What's the Long-Term Expected Return to Your Portfolio?

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Many investors and investment consultants use mean–variance optimizers to select efficient portfolios. Mean–variance optimization is inherently a single-period technique. Most investors use it in a multiyear context by assuming annual rebalancing and independence and stationarity of returns. These assumptions are often reasonable, and they enable investors to estimate a portfolio's future growth rate, or long-term expected return.

Investors such as defined-benefit plan sponsors and foundations have a keen interest in the long-term expected returns to their portfolios. Sponsor contributions and foundation budgets are often tied directly to this estimate of the future growth rate of assets. This focus on a future portfolio growth rate appears to lead many practitioners mistakenly to use past growth rates to asset classes as inputs to their mean–variance optimizers. The consequence of this error is an estimated long-term expected return to the portfolio that is likely to be at least 50 basis points lower than the correct estimate.

Correct estimation of a portfolio's future multiperiod growth rate is a two-step procedure. First, determine the single-period characteristics of portfolio returns. Then, estimate the multiperiod characteristics of those single-period returns.

A simple example illustrates this point. From Ibbotson Associates' *Stocks, Bonds, Bills, and Inflation 1994 Yearbook*, the compound annual growth rates to large-company stocks and long-term corporate bonds from December 31, 1925, through December 31, 1993, were 10.3 percent and 5.6 percent, respectively. These numbers could be used as estimates of future long-term expected returns to stocks and bonds. To estimate the future growth rate to a portfolio of stocks and bonds, rebalanced annually to 50/50, one might be tempted simply to take the arithmetic average of the stock and bond growth rates. This calculation is quick and easy, and it yields an estimated future portfolio growth rate of 7.9 percent. One might think 7.9 percent is a reasonable estimate, but it is not. Using the two-step procedure outlined in the preceding paragraph, first calculate all 68 annual portfolio returns to stocks and bonds. In the second step, calculate the annual growth rate from these 68 annual portfolio returns. That growth rate is 8.4 percent, 50 basis points higher than the quick and easy estimate of 7.9 percent!

The stock/bond version of the erroneous calculation is given in Equation 1. Equation 1 is appropriate for calculating the single-period, say annual, expected return to a portfolio, because it holds for actual annual rates of return. Skipping Step 1 by using growth rates in Equation 1 is a mistake, however, because a portfolio growth rate is not a linear combination of the growth rates of the assets in the portfolio.

An annualized growth rate calculated from a series of annual returns will be lower than the arithmetic average of the returns because of volatility. A simple approximation of this relationship is given in Equation 4. Use of Equation 1 with long-term expected returns ignores the reduction in volatility coming from diversification between the two assets and yields a long-term expected return that is too low. The more diversified the portfolio, the larger the error. To convince yourself about this, just follow through the details of the stock and bond example below. This is the same example as above, except that Equation 4 is used for Step 2 rather than a calculation of all 68 annual portfolio returns.

EXAMPLE

Suppose a portfolio consists of two assets: stocks and bonds. Let R_s and R_b represent annual returns to the assets, and let P_s and P_b represent the proportion of assets invested in stocks and bonds, respectively. The expected return to the portfolio for some period, say one year, $E(R_p)$, is a weighted sum of the expected returns to the two assets; that is,

$$E(R_p) = P_s \times E(R_s) + P_b \times E(R_b). \quad (1)$$

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The variance of returns to the portfolio, $V(R_p)$, is a weighted sum of the variance of returns to stocks, the variance of returns to bonds, and the covariance between stock returns and bond returns; these numbers are also stated in annual terms. Formally,

$$V(R_p) = P_s^2 \times V(R_s) + P_b^2 \times V(R_b) + 2 \times P_s \times P_b \times COV(R_s,R_b). \quad (2)$$

Investors usually think in terms of standard deviation rather than variance. The annual standard deviation is the square root of the variance and is in the same units as expected return, percentages; that is,

$$S(R_p) = V(R_p)^{1/2}. \quad (3)$$

To use Equations 1, 2, and 3, we need to know the investment proportions P_s and P_b, as well as the expected returns, standard deviations, and the covariance. Past returns contain a good deal of information about future expected returns. Using only past returns to obtain estimates of expected returns is not recommended, but for simplicity, we will base our expected returns on past returns from 1926 through 1993 taken from the Ibbotson's *1994 Yearbook*. The goal is to calculate a long-term expected return to a portfolio that is annually rebalanced to 50 percent stocks and 50 percent bonds.

To arrive at the portfolio's expected growth rate, first estimate its annual expected return and standard deviation. Equation 1 and the arithmetic average returns are used as estimates of annual expected returns. The sample annual standard deviations and covariance are used as estimates of the annual standard deviations and covariance in Equations 2 and 3. The means and standard deviations are reported in Ibbotson's *1994 Yearbook*, and the covariance can be calculated from additional information in that source. These calculations produce the following values:

$E(R_s)$ = 0.123, or 12.3 percent
$E(R_b)$ = 0.059
$S(R_s)$ = 0.205
$S(R_b)$ = 0.084
P_s = 0.500
$COV(R_s,R_b)$ = 0.003788
P_b = 0.500

With these inputs, Equation 1 gives an annual expected portfolio return equal to 9.1 percent and Equations 2 and 3 give an annual standard deviation equal to 11.9 percent.

