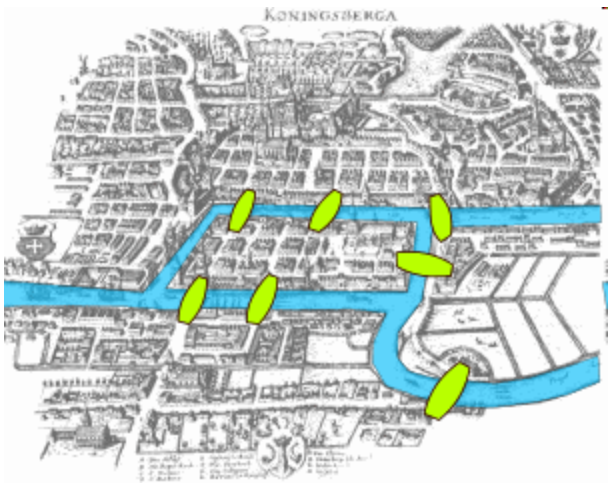
Topological Data Analysis and Neuroscience

Lecture 0: Introduction

Instructor: Alex McCleary

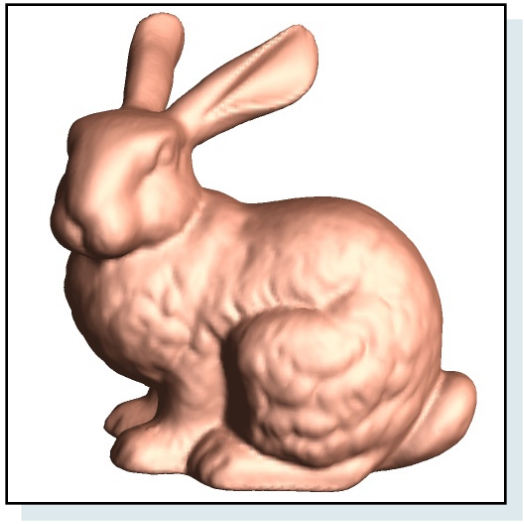
Lecture 0: Introduction

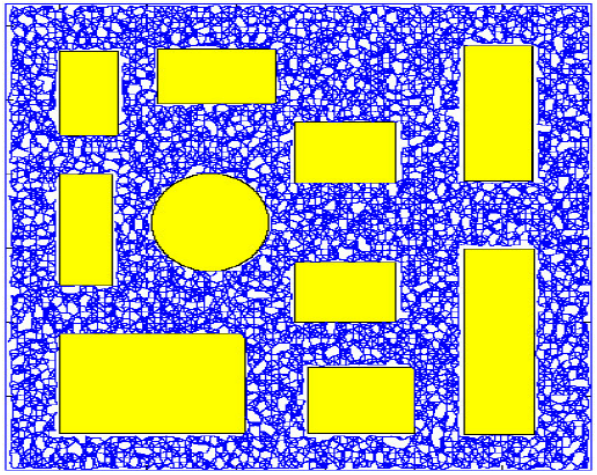
- Why should we be interested in topology
- What is topology
- What to expect from this course

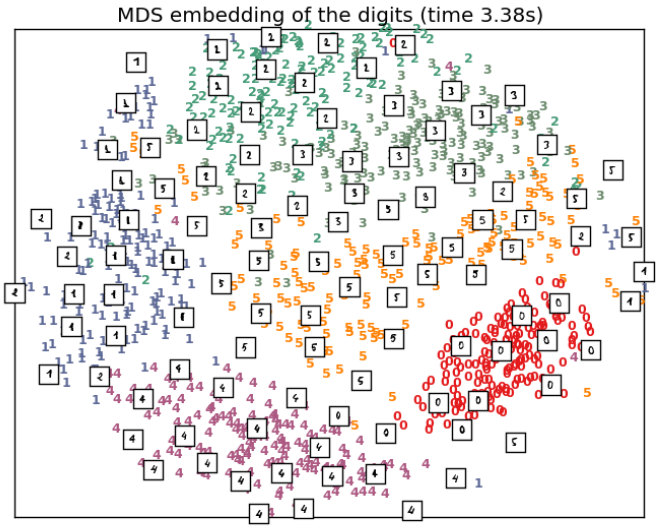


Applications of Topology

- Topology has countless applications to data science
- Focusing on topological, rather than geometric information can reduce excess information, allowing us to focus on just the essential features

