

## A. Invariant Feature Identifiability

We take the simulation study with  $(p_s^-, p_s^+, p_v(t)) = (0.999, 0.9, 0.8)$  as an example to show how IRM-TV- $\ell_1$  improves identifiability of invariant features over IRM-TV- $\ell_2$ . There are 15 features in this experiment, where **No. 1~5** are **invariant** features and **No. 6~15** are **spurious** features. We run 10 times of this experiment with different simulated samples and compute the normalized average absolute values of the feature weights, shown in Table 1 and Figure 1. The invariant features of IRM-TV- $\ell_1$  and Minimax-TV- $\ell_1$  take up 64.88% and 65.08% of the total feature weights, respectively. In contrast, the invariant features of IRM-TV- $\ell_2$  and ZIN take up only 58.88% and 55.90% of the total feature weights, respectively. Hence the TV- $\ell_1$  models extract more invariant features than the TV- $\ell_2$  models. Moreover, the gap between invariant and spurious features in Figures 1(b) or 1(d) is larger than that in Figures 1(a) or 1(c), respectively. For example, the invariant feature weight  $w_2 = 0.0884$  and the spurious feature weights  $w_6 = 0.0679$  and  $w_8 = 0.0586$  for ZIN, while  $w_2 = 0.1210$ ,  $w_6 = 0.0403$ , and  $w_8 = 0.0446$  for Minimax-TV- $\ell_1$ , respectively. It indicates that Minimax-TV- $\ell_1$  enhances the invariant feature  $w_2$  while suppresses the spurious features  $w_6$  and  $w_8$ .

Table 1: Normalized absolute values of feature weights for different methods in simulation study with  $(p_s^-, p_s^+, p_v(t)) = (0.999, 0.9, 0.8)$ . **w1~w5** correspond to **invariant** features and **w6~w15** correspond to **spurious** features.

METHODS	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15
IRM-TV- $\ell_2$	0.1091	0.1143	0.1452	0.0912	0.1290	0.0412	0.0439	0.0454	0.0323	0.0483	0.0361	0.0438	0.0282	0.0526	0.0395
IRM-TV- $\ell_1$	0.1232	0.1292	0.1548	0.1090	0.1326	0.0329	0.0329	0.0446	0.0249	0.0371	0.0317	0.0386	0.0262	0.0482	0.0341
ZIN	0.1253	0.0884	0.1118	0.1336	0.0999	0.0679	0.0454	0.0586	0.0353	0.0261	0.0267	0.0376	0.0510	0.0505	0.0419
MINIMAX-TV- $\ell_1$	0.1320	0.1210	0.1410	0.1353	0.1215	0.0403	0.0356	0.0446	0.0221	0.0279	0.0285	0.0341	0.0352	0.0395	0.0412

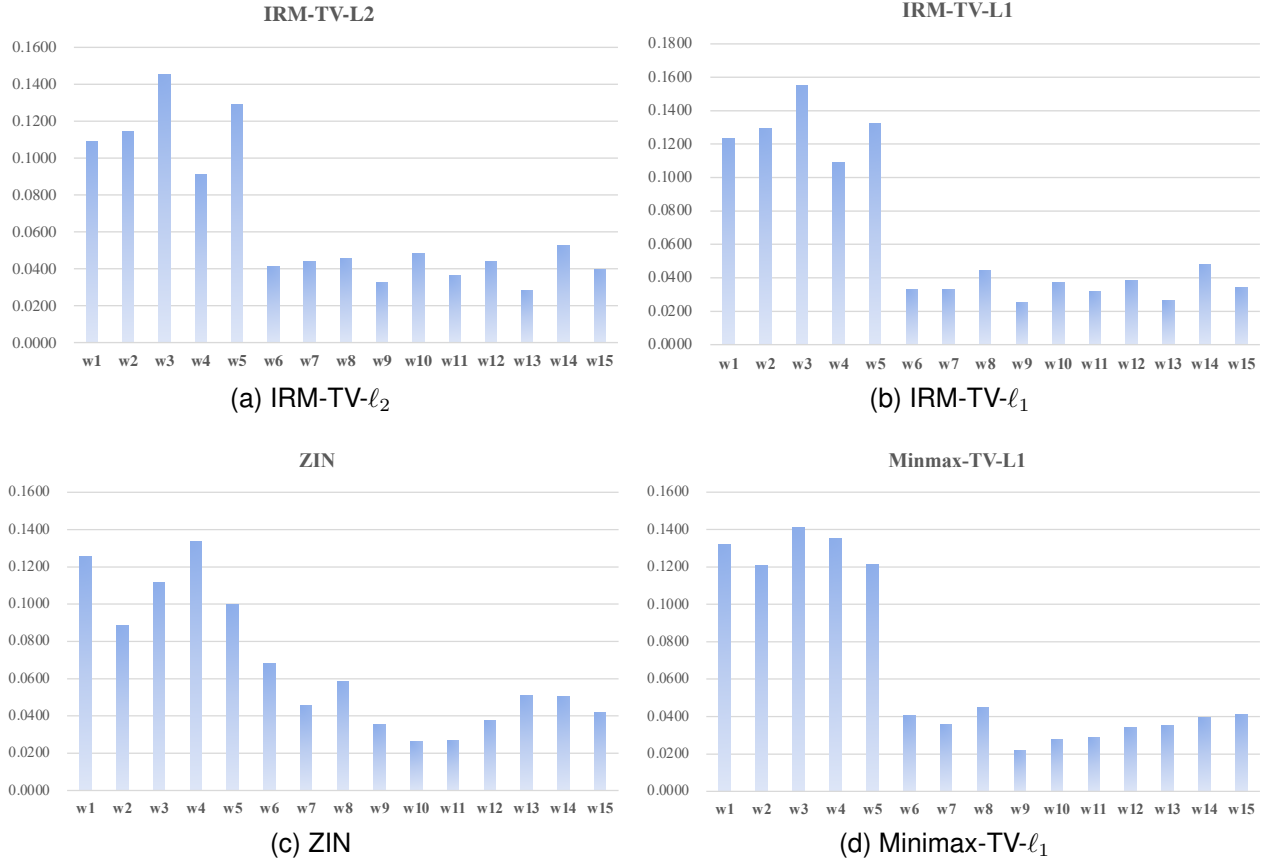


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