AP STATISTICS

UNIT 8

Inference for Categorical Data: Chi-Square

2–5%
AP EXAM WEIGHTING

~10–11
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topics and skills.

Personal Progress Check 8

Multiple-choice: ~30 questions
Free-response: 2 questions

- Inference
- Inference and Exploring Data/ Collecting Data

UNIT 8

2–5% AP EXAM WEIGHTING | **~10–11** CLASS PERIODS

Inference for Categorical Data: Chi-Square

BIG IDEA 1
Variation and Distribution VAR
- How does increasing the degrees of freedom influence the shape of the chi-square distribution?

BIG IDEA 3
Data-Based Predictions, Decisions, and Conclusions DAT
- Why is it inappropriate to use statistical inference to justify a claim that there is *no association* between variables?

Developing Understanding

Unit 6 introduced inference for proportions of categorical data. Unit 8 introduces chi-square tests, which can be used when there are two or more categories. Students need to understand how to select from the following tests: the chi-square test for goodness of fit (for a distribution of proportions of one categorical variable in a population), the chi-square test for independence (for associations between categorical variables within a single population), or the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding, teachers can make connections between frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between observed and expected counts relative to expected counts.

Building Course Skills

1.E 3.E 4.C 4.E

In Unit 8, students should continue applying the same problem-solving structure to chi-square significance testing: State the hypotheses in words, explicitly identify the correct procedure, verify conditions, calculate the test statistic and the p-value, and then draw a conclusion in context that is directly linked to the p-value. Students should have opportunities to practice the distinctive elements for each type of chi-square test, such as analysis of expected counts, degrees of freedom, verbally stated hypotheses, and two-way tables.

When the p-value is large, drawing an appropriate conclusion is challenging for students. Saying there is "no association" between two variables is equivalent to incorrectly "accepting the null hypothesis." Instead, teachers can teach students to use nondeterministic language in their conclusions, that is, "The data do not provide strong enough evidence to conclude that the variables are associated." Students should have frequent opportunities to practice writing, with detailed feedback to help them improve.

Preparing for the AP Exam

When writing hypotheses, students should refer to the population, using language from the question. For example, "The null hypothesis is that the age group at diagnosis and gender are independent (i.e., they are not associated) for the population of people currently being treated for schizophrenia" (see **Scoring Guidelines for 2017 FRQ 5**). As always, students should name the test and provide evidence verifying appropriate conditions. For chi-square tests, the conditions are (1) random selection or randomized experiment and (2) large counts. Students should be sure to say that all expected counts (rather than actual counts) are at least 5. Students need to clearly present calculations and state the conclusion in context with linkage to p-values. Students should avoid tacitly accepting the null hypothesis. If the p-value is greater than conventional significance levels, the correct conclusion of a chi-square test for independence would be that there is insufficient evidence that there is an association.

