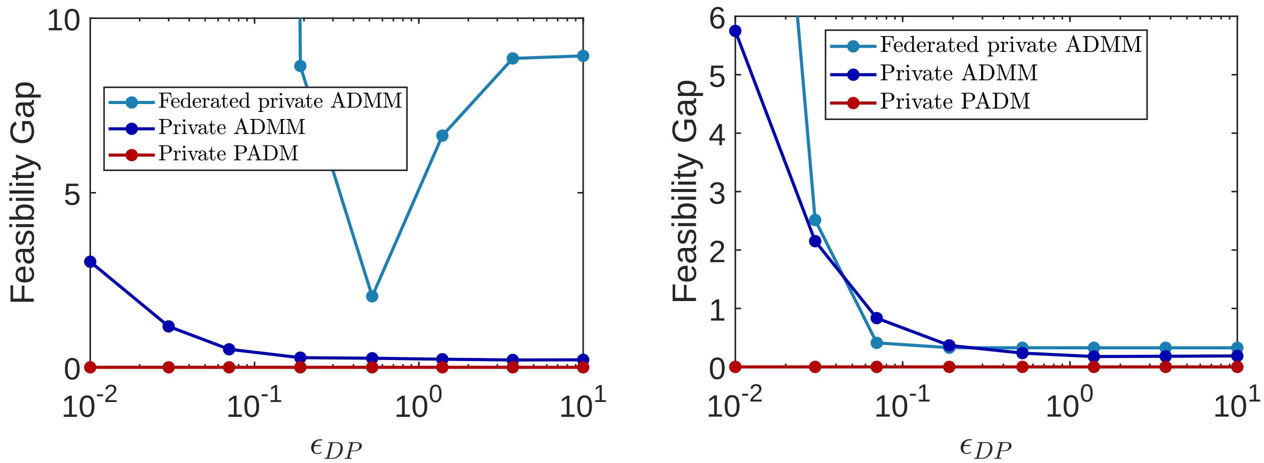


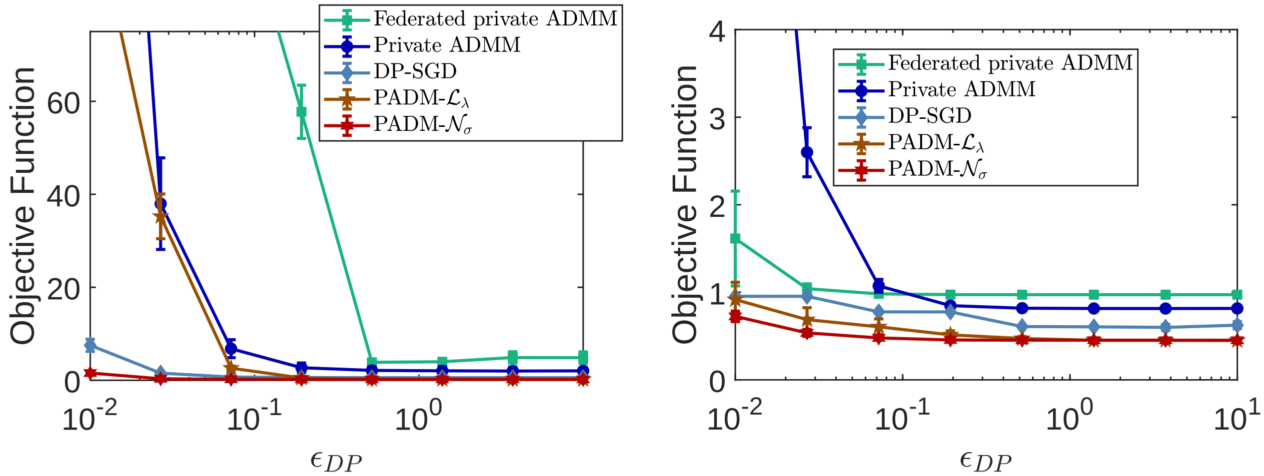
(a) Synthetic Experiment

(b) Real-world Experiment

Figure 1: Feasibility gaps of federated private ADMM [Cyffer et al. 2023], private ADMM [Chan et al. 2024] and private PADM (ours) at 1000-th iteration under eight privacy budgets ϵ_{DP}. The real-world experiment is conducted with the following model $F(\boldsymbol{x}) := \frac{1}{n}\sum_{k=1}^{n} \ln\left(1+\exp(-b^{(k)} * \boldsymbol{R}^{(k)}\boldsymbol{x})\right)$, $g(\boldsymbol{y}) := \kappa_1\|\boldsymbol{y}\|_1 + \frac{\kappa_2}{2}\|\boldsymbol{y}\|_2^2$ on the Adult data set from the UCI Machine Learning Repository: `https://archive.ics.uci.edu/dataset/2/adult`. **The two ADMM methods are infeasible while our PADM is always feasible in both cases.**

Table 1: Feasibility gaps of federated private ADMM [Cyffer et al. 2023], private ADMM [Chan et al. 2024] and private PADM (ours) at 1000-th iteration under eight privacy budgets ϵ_{DP}. **The two ADMM methods are infeasible while our PADM is always feasible in both cases.**

Synthetic Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
Federated private ADMM	25771.13	3375.37	295.64	8.63	2.03	6.64	8.85	8.92
Private ADMM	3.02	1.17	0.52	0.28	0.26	0.23	0.21	0.22
Private PADM	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Real-world Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
Federated private ADMM	18.27	2.51	0.41	0.33	0.33	0.33	0.33	0.33
Private ADMM	5.73	2.16	0.85	0.38	0.21	0.19	0.19	0.15
Private PADM	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**



(a) Synthetic Experiment (b) Real-world Experiment

Figure 2: Final objective function values (mean $\pm$ STD) of federated private ADMM [Cyffer et al. 2023], private ADMM [Chan et al. 2024], DP-SGD [Feldman et al. 2018], PADM-$\mathcal{L}_\lambda$ (ours), and PADM-$\mathcal{N}_\sigma$ (ours). **Our PADM-$\mathcal{N}_\sigma$ outperforms the three competitors in all the cases, while the outputs of the two ADMM methods are even infeasible, and DP-SGD directly drops the public variable y and cannot solve the private-public joint optimization problem.**

