Lab 9
CALCULUS FOR IT 501031

1 Exercises

Exercise 1: Calculate the definite integral of functions:

(a) $\int_1^2 x^3 + 2x^2 + 3dx$

(b) $\int_1^4 \frac{1}{x^3} + \frac{1}{x^2} + x\sqrt{x}dx$

(c) $\int_1^4 \frac{x^3 + x\sqrt{x} + x}{x^2}dx$

(d) $\int_1^2 (\frac{2}{x} + x^3)dx$

(e) $\int_1^2 x^2(\frac{1}{x} + 2x)dx$

(f) $\int_0^1 (\sqrt{x} - 1)(x + \sqrt{x} + 1)dx$

(g) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 - \frac{2}{sin^2x})dx$

(h) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{sin^2x.cos^2x}dx$

(i) $\int_0^{\frac{\pi}{4}} e^x(1 - \frac{e^{-x}}{cos^2x})dx$

(j) $\int_0^{ln2} e^x(2 + \frac{e^{-x}}{e^x})dx$

(k) $\int_1^2 (2^x + \frac{2}{x})dx$

(l) $\int_0^1 x^2(x-1)^2dx$

(m) $\int_1^2 \frac{1}{x.(x+1)}dx$

(n) $\int_0^2 \mid 1 - x \mid dx$

(o) $\int_0^3 \mid 2x - x^2 \mid dx$

(p) $\int_2^4 \sqrt{x^2 - 3x + 2}dx$

(q) $\int_0^{\pi} \sqrt{1 + cos2x}dx$

Exercise 2: Calculate the definite integrals and plot graphs of functions:

(a) $\int_0^{\frac{\pi}{2}} x^3 - 3sin(x)cos(x)dx$

(b) $\int_0^1 sin(x^2)^2dx$

(c) $\int_0^3 \sqrt{1 + g(x^2) + g(x)^2}dx, g(x) = x + 1$

(d) $\int_1^2 \int_0^3 x^2ydxdy$

Exercise 3: Graph the function and find its average value over given interval.

(a) $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$

(b) $f(x) = -\frac{x^2}{2}$ on $[0, 3]$

(c) $f(x) = -3x^2 - 1$ on $[0, 1]$

(d) $f(x) = x^2 - x$ on $[-2, 1]$

> **Hint:** Recall that:
> $$avg(f) = \frac{1}{b-a}\int_a^b f(x)dx$$

Exercise 4: Graphing of these functions and calculating the area under curve of each function $f(x)$:

(a) $f(x) = x^2 cos(x)$ với $-4 \leq x \leq 9$

(b) $f(x) = \int_{-\infty}^{\infty} e^{-ax^2} dx$ with $a = \frac{1}{2}$.

> **Note:** *Define interval value to plot graph*

Exercise 5: Write a program to compute the displacement of the rock during the time period $0 \leq t \leq 8$. Suppose that the velocity of the rock at any time t during its motion was given as $v(t) = 160 - 32t$ ft/sec.

Exercise 6: The margin cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}} \text{ (dollars)}$$

Write a program to find $c(100) - c(1)$, the cost of printing posters 2-100.

Exercise 7: The height $H(ft)$ of a palm tree after growing for t year is given by

$$H = \sqrt{t+1} + 5t^{1/3} \text{ for } 0 \leq t \leq 8$$

(a) Find the tree's height when $t = 0, t = 4$, and $t = 8$.

(b) Find the tree's average height for $0 \leq t \leq 8$

Exercise 8: Write a program to perform the following steps:

1. Plot the functions over the given interval.
2. Partition the interval into $n = 4, 100, 200,$ and 1000 subinter-vals of equal length, and evaluate the function at the midpoint of each subinterval.
3. Compute the average value of the function values generated in part 2

Evaluate this program by the following function:

(a) $f(x) = \int_0^1 (1-x)dx = \frac{1}{2}$

(b) $f(x) = \int_0^1 (x^2+1)dx = \frac{4}{3}$

(c) $f(x) = \int_{-\pi}^{\pi} cosxdx = 0$

(d) $f(x) = \int_{-1}^{1} \mid x \mid dx = 1$

Exercise 9: Write a program to implement the numerical integration with the composite Trapezoidal rule to solve the integral of these functions:

(a) $y = e^{-x^2}$ on the interval $[0, 1]$, with $n = 3$ segments.

(b) $y = 2x^2 + 5x + 12$ on the interval $[-1, 5]$, with $n = 1, 3, 4$ and 6 segments, respectively.

(c) $y = x^3 + 2x^2 - 5x - 2$ on the interval $[0, 2]$ with $n = 2, 4, 6$ and 8 segments, respectively.

(d) $y = xe^{-x}$ on the interval $[0.2, 3.8]$ with $n = 2, 4, 6$ and 8 segments, respectively.

> **Hint:**
>
> $$Area \approx \triangle x(\frac{y_1}{2} + y_1 + y_2 + y_3 + ... + \frac{y_n}{2})$$

Exercise 10: Write a program to implement the numerical integration with the composite Simpson's rule to solve the integral of these functions:

(a) $y = e^{-x^2}$ on the interval $[0, 1]$, with $n = 3$ segments.

(b) $y = 2x^2 + 5x + 12$ on the interval $[-1, 5]$, with $n = 1, 3, 4$ and 6 segments, respectively.

(c) $y = x^3 + 2x^2 - 5x - 2$ on the interval [0, 2], with $n = 2, 4, 6$ and 8 segments, respectively.

(d) $y = xe^{-x}$ on the interval $[0.2, 3.8]$, with $n = 2, 4, 6$ and 8 segments, respectively.

Hint:

$$Area \approx \frac{\triangle x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 ... + 4y_{n-1} + y_n)$$