TEHREEM ZUBAIR

BYTEWISE FELLOWSHIP

TASK 9

MEAN, MEDIAN AND MODE

PROBLEM 1:

1. Find the mean of the first 10 odd integers.

SOLUTION:

Mean, Median, Mode Questions:

First 10 odd integers: 1,3,5,7,9,11,13,15,17,19.

Mean = 1+3+5+7+9+11,+15+17+19 / 10
= 100 / 10
= 10

PROBLEM 2:

2. What is the median of the following data set?

32, 6, 21, 10, 8, 11, 12, 36, 17, 16, 15, 18, 40, 24, 21, 23, 24, 24, 29, 16, 32, 31, 10, 30, 35, 32, 18, 39, 12, 20

SOLUTION:

The ascending order of given data is:

6,8,10,10,11,12,12,15,16,16,17,18,18,20,21,21,23,24,24,
24,29,30,31,32,32,35,36,39,40

Number of values = n = 30
n/2 = 15
15th value = 21
n/2 + 1 = 16
16th value = 21
Meadian = 21+21/2
= 21

PROBLEM 3:

3. Identify the mode for the following data set:

21, 19, 62, 21, 66, 28, 66, 48, 79, 59, 28, 62, 63, 63, 48, 66, 59, 66, 94, 79, 19 94

SOLUTION:

The data in ascending order is
19, 19, 21, 21, 28, 28, 48, 48, 59, 59, 62, 62, 63, 63, 66, 66, 66,
66, 66, 79, 79, 94, 94

Most frequent number = 66.

PROBLEM 3:

4. Consider the following frequency distribution. Calculate the mean weight of students.

Weight (in kg)	31-35	36 – 40	41 – 45	46 – 50	51 – 55	56 – 60	61 – 65	66 – 70	71 – 75
Number of Students	9	6	15	3	1	2	2	1	1

SOLUTION:

Class Interval	Number of Students (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$f_i d_i$
30.5 - 35.5	9	33	-10	-90
35.5 - 40.5	6	38	-5	-25
40.5 - 45.5	15	43 = a	0	0
45.5 - 50.5	3	48	5	15
50.5 - 55.5	1	53	10	10
55.5 - 60.5	2	58	15	30
60.5 - 65.5	2	63	20	40
65.5 - 70.5	1	68	25	25
70.5 - 75.5	1	73	30	30
~~75.5~~ Total	$\Sigma f_i = 40$			$\Sigma f_i d_i = 35$

By assumed mean method.

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 43 + \frac{35}{40} = 43.875$$

PROBLEM 5:

5. Find the mean for the following distribution.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

SOLUTION:

x_i	f_i	$f_i x_i$
15	3	45
21	5	105
27	6	162
30	7	210
35	8	280
Total	$\Sigma f_i = 29$	$\Sigma f_i x_i = 802$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{802}{29} = 27.655$$

PROBLEM 6

6. Calculate the median marks of students from the following distribution.

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of Students	7	10	10	20	20	15	8

SOLUTION

Class Interval	Number of Students	Cumulative frequency
10 - 20	7	7
20 - 30	10	17
30 - 40	10	27 = cf
40 - 50	20 = f	47
50 - 60	20	67
60 - 70	15	82
70 - 80	8	90

$N/2 = 90/45$

Median class is 40 – 50

Lower Limit = 40

Class size = 10

frequency = 20

Cumulative frequency = 27

$$\text{Median} = 40 + \left[\frac{45-27}{20}\right] \times 10$$

$$= 40 + \frac{18}{2} = 49$$

PROBLEM 7:

8. If the median of a distribution given below is 28.5, then find the value of x and y.

CI	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	Total
Frequency	5	x	20	15	y	5	60

SOLUTION:

From the data,

$N/2 = 30$

Median $= 28.5$

Midian class $= 20 - 30$

cumulative frequency $= 25 + x$

lower limit $= l = 20$

frequency $= f = 20$

cumulative frequency of preceding class $= 5 + x$

Class size $= h = 10$

$$\text{Median} = l + \left(\frac{N/2 - c_f}{f}\right) \times h$$

$$28.5 = 20 + \left(\frac{30 - 5 - x}{20} \times 10\right)$$

$$28.5 = 20 + \frac{30 - 5 - x}{2}$$

$$2(8.5) = 25 - x$$

$$\boxed{x = 8}$$

Also

$$60 = 5 + 20 + 15 + 5 + x + y$$

$$y = 60 - 53$$

$$y = 7$$

PROBABILITY

PROBLEM 1:

Two coins are tossed 500 times, and we get:

Two heads: 105 times

One head: 275 times

No head: 120 times

Find the probability of each event to occur.

SOLUTION:

Probability

1. Let's say the ~~event~~ of getting two heads, one head and no head by E_1, E_2, E_3 respectively.

$$P(E_1) = \frac{105}{500} = 0.21$$

$$P(E_2) = \frac{275}{500} = 0.55$$

$$P(E_3) = \frac{120}{500} = 0.24$$

The sum of all the probability should be 1.

$$P(E_1) + P(E_2) + P(E_3)$$
$$= 0.21 + 0.55 + 0.24$$
$$= 1.0$$

PROBLEM 2:

A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	Less than 4000	4000 to 9000	9001 to 14000	More than 14000
Frequency	20	210	325	445

If a tyre is bought from this company, what is the probability that :

(i) it has to be substituted before 4000 km is covered?

