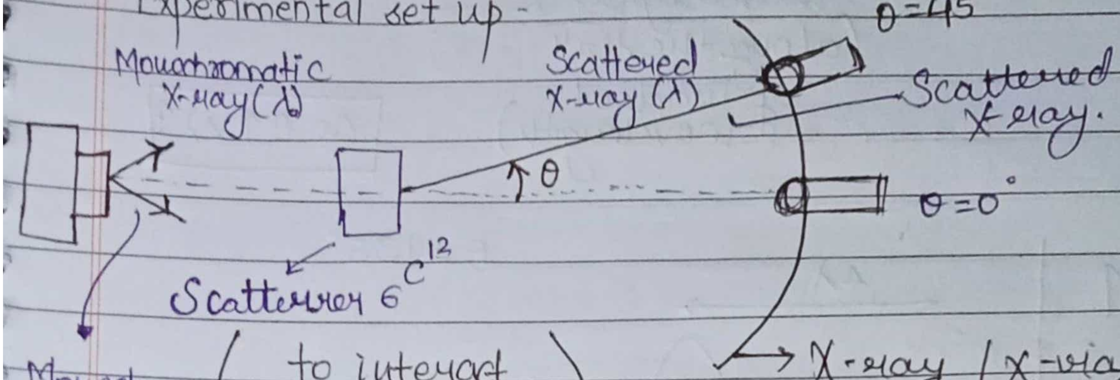
7/11/23

Quantum Physics

Compton Effect -

Varies the particle nature of Radiation

Experimental set up -



(to interact we have to select EMR which has very high energy)

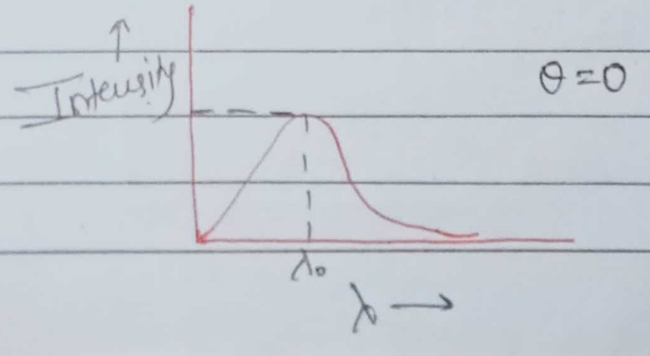
(Checks intensity of X-rays)

* Classical approach ($\lambda = \lambda_0$)
λ_0 - incident wavelength.
λ = scattered wavelength.

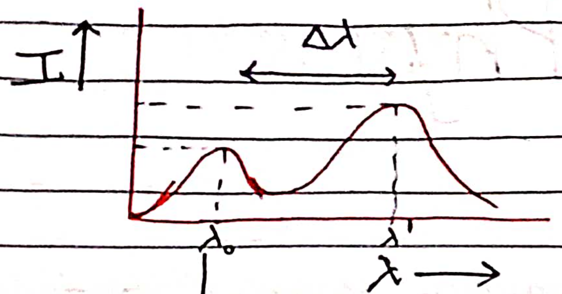
* Classical theory - I max. corresponding to λ_0 will be same for any θ.

In Higher atoms as no. of electrons increase no. of bound electrons increase due to screening due to outer electron. Outermost electrons are free and innermost e⁻s are bound.

Observations :-



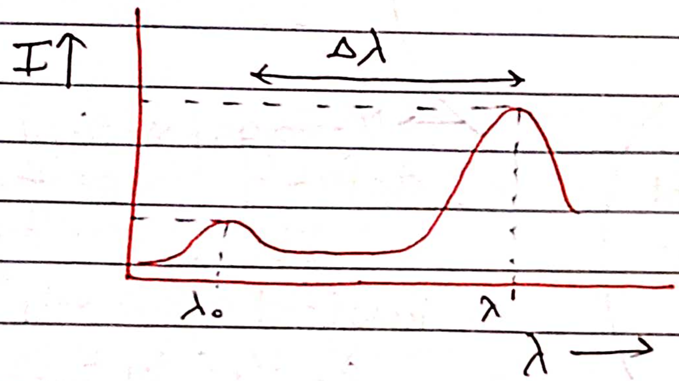
$\theta = 45°$



Unscattered wavelength (along the dir^n of incident wavelength)

$\boxed{\text{As } \theta \uparrow \Delta\lambda \uparrow}$

$\theta = 135°$



① Intensity of scattered x-ray doesn't depend on intensity of incident x-ray.

