Challenge 2: Surviving the zombie apocalypse

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1 Introduction

Nowadays, graphs are present in many aspects of everyday life such as: social networks [1, 2], internet and web [3], biological structures [4, 5], organization of individuals [6, 7] and energy transport [8]. By representing an object (individuals, documents, populations) as network nodes [2, 4, 7, 8] it is not only possible to determine their close relationships with graph theory, but also simulations [9], predictions [10] and modeling [11] of dynamics that spread over the network can be carried out.

A dynamic in a network can be understood as a force or property that stimulates, grows or develops over a network, effecting changes in it [12]. We find multiple types of dynamics in a network, such as the spread of a comment, the change of opinions and the spread of diseases. Moreover, these dynamics have both global and local repercussions throughout the network. Sometimes, depending on the dynamics, it can be observed how the network completely changes its states, depending on the origin of the event.Thus, different types of actions can be taken into account, from actions that exert control over the propagation of the dynamics, to actions that completely change the states of each node. As required, the actions could be predicted using graph theory.

Therefore, the paper presents the use of graph theory tools to take control actions on the spread of epidemics. For this, we will model a hypothetical dynamic such as a zombie epidemic. Consequently, the document is structured as follows: in Section 2 we will present in depth the problem to attack, showing the methodology and tasks to be performed in Section 3 and Section 4, respectively.

2 Problem presentation

As an illustrative example of how an action can exert control over the global state of a process in a community, let us assume the hypothetical case of a zombie apocalypse. We define the dynamics to be modeled as the geopositional interactions of groups of people on the European continent. In order to describe the dynamics to be implemented, let's look at the whole context.

The apocalypse has a start date of August 18th, 2019, originating in the town Rize, Turkey, and which spreads in Europe. At that time, the European continent had a population density shown in the map in the Figure 1a, where each pixel represents a population density between $[0, 3000]$ **inhabitants per Km**^{2} (`in/Km`^{2}), where white color represent the maximum population density and black the minimum.

(a) (b)

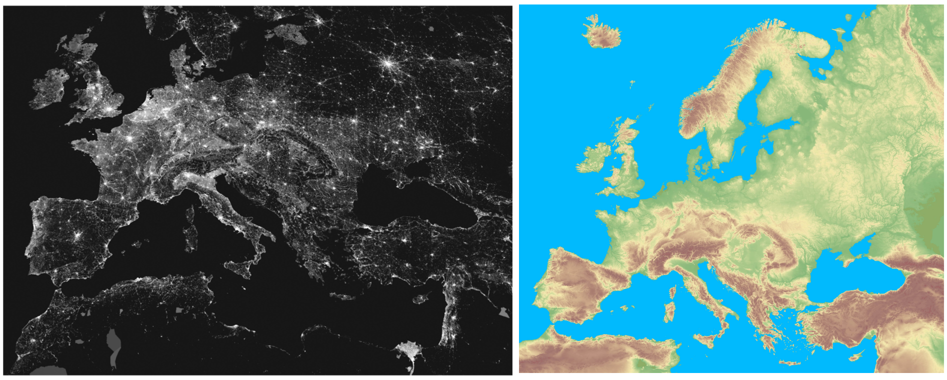

Figure 1: Maps with the description of (a) initial population density expressed in grayscale on August 18, 2019 and (b) elevation of the terrain where the apocalypse takes place.

Studies related to the apocalypse showed that zombies prefer to move in areas with low slope to areas with high slope, represented by the geography of the terrain (Figure 1b). Slopes steeper than 10 degrees are not traversable by zombies. Additionally, scientists demonstrated that zombie communities spread omnidirectionally, depending on human population density. More humans in an area will then imply a greater transition of zombies to that location, compared to an area with low (or null) density of humans per surface area.