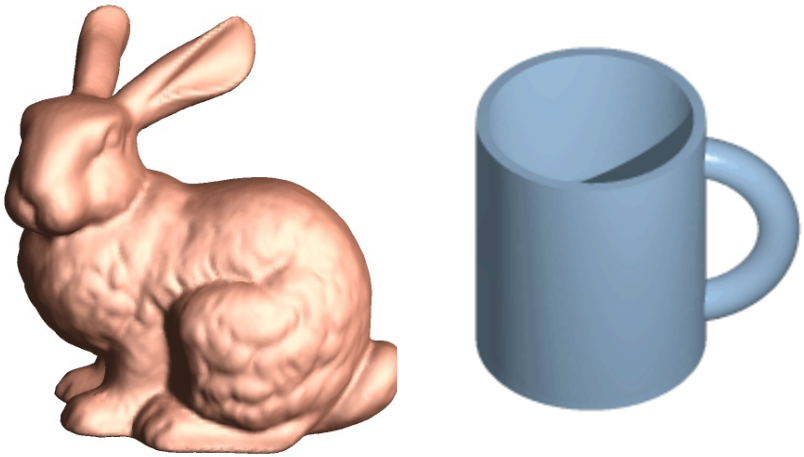






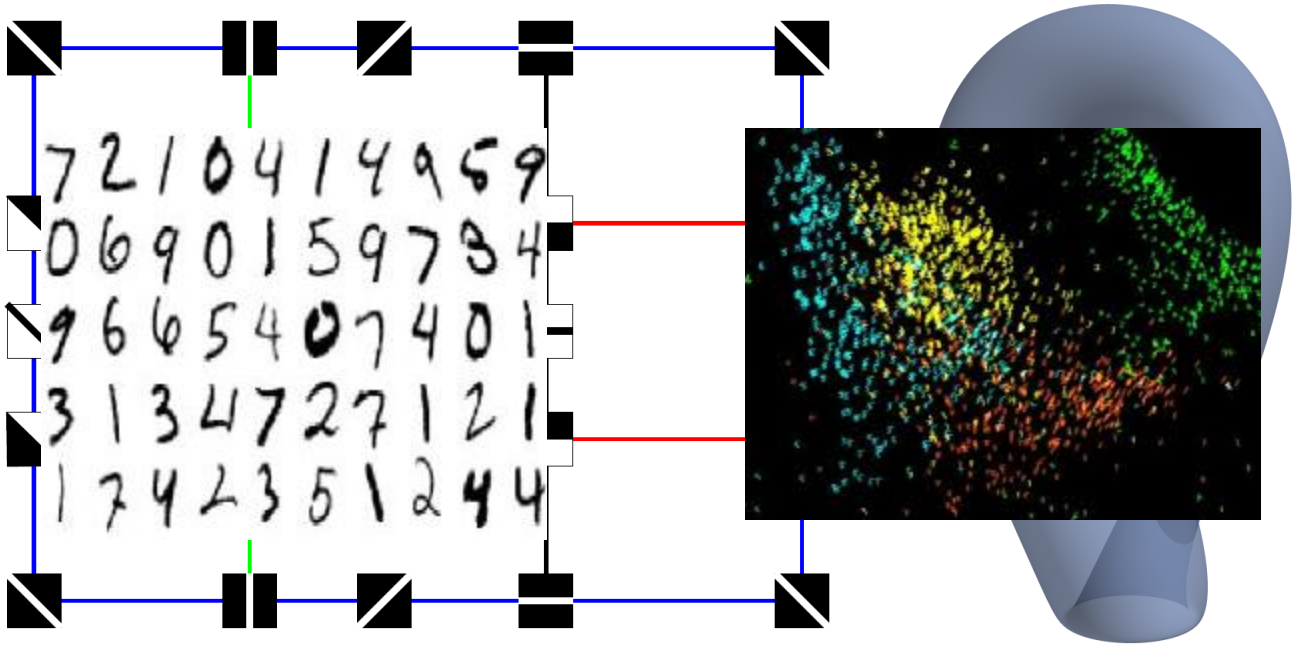
Motivating Example I

- Shape recognition
  - Classifying objects
  - Quantifying the difference between objects



Motivating Example II

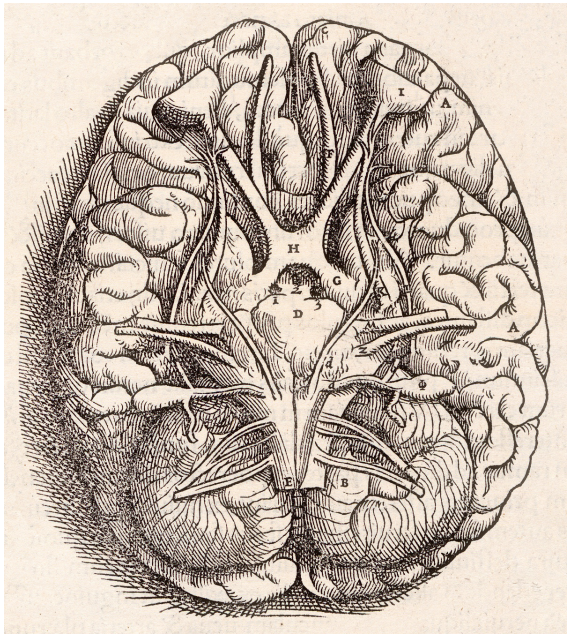
- Computer Vision
  - Clustering
  - Shape space

