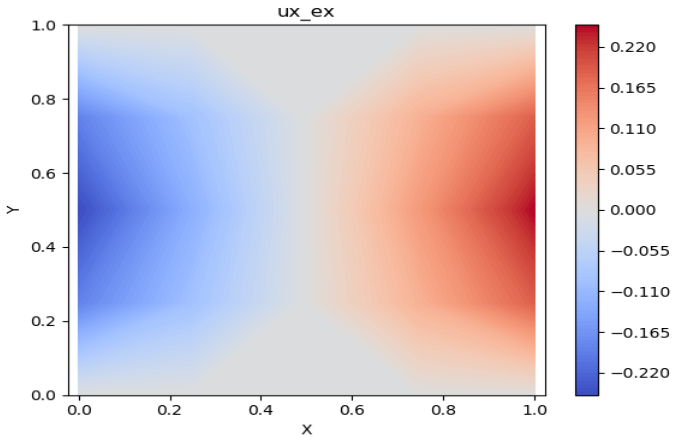
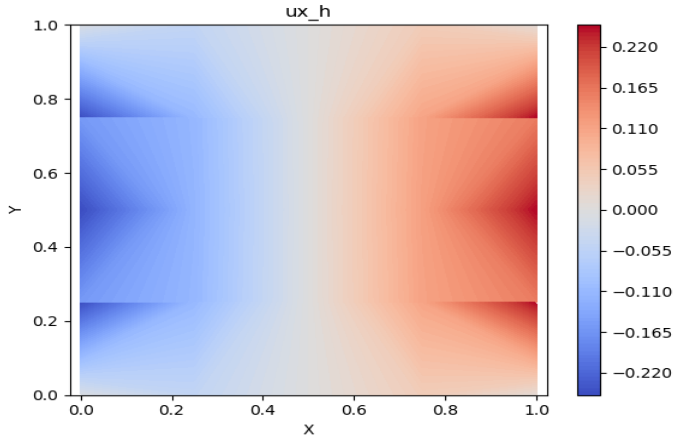
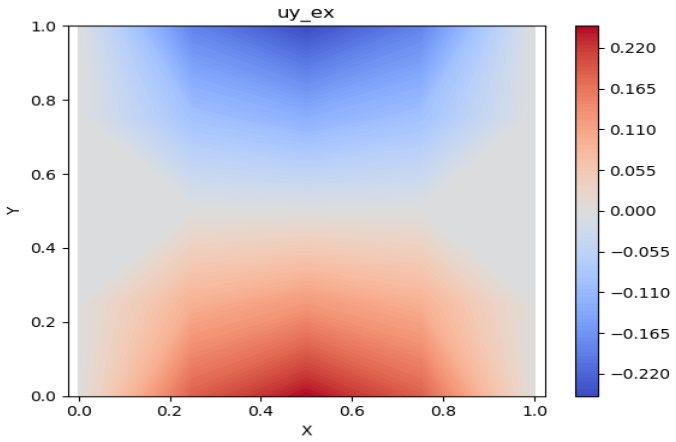
The results of MMS solution with $p = x(1-x)y(1-y)$ on 4x4 uniform element distribution is as follow. Note that the exact solution is project to AC space i.e., $\boldsymbol{u}(\boldsymbol{x}(\boldsymbol{X}_i)) \cdot \nu_i$



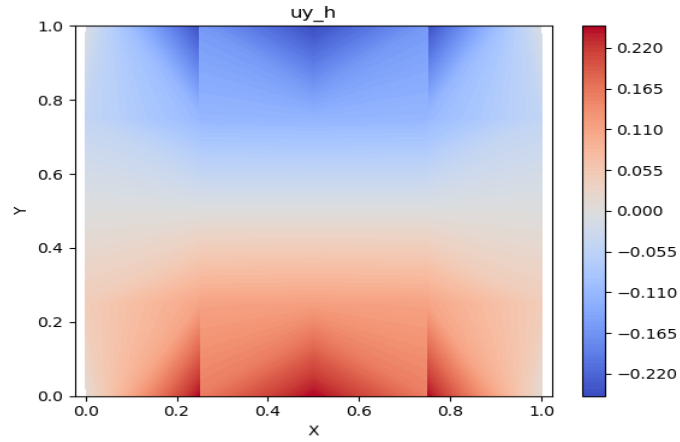
(a) exact solution $u_x = -y(1-y)(1-2x)$