In addition, the scientists state that the zombies move at night, eat and infect the population they encounter in the morning hours and cease activities in the afternoon, a period that the survivors take advantage of to annihilate as many zombies as possible. The report shows that the physical capabilities of the zombies increase considerably after "being converted", allowing them to travel distances of up to 15 Km per day. Unfortunately for them, the virus does not allow them to live for more than 15 days after their transformation.

Finally, the documents submitted to the **European Union** (EU) show that the maximum infection capacity of a zombie is 10 humans per day. Every infected human becomes a zombie. Likewise, a survivor is capable of annihilating a total of 10 zombies per day.

Following this tragic report, the EU decided to deploy 20 military troops to contain the epidemic within 2 months of the first reported case of infection. The soldiers were trained to protect the surviving population of the area to which they were assigned, as well as to prevent the arrival of more zombies in the area. EXPLICAR MÁS SOBRE LA HIPOTESIS DE ESTA ACCION.

If the emergency measures do not completely stop the invasion, the EU would be ready to bombard the European continent with nuclear bombs 4 months after the beginning of the epidemic, eradicating all forms of life (except for salamanders) where the bomb will hit. The after-effects of the radiation would not allow any organism to transit the area again. The drastic measures would be executed in order to save as many survivors as possible.

To accomplish these tasks, the EU decided to hire a group of engineers located in Brest, France in order to make the decision of the appropriate locations to position the military troops and deploy the bombs (if necessary). The agreement reached between the engineers and the EU privileges the salvation of the engineers, so Brest will be considered as a priority city. It will be important to know the dates on which the zombies arrive in this city.

In order to present the actions taken, Section 3 explains the methodology to be implemented for the development of the project.

3 Methodology

The problematic follows a model of a zombie infection, wherein the infection spreads rapidly among the human population converting them to zombies in the process. We use networks as the topology and allow our human and zombies agents to act on and change this topology. Additionally, we take a dynamic **Suceptible - Infected - Recovered** (SIR) as referential propagation model, with three possible scenarios:

1. Humans are susceptible to becoming zombies.
2. Zombies can die.
3. Dead people cannot revive.

As a first objective we must capture the population density from an image and translate it into a graph network, this will be done in order to determine the number of infected humans and how will be the behavior of the zombies to move from cell to cell. From the movements of the zombies, the contribution of the neighboring cells and their current state at a selected time instant will be determined.

At the same time it will be interesting to analyze the movement of the zombies since not all zombies will be able to move due to slope constraints, therefore it is necessary to analyze how many zombies can move in a given cell.

4 Tasks to perform

In order to answer the questions formulated in the challenge, the following tasks must be performed:

1. **Data understanding:** perform a statistical analysis of the information in order to understand the distribution of the data of population and altitude maps
2. **Data preparation:** In order to apply graph strategies, it is necessary to translate the information from the maps to graphs. Each node corresponds to an area of 15 x 15 pixels where there are *attributes of human population and zombie population*, the connections between nodes (edges) will be defined by the possibility of a zombie to move from one cell to another, a weight between 0 and 1, where 0 indicates the impossibility of moving from one cell to another and 1 indicates complete ease of movement.
3. **Modeling - definition:** definition of techniques to be used as solution models, and the metrics to evaluate the performance of each model.
4. **Modeling - develop:** implementation of the different models stipulated, with the evaluation of the metrics established above.
5. **Evaluation:** Extract the best model able to response to the questions of the challenge: 1. Find the 20 cells in which it would be most appropriate to send soldiers two months after the start of the pandemic and 2. Find the number and location where nuclear bombs should be sent to stop the zombies and save as many people as possible.

5 Definition of the epidemic propagation model

After having modeled the geographic space as a network, with the definition of the nodes, their attributes and connections, the purpose of this section is to define and implement the interaction dynamics between nodes, which will be defined with human/zombie population as attributes on each node.