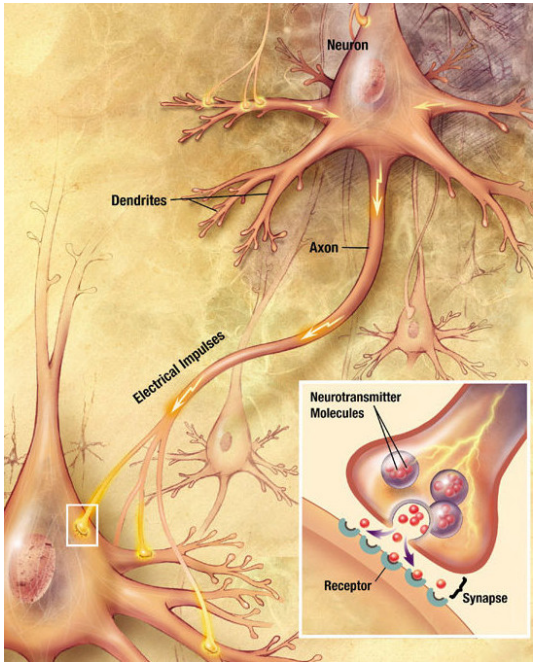


Courtesy of Carlsson et al, *On the local behavior of spaces of natural images*

Motivating Example III

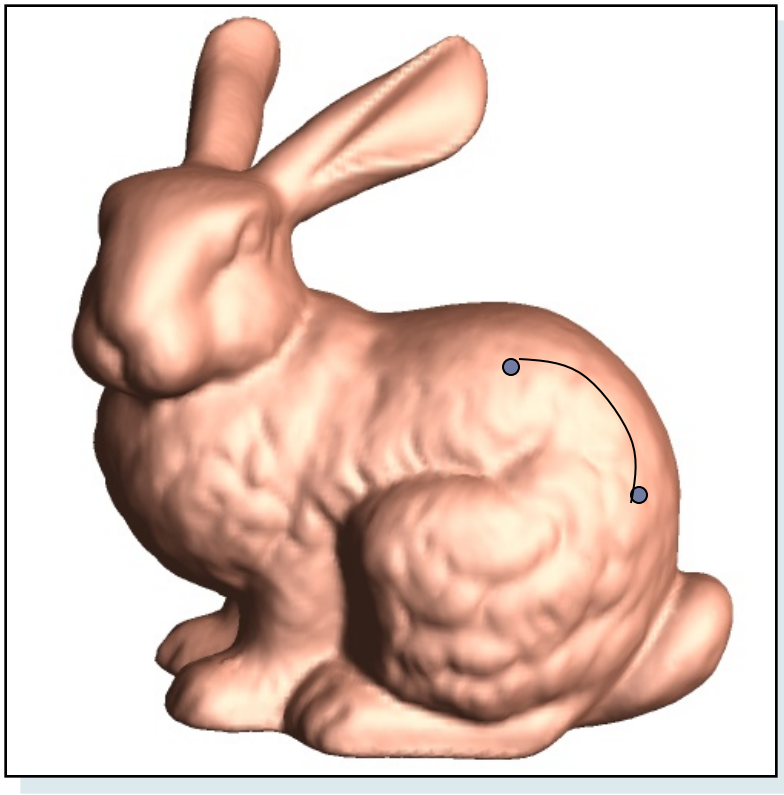
- Neuroscience
  - Topology is a powerful for modeling the brain.
  - It's how parts of the brain are connected that matters, not their proximity.





Geometry

- All about distances and angles
  - area, volume, curvatures, etc
- Euclidean geometry
- Riemannian geometry
- Hyperbolic geometry
- …



Topology

- Detailed geometric information not sufficient
  - Or not necessary
  - Or may even be harmful
  - Wish to identify key information, "qualitative" structure

- Topology
  - Connectivity
  - Holes

Overview

- In general, topology
  - Coarser yet essential information
  - Characterization, feature identification
  - General, powerful tools for both space and functions defined on a space
  - Elegant mathematical understanding available
- Indeed, topological ideas / methods have been used in many applications fields, including:
  - Graphics, visualization, medical image processing, computer vision, computational neuron science, computational biology, material science
  - Framework to combine persistence idea + machine learning pipelines
- However
  - Difficult mathematical language

This Course

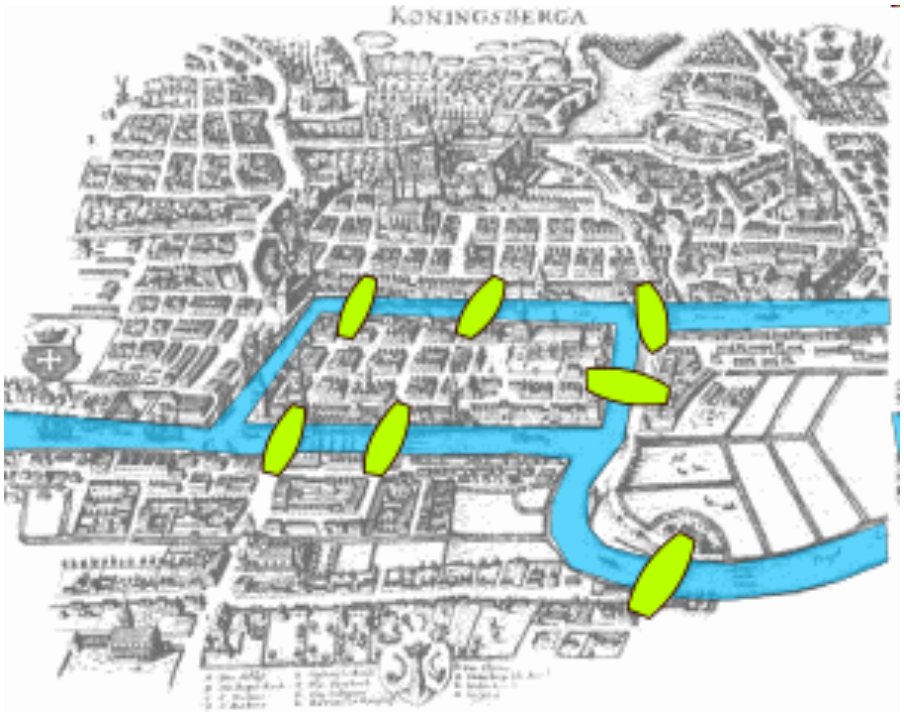
- Introduce basics and recent developments in computational topology
- Goals:
  - Understand basic language in computational topology
  - Study applications of topology to neuroscience
  - Apply topological methods to a shape recognition problem

References

- *Computational Topology: An Introduction,* by H. Edelsbrunner and J. Harer, AMS Press, 2009.
  - Online course notes by Herbert Edelsbrunner on computational topology
- *Algebraic Topology,* by A. Hatcher, Cambridge University Press, 2002. (Online version available)
- *Elements of Algebraic Topology,* by J. R. Munkres, Perseus, Cambridge, Massachusetts, 1984.

