

$$\text{l2norm}[\mathbf{x}_-] := \left(\int_0^1 \mathbf{x}^2 dt \right)^{1/2}$$

$$\text{dotpr}[\mathbf{x}_-, \mathbf{y}_-] := \left(\int_0^1 \mathbf{x} \mathbf{y} dt \right)$$

$$\mathbf{e0} = 1$$

$$1$$

$$\text{dotpr}[\mathbf{e0}, t] / \text{dotpr}[\mathbf{e0}, \mathbf{e0}]$$

$$\frac{1}{2}$$

$$\mathbf{e1} = t - 1/2$$

$$-\frac{1}{2} + t$$

$$\text{l2norm}[t - 1/2]$$

$$\frac{1}{2\sqrt{3}}$$

$$\mathbf{a20} = \text{dotpr}[t^2, \mathbf{e0}] / \text{dotpr}[\mathbf{e0}, \mathbf{e0}]$$

$$\frac{1}{3}$$

$$\mathbf{a21} = \text{dotpr}[t^2, \mathbf{e1}] / \text{dotpr}[\mathbf{e1}, \mathbf{e1}]$$

$$1$$

$$\mathbf{e2} = t^2 - \mathbf{a20} \mathbf{e0} - \mathbf{a21} \mathbf{e1}$$

$$\frac{1}{6} - t + t^2$$

$$\mathbf{a30} = \text{dotpr}[t^3, \mathbf{e0}] / \text{dotpr}[\mathbf{e0}, \mathbf{e0}]$$

$$\frac{1}{4}$$

$$\mathbf{a31} = \text{dotpr}[t^3, \mathbf{e1}] / \text{dotpr}[\mathbf{e1}, \mathbf{e1}]$$

$$\frac{9}{10}$$

$$\mathbf{a32} = \text{dotpr}[t^3, \mathbf{e2}] / \text{dotpr}[\mathbf{e2}, \mathbf{e2}]$$

$$\frac{9}{10}$$

$$\frac{3}{2}$$

$$\mathbf{a32} = \text{dotpr}[t^3, \mathbf{e2}] / \text{dotpr}[\mathbf{e2}, \mathbf{e2}]$$

$$\frac{3}{2}$$

$$\mathbf{e3} = t^3 - \mathbf{a30} \mathbf{e0} - \mathbf{a31} \mathbf{e1} - \mathbf{a32} \mathbf{e2}$$

$$-\frac{1}{4} - \frac{9}{10} \left(-\frac{1}{2} + t \right) + t^3 - \frac{3}{2} \left(\frac{1}{6} - t + t^2 \right)$$

$$a40 = \text{dotpr}[t^4, e0] / \text{dotpr}[e0, e0]$$

$$\frac{1}{5}$$

$$a41 = \text{dotpr}[t^4, e1] / \text{dotpr}[e1, e1]$$

$$\frac{4}{5}$$

$$a42 = \text{dotpr}[t^4, e2] / \text{dotpr}[e2, e2]$$

$$\frac{12}{7}$$

$$a43 = \text{dotpr}[t^4, e3] / \text{dotpr}[e3, e3]$$

$$2$$

$$p0 = e0 / \text{l2norm}[e0]$$

$$1$$

$$p1 = e1 / \text{l2norm}[e1]$$

$$2\sqrt{3} \left(-\frac{1}{2} + t \right)$$

$$p2 = e2 / \text{l2norm}[e2]$$

$$6\sqrt{5} \left(\frac{1}{6} - t + t^2 \right)$$

$$p3 = e3 / \text{l2norm}[e3]$$

$$20\sqrt{7} \left(-\frac{1}{4} - \frac{9}{10} \left(-\frac{1}{2} + t \right) + t^3 - \frac{3}{2} \left(\frac{1}{6} - t + t^2 \right) \right)$$

$$e4 = t^4 - a40 e0 - a41 e1 - a42 e2 - a43 e3$$

$$-\frac{1}{5} - \frac{4}{5} \left(-\frac{1}{2} + t \right) + t^4 - \frac{12}{7} \left(\frac{1}{6} - t + t^2 \right) - 2 \left(-\frac{1}{4} - \frac{9}{10} \left(-\frac{1}{2} + t \right) + t^3 - \frac{3}{2} \left(\frac{1}{6} - t + t^2 \right) \right)$$

$$p4 = \text{FullSimplify}[e4 / \text{l2norm}[e4]]$$

$$3(1 + 10(-1 + t)t(2 + 7(-1 + t)t))$$

$$\text{ans} = \text{dotpr}[t^5, p0] * p0 + \text{dotpr}[t^5, p1] * p1 + \text{dotpr}[t^5, p2] * p2 + \text{dotpr}[t^5, p3] * p3 + \text{dotpr}[t^5, p4] * p4$$

$$\frac{1}{6} + \frac{5}{7} \left(-\frac{1}{2} + t \right) + \frac{25}{14} \left(\frac{1}{6} - t + t^2 \right) +$$

$$\frac{1}{28} (1 + 10(-1 + t)t(2 + 7(-1 + t)t)) + \frac{25}{9} \left(-\frac{1}{4} - \frac{9}{10} \left(-\frac{1}{2} + t \right) + t^3 - \frac{3}{2} \left(\frac{1}{6} - t + t^2 \right) \right)$$

$$\text{ans} = \text{FullSimplify}[\text{ans}]$$

$$\frac{1}{252} (1 + 10t(-3 + 7t(3 + t(-8 + 9t))))$$