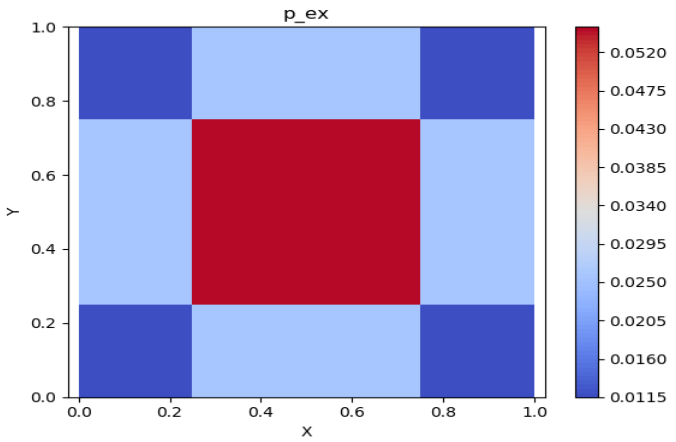
(b) numerical solution u_x^h



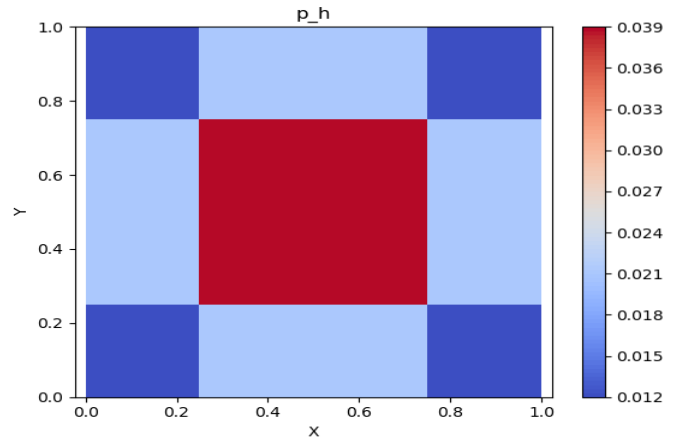
(c) exact solution $u_y = -x(1-x)(1-2y)$