Introduction to Topology

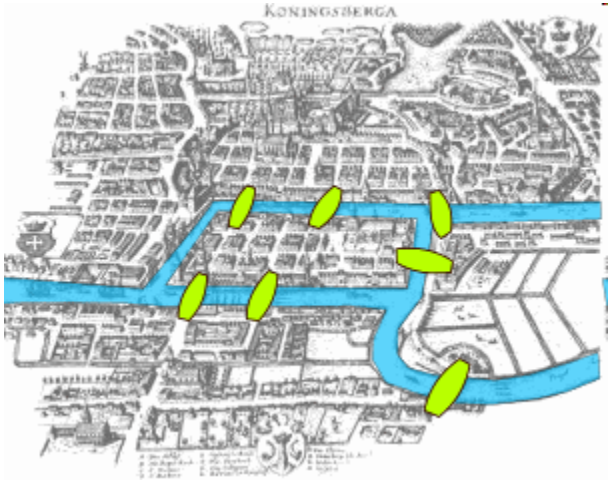
Seven Bridges of Königsberg

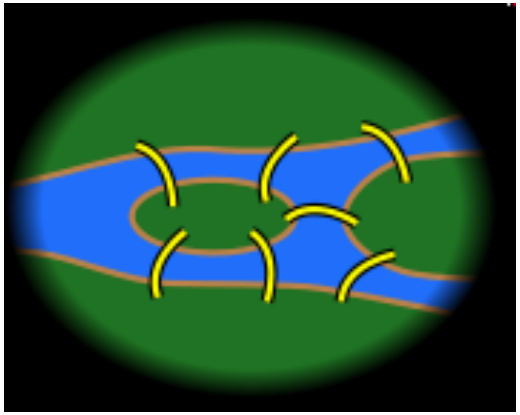


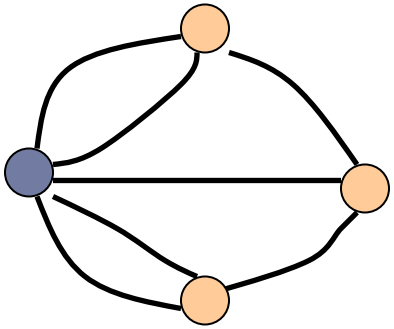
Can we find a path through the city crossing each bridge once and only once?

"If you can't solve a problem, then there is an easier problem you can solve: find it."
-George Pólya

Seven Bridges of Königsberg



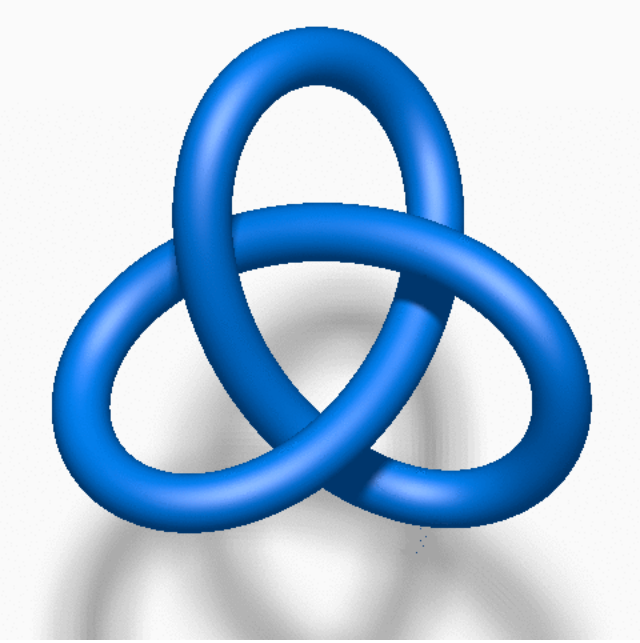




The shape of the land masses, and even the positions of the land masses do not matter. All that matters is the way they are connected.

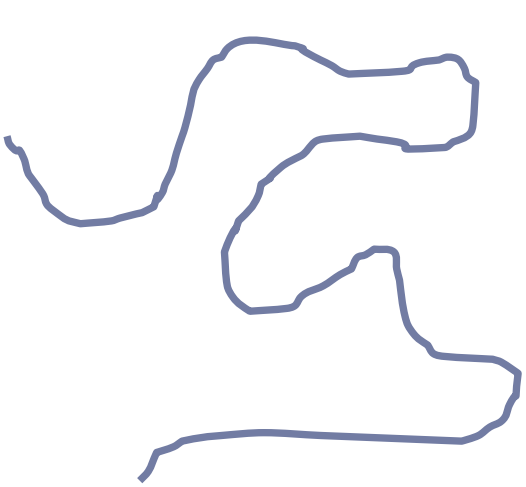
Topology

- Getting rid of extraneous information can help us reduce geometric problems to simpler problems.
- In topology we can study geometric objects without any embedding into Euclidean space (or any other ambient space.)

