Lab 8
CALCULUS FOR IT 501031

1 Exercises

Exercise 1: Find the critical numbers (C.N) of $f(x)$ for the following cases:

(a) $f(x) = 3x^4 - 16x^3 + 18x^2 - 9$

(b) $f(x) = \dfrac{x+2}{2x^2}$

(c) $f(x) = -\dfrac{x^2}{3} + x^2 + 3x + 4$

(d) $f(x) = \dfrac{5x^2+5}{x}$

Exercise 2: Find the relative extrema using the second derivative test for the following cases:

(a) $f(x) = 3x^4 - 16x^3 + 18x^2 - 9$

(b) $f(x) = \dfrac{x+2}{2x^2}$

(c) $f(x) = -\dfrac{x^2}{3} + x^2 + 3x + 4$

(d) $f(x) = \dfrac{5x^2+5}{x}$

Exercise 3: Given $f(x)$ over a closed interval $[a, b]$, find the absolute maximum and the absolute minimum for the following cases:

(a) $f(x) = x^3 - 27x, [0, 5]$

(b) $f(x) = \dfrac{3}{2}x^4 - 4x^3 + 4, [0, 3]$

(c) $f(x) = \dfrac{1}{2}x^4 - 4x^2 + 5, [1, 3]$

(d) $f(x) = \dfrac{5}{2}x^4 - \dfrac{20}{3}x^3 + 6, [-1, 3]$

Exercise 4: Determine the minima or maxima of the functions $f(x)$ following:

(a) $f(x) = x^2 - 2x - 5, a = 0, b = 2$

(b) $f(x) = 3x + x^3 + 5, a = -4, b = 4$

(c) $f(x) = sin(x) + 3x^2, a = -2, b = 2$

(d) $f(x) = e^{x^2} + 3x, a = -1, b = 1$

(e) $f(x) = x^3 - 3x, a = -3, b = 0$

(f) $f(x) = x^3 - 3x, a = 0, b = 3$

(g) $f(x) = sin(x), a = 0, b = \pi$

(h) $f(x) = sin(2x), a = 0, b = 2$

(i) $f(x) = cos(x), a = \dfrac{\pi}{2}, b = \dfrac{3\pi}{2}$

(j) $f(x) = tan^2(x), a = \dfrac{-\pi}{4}, b = \dfrac{\pi}{4}$

(k) $f(x) = e^x sin(x), a = 0, b = \pi$

(l) $f(x) = x^4 - 3x^2, a = -4, b = 0$

(m) $f(x) = x^4 - 3x^2, a = 0, b = 4$

(n) $f(x) = x^5 - 5x^3, a = -4, b = 0$

(o) $f(x) = x^6 - 5x^2, a = -1, b = 1$

(p) $f(x) = x^3 - 9x, a = -3, b = 0$

(q) $f(x) = x^3 - 9x, a = 0, b = 3$

(r) $f(x) = x^3 + 9x, a = -1, b = 1$

Graph $f(x)$ and mark the maximum point.

Algorithm 1 Golden Search

Input: Objective function $f(x)$, boundaries a and b, and tolerance ϵ

$d = b - a$
while $b - a \geq \epsilon$ **do**
$d \longleftarrow 0.618 \times d$
$x_1 \longleftarrow b - d$
$x_2 \longleftarrow a + d$
if $f(x_1) \leq f(x_2)$ **then**
$b \longleftarrow x_2$
else
$a \longleftarrow x_1$
end if
end while
Output: Reduced interval [a, b]

Exercise 5: Write a program to implement **Golden Search** and apply to determinate minimum value of $f(x) = x^2$ in $[-2, 1]$, with a tolerate $\epsilon = 0.3$, , and illustrate on the graph/ table for each iteration.

Exercise 6: Implement **Fibonacci Search** and apply to determinate minimum value of $f(x) = x^2$ in $[-2, 1]$, with a tolerate $\epsilon = 0.3$, and illustrate on the graph/ table for each iteration.

Algorithm 2 Fibonacci Search

Input: Objective function $f(x)$, boundaries a and b, and tolerance ϵ

$F_1 = 2, F_2 = 3$

$n = 2$ **while** $b - a \geq \epsilon$ **do**
$d \longleftarrow b - a$
$x_1 \longleftarrow b - d\frac{F_{n-1}}{F_n}$
$x_2 \longleftarrow a + d\frac{F_{n-1}}{F_n}$
if $f(x_1) \leq f(x_2)$ **then**
$b \longleftarrow x_2$
else
$a \longleftarrow x_1$
end if

$n = n + 1$
$F_n = F_{n-1} + F_{n-2}$ **end while**
Output: Reduced interval [a, b]

Exercise 7: Determine m to $y = x^3 - 3mx^2 + 3(m^2 - 1)x - (m^2 - 1)$ maximize at $x_0 = 1$

Exercise 8: Optimization for $f(x)$ functions and plot on the graphs.

(a) $f(x) = -2x^2 + x + 4$, in $[-5, 5]$, and $\epsilon = \frac{1}{9}$
(b) $f(x) = -4x^2 + 2x + 2$, in $[-6, 6]$, and $\epsilon = \frac{1}{10}$
(c) $f(x) = x^3 + 6x^2 + 5x - 12$, in $[-5, -2]$, and $\epsilon = \frac{1}{10}$
(d) $f(x) = 2x - x^2$, in $[0, 3]$, and $\epsilon = \frac{1}{100}$

(e) $f(x) = x^2 - x - 10$, in $[-10, 10]$, and $\epsilon = \frac{1}{5}$

(f) $f(x) = -(x+6)^2 + 4$, in $[-10, 10]$, and $\epsilon = \frac{1}{8}$

(g) $f(x) = -2x^2 + 3x + 6$, in $[-3, 5]$, and $\epsilon = \frac{1}{8}$

Exercise 9: Find minimum of root of $f(x_1, x_2)$ function

(a) $f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 4x_2$ in [0 1]

(b) $f(x_1, x_2) = 3x_1^2 + 2x_2^4 - x_2 + x_2^2 + 1$ in [1 2]

(c) $f(x_1, x_2) = e^{x_1^2} + x_2^2 - 3 + 2 * x_2$ in [1 2]

(d) $f(x_1, x_2) = (x_1^2 - 1)^2 + x_2^2 - 3x_2 + 1$ in [0 0]

(e) $f(x_1, x_2) = x_1^2 e^{x_1} + 51x_2 + x_2^4 + 3$ in [0 1]

(f) $f(x_1, x_2) = e^{x_1^2 - 3} + x_2^2 - 3x_2$ in [1 2]

(g) $f(x_1, x_2) = x_1 x_2^3 + 2x_1^2 + 2x_2^4 - 5$ in [-1 1]