(d) numerical solution u_y^h



(e) exact solution $p = x(1-x)y(1-y)$



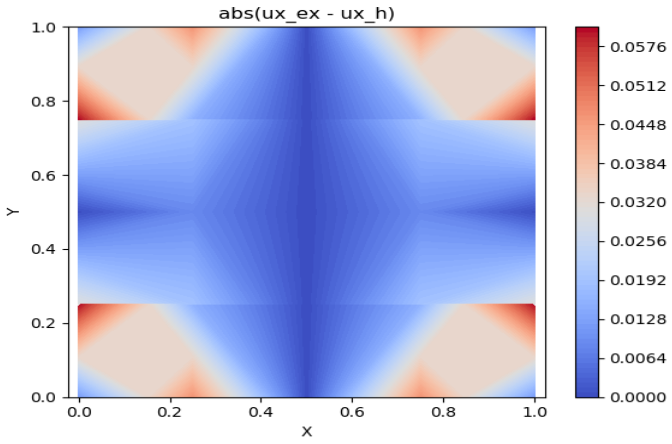
(f) numerical solution p^h

Figure 1: MMS results of quartic solution on 4x4 uniform element distribution

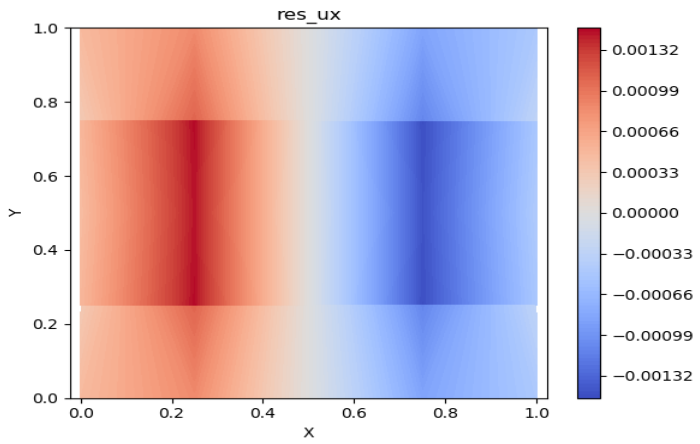
Then residual

$$R = \begin{bmatrix} \boldsymbol{M} & \boldsymbol{B}^T \\ \boldsymbol{B} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{ex} \\ \boldsymbol{P}_{ex} \end{bmatrix}$$

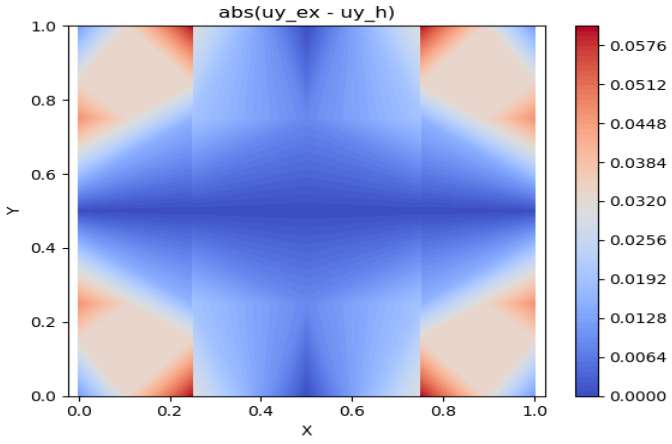
and the difference of the numerical and exact solution are