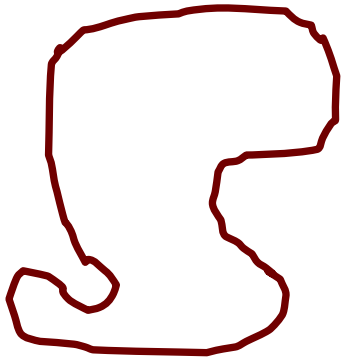


Homeomorphism

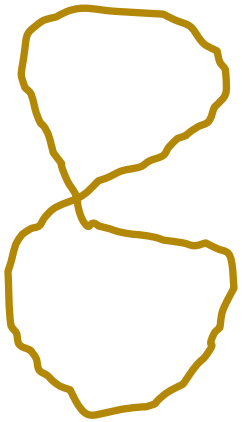
Intuitively, two spaces have the same topology if one can continuously deform one to the other without breaking, gluing, and inserting new things



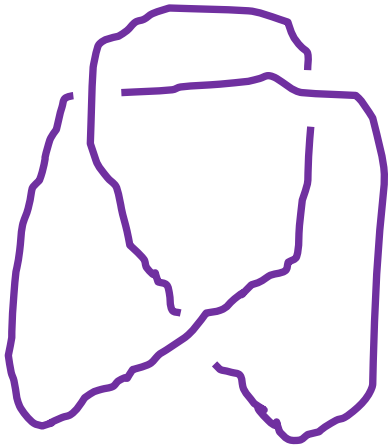
open curve



closed curve



self-intersecting curve

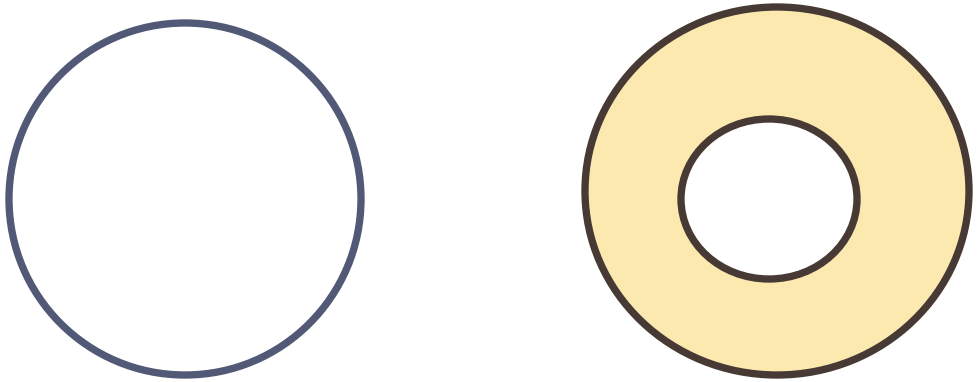


Trefoil knot

Two spaces with the same topology are ***homeomorphic***

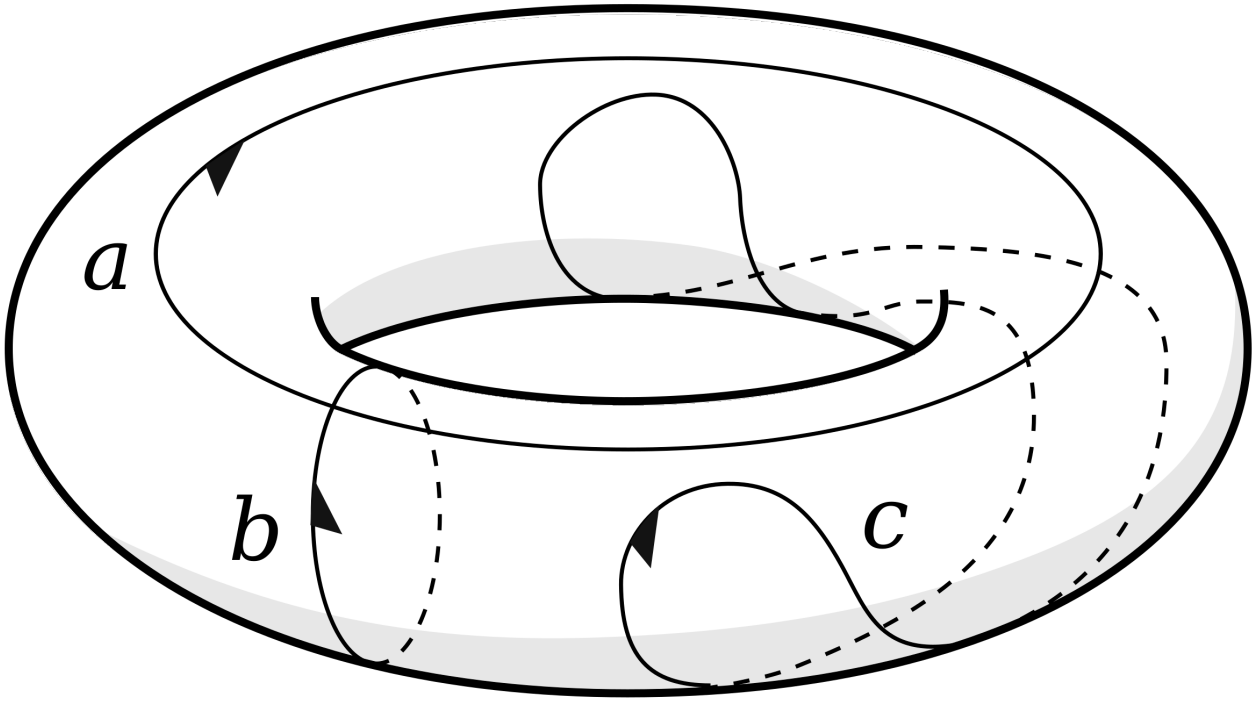
Relaxation of Homeomorphism

- Homotopy equivalent



Homology

- Homology counts "holes" in a space.
- This is the main topological tool we will use in this course.