(ii) it will last more than 9000 km?

(iii) it has to be replaced after 4000 km and 14000 km is covered by it?

SOLUTION:

2. (i) Total number of trials = 1000

The frequency of a tyre required to be replaced before covering 4000 km = 20

So $P(E_1) = \frac{20}{1000} = 0.02$

(ii) Frequency that tyre will last more than 9000 km $= 325 + 445 = 770$

So, $P(E_2) = \frac{770}{1000} = 0.77$

(iii) frequency that tyre requires replacement b/w 4000 km and 14000 km $= 210 + 325 = 535$

So, $P(E_3) = \frac{535}{1000} = 0.535$

PROBLEM 3:

3. The percentage of marks obtained by a student in the monthly tests are given below:

Test	1	2	3	4	5
Percentage of marks obtained	69	71	73	68	74

Based on the above table, find the probability of students getting more than 70% marks in a test.

SOLUTION:

3. The total number of test conducted is 5.
number of tests when % > 70 = 3

$P(\text{scoring more than } 70\%) = \frac{3}{5} = 0.6.$

PROBLEM 4:

4. One card is drawn from a deck of 52 cards, well-shuffled. Calculate the probability that the card will

(i) be an ace,

(ii) not be an ace.

SOLUTION:

4. (i) There are 4 aces in a deck.
Let E be the Event that card is an ace.
number of favourable outcomes = 4
Number of possible outcomes = 52

$P(E) = \cancel{\frac{52}{4}} \quad \frac{4}{52} = \frac{1}{13}$

(ii) Let F is event that card is not ace.
$\therefore F = 52 - 4 = 48$

$P(F) = \frac{48}{52} = \frac{12}{13}$

PROBLEM 5:

6. Two coins (a one rupee coin and a two rupee coin) are tossed once. Find a sample space.

SOLUTION:

5. Either Head (H) or Tail (T) can be the outcome.

Head on both coins = (H, H) = HH

Head on 1st Tail on Second = HT

Or Tail on 1st Head on 2nd = TH

Tail on both = TT

Sample space = {HH, HT, TH, TT}

PROBLEM 6:

10. If $P(A) = 7/13$, $P(B) = 9/13$ and $P(A \cap B) = 4/13$, evaluate $P(A|B)$.

SOLUTION:

6. If $P(A) = 7/13$

$P(B) = 9/13$

$P(A \cap B) = 4/13$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = 4/13 / 9/13 = \frac{4}{9}$

PROBLEM 7:

What is the probability of getting a sum of 7 when two dice are thrown?

7. Total number of ways $= 6 \times 6 = 36$

Favourable cases = (1,6), (6,1), (2,5), (5,2) (4,3), (3,4)

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

SOLUTION:

PROBLEM 8:

1 card is drawn at random from the pack of 52 cards.

(i) Find the Probability that it is an honor card.

(ii) It is a face card

SOLUTION:

8. There are 4 honour card in each suit.

so in 4 suits honour cards are $= 4 \times 4 = 16$

Total cards $= 52$

$$P(\text{honor card}) = \frac{4}{52} = \frac{1}{13}$$

There are 3 face card in a single suit.

In the 4 suits = 12 face cards

$$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$$

VARIANCE AND STANDARD DEVIATION

PROBLEM 1:

Find the variance and standard deviation of the following scores on an exam:

92, 95, 85, 80, 75, 50 **SOLUTION:**

Variance and Standard Deviation

1. Mean = 92+95 + 85 + 80+75+50 / 6
= 477 / 6 = 79.5

Then we find difference between score and mean (deviation).

Score	Score - Mean	
92	92 - 79.5	= 12.5
95	95 - 79.5	= 15.5
85	85 - 79.5	= 5.5
80	80 - 79.5	= 0.5
75	75 - 79.5	= -4.5
50	50 - 79.5	= -29.5

Now we square difference and sum them

Difference	$(\text{Difference})^2$
12.5	156.25
15.5	240.25
5.5	30.25
0.5	0.25
-4.5	20.25
-29.5	870.28
Sum	1317.50

$$\text{Variance} = \frac{1317.50}{5} = 263.5$$

$$\text{Standard deviation} = \sqrt{263.5} = 16.2$$

PROBLEM 2:

Find the standard deviation of the average temperatures recorded over a five-day period last winter:

18, 22, 19, 25, 12 **SOLUTION:**

Temp	Temp - Mean	Deviation squared.
18	-1.2	1.44
22	2.8	7.84
19	-0.2	0.04
25	5.8	33.64
12	-7.2	51.84
96/5 = 19.2		Sum = 94.80

$$\text{Variance} = \frac{94.8}{5} = 23.7$$

$$\text{Standard deviation} = \sqrt{23.7} = 4.9$$

NORMAL DISTRIBUTION

PROBLEM 1:

1. Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?