UNIT AT A GLANCE

Enduring Understanding	Topic	Skills	Class Periods ~10–11 CLASS PERIODS
VAR-1	**8.1 Introducing Statistics: Are My Results Unexpected?**	1.A Identify the question to be answered or problem to be solved *(not assessed)*.	
VAR-8	**8.2 Setting Up a Chi-Square Goodness of Fit Test**	3.C Describe probability distributions. 1.F Identify null and alternative hypotheses. 1.E Identify an appropriate inference method for significance tests. 3.A Determine relative frequencies, proportions, or probabilities using simulation or calculations. 4.C Verify that inference procedures apply in a given situation.	
VAR-8, DAT-3	**8.3 Carrying Out a Chi-Square Test for Goodness of Fit**	3.E Calculate a test statistic and find a p-value, provided conditions for inference are met. 4.B Interpret statistical calculations and findings to assign meaning or assess a claim. 4.E Justify a claim using a decision based on significance tests.	
VAR-8	**8.4 Expected Counts in Two-Way Tables**	3.A Determine relative frequencies, proportions, or probabilities using simulation or calculations.	
	8.5 Setting Up a Chi-Square Test for Homogeneity or Independence	1.F Identify null and alternative hypotheses. 1.E Identify an appropriate inference method for significance tests. 4.C Verify that inference procedures apply in a given situation.	
VAR-8, DAT-3	**8.6 Carrying Out a Chi-Square Test for Homogeneity or Independence**	3.E Calculate a test statistic and find a p-value, provided conditions for inference are met. 4.B Interpret statistical calculations and findings to assign meaning or assess a claim. 4.E Justify a claim using a decision based on significance tests.	
	8.7 Skills Focus: Selecting an Appropriate Inference Procedure for Categorical Data	N/A	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 8. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate various instructional approaches into the classroom. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 207 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	8.1	**Simulation** Prepare several bags with an identical mix of at least 250 chips or beads of three colors in different proportions (e.g., red = 0.5, white = 0.3, blue = 0.2). Have each student take a random sample of 25 chips or beads from the bag, calculate $\sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$, and plot their value on a class dotplot. Use this graph to introduce the chi-square distribution with $df = 2$.
2	8.5	**Discussion Groups** Give each group of three to four students an example of a chi-square test involving a two-way table. Have students work together to state appropriate hypotheses, describe a Type 1 and Type 2 error in context, and give a possible consequence of each of those errors.
3	8.7	**Graphic Organizer** Have students work in teams of two to three to develop a chart that summarizes the three types of chi-square tests, including when each is appropriate, as well as the hypotheses, conditions, and degrees of freedom.

SKILL
Selecting Statistical Methods

1.A
Identify the question to be answered or problem to be solved.

TOPIC 8.1
Introducing Statistics: Are My Results Unexpected?

Required Course Content

ENDURING UNDERSTANDING

VAR-1
Given that variation may be random or not, conclusions are uncertain.

LEARNING OBJECTIVE

VAR-1.J
Identify questions suggested by variation between observed and expected counts in categorical data. **[Skill 1.A]**

ESSENTIAL KNOWLEDGE

VAR-1.J.1
Variation between what we find and what we expect to find may be random or not.

TOPIC 8.2
Setting Up a Chi-Square Goodness of Fit Test

Required Course Content

ENDURING UNDERSTANDING

VAR-8

The chi-square distribution may be used to model variation.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
VAR-8.A Describe chi-square distributions. **[Skill 3.C]**	VAR-8.A.1 Expected counts of categorical data are counts consistent with the null hypothesis. In general, an expected count is a sample size times a probability.
	VAR-8.A.2 The chi-square statistic measures the distance between observed and expected counts relative to expected counts.
	VAR-8.A.3 Chi-square distributions have positive values and are skewed right. Within a family of density curves, the skew becomes less pronounced with increasing degrees of freedom.
VAR-8.B Identify the null and alternative hypotheses in a test for a distribution of proportions in a set of categorical data. **[Skill 1.F]**	VAR-8.B.1 For a chi-square goodness-of-fit test, the null hypothesis specifies null proportions for each category, and the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis.

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SKILLS

Using Probability and Simulation

3.C
Describe probability distributions.

Selecting Statistical Methods

1.F
Identify null and alternative hypotheses.

1.E
Identify an appropriate inference method for significance tests.

Using Probability and Simulation

3.A
Determine relative frequencies, proportions, or probabilities using simulation or calculations.

Statistical Argumentation

4.C
Verify that inference procedures apply in a given situation.

AVAILABLE RESOURCE

- Classroom Resource > Inference

UNIT 8

Inference for Categorical Data: Chi-Square

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
VAR-8.C Identify an appropriate testing method for a distribution of proportions in a set of categorical data. **[Skill 1.E]**	**VAR-8.C.1** When considering a distribution of proportions for one categorical variable, the appropriate test is the chi-square test for goodness of fit.
VAR-8.D Calculate expected counts for the chi-square test for goodness of fit. **[Skill 3.A]**	**VAR-8.D.1** Expected counts for a chi-square goodness-of-fit test are (sample size) (null proportion).
VAR-8.E Verify the conditions for making statistical inferences when testing goodness of fit for a chi-square distribution. **[Skill 4.C]**	**VAR-8.E.1** In order to make statistical inferences for a chi-square test for goodness of fit we must check the following: a. To check for independence: i. Data should be collected using a random sample or randomized experiment. ii. When sampling without replacement, check that $n \leq 10\%N$. b. The chi-square test for goodness of fit becomes more accurate with more observations, so large counts should be used (shape). i. A conservative check for large counts is that all expected counts should be greater than 5.

