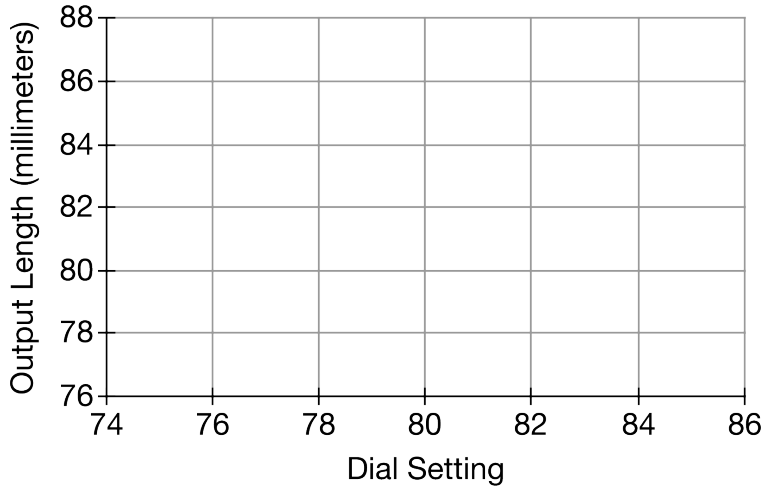
Unit 9 Progress Check: FRQ

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

At a plant that manufactures bars of steel, a machine is used to cut the bars to specific lengths. The machine has a dial that sets the length of the bars to be cut. However, the dial is currently out of alignment and the plant manager is collecting data to assess the situation. The following table shows 8 trials at different dial settings along with the actual output length of the bars that were cut. All measurements are in millimeters.

Dial Setting	Output Length
75	78
77	79
79	82
80	83
81	85
82	83
83	86
85	88

(a) Use the following grid to construct a scatterplot in which dial setting is the explanatory variable and output length is the response variable. Based on your graph, does a linear model seem appropriate? Justify your answer.



(b) Use the data to construct a least-squares regression line to predict output length from dial setting.

(c) Assume that all conditions for inference are met. Indicate the hypotheses appropriate to test whether there is a linear relationship between output length and dial setting.

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(d) The test statistic for the appropriate test is $t = 9.018$. Do the data provide convincing statistical evidence that there is a linear relationship between output length and dial setting?

Part A, B, C, and D

The primary goals of this question are to assess a student's ability to (1) construct a scatterplot and describe the form of the association shown in the scatterplot; (2) create a least-squares regression equation from data; (3) construct the hypotheses for a test for the slope of a regression line; and (4) draw a conclusion for a test for the slope of a regression line.

Each essentially correct (E) part counts as 1 point.

Each partially correct (P) part counts as $1/2$ point.

Scoring

Parts (a), (b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

If a response is between two scores (for example, $2\,1/2$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

Reasons to score up:

· *All notation is correct and clearly marked*

· *All explanations are clear*

· *No wrong information is included that was not part of the scoring (for example, saying sample size must be greater than 30 when that has nothing to do with the problem)*

· *No minor calculation errors are made, if they are not part of the scoring*

· *Interpretation parts are especially strong*

Reasons to score down:

· *Notation is not wrong, but is spotty and not clearly marked*

· *Explanations are not wrong, but are hard to follow*

· *Wrong or extraneous information is included but not part of scoring*

· *Minor calculation errors that are not part of the scoring are made*

· *Interpretation parts are scored an E but are considered a weak E*

0	1	2	3	4

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Parts (a) through (d) sum to 4 points