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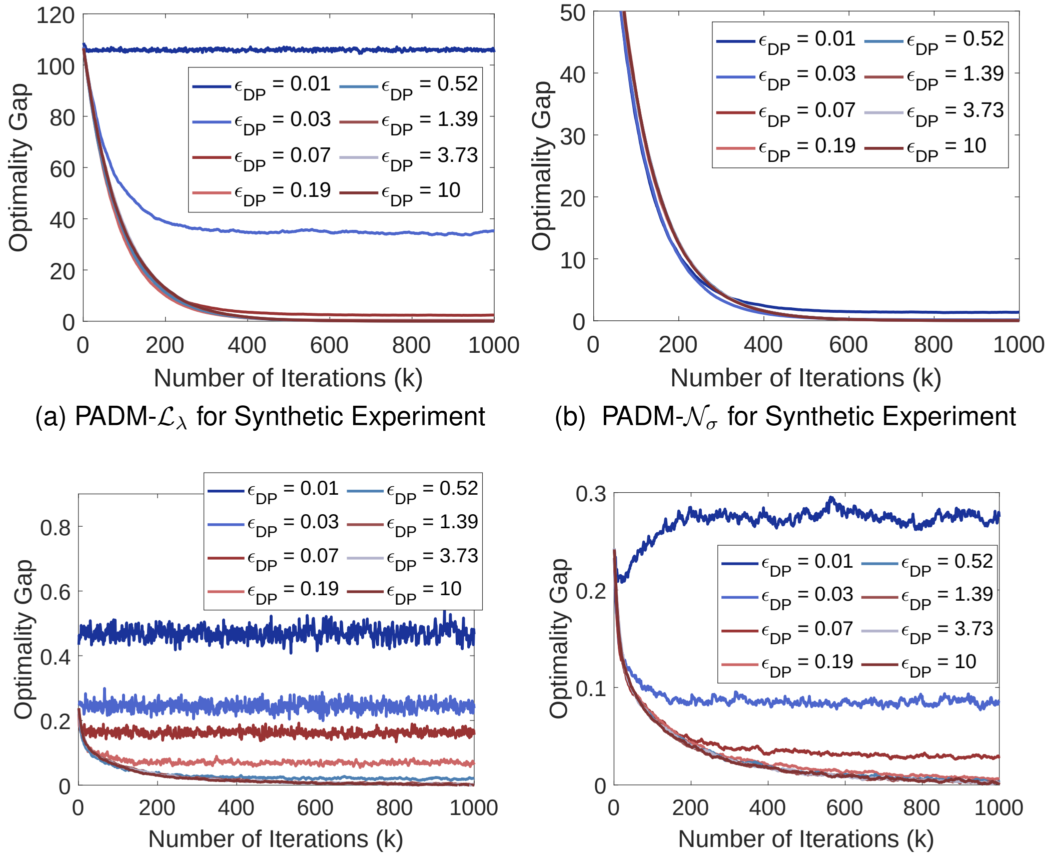
Synthetic Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
Federated Private ADMM	27785.87 ± 5126.50	617.35 ± 109.51	110.23 ± 9.86	57.74 ± 5.72	3.87 ± 0.51	3.99 ± 0.66	4.89 ± 1.25	4.87 ± 1.27
Private ADMM	253.29 ± 58.27	37.99 ± 9.85	6.80 ± 1.94	2.70 ± 1.02	2.12 ± 0.76	2.04 ± 0.87	1.99 ± 0.81	2.03 ± 0.70
DP-SGD	7.78 ± 1.34	1.51 ± 0.22	0.70 ± 0.07	0.58 ± 0.04	0.56 ± 0.02	0.57 ± 0.02	0.56 ± 0.02	0.56 ± 0.02
PADM-$\mathcal{L}_\lambda$	105.20 ± 4.39	35.25 ± 4.81	2.61 ± 0.45	$\mathbf{0.50 \pm 0.06}$	$\mathbf{0.25 \pm 0.01}$	$\mathbf{0.21 \pm 0.005}$	$\mathbf{0.20 \pm 0.004}$	$\mathbf{0.20 \pm 0.004}$
PADM-$\mathcal{N}_\sigma$	$\mathbf{1.56 \pm 0.32}$	$\mathbf{0.33 \pm 0.03}$	$\mathbf{0.22 \pm 0.007}$	$\mathbf{0.20 \pm 0.004}$	$\mathbf{0.20 \pm 0.004}$	$\mathbf{0.20 \pm 0.004}$	$\mathbf{0.20 \pm 0.004}$	$\mathbf{0.20 \pm 0.004}$
