Inverse Kinematics.

— Robo Analyzer Online (competition CROC)

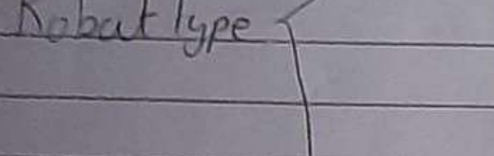
Robot → Device that can carry out task

→ motion → Study → Kinematics

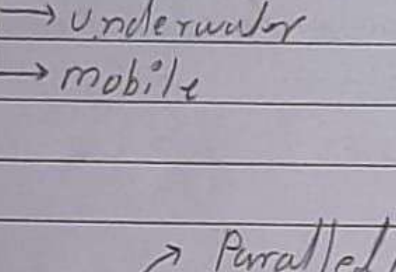
Robot type → moving base →
- → Areal
- → underwater
- → mobile

→ Fixed base
- → Parallel Robot
- → Serial Robot

Serial Robot



F_{kin} (forward kinematics) of Serial Planar Manipulator.



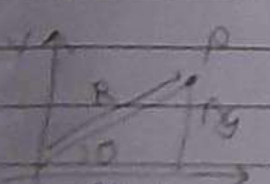
If θ_1 & θ_2 are given. We can find P_x and P_y.

I_{kin} (Inverse kinematics)

If P_y and P_x are given then find θ_1 & θ_2.

P = Endeffector point or (Pose) → In simple : tip of point

For F_{kin} of 2R Plane Manipulator.



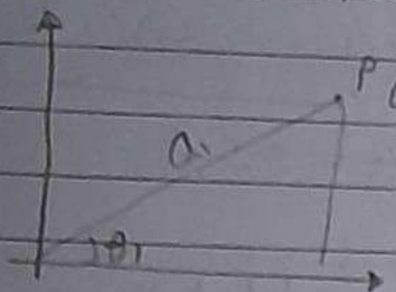
$\vec{OP} = \vec{OA} + \vec{AP}$ $\quad OP = (P_x)\hat{i} + (P_y)\hat{j} = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$

If $|\vec{OP}| = R$ & $\angle POA = \theta$

$\vec{OP} = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$ $= \vec{OP} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$