The long-term expected return, the expected growth rate for an investment horizon of many years, can be estimated from the annual expected return and standard deviation. Chapter 9 of Ibbotson's *Yearbook* goes into this topic in detail using the lognormal probability distribution. For the purpose of this illustration, a reasonable approximation for the annualized growth rate is the annual expected return less half the annual variance, or

$$R_{gp} = E(R_p) - V(R_p)/2. \quad (4)$$

For stocks, this approximation is:

$$R_{gs} = E(R_s) - V(R_s)/2.$$

$$0.102 = 0.123 - (0.205)^2/2.$$

So, 10.2 percent is a reasonable estimate of the expected growth rate to a stock portfolio over, say, the next 20 years. Equation 4 gives an estimate of 5.6 percent for bonds. If you are comfortable assuming an annual expected return for bonds of 5.9 percent, then 5.6 percent is a reasonable estimate of the long-term expected return to a bond portfolio.[1] These estimates of future growth rates are almost identical to the actual growth rates for stocks, 10.3 percent, and bonds, 5.6 percent, from December 31, 1925, through December 31, 1993.

Equation 4 can be used to achieve the goal of this exercise: an estimate of the growth rate for a 50/50 stock/bond portfolio over the next 20 years. The estimate is

$$R_{gp} = 0.091 - (0.119)^2/2 = 0.084,$$

that is, 8.4 percent. Equation 1 with the long-term expected returns to stocks and bonds would have produced a lower growth rate; that is,

$$R^{\text{wrong}}_{gp} = 0.5 \times 0.102 + 0.5 \times 0.056 = 0.079.$$

This number is 50 basis points lower than the estimate obtained from Equation 4.

A skeptical reader might ask why we recommend the 8.4 percent estimate over the 7.9 percent estimate. First, 8.4 percent is based upon sound statistical analysis, whereas 7.9 percent is the result of a logical error. Indeed, recall that the actual growth rate to an annually rebalanced 50/50 stock/bond portfolio from December 31, 1925, through December 31, 1993, was also 8.4 percent.

With more securities and even greater risk reduction from diversification, the difference between the reasonable estimate of a long-term expected return and the estimate based upon the application of Equation 1 would have been greater than 50 basis points.

IMPLICATIONS AND CONCLUSIONS

The pervasiveness of this mistake is difficult to gauge. On the basis of anecdotal evidence, we believe it is far too common. At least two well-known pension consulting firms have been committing this error for years.[2] More than one popular mean–variance optimization software package does not even allow the user to use average returns in the calculation of optimal portfolios. The programs simply use growth rates calculated over some sample of past returns, or they ask the user to input growth rates.

Take a look at your most recent asset/liability or asset allocation analysis. If you do not find any differentiation between single-period (annual) expected returns and long-term expected returns, the chance is very good that long-term expected returns and Equation 1 were used to estimate the long-term expected return to your portfolio. You can easily verify this by taking a weighted sum of the long-term expected returns with the multiasset version of Equation 1 and the investment proportions for the asset classes represented in your portfolio. If that calculation produces a number that matches the long-term expected portfolio return in your report, then your report is in error. If your portfolio is at all diversified, its long-term expected return is very likely to be 50 to 70 basis points higher than the number in your report.

You might ask, "So what? The cost of a defined-benefit plan will depend upon actual returns not assumed expected returns. Fussing about this assumption isn't important." But a sponsor's contributions to a defined-benefit plan are determined largely by the actuarial valuation rate. A sponsor and its actuary should give considerable weight to the consultant's estimate of the long-term expected return to plan assets in setting the valuation rate. Logically, the valuation rate should equal the long-term expected return to plan assets. A 50 basis point change in the valuation rate can easily change the accrued liability by 10 percent or more. A plan sponsor using a valuation rate that is 50 basis points too low is probably contributing more to the plan than is necessary. For that matter, any long-term investor who believes the long-term expected return to his or her portfolio is x percent, when in reality, it is x percent plus 50 basis points, is very likely not managing his or her assets efficiently.[3]

NOTES

1. We believe this estimate based solely on past bond returns is too low.
2. See "State Fund Returns Lag Actuarial Rates," *Pensions & Investments* (March 20, 1995):1, for a review of a recent study in which this error occurred. The error is not apparent unless one knows the investment proportions of the funds identified in the study. Using that information, we have ascertained that the long-term expected return to the 84 state funds analyzed is understated by an average of 49 basis points. Contact the author to receive a version of this paper that contains the details.
3. I wish to acknowledge helpful comments from David Emanuel, Charles Gabriel, and Klaus Truemper.