Real-world Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
Federated Private ADMM	1.61 ± 0.54	1.04 ± 0.05	0.98 ± 0.01	0.97 ± 0.00	0.97 ± 0.00	0.97 ± 0.00	0.97 ± 0.00	0.97 ± 0.00
Private ADMM	11.83 ± 1.43	2.60 ± 0.28	1.07 ± 0.07	0.85 ± 0.03	0.82 ± 0.02	0.82 ± 0.02	0.81 ± 0.03	0.82 ± 0.02
DP-SGD	0.95 ± 0.00	0.95 ± 0.00	0.78 ± 0.00	0.78 ± 0.00	0.61 ± 0.01	0.61 ± 0.01	0.60 ± 0.04	0.62 ± 0.04
PADM-$\mathcal{L}_\lambda$	$\mathbf{0.92 \pm 0.20}$	$\mathbf{0.69 \pm 0.14}$	$\mathbf{0.61 \pm 0.09}$	$\mathbf{0.51 \pm 0.05}$	$\mathbf{0.47 \pm 0.02}$	$\mathbf{0.45 \pm 0.01}$	$\mathbf{0.45 \pm 0.02}$	$\mathbf{0.45 \pm 0.01}$
PADM-$\mathcal{N}_\sigma$	$\mathbf{0.73 \pm 0.06}$	$\mathbf{0.54 \pm 0.04}$	$\mathbf{0.48 \pm 0.01}$	$\mathbf{0.46 \pm 0.01}$	$\mathbf{0.45 \pm 0.01}$	$\mathbf{0.45 \pm 0.01}$	$\mathbf{0.45 \pm 0.01}$	$\mathbf{0.45 \pm 0.01}$