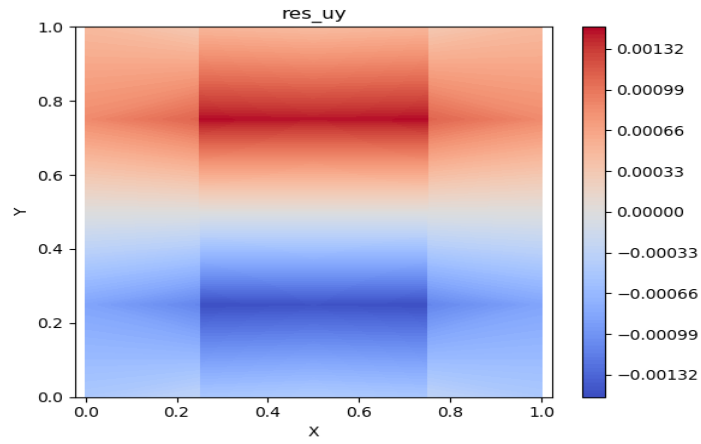
(a) $|u_x - u_x^h|$



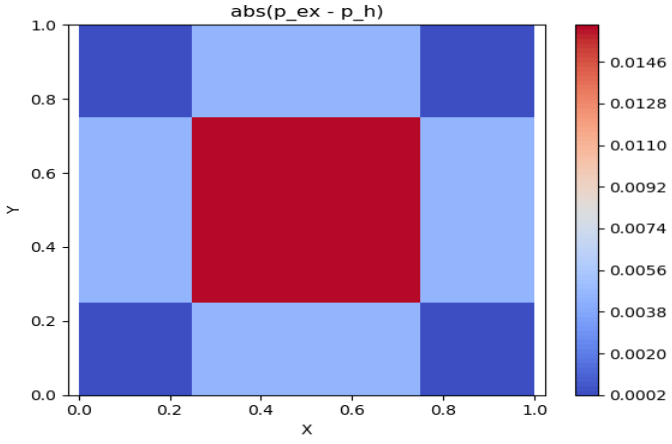
(b) residual of u_x



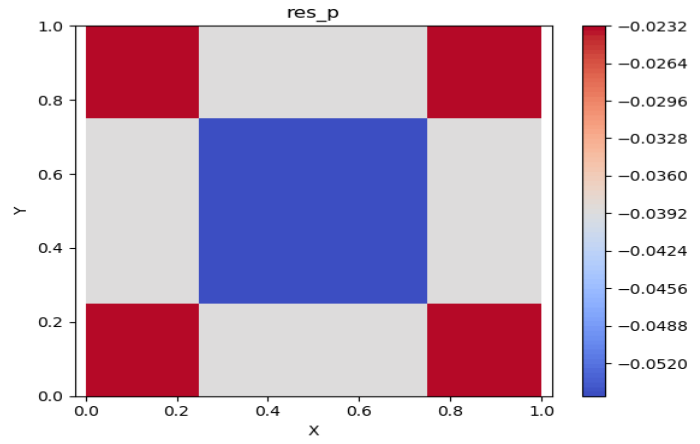
(c) $|u_y - u_y^h|$



(d) residual of u_y



(e) $|p - p^h|$



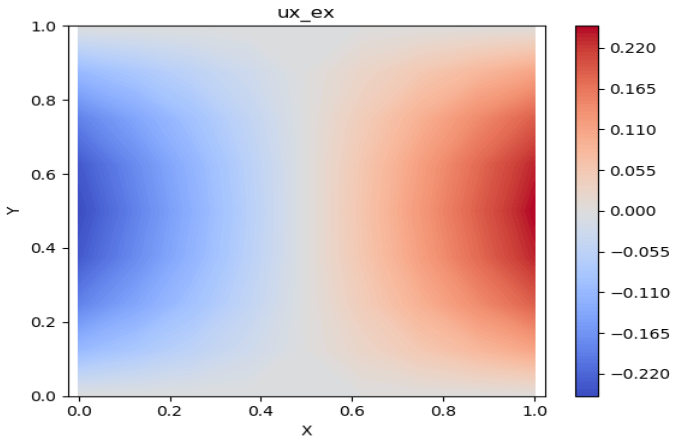
(f) residual of p

Figure 2: Difference of the numerical and exact solution and residual

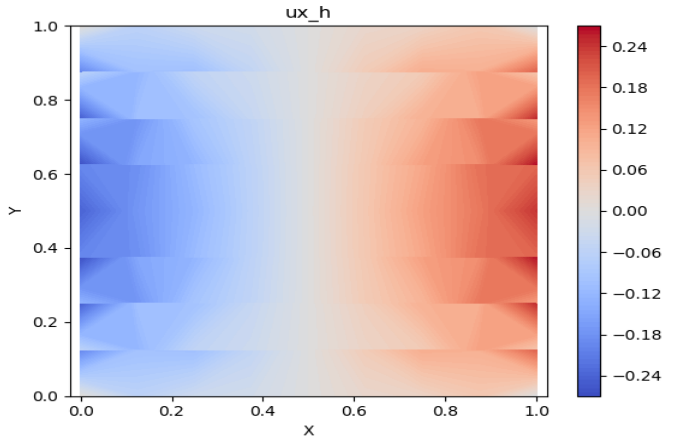
Finally the eigenvalues of $\boldsymbol{B}\boldsymbol{M}^{-1}\boldsymbol{B}^T$ is

$$
\begin{array}{lll}
1.6868000238099512 & 5.839520107976473 & 5.839520107976485 \\
12.024851300573168 & 16.6672183277769 & 19.023041474654367 \\
26.370043516152126 & 26.37004351615215 & 30.370177530853077 \\
35.86130955134691 & 41.48914644357674 & 44.02386183274496 \\
51.873627485123436 & 51.87362748512349 & 56.42354438231801
\end{array} \tag{1}
$$

