The Mighty Big Problem:
Precession of the Perihelion of Mercury

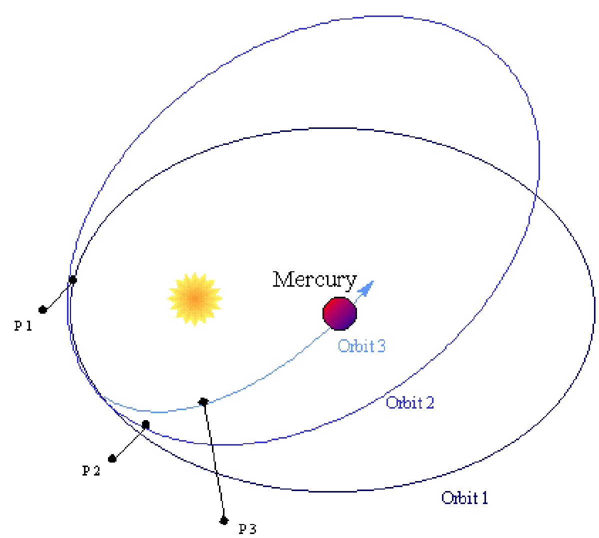
The Problem:

- Simulate the orbit of Mercury and measure its precession.
- Test whether your simulation agrees with the observed precession of 573 arcseconds/century.
- Turn in a PDF writeup and your Jupyter Notebook code on Canvas by 11:59 pm on Thursday, 11/16. For fairness to all, no extensions on this project, please.

Problem Statement: In 1859, it was recognized that Mercury's orbit does not appear obey Newton's laws and the universal law of gravitation. It took Einstein's General Relativity revolution of 1916 to resolve this problem, which first threatened to and then did topple Newtonian gravitation.

Here's the problem. Mercury's orbit is approximately elliptical, but the point of closest approach of Mercury to the Sun does not always occur at the same place. Instead, this 'perihelion' (closes approach) slowly moves around the sun. This rotation of the orbit is called *precession*.

Newton's theory predicts that these effects result from the pull of the planets on one another. The question is whether or not Newton's predictions agree with the *amount* that an orbit precesses. It is not enough to qualitatively understand the origin of the effect; arguments must be backed by hard numbers to give them credence. The precession of the orbits of all planets *except* for Mercury's can, in fact, be understood using Newton's equations. **Mercury is the exception.**



The figure above shows a greatly exaggerated version of the precession of Mercury's orbit. The points p1, p2, and p3 are the points of closest approach (perihelion) during orbits 1, 2, and 3.

Mercury has its perihelion precess at a rate of 575.19 arcsecconds per century (1 arcsecond =1/3600 of a degree). The precession is a result of effects that can be explained by including the forces on Mercury exerted by the other planets as well as modifications in the force of gravity from the sun that are present because the sun is not perfectly spherical in shape. When all of these effects are considered there is still a discrepancy of **43 arcseconds/century in the prediction**. The discrepancy was ultimately explained by the theory of general relativity. You've got the opportunity to see if you can **explain the precession quantitatively.**

Things to keep in mind:

1. **Read this entire document before starting your work.** If there is something you don't understand, talk to me.
2. **Read this entire document *before* starting your work.** Leave Jupyter Notebook closed for now.
3. **Read this *entire* document before starting your work.** Don't stop at the first comma.
4. ***Read* this entire document before starting your work.** Skimming it for the gist will guarantee you miss something important.
5. Use me during class time and office hours. Ask if you need to meet outside those hours if you need to arrange a different time!
6. Hand in a PDF paper describing your investigation, including plots. Plots should have titles, axis labels, figure captions. Refer to figures by number in your text. Upload an electronic version of your paper and the Jupyter Notebook containing your code to the Canvas site by the deadline.
7. Make sure everything is reasonably documented using # for comments.
8. **This is a solo project.** You may use the class books, notes, and problems as resources. Ask me questions. Use the Python, SciPy, NumPy, and matplotlib documentation for coding help. All other internet help is not for this problem.
9. Remember to reaffirm the honor code.
10. Feel free to explore the problem in full. Have fun. Fail often to succeed sooner. You are working on a problem a Nobel Laureate sweated over, and you will be doing it BETTER than he did. HOW AWESOME IS THAT? (+1 EC if you answer that question in your paper)