5.1 Description of the spreading dynamics

To describe the dynamics of epidemic spread, take $H_j(c_i)$ to be the population of humans and $Z_j(c_i)$ the population of zombies in node c_i and at day j. Each day, the zombie population on c_i is distributed to its 8 neighbors. That is, the possible movements of $Z_j(c_i)$ would be given by Figure 2.

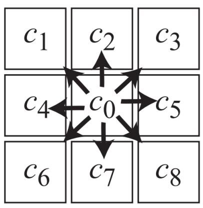

Figure 2: Set of possible movements of the zombie population given its initial node (c_0) to its neighboring (c_i).

As mentioned in Section 2, every day the zombies move to their neighbors (omnidirectionally) according to the geographical elevation of the terrain (given by $\lambda_{0,i}$ factor) and the human population density. In principle, all zombies are forced to move from their node of origin, unless there is no human population around them or it is not possible for them to reach it, which would force them to stay at the node. We can then define the contribution of zombies $C_{j+1}(c_0, c_i)$ (Equation 1) from node c_0 to c_i as the number of zombies that move between these cells. This relationship could be posed as a weighted sum with $\lambda_{0,i}$ factor.

$$C_{j+1}(c_0, c_i) = \begin{cases} \left\lfloor \dfrac{\lambda_{0,i} H_j(c_i)}{\sum_{k=1}^{8} \lambda_{0,k} H_j(c_k)} Z_j(c_0) \right\rfloor & \text{if } i \neq 0 \text{ and } \sum \lambda_{0,k} H_j(c_k) > 0 \\ Z_j(c_0) & \text{if } i = 0 \text{ and } \sum \lambda_{0,k} H_j(c_k) = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

We clarify that $\lambda_{0,i}$ is a factor of geographical slope between c_0 and c_i: is zero for a slope higher than 10 degrees, it is one for a slope of zero, and linear between these two values for slopes between 0 and 10 degrees. On the other hand, when handling decimal values it may be the case that decimal contributions are obtained, giving a problem (it is not possible for 1.5 zombies to move nodes!). To solve this, we round the contribution result. The remaining number of zombies should remain on the same node (assuming that they did not change nodes). For this, we define the set of remaining zombies $\hat{Z}_{j+1}(c_0)$ in Equation 2.

$$\hat{Z}_{j+1}(c_0) = Z_j(c_0) - \sum_{k=0}^{8} C_{j+1}(c_0, c_k) \tag{2}$$

Ideally, $\hat{Z}_{j+1}(c_0) = 0$ if the entire zombie population spreads to its neighbors as a whole. Now, we define $z_j^l(c_i)$ as the zombies sub-population with age l, gives in days. We know that a zombie can be no more than 15 days old, so $l = \{0, 1, 2, \cdots, 14\}$, and thus $Z_j(c_i) = \sum_{l=0}^{14} z_j^l(c_i)$. This allows, together with $C_{j+1}^l(c_i, c_k)$ as the zombie contribution from nodes c_i to node c_k ($c_k \in \mathcal{N}_i$ as the neighbors of c_i) and age l, to define the propagation dynamics by means of the following algorithm:

Step 0. Updating zombies ages, eliminating those zombies older than 15 days.

$$z_j^l(c_i) = \begin{cases} z_j^{l-1}(c_i) & \text{if } 0 < l \leq 15 \\ 0 & \text{otherwise} \end{cases}, \qquad \text{with } l = \{0, 1, 2, \cdots, 15\}$$

Step 1. Estimate the contribution of zombies $C_{j+1}^l(c_i, c_k)$ in the entire network as well as the remaining subsets $\hat{z}_{j+1}^l(c_i)$. This enables the calculation of the zombie population on c_i.

$$z_{j+1}^l(c_i) = \sum_{c_k \in \{\mathcal{N}_i, c_i\}} C_{j+1}^l(c_k, c_i) + \hat{z}_{j+1}^l(c_i)$$

Step 2. Calculate the number of human deaths: each zombie is capable of killing 10 humans, who will become zombies. The new zombies will clearly have an age of zero.