Topological Quantities

- Homeomorphism —› homotopy equivalence —› homology
  - Describe the qualitative structure of input space at different levels

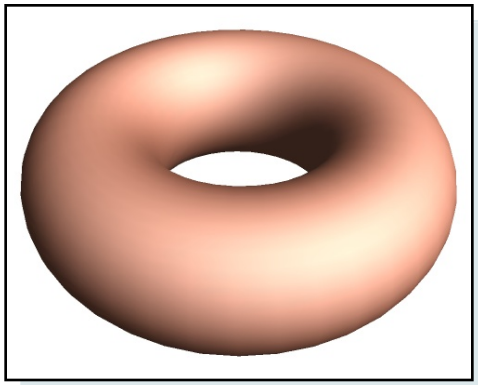
- Quantities invariant under them (topological quantities)

  => *(Essential) features*

  - Make topologic objects powerful for feature identification and characterization
  - This course will give
    - Definition, intuition, and their computation
    - Also examples of applications

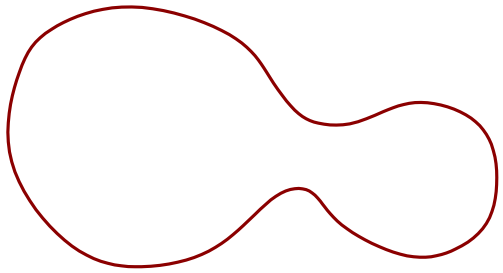
One Important Point

- Topological concepts and methods are not just about describing the topology of the domain itself
- It can be very powerful by
  - Say, considering topology associated to maps on a domain



Topological objects

- Not just features of space, one can also incorporate maps of a space to capture more sophisticated features



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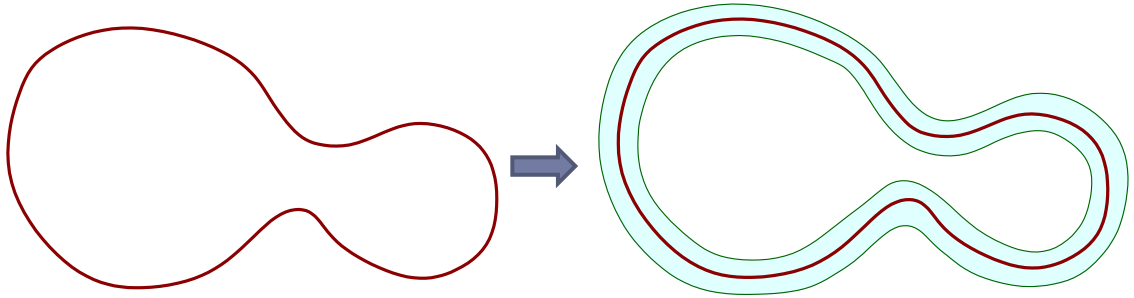

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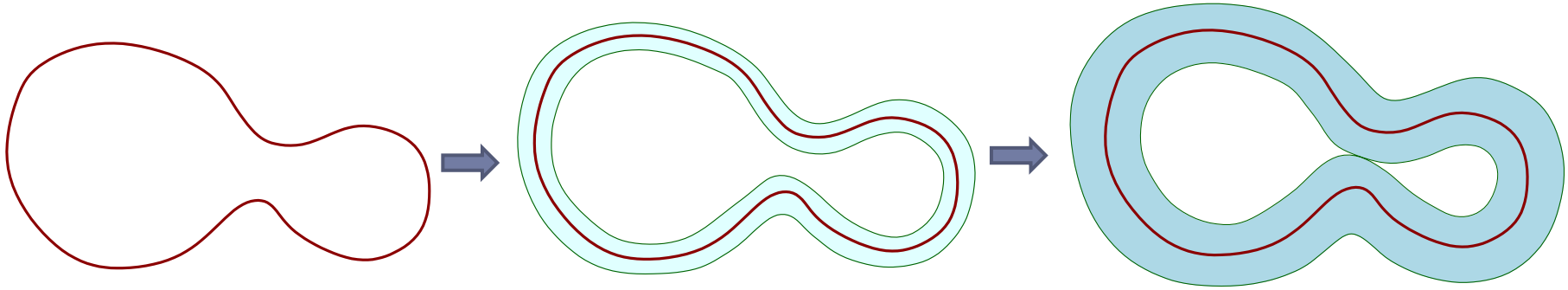

