Lab 6
CALCULUS FOR IT 501031

1 Exercises

Exercise 1: Write a program to generate n numbers in special sequences as follows

(a) Arithmetic sequence: $x_n = 4n + 1$

(b) Geometric sequence: $x_n = 3^n$

(c) Cubic sequence: $x_n = n^3$

(d) Fibonacci sequence: $x_n = x_{n-1} + x_{n-2}$

Exercise 2: Write a program to find the parameters of the corresponding sequences

(a) Given the sequence arithmetic sequence: $5, 20, 35, 50, 65, ...$ find d, a_n, a_{55} and which term equals 230?

(b) Given the geometric sequence: $120, 60, 30, 15, \frac{15}{2}, ...$ find r, a_n, a_{10} and which term equals $\frac{15}{32}$

Hint:

- Definition of an Arithmetic sequence: $a_2 - a_1 = d; a_7 - a_6 = d$ and so on. Therefore, the n^{th} term of an arithmetic sequence is $a_n = a_1 + (n-1)d$
- Definition of a Geometric sequence: $r = \frac{a_2}{a_1}; r = \frac{a_9}{a_8}$ and so on. Therefore, the n^{th} term of a geometric sequence is $a_n = a_1(r)^{n-1}$

Exercise 3: Find the Taylor series expension of these function:

(a) $f(x) = cos(x)$ at $x = \frac{\pi}{3}$ and the order is 6

(b) $f(x) = ln(x)$ at $x = 2$ and the order is 10

(c) $f(x) = e^x$ at $x = 3$ and the order is 12

Exercise 4: Find the Maclaurin series expansion of these function:

(a) $f(x) = cos(x)$ with the order is 6

(b) $f(x) = e^x$ with the order is 12

(c) $f(x) = \frac{1}{1-x}$ with the order is 12

(d) $f(x) = tan^{-1}(x)$ with the order is 12

Exercise 5: Find the limit of the following sequences:

(a) $\lim_{n\to\infty} \frac{4n^2+1}{3n^2+2}$

(b) $\lim_{n\to\infty} (\sqrt{n^2+1}-n)$

(c) $\lim_{n\to\infty} (\sqrt{2n+\sqrt{n}}-\sqrt{2n+1})$

(d) $\lim_{n\to\infty} \frac{3(5)^n-2^n}{4^n+2.5^n}$

(e) $\lim_{n\to\infty} \frac{nsin\sqrt{n}}{n^2+n-1}$

Exercise 6: Determine whether the sequence converges or diverges:

(a) $a_n = 1-(0.2)^n$

(b) $a_n = \frac{n^3}{n^3+1}$

(c) $a_n = \frac{3+5n^2}{n+n^2}$

(d) $a_n = \frac{n^3}{n+1}$

(e) $a_n = e^{\frac{1}{n}}$

(f) $a_n = \sqrt{\frac{n+1}{9n+1}}$

(g) $a_n = \frac{(-1)^{n+1}n}{n+\sqrt{n}}$

(h) $a_n = tan(\frac{2n\pi}{1+8n})$

(i) $a_n = \frac{(2n-1)!}{(2n+1)!}$

(j) $a_n = ln(2n^2+1) - ln(n^2+1)$

Exercise 7: Find the first five terms of the sequence following:

(a) $a_n = 1-(0.2)^n$

(b) $a_n = \frac{2n}{n^2+1}$

(c) $a_n = \frac{(-1)^{n-1}}{5^n}$

(d) $a_n = \frac{1}{(n+1)!}$

(e) $a_1 = 1, a_{n+1} = 5a_n - 3$

(f) $a_1 = 2, a_{n+1} = \frac{a_n}{a_n+1}$

and show the result graphically

Exercise 8: Using a graph of the sequence to determine whether the sequence is convergent or divergent.

(a) $a_n = 1-(\frac{-2}{e})^n$

(b) $a_n = \sqrt{n}sin(\frac{\pi}{\sqrt{n}})$

(c) $a_n = \sqrt{\frac{3+2n^2}{8n^2+n}}$

(d) $a_n = \frac{n^2cos(n)}{(1+n^2)}$

(e) $a_n = \frac{1.3.5...(2n-1)}{n!}$

(f) $a_n = \frac{1.3.5...(2n-1)}{(2n)^n}$

Exercise 9: Determinate if the following series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} 4^n = 4+16+64+256+1024+...$

(b) $\sum_{n=1}^{\infty} \frac{5}{2^n} = \frac{5}{2^1}+\frac{5}{2^2}+\frac{5}{2^3}+\frac{5}{2^4}+\frac{5}{2^5}...$

Exercise 10: Write a program to find the i^{th} Fibonacci number in Fibonacci sequence. By

(a) $x_i = x_{i-1} + x_{i-2}$

(b) $x_i = \frac{\phi^i - (1-\phi)^i}{\sqrt{5}}$, where $\phi = 1.618034$ is Golden Ratio.

(c) $x_i = [x_{i-1}\phi]$

Exercise 11: An employee has an initial salary of \$28000. The salary increase 3% per year. Use the n^{th} term $a_n = P[1+i]^n$ where P is the initial salary, i is the rate of increase in decimal, n is yearly term. Find a sequence of the first 3 years salaries.

Exercise 12: Write a program to illustrate Fractal sequences in Lindenmayer Systems (L-Systems). Where

- F: to represent the turtle moving forward by a certain distance d.
- +: to represent the turtle turning right by a certain angle α.
- -: to represent the turtle turning left by a certain angle.
- [Push current state of the turtle into a pushdown stack.
-] Pop a state from the stack, and make it the current state of the turtle. No line is drawn, although the position of the turtle may change. Show these Fractal sequences with $n = 5$.
  (a) *axiom:* $F \leftarrow$ *F+F-F-F+F*, if $n = 0 : F, \alpha = 90^0, d = 2$
  (b) *axiom:* $F \leftarrow$ *F[+F][-F]*, if $n = 0 : F, \alpha = 45^0, d = 2$