Table 3: Optimality gaps of PADM-$\mathcal{L}_\lambda$ and PADM-$\mathcal{N}_\sigma$ at 1000-th iteration under eight privacy budgets ϵ_{DP}. **PADM achieves optimality with sufficiently large privacy budgets.**

Synthetic Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
PADM-$\mathcal{L}_\lambda$	105.0202	35.0743	2.4273	0.3246	0.0682	0.0287	0.0227	0.0220
PADM-$\mathcal{N}_\sigma$	1.3808	0.1470	0.0417	0.0245	0.0220	0.0221	0.0214	0.0227
Real-world Experiment								
ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
PADM-$\mathcal{L}_\lambda$	0.4669	0.2379	0.1559	0.0641	0.0221	0.0024	0.0031	0.0006
PADM-$\mathcal{N}_\sigma$	0.2751	0.0867	0.0290	0.0059	0.0029	0.0033	0.0021	0.0022



(a) PADM-$\mathcal{L}_\lambda$ for Synthetic Experiment (b) PADM-$\mathcal{N}_\sigma$ for Synthetic Experiment

(c) PADM-$\mathcal{L}_\lambda$ for Real-world Experiment (d) PADM-$\mathcal{N}_\sigma$ for Real-world Experiment

Figure 3: Optimality gaps of PADM-$\mathcal{L}_\lambda$ (left) and PADM-$\mathcal{N}_\sigma$ (right) at 1000-th iteration under eight privacy budgets ϵ_{DP}. **PADM achieves optimality with sufficiently large privacy budgets.**

Table 4: Accuracies (mean $\pm$ STD) of federated private ADMM [Cyffer et al. 2023], private ADMM [Chan et al. 2024], DP-SGD [Feldman et al. 2018], PADM-$\mathcal{L}_\lambda$ (ours), and PADM-$\mathcal{N}_\sigma$ (ours) for real-world experiment on the Adult data set. The model is trained on the training set and the accuracy is obtained on the test set, which is the ratio of correctly classified samples to the total test samples.

ϵ_{DP}	0.01	0.03	0.07	0.19	0.52	1.39	3.73	10
Federated Private ADMM	$39.94 \pm 15.34\%$	$41.04 \pm 15.76\%$	$72.99 \pm 3.85\%$	$75.00 \pm 0.00\%$	$75.00 \pm 0.00\%$	$75.00 \pm 0.00\%$	$75.00 \pm 0.00\%$	$75.00 \pm 0.00\%$
Private ADMM	$68.52 \pm 3.99\%$	$68.75 \pm 2.42\%$	$74.61 \pm 1.01\%$	$76.54 \pm 1.83\%$	$77.45 \pm 1.29\%$	$78.03 \pm 0.82\%$	$77.15 \pm 1.08\%$	$76.91 \pm 2.61\%$
DP-SGD	$25.00 \pm 0.00\%$	$25.00 \pm 0.00\%$	$73.59 \pm 0.46\%$	$73.81 \pm 0.36\%$	$78.46 \pm 1.29\%$	$78.48 \pm 1.29\%$	$76.23 \pm 4.00\%$	$76.08 \pm 5.62\%$
PADM-$\mathcal{L}_\lambda$	$53.74 \pm 12.88\%$	$\mathbf{71.96 \pm 6.30\%}$	$74.27 \pm 1.60\%$	$\mathbf{77.64 \pm 1.76\%}$	$\mathbf{79.16 \pm 1.41\%}$	$\mathbf{79.85 \pm 1.17\%}$	$\mathbf{80.00 \pm 1.06\%}$	$\mathbf{78.79 \pm 2.25\%}$
PADM-$\mathcal{N}_\sigma$	$\mathbf{73.43 \pm 3.71\%}$	$\mathbf{75.65 \pm 0.80\%}$	$\mathbf{77.44 \pm 2.16\%}$	$\mathbf{78.95 \pm 1.71\%}$	$\mathbf{79.47 \pm 1.51\%}$	$\mathbf{79.03 \pm 1.81\%}$	$\mathbf{79.77 \pm 1.04\%}$	$\mathbf{78.47 \pm 1.62\%}$