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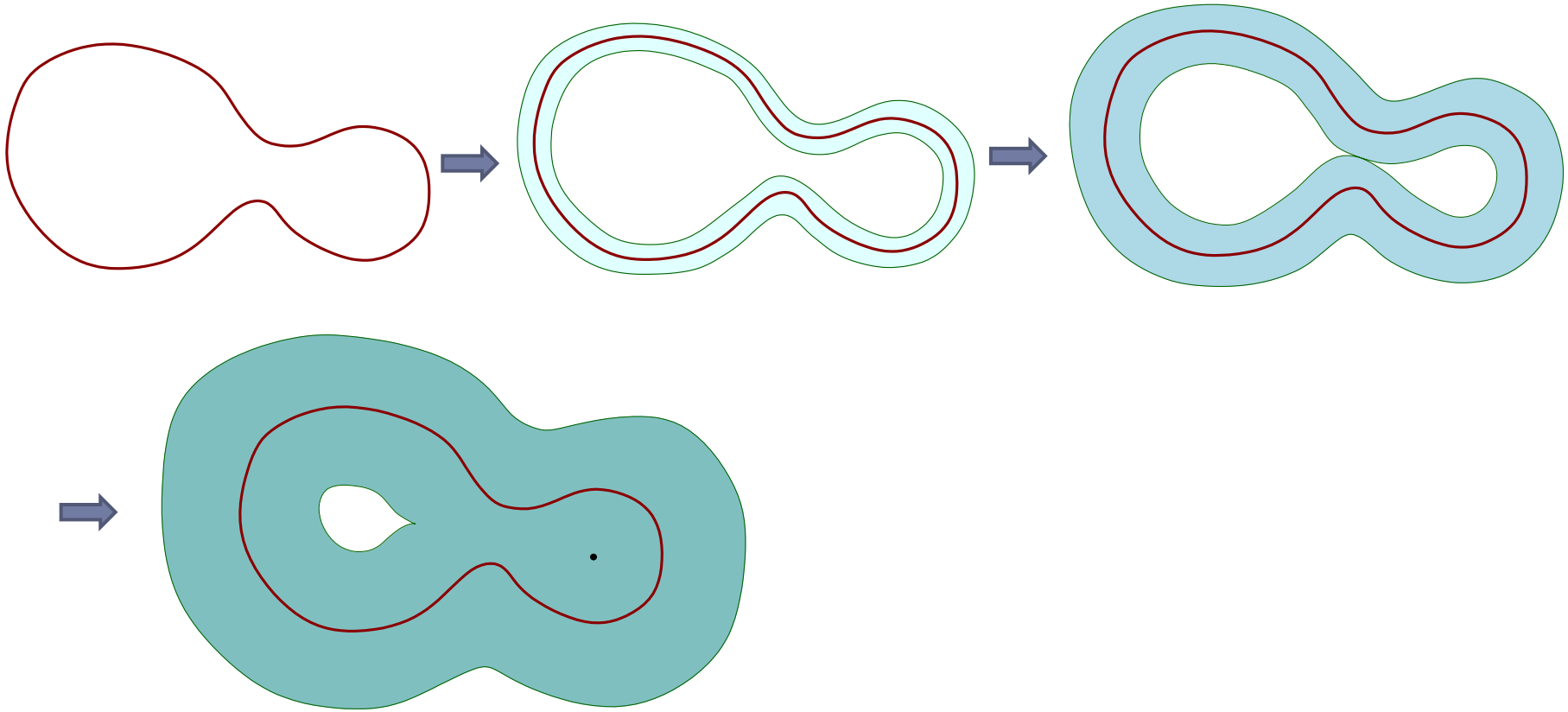

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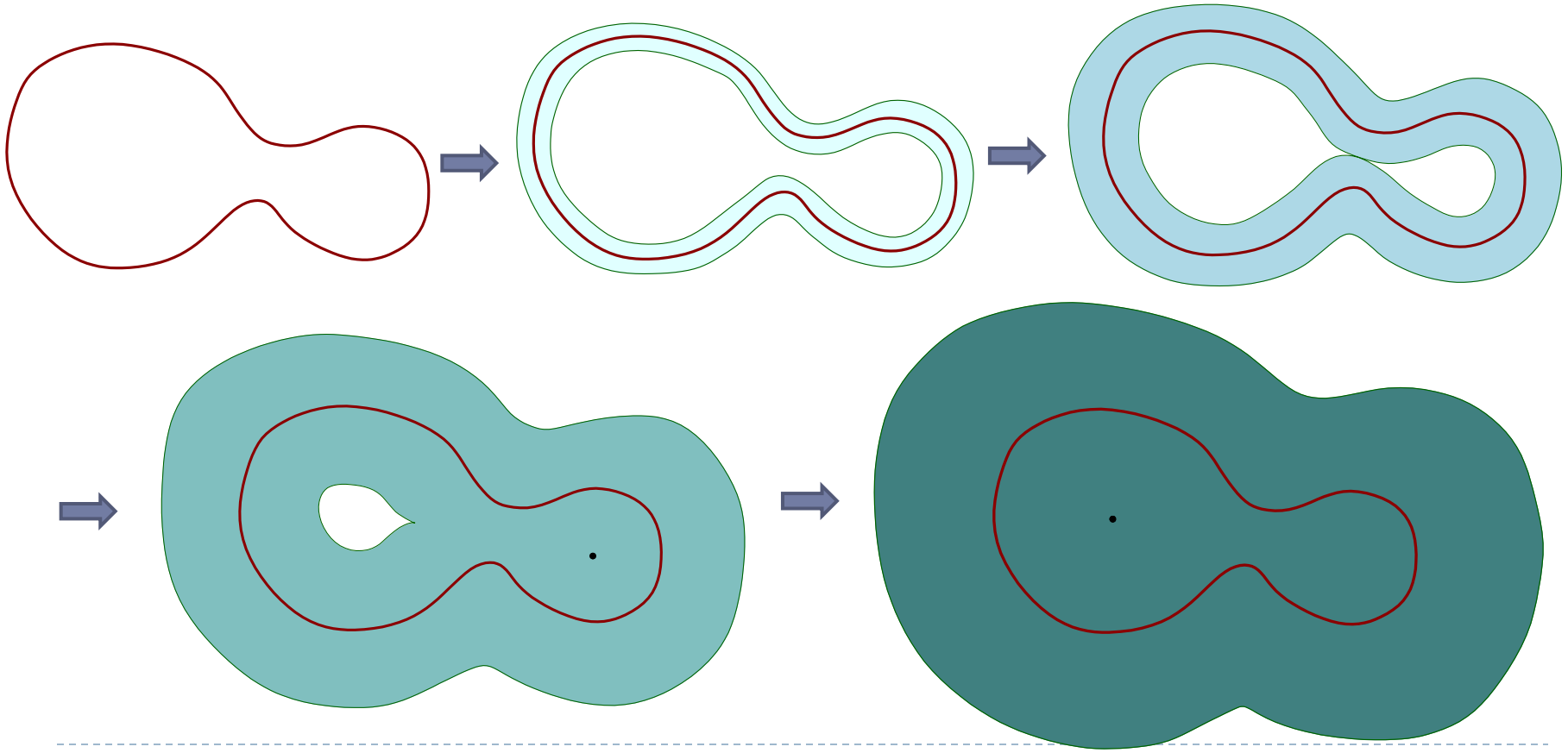

