Challenge #3: Surviving the zombie apocalypse

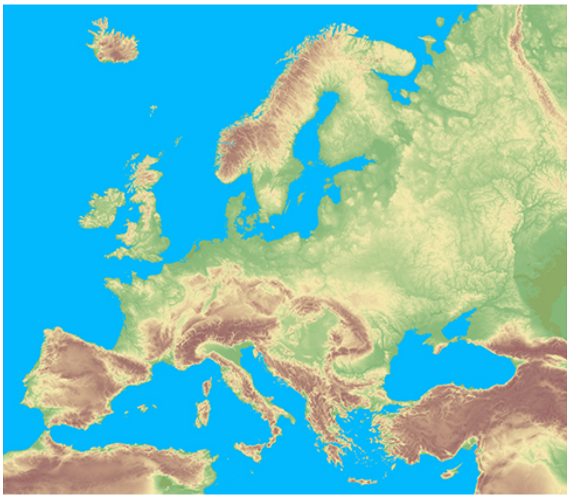
Yannis Haralambous

May 18, 2021

The zombie apocalypse after having been prophecied by TV serials like The Walking Dead or Z Nation and movies like Pride and Prejudice and Zombies, will happen on August 18th, 2019, starting from the Turkish town of Rize (district of Güneysu). You will be sitting in a bunker in Brest with a computer and access to the following resources:

1. a BMP file representing population density on the scale 1 pixel = 1 km^2

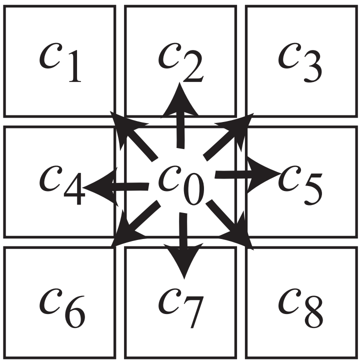
2. a BMP file representing elevation on the scale 1 pixel = 1 km^2



Here is some information about zombies that may be useful:

We will consider cells of 15×15 pixels based on the population density map, all values (population density, elevation) will be averages over the 225 pixels of the cell.

Zombies can move into 8 directions from a cell to another cell (cells are denoted by c_i):



If the altitude difference between two edges of a cell is higher or equal to 10 degrees and this edge (the one on the higher side) cannot be used by zombies to move to another cell.

Let $H_j(c_i)$ be the human population at cell c_i on day j. Let $Z_j(c_i)$ be the zombie population at cell c_i on day j.

Here is the algorithm of daily zombie spreading, from day j to day $j+1$:

STEP 1 we define the contribution $C_{j+1}(c_0, c_i)$ of cell c_0 to cell c_i by

$$
\begin{aligned}
C_{j+1}(c_0, c_i) &= \frac{H_j(c_i)}{\sum_{k=1}^{8} \lambda_{0,k} H_j(c_k)} Z_j(c_0)\lambda_{0,i} \quad \text{if } i \neq 0 \text{ and } \sum \lambda_{0,k} H_j(c_k) > 0 \\
C_{j+1}(c_0, c_i) &= 0 \quad \text{if } i \neq 0 \text{ and } \sum \lambda_{0,k} H_j(c_k) = 0 \\
C_{j+1}(c_0, c_0) &= 0 \quad \text{if } \sum \lambda_{0,k} H_j(c_k) > 0 \\
C_{j+1}(c_0, c_0) &= Z_j(c_0) \quad \text{if } \sum \lambda_{0,k} H_j(c_k) = 0.
\end{aligned}
$$

At step $j+1$ the zombie population at a given cell is obtained as the sum of contributions from neighboring cells. This is independent of the order in which cells are processed because each calculation at time $j+1$ depends *only* on data from time j.

Here $\lambda_{0,i}$ is a factor of geographical slope between c_0 and c_i;

STEP 2 after STEP 1 has been done for the whole graph, we calculate zombies killing humans: when N zombies have entered a cell they kill $10 \times N$ humans in the cell (if there are less humans in the cell, then they are all killed). Killed humans become zombies and are added to the zombie population of the cell;

STEP 3 once STEP 2 has been completed for the whole graph, we calculate humans destroying zombies: the M humans that survive the zombie attack destroy $10 \times M$ zombies in the cell (if there are less zombies in the cell then they are all destroyed), these are removed from the zombie population of the cell. Zombies are killed regardless of their age (whether a zombie is 0 day old or 14 days old, it has the same chances of being killed).

Notice that zombies are active for 15 days, after that they are auto-destroyed.

The formula for λ_d is as follows: λ_d is zero for a slope higher than 10 degrees, it is one for a slope of zero, and linear between these two values for slopes between 0 and 10 degrees.

The population density file has a red pixel for the center of Rize and a green pixel for the center of Brest, all other pixels are black and white. Maximum density (absolute white color) is 3000 inhabitants per km^2 (every pixel is a square kilometer).

The elevation file has blue color for sea, green/yellow/brown colors for elevation. To find absolute values, find the pixel of the Mont Blanc (4810 m) summit and create a correspondence map between colors and altitudes.

The epidemics starts by the entire population of the town Rize becoming zombies (`https://fr.wikipedia.org/wiki/Rize`) on day 0 (August 18, 2019).

Here are the challenge questions:

QUESTION 0 Will zombies arrive to Brest, and if yes, when?

QUESTION 1 Two months after the start of the epidemics, the European Union is at last able to send military troops to fight the zombies and secure 20 cells. Securing a cell means that all zombies are destroyed and no zombie can enter the cell anymore. Find, by using an graph algorithm, the optimal twenty cells to secure (optimal in the sense that they will allow protection of a maximum amount of persons). Consider that troops are transfered instantly to their locations. What will be the impact of your choice of 20 cells to the spreading of the epidemics? Will Brest be preserved?

QUESTION 2 Four months after the start of the epidemics, the European Union takes the decision of using nuclear weapons to save the rest of humanity. Nuclear bombs destroy entire cells (they kill all life forms except salamanders [which 50 years later become godzillas] and destroy all zombies). After a cell has been bombed it cannot be traversed by zombies anymore. The number of nuclear bombs is unlimited. How many and which cells should be bombed to obtain the best possible ratio of (humans killed by bombs) / (humans saved by total destruction of zombies)? Will Brest be preserved?