$\boxed{R = \text{One Rotatory arm}}$

for 2R Manipulator



$P_x = a_1 \cos\theta = a\,c_\theta$

$P_y = a_1 \sin\theta = a\,s_\theta$

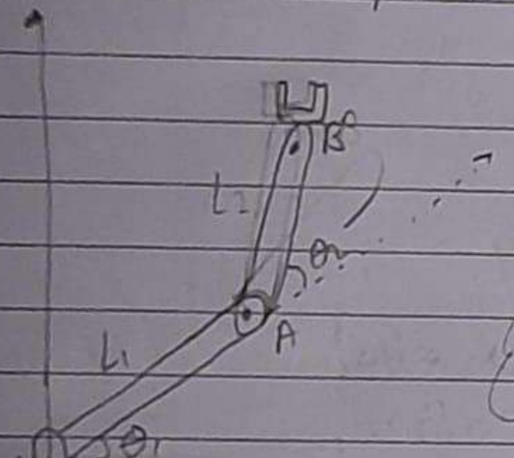
F.kin ⟶ If we know θ find P_x & P_y

I.kin ⟶ If we know (P_x, P_y) determine θ_1

$$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

for 2R Manipulator

L_1 & L_2 = Link

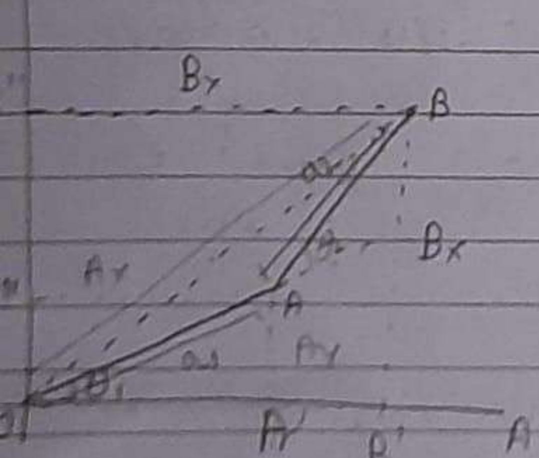


θ_1 is Inclination of Link 1 w.r.t 0

θ_2 is Inclination (Relative) of Link 2 w.r.t θ_1

Coordinate of Point B as function joint angle θ_1 and θ_2

→ Geometry Methode



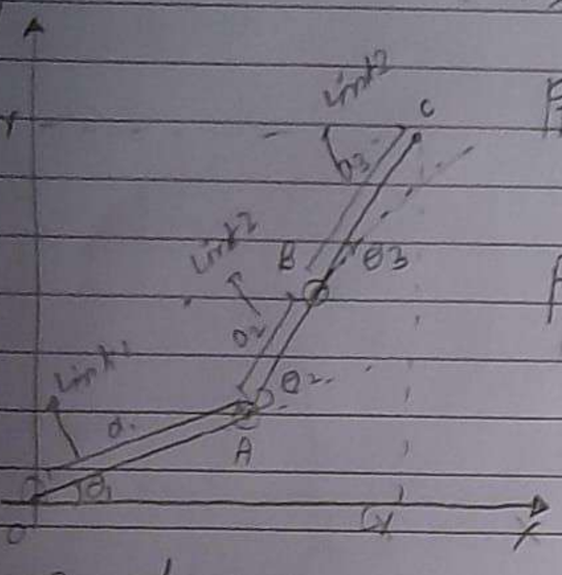
$$\vec{OB} = \begin{bmatrix} OB' \\ OB'' \end{bmatrix} = \begin{bmatrix} OA' + A'B' \\ OA'' + A''B'' \end{bmatrix}$$

$$\boxed{\begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} a_1 C\theta & + a_2\, C(\theta_1 + \theta_2) \\ a_1 S\theta & a_2\, S(\theta_1 + \theta_2) \end{bmatrix}}$$

for θ_1 & θ_2 determine B_x and B_y

Forward kinematics of 3R Plane Manipulator

3R: 3 Revolute → Planer → (We will Study)
→ Spatial → Articulated Arm



If θ_1, θ_2 & θ_3 are given determine the pose of the end effector (c)

Pose ≅ Configuration (C) → Position
→ Orientation

for link 3 pose is (C_x, C_y) and $\theta = \theta_1 + \theta_2 + \theta_3$

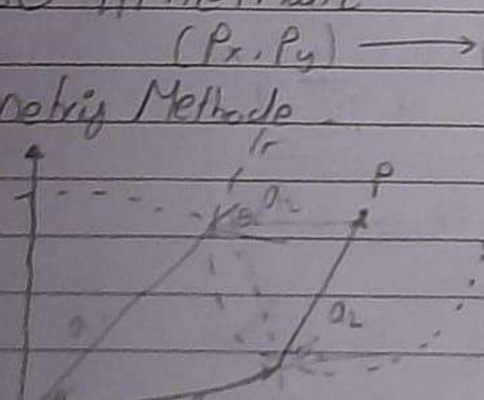
$$\vec{OC} = \vec{OA} + \vec{AB} + \vec{BC}$$

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} a_1 C\theta \\ a_1 S\theta \end{bmatrix} + \begin{bmatrix} a_2 C(\theta_1+\theta_2) \\ a_2 S(\theta_1+\theta_2) \end{bmatrix} + \begin{bmatrix} a_3 C[\theta_1+\theta_2+\theta_3] \\ a_3 S[\theta_1+\theta_2+\theta_3] \end{bmatrix}$$

Inverse Kinematic

$(P_x, P_y) \longrightarrow (\theta_1 + \theta_2)$ and rule as & are known (Fixed Dimension Thegare the Radius).