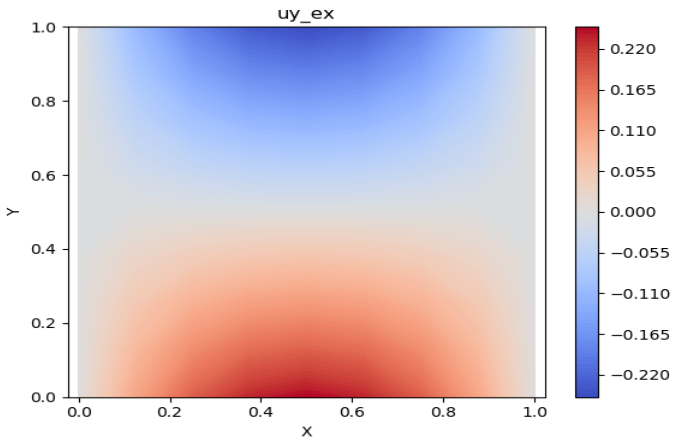
The results of MMS solution with $p = x(1-x)y(1-y)$ on 8x8 uniform element distribution is as follow. Note that the exact solution is project to AC space i.e., $\boldsymbol{u}(\boldsymbol{x}(\boldsymbol{X}_i)) \cdot \nu_i$



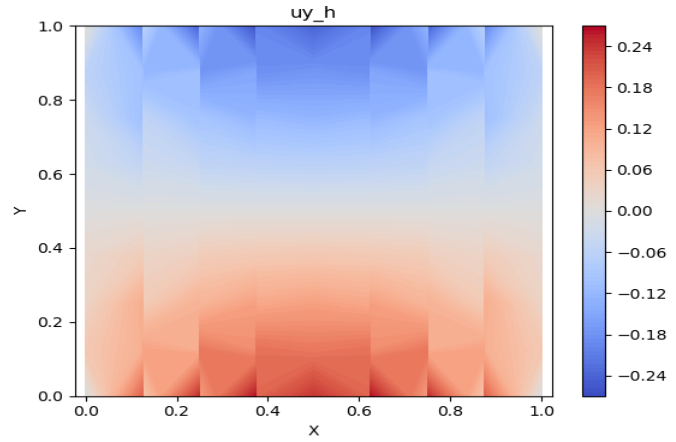
(a) exact solution $u_x = -y(1-y)(1-2x)$



