

## Lab 3

### CALCULUS FOR IT - 501031

## 1 Exercises

**Exercise 1:** Write a computer program to find the limit of functions

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|--|---|---|
| (a) $\lim_{x \rightarrow 3}  x^2 - x - 7 $                       | (f) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{x^2 + 7} - 4}$   | (k) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{3+x}\right)^x$                            |
| (b) $\lim_{x \rightarrow 1} \frac{ x-1 }{x^2-1}$                 | (g) $\lim_{x \rightarrow 1} \frac{ x }{\sin(x)}$                  | (l) $\lim_{x \rightarrow \infty} \sqrt[x]{\frac{1}{x}}$                                       |
| (c) $\lim_{x \rightarrow 1} \sqrt[x]{e}$                         | (h) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$          | (m) $\lim_{x \rightarrow \infty} \frac{-\sqrt[3]{x} + \sqrt[3]{1+x}}{-\sqrt{x} + \sqrt{1+x}}$ |
| (d) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$              | (i) $\lim_{x \rightarrow 0} \frac{2x^2}{3 - 3\cos x}$             | (n) $\lim_{x \rightarrow \infty} \frac{x!}{x^x}$  |
| (e) $\lim_{x \rightarrow -1} \frac{x^3 - x^2 - 5x - 3}{(x+1)^2}$ | (j) $\lim_{x \rightarrow \infty} \left(\frac{3+x}{-1+x}\right)^x$ |   |

**Exercise 2:** Graph the functions which were defined in the previous exercise, and then show the limit points on the graph if possible.

**Exercise 3:** The  $f$  functions are defined as

$$1. f(x) = \frac{1}{1 + 2^{\frac{1}{x}}} \qquad 2. f(x) = \frac{x^2 + x}{\sqrt{x^3 + x^2}}$$

Find  $\lim_{x \rightarrow 0^+} f(x)$ ,  $\lim_{x \rightarrow 0^-} f(x)$ ,  $\lim_{x \rightarrow 0} f(x)$  if they exist then show on the graph.

**Exercise 4:** Let  $f(x) = \begin{cases} 0, & x \leq 0 \\ \sin(\frac{1}{x}) & x > 0 \end{cases}$

1. Does  $\lim_{x \rightarrow 0^+} f(x)$  exist? If so, what is it? If not, show on the screen to explain
2. Does  $\lim_{x \rightarrow 0^-} f(x)$  exist? If so, what is it? If not, show on the screen to explain
3. Does  $\lim_{x \rightarrow 0} f(x)$  exist? If so, what is it? If not, show on the screen to explain

**Exercise 5:** Prove that the function is continuous at  $c$ .

$$(a) f(x) = x^2 - 7, c = 1 \qquad (b) f(x) = \sqrt{2x-3}, c = 2$$

**Exercise 6:** Write a program computer to verify at what points are the functions following continuous?

$$(a) \ g(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3} & x \neq 0 \\ 5 & x = 0 \end{cases}$$

$$(c) \ f(x) = \begin{cases} \frac{x^2 - x - 2}{x - 2} & x \neq 2 \\ 1 & x = 2 \end{cases}$$

$$(b) \ f(x) = \begin{cases} \frac{x^3 - 8}{x^2 - 4} & x \neq 2, x \neq -2 \\ 3 & x = 2 \\ 4 & x = -2 \end{cases}$$

$$(d) \ f(x) = \begin{cases} \frac{1}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

**Exercise 7:** Write a program computer to verify where are each of the following functions discontinuous?

$$1. \ f(x) = \frac{x^2 - x - 2}{x - 2}$$

$$2. \ f(x) = \frac{x^2 - 2x - 3}{2x - 6}$$

**Exercise 8:** Write a program computer to verify that the function  $f(x) = 1 - \sqrt{1 - x^2}$  is continuous on the interval  $[-1, 1]$  or not.

**Hint:**

- Find the limit of function  $\lim_{x \rightarrow -1} f(x)$
- Find the limit of function  $\lim_{x \rightarrow 1} f(x)$
- Check  $\lim_{x \rightarrow -1} f(x)$  equals  $\lim_{x \rightarrow 1} f(x)$  or not.

**Exercise 9:** Given  $P(1, 0)$  lies on  $y = \sin(10\pi/x)$ .  $Q$  has  $(x, \sin(10\pi/x))$ , finding slope of secant  $PQ$  with  $x = 2, 1.5, 1.4, 1.3, 1.2, 1.1, 0.5, 0.6, 0.7, 0.8, 0.9$ . Write a computer program to show the result.

**Exercise 10:** Define  $L$  so that the functions are continuous

$$(a) \ f(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0 \\ L, & x = 0 \end{cases}$$

$$(b) \ f(x) = \begin{cases} \frac{x^2 + x - 6}{x^2 - 4} & x \neq 2 \\ L & x = 2 \end{cases}$$