1093 | 341 | 278 |

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|---------|------|--------|--------|---------|--------|--------|---------|
| i | 0.002 | 0.33 | 0 | 0.0036 | 0 | 0 | 0 | 0.00079 |
| want | 0.0022 | 0 | 0.66 | 0.0011 | 0.0065 | 0.0065 | 0.0054 | 0.0011 |
| to | 0.00083 | 0 | 0.0017 | 0.28 | 0.00083 | 0 | 0.0025 | 0.087 |
| eat | 0 | 0 | 0.0027 | 0 | 0.021 | 0.0027 | 0.056 | 0 |
| chinese | 0.0063 | 0 | 0 | 0 | 0 | 0.52 | 0.0063 | 0 |
| food | 0.014 | 0 | 0.014 | 0 | 0.00092 | 0.0037 | 0 | 0 |
| lunch | 0.0059 | 0 | 0 | 0 | 0 | 0.0029 | 0 | 0 |
| spend | 0.0036 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 |

Bigram Estimates of Sentence Probabilities

- $P(<s> \text{ I want english food } </s>) =$
 $P(i | <s>)^*$
 $P(\text{want} | \text{I})^*$
 $P(\text{english} | \text{want})^*$
 $P(\text{food} | \text{english})^*$
 $P(</s> | \text{food})^*$
 $= .000031$

Kinds of Knowledge

- As crude as they are, N -gram probabilities capture a range of interesting facts about language.

- $P(\text{english} | \text{want}) = .0011$

World knowledge

- $P(\text{chinese} | \text{want}) = .0065$

- $P(\text{to} | \text{want}) = .66$

- $P(\text{eat} | \text{to}) = .28$

Syntax

- $P(\text{food} | \text{to}) = 0$

- $P(\text{want} | \text{spend}) = 0$

- $P(i | \langle s \rangle) = .25$

Discourse

Shannon's Method

- Assigning probabilities to sentences is all well and good, but it's not terribly illuminating.
- A more entertaining (and very useful) task is to turn the model around and use it to *generate* random sentences that are *similar to* the sentences from which the model was derived.
- Idea enerally attributed to Claude Shannon.



Shannon's Method (Autoregressive Generation)

- Sample a random bigram ($\langle s \rangle, w_i$) according to the model's probability distribution over bigrams
- Now sample a new random bigram (w_i, x) according to its probability. Where the prefix w matches the suffix of the first bigram chosen.
- And so on until we randomly choose a ($w_i, \langle /s \rangle$)
- Then string them together
- $\langle s \rangle$ |

I want

 want to

 to eat

 eat Chinese

 Chinese food

 food $\langle /s \rangle$

Shakespeare

| | |
|------------|--|
| Unigram | <ul style="list-style-type: none"> • To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have • Every enter now severally so, let • Hill he late speaks; or! a more to leg less first you enter • Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like |
| Bigram | <ul style="list-style-type: none"> • What means, sir. I confess she? then all sorts, he is trim, captain. • Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. • What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman? • Enter Menenius, if it so many good direction found'st thou art a strong upon command of fear not a liberal largess given away, Falstaff! Exeunt |
| Trigram | <ul style="list-style-type: none"> • Sweet prince, Falstaff shall die. Harry of Monmouth's grave. • This shall forbid it should be branded, if renown made it empty. • Indeed the duke; and had a very good friend. • Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. |
| Quadrigram | <ul style="list-style-type: none"> • King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; • Will you not tell me who I am? • It cannot be but so. • Indeed the short and the long. Marry, 'tis a noble Lepidus. |

Shakespeare as a Corpus

- $N=884,647$ tokens, $V=29,066$
- Shakespeare produced 300,000 bigram types out of $V^2= 844$ million possible bigrams...
 - So, 99.96% of the possible bigrams were never seen (have zero entries in the table)
 - This is the biggest problem in language modeling; we'll come back to it.
- Quadrigrams are worse: What's coming out looks like Shakespeare because it is Shakespeare



Model Evaluation

- How do we know if our models are any good?
 - And in particular, how do we know if one model is better than another.
- Well Shannon's game gives us an intuition.
 - The generated texts from the higher order models surely sound better.
 - That is, they sound more like the text the model was obtained from.
 - The generated texts from the WSJ and Shakespeare models look very different
 - That is, they look like they're based on different underlying models.
- But what does that mean? How can we make that notion operational?

Evaluating N -Gram Models

- Best evaluation for a language model
 - Put model A into an application
 - For example, a machine translation system
 - Evaluate the performance of the application with model A
 - Put model B into the application and evaluate
 - Compare performance of the application with the two models
 - *Extrinsic evaluation*
 - *A/B Testing*

Evaluation

- Extrinsic evaluation

- This is quite time consuming and expensive
- Not something you want to do unless you're pretty sure you've got a good solution

- So

- As an intermediate evaluation, in order to run rapid experiments, we evaluate N-grams with an *intrinsic* evaluation
- An evaluation that tries to capture how good the model is intrinsically, not how much it improves performance in some larger system

Evaluation

- **Standard method**
 - Train parameters of our model on a *training set*.
 - Evaluate the model on some new data: a *test set*.
 - A dataset which is different than our training set, but is drawn from the same source

Perplexity

- Perplexity is just the probability of a test set (assigned by the language model), as normalized by the number of words:
$$\begin{aligned} \text{PP}(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned}$$

- Chain rule:
$$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$$

- For bigrams:
$$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_{i-1})}}$$

- Minimizing model perplexity is the same as maximizing probability of a test set

Perplexity

- The intuition behind perplexity is as a measure is the notion of surprise.
 - How surprised is the language model when it sees the test set?
 - Where surprise is a measure of...
 - Gee, I didn't see that coming...
 - The more surprised the model is, the higher the perplexity
 - The lower the perplexity, the less surprised it was

Lower perplexity is better

- Training 38 million words, test 1.5 million words, WSJ

| <i>N</i> -gram Order | Unigram | Bigram | Trigram |
|----------------------|---------|--------|---------|
| Perplexity | 962 | 170 | 109 |

Practical Issues

- Once we start looking at test data, we'll run into words that we haven't seen before. So, our models won't work. Standard non-subword solution:
 - Given a corpus
 - Create a fixed lexicon L , of size V
 - Say from a dictionary or
 - A subset of terms from the training set
 - At text normalization phase, any training word not in L is changed to $\langle \text{UNK} \rangle$
 - Collect counts for that as for any normal word
 - At test time
 - Use UNK counts for any word not seen in training

Practical Issues

- Multiplying a bunch of really small probabilities is a really bad idea.
 - Underflow is likely
- So do everything in log space
 - Avoids underflow (and adding is faster than multiplying)

$$p_1 \times p_2 \times p_3 \times p_4 = \exp(\log p_1 + \log p_2 + \log p_3 + \log p_4)$$