$$H_{j+1}(c_i) = \begin{cases} H_j(c_i) - 10Z_{j+1}(c_i) & \text{if } 10Z_{j+1}(c_i) \leq H_j(c_i) \\ 0 & \text{otherwise} \end{cases}$$

$$z_{j+1}^0(c_i) = H_j(c_i) - H_{j+1}(c_i) = \begin{cases} 10Z_{j+1}(c_i) & \text{if } 10Z_{j+1}(c_i) \leq H_j(c_i) \\ H_j(c_i) & \text{otherwise} \end{cases}$$

Step 3. Calculate the amount of human resistance: each surviving human is able to annihilate 10 zombies, which will never revive again. Each zombie has the same probability of dying, regardless of its age.

$$z_{j+1}^l(c_i) = \begin{cases} \left\lfloor z_{j+1}^l(c_i) \left(1 - 10\dfrac{H_{j+1}(c_i)}{Z_{j+1}(c_i)}\right) \right\rfloor & \text{if } 10H_{j+1}(c_i) \leq Z_{j+1}(c_i) \text{ and } Z_{j+1}(c_i) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Step 4. Updating current day in stocks for future iterations.

$$z_j^l(c_i) = z_{j+1}^l(c_i)$$
$$H_j(c_i) = H_{j+1}(c_i)$$

Note: It is easy to demonstrate in step 3 that $Z_{j+1}(c_i) = Z_{j+1}(c_i) - 10H_{j+1}(c_i)$ if $10H_{j+1}(c_i) \leq Z_{j+1}(c_i)$ else 0, which shows that the zombie population is being reduced in proportion to 10 times the human population.

5.2 Description of actions set

As mentioned in the Section 2, two possible actions can be implemented as preventive measures to control the epidemic. Each measure has repercussions on the propagation dynamics, directly affecting the behavior of the equations described in Sub-section 5.1. To determine the changes they produce, let us take $c_i^* \in C_i^*$ as the set of nodes directly affected by the actions.

1. **Reinforcements by military troops:** Two months after the beginning of the epidemic, the `EU` decides to send military troops to 20 nodes. Each troop is in charge of eliminating the zombie population of that node, so $z_{j_m}^l(c_i^*) = 0 \; \forall l$ and for j_m = October 19th, 2019. In addition, the troops are in charge of making it impossible for zombies to pass through the node. This measure can be interpreted as a change in the elevation factor, given by Equation 3.

$$\lambda_{k,i}^* = \begin{cases} \lambda_{k,i} & \text{if } j < j_m \text{ or } c_i \notin C_i^* \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

   Given $\lambda_{k,i}^*$ the new λ factor.

2. **Nuclear bombs:** Four months after the start of the epidemics, the `EU` takes the decision of using nuclear weapons to save the rest of humanity. The development of this action is the same as the reinforcement with military troops, except that on the day they explode (j_b = December 19th, 2019) they also eliminate the human population in the affected nodes, i.e. $H_{j_b}(c_i^*) = 0$.

After defining the mathematical scheme, the results after implementing the algorithm will be presented.

5.3 Propagation simulation

6 Spreading Blockers Nodes

The structural properties of a dynamic network usually reveal important information about its dynamics and function. This is particularly true if we take into account the relationship between the position occupied by a node in a static graph and the role played by the node for the evolution of a dynamic process. For instance, not all nodes have the same impact on the transmission of a disease or virus over a network: intuitively, the nodes having a higher number of neighbours should contribute much more to the spreading than nodes having few connections. However, we observe that not just the number of edges is important to identify good spreaders, since also the actual organisation of these edges has an impact on the speed of the spreading process. In fact, nodes mediating a large number of shortest paths are indeed those that contribute the most to the transmission of diseases and information over a network. The identification of nodes that play a central role, i.e., nodes having high centrality is a good approach to find the spreading blocker nodes that could help us to stop the virus in Brest.

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