TOPIC 8.3
Carrying Out a Chi-Square Test for Goodness of Fit

Required Course Content

ENDURING UNDERSTANDING

VAR-8

The chi-square distribution may be used to model variation.

LEARNING OBJECTIVE

VAR-8.F

Calculate the appropriate statistic for the chi-square test for goodness of fit.
[Skill 3.E]

ESSENTIAL KNOWLEDGE

VAR-8.F.1

The test statistic for the chi-square test for goodness of fit is

$$\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count},$$

with

$$degrees\ of\ freedom = number\ of\ categories - 1.$$

VAR-8.F.2

The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or, when a probability model is assumed to be true, a theoretical distribution (chi-square).

VAR-8.G

Determine the p-value for chi-square test for goodness of fit significance test.
[Skill 3.E]

VAR-8.G.1

The p-value for a chi-square test for goodness of fit for a number of degrees of freedom is found using the appropriate table or computer generated output.

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SKILLS

Using Probability and Simulation

3.E

Calculate a test statistic and find a p-value, provided conditions for inference are met.

Statistical Argumentation

4.B

Interpret statistical calculations and findings to assign meaning or assess a claim.

4.E

Justify a claim using a decision based on significance tests.

AVAILABLE RESOURCE

- Classroom Resource > Inference

UNIT 8

Inference for Categorical Data: Chi-Square

ENDURING UNDERSTANDING

DAT-3

Significance testing allows us to make decisions about hypotheses within a particular context.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
DAT-3.I Interpret the p-value for the chi-square test for goodness of fit. **[Skill 4.B]**	DAT-3.I.1 An interpretation of the p-value for the chi-square test for goodness of fit is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
DAT-3.J Justify a claim about the population based on the results of a chi-square test for goodness of fit. **[Skill 4.E]**	DAT-3.J.1 A decision to either reject or fail to reject the null hypothesis is based on comparison of the p-value to the significance level, α. DAT-3.J.2 The results of a chi-square test for goodness of fit can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.

TOPIC 8.4
Expected Counts in Two-Way Tables

SKILL

Using Probability and Simulation

3.A

Determine relative frequencies, proportions, or probabilities using simulation or calculations.

Required Course Content

ENDURING UNDERSTANDING

VAR-8

The chi-square distribution may be used to model variation.

LEARNING OBJECTIVE

VAR-8.H

Calculate expected counts for two-way tables of categorical data. **[Skill 3.A]**

ESSENTIAL KNOWLEDGE

VAR-8.H.1

The expected count in a particular cell of a two-way table of categorical data can be calculated using the formula:

$$expected\ count = \frac{(row\ total)(column\ total)}{table\ total}.$$

SKILLS

Selecting Statistical Methods

1.F
Identify null and alternative hypotheses.

1.E
Identify an appropriate inference method for significance tests.

Statistical Argumentation

4.C
Verify that inference procedures apply in a given situation.

AVAILABLE RESOURCE
- Classroom Resource > **Inference**

TOPIC 8.5
Setting Up a Chi-Square Test for Homogeneity or Independence

Required Course Content

ENDURING UNDERSTANDING

VAR-8
The chi-square distribution may be used to model variation.

LEARNING OBJECTIVE

VAR-8.I
Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence. **[Skill 1.F]**

ESSENTIAL KNOWLEDGE

VAR-8.I.1
The appropriate hypotheses for a chi-square test for homogeneity are:

H_0: There is no difference in distributions of a categorical variable across populations or treatments.