Some relevant information:

Object	Mass (kg)	Semimajor Radius of Orbit (AU)	Eccentricity	Period (years)
Sun	1.98844×10^{30}	----------	----------	---------
Mercury	3.30104×10^{23}	0.387098261	0.20563593	0.2408425
Venus	4.86732×10^{24}	0.723333771	0.00677672	0.6151866
Earth	5.97219×10^{24}	1.00	0.01671123	1.00
Mars	6.41693×10^{23}	1.523706365	0.0933941	1.8808149
Jupiter	1.89813×10^{27}	5.20287342	0.04838624	11.862409
Saturn	5.68319×10^{26}	9.536651047	0.05386179	29.446986

Part A:

i. Starting from Newton's Law of universal gravitation write, the differential equations governing the motion of Mercury around the Sun. Given that the data we have is in years and AU's (the semimajor axis of the earth's orbit), choose the AU and the year as your units. *Note*: these will be equations with units, not dimensionless equations.

ii. Simulate Mercury's orbit around the Sun alone and plot this motion. Then *find* the perihelion of the orbit, and *track* the orientation of the perihelion over time. Plot the trajectory of the perihelion in polar coordinates. *Do this with arcsecond accuracy*.

Hint: Show that Newton's 2nd Law applied to Mercury and the sun gives the following equation, and use it as the basis of your simulations.

$$M_m \frac{d^2\vec{r}_m}{dt^2} = -GM_sM_m \frac{\vec{r}_m}{r_m^3}$$

Part B:

Once you have confirmed that you can reproduce Mercury's orbit around the Sun in the absence of other planets, try adding in more planets **one at a time** to estimate the effect on Mercury's orbit. Show that Newton's 2nd Law applied to Mercury gives:

$$M_m \frac{d^2\vec{r}_m}{dt^2} = -GM_sM_m \frac{\vec{r}_m}{r_m^3} - GM_pM_m \frac{\vec{r}_m - \vec{r}_p}{\left|\vec{r}_m - \vec{r}_p\right|^3}$$

when another planet is added to the equation of motion. <u>You will also need to an equation of motion for the other planet to track *its* non-constant position.</u> **Estimate which planets you expect to have the largest effect on Mercury before starting your numerical simulation**. *Do not blindly take the top 3 on the list.*

Part C:

Now consider the effect of general relativity. An approximate form of the modification to Newton's law of gravitation is

$$M_m \frac{d^2\vec{r}_m}{dt^2} = -GM_sM_m \frac{\vec{r}_m}{r_m^3}\left(1 + \frac{\alpha}{r_m^2}\right)$$

where $\alpha \approx 1.1 \times 10^{-8}$ (AU^2). This tiny correction is difficult to measure in a simulation so evaluate the effect for several larger values of α and then convince yourself (and me) that the effect is directly proportional to α and then make a linear extrapolation back to the actual value of α. Be sure to include any other planets you've included from Part B.

Words of Advice (before you even start working)

1. Think carefully about the problem before starting your simulation. Write out your plans on paper, including the relevant equations. Write out, on paper, how you'll solve the differential equation(s). Basically, do part A(i) first.
2. Focus on Mercury. Use simplifying assumptions about the orbits of the other planets to streamline computation. Make sure that you explain those assumptions in the text of your paper.
3. Since the effect that you are modeling is small, you will need to pay attention to accuracy. Use double precision floating point numbers in your calculations to minimize roundoff errors.
4. Find a creative way to track the perihelion. Remember that you are doing a computational experiment so some tactics that you use in analyzing lab data may be useful.