Geometric Method



We can find it by drawing Circle

Alebra Methode

Manipulate $\longrightarrow$ Juggling

$P_x = a_1 c\theta_1 + a_2 c(\theta_1 + \theta_2)$ - equat^n 1

$P_y = a_1 s\theta_1 + a_2 s(\theta_1 + \theta_2)$ - equation 2

Squaring 1 & 2 sdoing

$$\cos\theta_2 = \frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

Note There is proplem of sign:

$\sin\theta_2 = \pm\sqrt{1-\cos\theta_2}$ → (+) / (−)

S	All
T	C

rule.

We can't use $\theta_2 = \operatorname{atan}\left(\frac{S_2}{C_2}\right)$ because sign

So use $\boxed{\theta_2 = \operatorname{atan2}(\sin\theta_2, \cos\theta_2)}$ ← Methode Build in Matlab of python

↳ it will give 2 Solution $\theta_{2(i)}$ & $\theta_{2(-i)}$

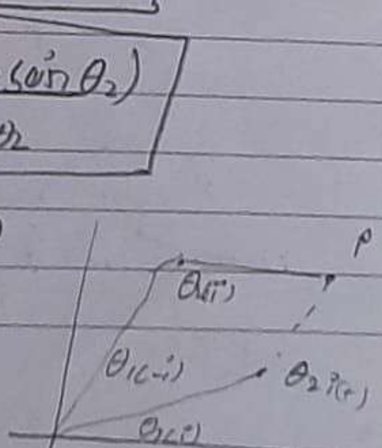
$\boxed{\text{Note } |\cos\theta_2| = 1}$

For θ_1:

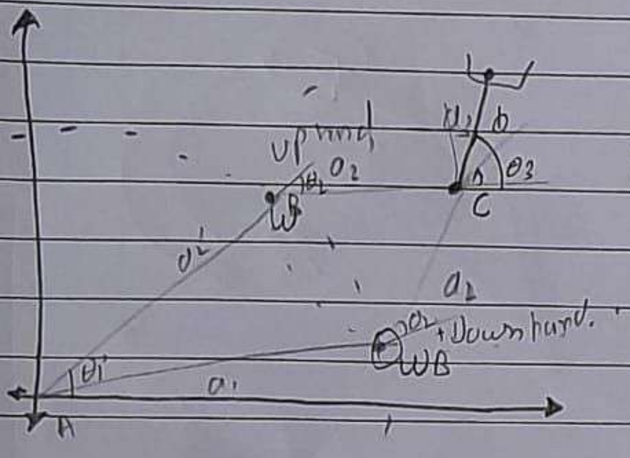
$$\boxed{\sin\theta_1 = \frac{P_y(a_1 + a_2\cos\theta) - P_x(a_2\cos\theta_2)}{(a_1^2 + a_2^2 + 2a_1a_2\cos\theta_2)}}$$

$$\boxed{\cos\theta_2 = \frac{P_x(a_1 + a_2\cos\theta) + P_y(a_2\sin\theta_2)}{(a_1^2 + a_2^2 + 2a_1a_2\cos\theta_2)}}$$

$\theta_1 = \operatorname{atan2}(\sin\theta_1, \cos\theta_1)$ → $\theta_{1(i)}$ / $\theta_{1(-i)}$



for 3R Plane Manipulator inverse.



$P_x = a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) + a_3(\theta_1+\theta_2+\theta_3)$

$P_y = a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) + a_3\sin(\theta_1+\theta_2+\theta_3)$

$\phi = \theta_1+\theta_2+\theta_3$

$W(x', y')$ = Wrist point.

$C(C_x, C_y)$ = endaffector

Note: Maths here is hard, so f* it.

