Xenon-Iodine interaction programming assignment

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Abstract:
A simple ODE is used to describe the ^{135}Xe and ^{135}I transient behavior during steady operation of a reactor and directly after shutdown. We present the simplified equations and apply an appropriate temporal discretization that can be programmed.

Keywords: coupled ODE, explicit Euler

1 Introduction

The localized behavior of ^{135}Xe and ^{135}I can generally be described by the following:

$$\frac{\partial N_X}{\partial t} = \gamma_X N_{35} \sigma_f \phi - \lambda_X N_X - N_X \sigma_{a,X} \phi + \lambda_I N_I \tag{1a}$$

$$\frac{\partial N_I}{\partial t} = \gamma_I N_{35} \sigma_f \phi - \lambda_I N_I, \tag{1b}$$

$$N_X(t=0) = N_I(t=0) = 0.0 \tag{1c}$$

where the subscripts $_X$ and $_I$ denote quantities for ^{135}Xe and ^{135}I respectively, γ is the fission yield, λ is the decay constant, N is the atom density. N_{35} is the fuel atomic density, σ_f is the fuel's microscopic cross section for fission, $\sigma_{a,X}$ is ^{135}Xe's absorption microscopic cross section, finally ϕ is the thermal neutron flux.

2 Explicit Euler temporal discretization

The explicit Euler temporal discretization can be described in simple terms by regarding the above set of coupled ODEs as a simplified set described by:

$$\frac{\partial \mathbf{N}}{\partial t} = \mathbf{F}(\mathbf{N}, t) \tag{2}$$

where $\mathbf{N}$ is a list/vector of time-dependent unknowns and $\mathbf{F}$ is function dependent both on the unknowns and time, t. The **explicit Euler** method then becomes

$$\begin{aligned} \frac{\mathbf{N}^{n+1} - \mathbf{N}^n}{\Delta t} &= \mathbf{F}(\mathbf{N}^n, t^n) \\ \mathbf{N}^0 &= \mathbf{N}(t=0) \end{aligned} \tag{3}$$

where the indices n now denote the discrete time values of both time and all the unknowns.

Applying this scheme to our equations gives

$$\begin{aligned} N_X^{n+1} &= N_X^n + \Delta t \Big(\gamma_X N_{35} \sigma_f \phi - \lambda_X N_X^n - N_X^n \sigma_{a,X} \phi + \lambda_I N_I^n \Big) \\ N_I^{n+1} &= N_I^n + \Delta t \Big(\gamma_I N_{35} \sigma_f \phi - \lambda_I N_I^n \Big), \end{aligned} \tag{4}$$

3 Assigment

Part 1. Write a program that solves these equations for the following conditions:

$$\begin{aligned}
\gamma_X &= 0.00237 \\
\gamma_I &= 0.0475 \\
\lambda_X &= 2.09e{-5} \\
\lambda_I &= 2.87e{-5} \\
\sigma_f &= 500e{-24} \\
\sigma_a &= 1.0e6 * 1.0e{-24} \\
N_{35} &= 0.0077e24 \\
\phi &= 1.0e14 \\
\Delta t &= 2000.0 \\
\text{Total time} &= 300\Delta t
\end{aligned}$$

Additionally, $N_X(t{=}0) = N_I(t{=}0) = 0.0$, and at time $t > 150\Delta t$, $\phi = 0$.

Part 2. At each time step, including n=0, write the time t, the atomic densities N_X and N_I to a line in a text file. Each line of the file must contain 3 values separated by one or more spaces. For example:

```
0 0 0
2000 1.8249e+15 3.6575e+16
4000 5.30794e+15 7.10506e+16
6000 9.92769e+15 1.03547e+17
8000 1.52957e+16 1.34179e+17
10000 2.11239e+16 1.63052e+17
12000 2.72003e+16 1.90268e+17
14000 3.33695e+16 2.15921e+17
16000 3.95195e+16 2.40102e+17
18000 4.55705e+16 2.62896e+17
20000 5.14666e+16 2.8438e+17
etc.
```

Part 3. Write a python script that can read the output file and automatically create a plot of the following nature: