PYTHON FOR THE FINANCIAL ECONOMIST, ORDINARY EXAM 2021

Copenhagen Business School
17. December 2021

2 weeks, home assignment

The home assignment is to be answered in groups of two students (maximum of 25 A4-pages) or individually (maximum of 15 A4-pages). The students must individualize the assignment.

The take-home assignment should take the form of a report written in either Word or Latex.

The analysis should be performed using Python. Please attach code / Jupyter notebooks.

If you think that you do not have all the necessary information to answer a problem, make the necessary assumptions in order to proceed and state these assumptions in the solution.

Good luck!

Optimizing portfolios

Consider an universe of 10 assets provided in the attached zip-file. Each *csv* file contains the prices of a specific asset for year 0, 1, 2, 3, 4, 5 and for 10,000 simulations. The main task will be to perform a full scale asset allocation study. Below, the minimum required steps in the asset allocation study is listed.

Present relevant descriptive statistics for the 1-year and 5-year price and / or return distribution for all assets, e.g. mean, standard deviation, skewness, etc. How well does the 1-year and 5-year distributions compare to a log-normal or normal distribution?

Consider a buy-and-hold and an yearly rebalanced equally weighted investment strategy. Plot the price development for the two strategies using a fan chart. Present relevant descriptive statistics for the 1-year and 5-year return distributions including risk measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). Present plots of the 1-year and 5-year distributions illustrating the VaR and CVaR measures.

Discuss relevant approaches to finding optimal portfolios using the provided data, e.g. what risk measures to use, what are the relevant restrictions, etc. Perform portfolio optimizations at a 1-year and 5-year horizon using at least standard deviation and CVaR as risk measures and present results.

Enforce a view / views on the percentiles of the assets' price distribution on a 1-year and 5-year horizon, e.g. the view that the 10% percentiles for the first, second and third asset (prices) at the 1-year and 5-year horizon are smaller than some specific value(s). The views at the different horizons may be enforced at the same time or one horizon at the time. Describe carefully how you will implement the views (see Attilio Meucci (2008)). Quantify and visualize the consequences for the asset distributions and the optimal portfolios, e.g. how do allocations change and how does skewness and kurtosis of the distributions change.

The Cox-Ingersoll-Ross model

The Cox-Ingersoll-Ross (CIR) model specifies the dynamics for the short rate as (under the physical measure $\mathbb{P}$)

$$dr_t = \kappa[\theta - r_t]dt + \beta\sqrt{r_t}dz_t^{\mathbb{P}}$$

where κ, θ and β are positive constants. It is possible to show that

$$\mathrm{E}_t[r_s] = \theta + (r_t - \theta)e^{-\kappa[s-t]}$$

and

$$\mathrm{Var}_t[r_s] = \frac{\beta^2 r_t}{\kappa}\left(e^{-\kappa[s-t]} - e^{-2\kappa[s-t]}\right) + \frac{\beta^2\theta}{2\kappa}(1 - e^{-\kappa[s-t]})^2$$

This information is useful when simulating the short rate into the future. However, the distribution is not normal but non-central χ^2. An alternative approach would be to use the

Euler method. Assume a initial short rate of $r_t = 0.02$ and $\kappa = 0.36$, $\theta = 0.08$ and $\beta = 0.1$. Simulate 10,000 paths of the short rate for the next 10 years with weekly time steps. Visualize the short rate development using a fan chart. Do the formulas for expected value and variance match the simulated data?

For risk-neutral pricing, we need the dynamics under $\mathbb{Q}$ (the risk-neutral probability measure). Under the risk-neutral probability measure we have

$$dr_t = \hat{\kappa}[\hat{\theta} - r_t]dt + \beta\sqrt{r_t}dz_t^{\mathbb{Q}}$$

where $\hat{\kappa} = \kappa + \lambda$ and $\hat{\theta} = \kappa\theta/(\kappa + \lambda)$. Assume $\lambda = 0$. Are there any difference between the distribution under $\mathbb{P}$ and $\mathbb{Q}$?

The price of a zero-coupon bond maturing at time T can be written as

$$B(\tau) = e^{-a(\tau)-b(\tau)r_t}$$

with $\tau = T - t$ and

$$\begin{aligned} a(\tau) &= -\frac{2\hat{\kappa}\hat{\theta}}{\beta^2}\left(\ln(2\gamma) + \frac{1}{2}(\hat{\kappa} + \gamma)\tau - \ln\left((\gamma + \hat{\kappa})(e^{\gamma\tau} - 1) + 2\gamma\right)\right) \\ b(\tau) &= \frac{2(e^{\gamma\tau} - 1)}{(\gamma + \hat{\kappa})(e^{\gamma\tau} - 1) + 2\gamma} \end{aligned}$$

where $\gamma = \sqrt{\hat{\kappa}^2 + 2\beta^2}$. Based on the above information, plot the zero-coupon yield curve.

Assume that we have the cash flow provided in the attached excel file *cashflow.xlsx*. What is the present value of the cash flow at time $t = 0$?

Describe how you can calculate the accumulated cash flow + the present value of future cash flows along each simulated path. Plot the distribution over time of the accumulated cash flow + the present value of future cash flows. You may choose to use monthly or weekly time steps.