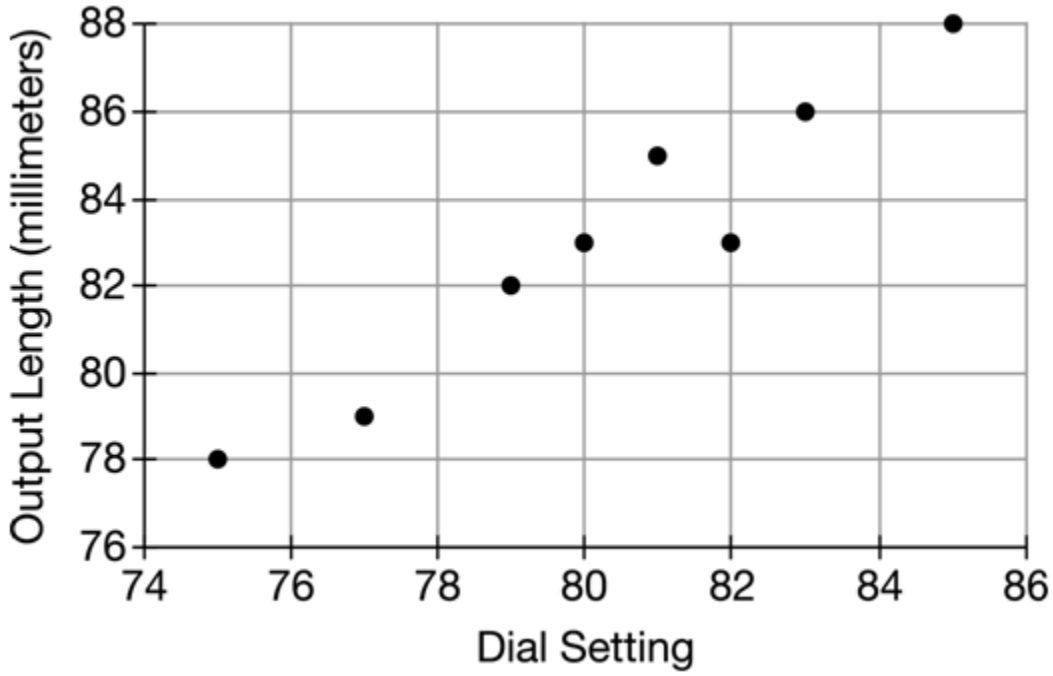
OR

Parts (a) through (d) sum to 3½ points AND a holistic approach is used to decide to score up

- [ ] Part (a) essentially correct
- [ ] Part (a) partially correct
- [ ] Part (a) incorrect
- [ ] Part (b) essentially correct
- [ ] Part (b) partially correct
- [ ] Part (b) incorrect
- [ ] Part (c) essentially correct
- [ ] Part (c) partially correct
- [ ] Part (c) incorrect
- [ ] Part (d) essentially correct
- [ ] Part (d) partially correct
- [ ] Part (d) incorrect

Solution

Part (a):



The relationship between Output Length and Dial Setting appears to be fairly strong and linear. There does not appear to be any substantial curvature nor do there seem to be any outliers, so a linear model seems appropriate in this situation.

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Scoring

Part (a) is scored as follows:

Essentially correct (E) if the response correctly constructs a scatterplot *AND* indicates, with justification, that a linear model seems appropriate

Partially correct (P) if the response fails to accurately construct a plot but indicates, with justification, that a linear model seems appropriate

OR

If the response constructs an appropriate scatterplot but fails to justify that a linear model seems appropriate

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (b):

The least-squares regression line, found using technology, is the following:
$$\text{Predicted Output Length} = 2.204 + 1.007(\text{Dial Setting})$$

Scoring

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· Indicates the estimate for the slope of the least-squares regression line is approximately 1

· Indicates the estimate for the intercept of the least-squares regression line is approximately 2.2

· The explanatory and response variables are clearly labeled in the least-squares regression line OR the variables are clearly defined

Partially correct (P) if the response satisfies two of the three components

OR

if the response reverses the explanatory and response variables and finds the equation
$$\text{Predicted Dial Setting} = 3.475 + 0.925(\text{Output Length})$$

Incorrect (I) if the response does not satisfy the criteria for E or P

Note:

The following are examples of clearly labeled variables that satisfy component three.

· x = Dial Setting and y =Output Length, where $\hat{y} = 2.204 + 1.007x$

· x = Dial Setting and $\hat{y}$ = predicted Output Length, where $\hat{y} = 2.204 + 1.007x$

Solution

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Part (c):

The hypotheses are as follows:

H_0 : The slope of the population regression line relating output length and dial setting is equal to 0.

H_a : The slope of the population regression line relating output length and dial setting is not equal to 0.

Scoring

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

· Provides a null hypothesis that states that there not is a linear relationship

· Provides an alternative hypothesis that states that there is a linear relationship

Partially correct (P) if the response satisfies one of the two components

Incorrect (I) if the response does not satisfy the criteria for E or P

Note:

The following are examples of acceptable hypotheses.

· $H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$

· H_0 : There is not a linear relationship between output length and dial setting in the population.

H_a : There is a linear relationship between output length and dial setting in the population.

Solution

Part (d):

The *p*-value is 0.0001 based on a *t*-test with $n - 2 = 6$ degrees of freedom. The *p*-value is less than any reasonable level of significance. Reject the null hypothesis. There is statistical evidence to support the claim that there is a linear relationship between output length and dial setting.

Scoring

Part (d) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· Comparison of the *p*-value to a predetermined level of significance, such as 0.01 or 0.05

OR

Comparison of the test statistic to a critical value, based on a predetermined level of significance, such as 0.01 or 0.05

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· Correctly rejects the null hypothesis

· States conclusion in context

Partially correct (P) if the response satisfies only two of the three components

Incorrect (I) if the response does not meet the criteria for E or P