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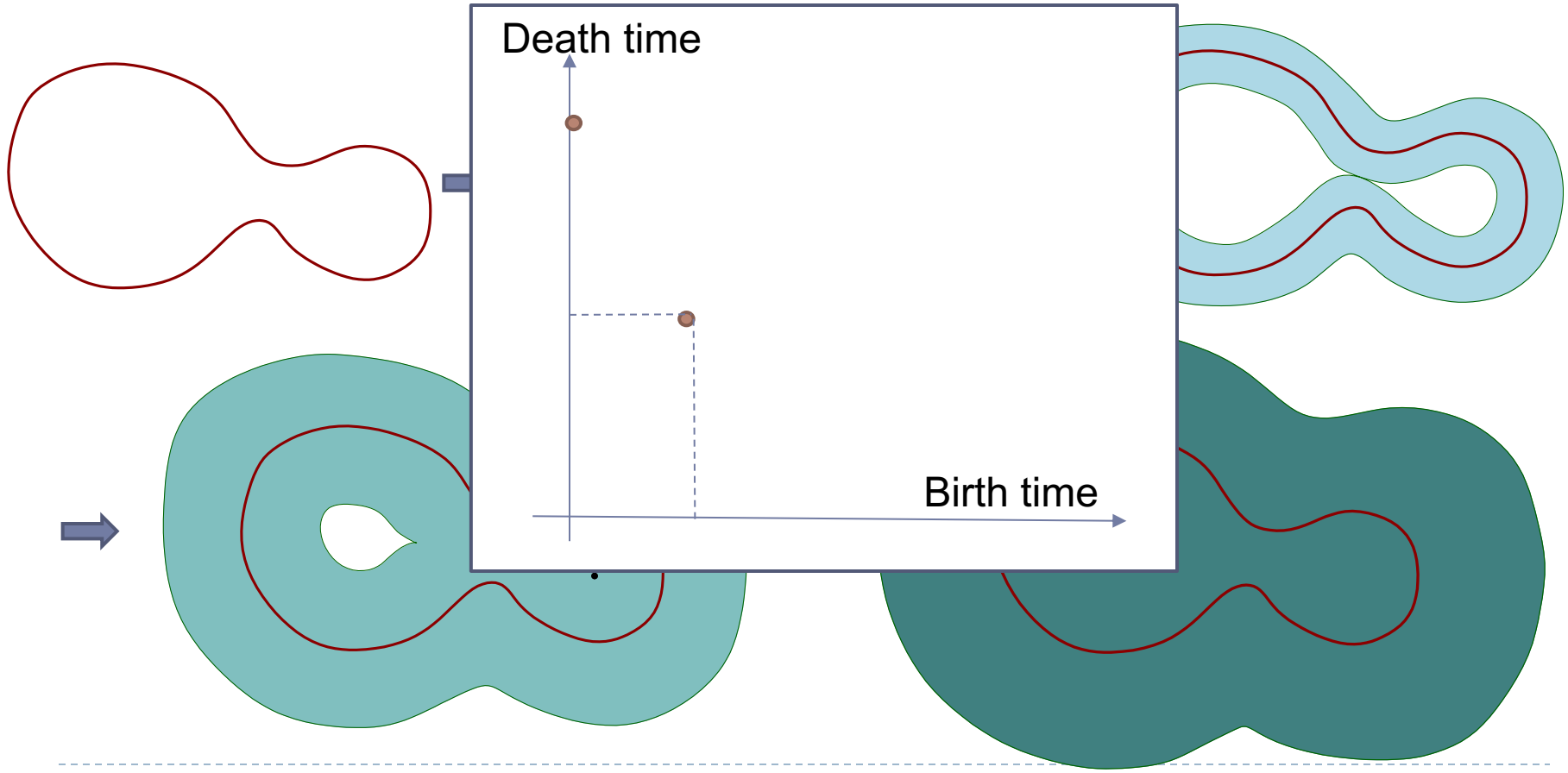

To summarize

- Topological concepts
  - Captures coarse yet essential information about data, space, or functions
- Topological objects / methods
  - Are powerful due to their generality
  - Are flexible, can describe geometric information or any other information of interests (modeled as functions / maps)
- They form natural tools for
  - Feature identification, characterization of space / data

Topics

- Basics in topology
- Simplicial complexes
- Homology
- Persistent homology
- Homology inference from data
- Scalar field analysis
  - Hierarchical clustering
  - Mapper and multiscale-mapper
- Subsampling
- Multiparameter persistent homology

Will focus on not only concepts, definitions, algorithms, also intuition why they work, and how they can be used.