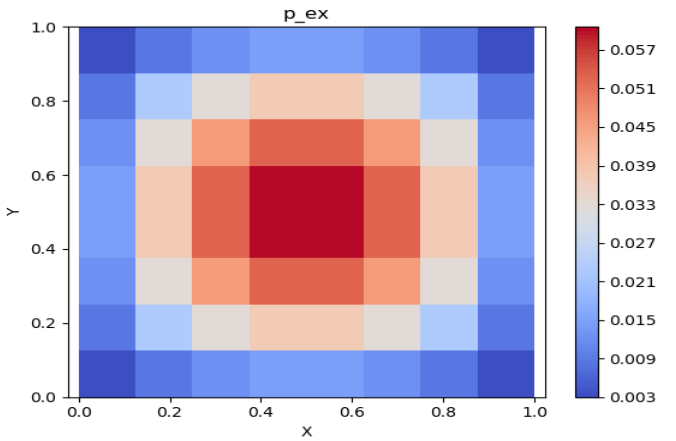
(b) numerical solution u_x^h



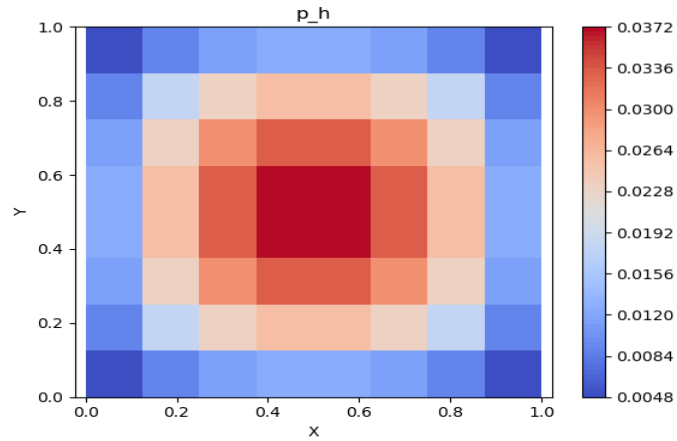
(c) exact solution $u_y = -x(1-x)(1-2y)$



(d) numerical solution u_y^h



(e) exact solution $p = x(1-x)y(1-y)$



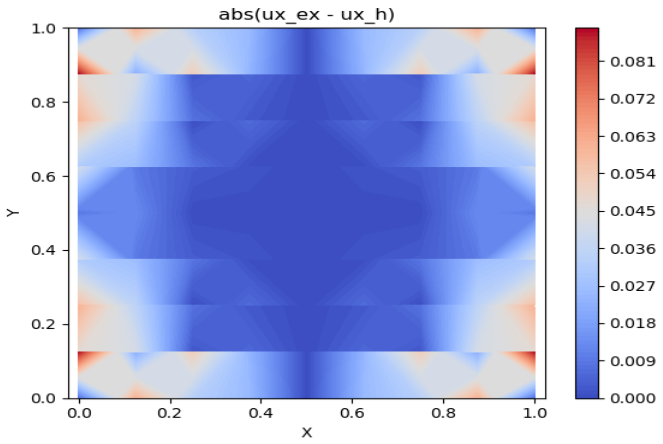
(f) numerical solution p^h

Figure 3: MMS results of quartic solution on 8x8 uniform element distribution

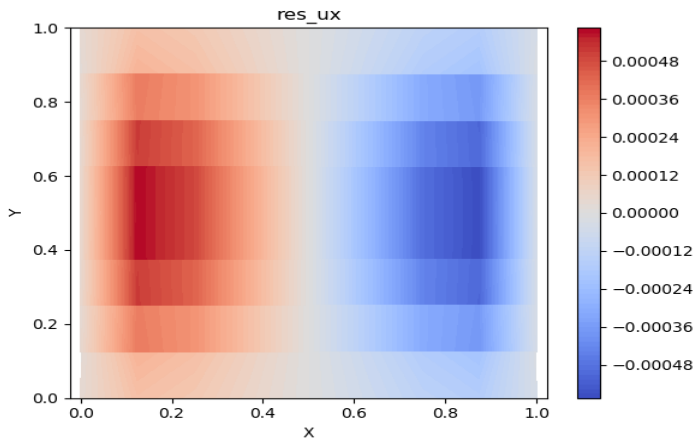
Then residual

$$R = \begin{bmatrix} \boldsymbol{M} & \boldsymbol{B}^T \\ \boldsymbol{B} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{ex} \\ \boldsymbol{P}_{ex} \end{bmatrix}$$

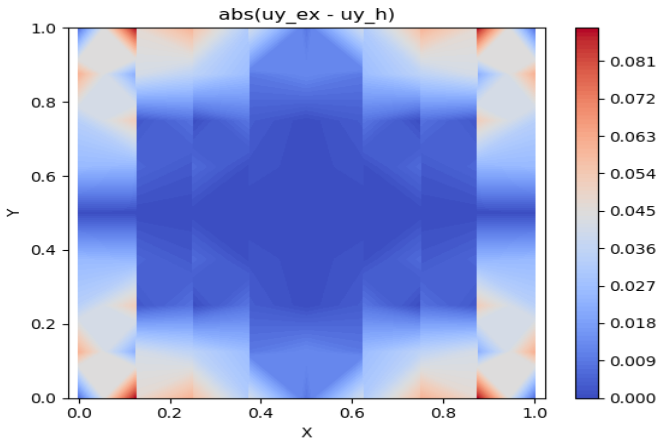
and the difference of the numerical and exact solution are