* Wrist Position $(x', y) = x - a_3\cos(\phi),\ y - l_3\sin(\phi)$

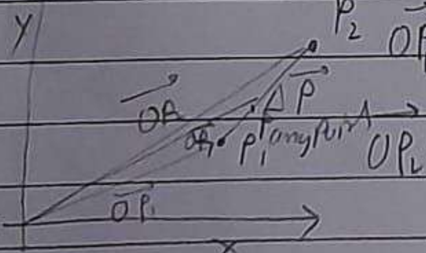
* $\theta_2 = \text{atan2}(\pm\sqrt{1-D})$

where $D = \dfrac{x^2+y^2-a_1^2 \mp a_2^2}{2a_1a_2}$

* $\theta_1 = \text{atan2}((y', x')) - \text{atan2}(a_2\sin\theta_2),\ a_1 + a_2\cos(\theta_2))$

$= \theta_3 = \phi - (\theta_1+\theta_2)$

Get in parametric equation of 2R serial Planar manipulator.



$\vec{OP_1} = P_1 = \begin{bmatrix} P_1x \\ P_1y \end{bmatrix}$ Colom vector formate

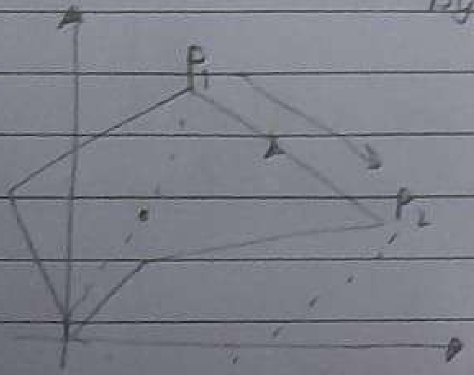
$OP_2 = P_2 = \begin{bmatrix} P_2X \\ P_2Y \end{bmatrix}$

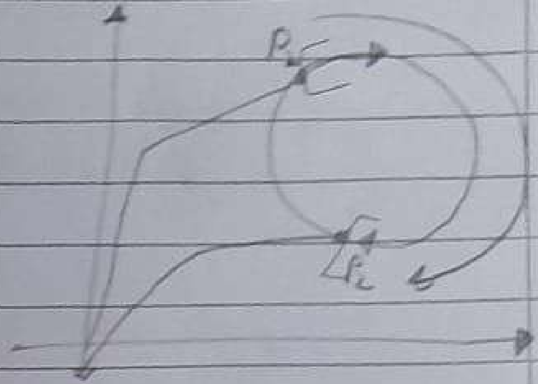
$\vec{\Delta P} = \vec{PO_2} - \vec{OP_1} = \begin{bmatrix} P_2x - P_1Y \\ P_2y \quad P_1y \end{bmatrix}$

Intermediate Point in Line (P)

$\boxed{\vec{P} = \vec{OP} = \vec{OP} + t\vec{\Delta P}}$

If we have perform Motion ($P_1 \longrightarrow P_2$)
by ~~~





Each via point we do I_{kin}

For Solution Chosing Up hand or down hand.
Choice were there is Neat motion in Joint

Complare θ_{new} & θ_{old}

$$\therefore \boxed{(\theta_{1new} - \theta_{1old})^2 + (\theta_{2_{New}} - \theta_2 old)^2 = \text{Minium}}$$

In Matlab

$\boxed{\text{Generate End Effector (Tip) Path} \begin{cases} \text{Line} \\ \text{Circle} \end{cases}}$ $\xrightarrow[\text{at each via Point}]{I\,kin}$ $\boxed{\text{Joint Angle Trajector}}$ $\xrightarrow{f\,kin}$ $\boxed{\text{Animation of arm}}$

Export Matlab data in CVS file.

$\longrightarrow$ Joint Trajectores $\begin{cases} \theta_1 \\ \theta_2 \end{cases}$ in Roboanalyzer
Path
$\dot{\theta}$ = velocity
$\ddot{\theta}$ = acceleration.

Note: Roboanalyzer requirement
(no column heading)

Simulation time	θ_1	$\dot{\theta}_1$	$\ddot{\theta}_1$	θ_2	$\dot{\theta}_2$	$\ddot{\theta}_2$
	↑	↑		↑	↑	
	0	0		0	0	