H_a: There is a difference in distributions of a categorical variable across populations or treatments.

VAR-8.I.2
The appropriate hypotheses for a chi-square test for independence are:

H_0: There is no association between two categorical variables in a given population or the two categorical variables are independent.

H_a: Two categorical variables in a population are associated or dependent.

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LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
VAR-8.J Identify an appropriate testing method for comparing distributions in two-way tables of categorical data. **[Skill 1.E]**	**VAR-8.J.1** When comparing distributions to determine whether proportions in each category for categorical data collected from different populations are the same, the appropriate test is the chi-square test for homogeneity. **VAR-8.J.2** To determine whether row and column variables in a two-way table of categorical data might be associated in the population from which the data were sampled, the appropriate test is the chi-square test for independence.
VAR-8.K Verify the conditions for making statistical inferences when testing a chi-square distribution for independence or homogeneity. **[Skill 4.C]**	**VAR-8.K.1** In order to make statistical inferences for a chi-square test for two-way tables (homogeneity or independence), we must verify the following: a. To check for independence: i. For a test for independence: Data should be collected using a simple random sample. ii. For a test for homogeneity: Data should be collected using a stratified random sample or randomized experiment. iii. When sampling without replacement, check that $n \leq 10\%N$. b. The chi-square tests for independence and homogeneity become more accurate with more observations, so large counts should be used (shape). i. A conservative check for large counts is that all expected counts should be greater than 5.

SKILLS

Using Probability and Simulation

3.E

Calculate a test statistic and find a p-value, provided conditions for inference are met.

Statistical Argumentation

4.B

Interpret statistical calculations and findings to assign meaning or assess a claim.

4.E

Justify a claim using a decision based on significance tests.

AVAILABLE RESOURCE

- Classroom Resource > Inference

TOPIC 8.6
Carrying Out a Chi-Square Test for Homogeneity or Independence

Required Course Content

ENDURING UNDERSTANDING

VAR-8

The chi-square distribution may be used to model variation.

LEARNING OBJECTIVE

VAR-8.L

Calculate the appropriate statistic for a chi-square test for homogeneity or independence. **[Skill 3.E]**

VAR-8.M

Determine the p-value for a chi-square significance test for independence or homogeneity. **[Skill 3.E]**

ESSENTIAL KNOWLEDGE

VAR-8.L.1

The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic:

$$\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count},$$

with degrees of freedom equal to:

$$(number\ of\ rows - 1)(number\ of\ columns - 1).$$

VAR-8.M.1

The p-value for a chi-square test for independence or homogeneity for a number of degrees of freedom is found using the appropriate table or technology.

VAR-8.M.2

For a test of independence or homogeneity for a two-way table, the p-value is the proportion of values in a chi-square distribution with appropriate degrees of freedom that are equal to or larger than the test statistic.

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Inference for Categorical Data: Chi-Square

ENDURING UNDERSTANDING

DAT-3

Significance testing allows us to make decisions about hypotheses within a particular context.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
DAT-3.K Interpret the p-value for the chi-square test for homogeneity or independence. **[Skill 4.B]**	DAT-3.K.1 An interpretation of the p-value for the chi-square test for homogeneity or independence is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
DAT-3.L Justify a claim about the population based on the results of a chi-square test for homogeneity or independence. **[Skill 4.E]**	DAT-3.L.1 A decision to either reject or fail to reject the null hypothesis for a chi-square test for homogeneity or independence is based on comparison of the p-value to the significance level, α. DAT-3.L.2 The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to a research question about the population that was sampled (independence) or the populations that were sampled (homogeneity).

TOPIC 8.7
Skills Focus: Selecting an Appropriate Inference Procedure for Categorical Data

AVAILABLE RESOURCE
- Classroom Resource > **Inference**

Required Course Content

This topic is intended to focus on the skill of selecting an appropriate inference procedure now that students have a range of options. Students should be given opportunities to practice when and how to apply all learning objectives relating to inference for categorical data.