② Wavelength of scattered of radiation is larger than the incident radiation

③ Change in wavelength $\Delta\lambda$ $(\lambda' - \lambda)$ increases with increasing the scattering angle.

Assumptions:-

(i) The interaction of EM radiation is with free electron.

(ii) Before scattering electrons were at rest.

↳ has rest mass energy

$$\boxed{m = \frac{m_0}{\sqrt{1-(v/c)^2}}}$$ Relativistic Mass

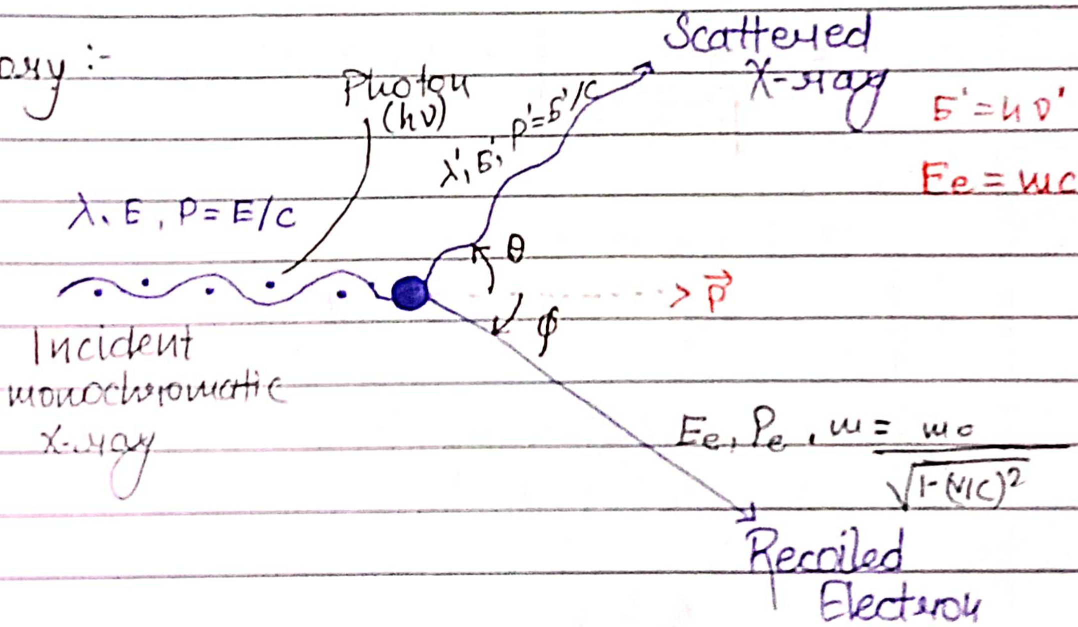
For photon :-

$m_0 = 0$

(iii) EM radiation → not a continuous
↳

Theory :-

$P = m_0 c$



$E' = h\nu'$

$E_e = mc^2$

E_e = Relativistic energy

P_e = Relativistic moment

Before collision :-

* e^- was at rest

$E_0 = m_0 c^2$ (Rest mass energy)

*

- The e^- of atom was at rest before collision.
- Collision was elastic.
- EM radiation is not a continuous spectrum but consists of stream of particles having definite energy $h\nu$ and moving in the free space with speed of light(c).