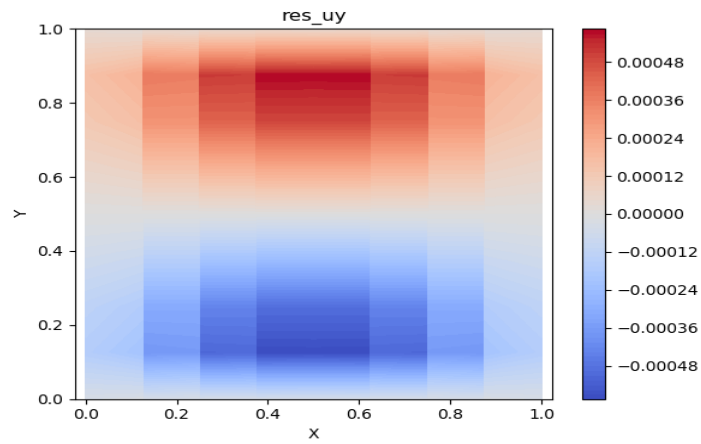
(a) $|u_x - u_x^h|$



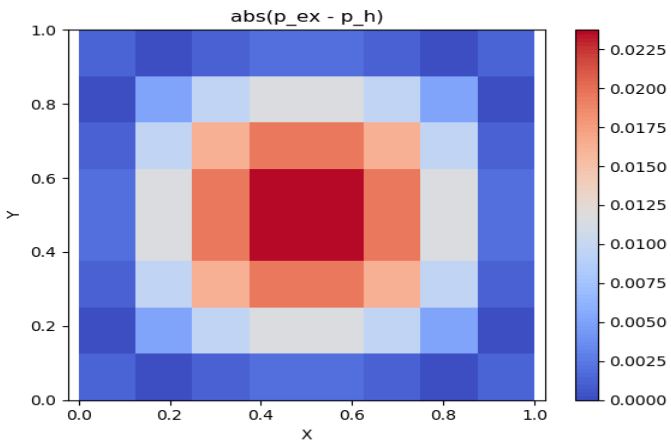
(b) residual of u_x



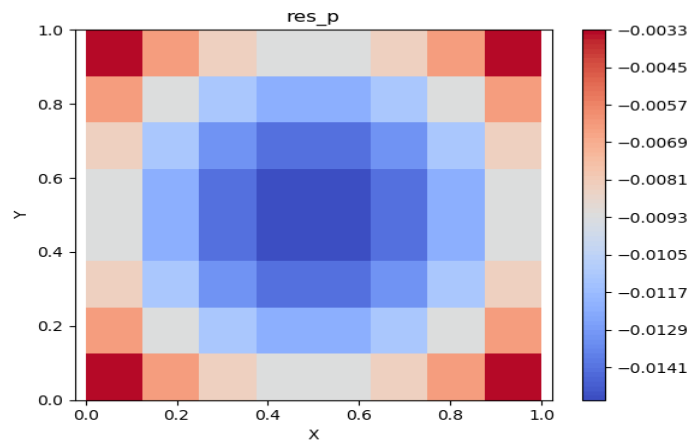
(c) $|u_y - u_y^h|$



(d) residual of u_y



(e) $|p - p^h|$



(f) residual of p

Figure 4: Difference of the numerical and exact solution and residual

Finally the eigenvalues of $\boldsymbol{BM}^{-1}\boldsymbol{B}^T$ is

$$
\begin{array}{l}
0.4766282118582078\ 1.2855185126412478\ 1.2855185126412607 \\
2.199139207156946\ 2.8486033469976784\ 3.96892299690448 \\
3.9689229969044857\ 5.612989546258567\ 5.6129895462585715 \\
6.036802036309303\ 6.82706840881192\ 7.081079610919463 \\
9.419350356852624\ 9.419350356852647\ ....
\end{array} \tag{2}
$$