According to conservation of energy.

$$E + m_0c^2 = E_e + E'$$
$$\& \quad E_e = mc^2$$

$$E_e^2 = m^2c^4$$
$$= \left(\frac{m_0}{\sqrt{1-(v/c)^2}}\right)^2 c^4$$

$$= \frac{m_0^2}{\left(\sqrt{1-(v/c)^2}\right)^2}\left[v^2 - v^2 + c^2\right]c^2$$

$$= \frac{m_0^2v^2c^2}{\left(\sqrt{1-v^2/c^2}\right)^2} + \frac{m_0^2c^2(c^2-v^2)}{\frac{\cancel{c^2-v^2}}{c^2}}$$

$$= m^2v^2c^2 + m_0^2c^4$$
$$= P_e^2c^2 + m_0^2c^4$$

$$\boxed{E_e = \sqrt{P_e^2c^2 + m_0^2c^4}}$$

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$$\boxed{E + m_0c^2 = \sqrt{P_e^2c^2 + m_0^2c^4} + E'} \quad \text{——— (1)}$$

Conservation of momentum.

$$\vec{P} = \vec{P}' + \vec{P_e}$$
$$\vec{P} - \vec{P}' = \vec{P_e}$$
$$\vec{P_e}\cdot\vec{P_e} = (\vec{P} - \vec{P}')(\vec{P} - \vec{P}')$$
$$P_e^2 = P^2 + P'^2 - 2PP'\cos\theta$$
$$P_e^2 = \left(\frac{E}{c}\right)^2 + \left(\frac{E'}{c}\right)^2 - \frac{2EE'}{c^2}\cos\theta \quad \text{——— (2)}$$

from (1)

$$(E - E') + m_0c^2 = \sqrt{P_e^2c^2 + m_0^2c^4}$$

Squaring both sides, we get

$$(E-E')^2 + m_0^2c^4 + 2(E-E')m_0c^2 = P_e^2c^2 + m_0^2c^4$$

$$\Rightarrow \quad -2EE' + 2(E-E')m_0c^2 = -2EE'\cos\theta$$

$$\Rightarrow \quad \cancel{2}(E-E')m_0c^2 = \cancel{2}(EE')(1-\cos\theta)$$

$$\left(\frac{1}{E'} - \frac{1}{E}\right)m_0c^2 = (1-\cos\theta)$$

$$\frac{1}{h\nu'} - \frac{1}{h\nu} = \frac{1}{m_0c^2}(1-\cos\theta)$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \underset{\text{shift}}{\text{Compton}} = \lambda_c(1-\cos\theta)}$$

λ_c ↳ Compton wavelength

$9.1\times10^{-31}\times 9\times10^{24}$

$\frac{81.9\times10^{-7}}{1.6\times10^{-19}} = 51.18\times10^{12}$

$\Delta\lambda = \quad (1-\cos\theta)$

Case ① :- $\theta^{\circ}=0^{\circ}$

$\Delta\lambda = \lambda_c(1-\cos\theta)$

$\lambda' = \lambda$

Case ② :- $\theta = 90^{\circ}$

$\Delta\lambda = \frac{h}{m_0 c}$

$\lambda' = \lambda + \frac{h}{m_0 c}$

Case ③ :- $\theta = 180^{\circ}$

$\Delta\lambda = \frac{2h}{m_0 c}$

$\lambda' = \lambda + \frac{2h}{m_0 c}$

$\lambda' \in [\lambda \ \cancel{to}, \ \lambda + 2h/m_0 c]$

Assignment

$$\boxed{\tan\phi = \frac{\cot(\theta/2)}{1+\frac{h\nu}{m_0 c^2}}}$$

Assignment

$$\boxed{E_e = E - E' = h(\nu - \nu')}$$

$$\boxed{E_e = h\nu\left[\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right]}$$

Assignment

$$\text{Fraction of energy} = \left(\frac{E-E'}{E}\right)\times100$$
$$= \left(\frac{\Delta\lambda}{\lambda+\Delta\lambda}\right)\times100$$