1-0.4052=

SOLUTION:

Normal Distribution.

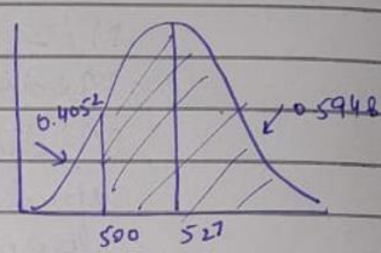
1.

$x = 500$

$\mu = 527$

$\sigma = 112$

$$z = \frac{500-527}{112} = -0.24107$$



$$P\{x > 500\} = P\{z > -0.24\} = 1 - 0.4052 = 0.5948$$

PROBLEM 2:

-0.24 0

2. How high must an individual score on the GMAT in order to score in the highest 5%?

SOLUTION:

2. $\mu = 527$

$\sigma = 112$

$P(X > ?) = 0.05 \longrightarrow P(Z > ?) = 0.05$

$P(Z < ?) = 1 - 0.05 = 0.95 \qquad Z = 1.645$

$X = 527 + 1.645(112)$

$= 527 + 184.24$

$X = 711.24$

PROBLEM 3:

7. The Edwards's Theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. What percentage of customers will spend less than \$3.00 on concessions?

SOLUTION:

3. $Z = \dfrac{3 - 4.11}{1.37} = 0.81021$

$\mu = 4.11$

$\sigma = 1.37$

$P(X < 3.00) = P(Z < -0.81) = 0.2090$

PROBLEM 4:

8. What spending amount corresponds to the top 87th percentile?

SOLUTION:

4. $\mu = 4.11$

$\sigma = 1.37$

$P(X > ?) = 0.87 \rightarrow P(Z > ?) = 0.87$

$P(Z > ?) = 0.87 \rightarrow P(Z < ?) = 1 - 0.87 = -1.13$

$X = 4.11 + (-1.13)(1.37)$

$= 4.11 - 1.5481$

$X = \$2.56$

PROBLEM 5:

The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?

SOLUTION:

5. $Z = 2500 - 4300 / 750 = -2.40$

$Z = 4200 - 4300 / 750 = -0.1333$

$\mu = 4300$

$\sigma = 750$

$P(2500 < X < 4200) = P(-2.40 < Z < -0.13)$

$P(-2.40 < Z < -0.13) = P(Z < -0.13) - P(Z < -2.40)$

$P(-2.40 < Z < -0.13) = 0.4483 - 0.0082$

$= 0.4401$

BINOMIAL DISTRIBUTION

PROBLEM 1:

Find the binomial distribution of getting a six in three tosses of an unbiased dice.

SOLUTION:

Binomial Distribution

Bernoulli's theorom

$$P(X=r) = {}^nC_r \, p^r \, q^{(n-r)}$$

1. Let x be random variable of getting 6.

Then X can be 0, 1, 2, 3.

Here, $n = 3$

P = probability of getting 6 $= 1/6$

Probability of not getting six $= q = 5/6$

$$P(x=0) = {}^3C_0 \left(1/6\right)^0 \left(5/6\right)^{3-0} = 1 \times 1 \times \frac{125}{216} = \frac{125}{216}$$

$$P(x=1) = {}^3C_1 \left(1/6\right)^1 \left(5/6\right)^{3-1} = 3 \times 1/6 \times \frac{25}{36} = \frac{25}{72}$$

$$P(x=2) = {}^3C_2 \left(1/6\right)^2 \left(5/6\right)^{3-2} = 3 \times 1/36 \times 5/6 = 5/72$$

$$P(x=3) = {}^3C_3 \left(1/6\right)^3 \left(5/6\right)^{3-3} = 1 \times \frac{1}{216} \times 1 = \frac{1}{216}$$

PROBLEM 2:

Find the probability of getting at least 5 times head-on tossing an unbiased coin for 6 times by using the binomial distribution.

SOLUTION:

P = of getting a head in single toss $= 1/2$

$q = 1/2$

X = successfully getting a head

$$P(x \geq 5) = P(x = 5) + P(x = 6)$$

$$= {}^6C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^{6-5} + {}^6C_6 \left(\tfrac{1}{2}\right)^6 \left(\tfrac{1}{2}\right)^{6-6}$$

$$= 6 \times \left(\tfrac{1}{2}\right)^6 + 1 \times \left(\tfrac{1}{2}\right)^6$$

$$= \frac{7}{24}$$

PROBLEM 3:

On average, every one out of 10 telephones is found busy. Six telephone numbers are selected at random. Find the probability that four of them will be busy.

SOLUTION:

$p = 1/10$, $q = 9/10$

$$P(x = 4) = {}^6C_4 \, p^4 q^{(6-4)}$$

$$= {}^6C_4 \left(\tfrac{1}{10}\right)^4 \left(\tfrac{9}{10}\right)^2$$

$$= 15 \times \frac{1}{10^4} \times \frac{81}{100}$$

$$= 0.001215$$

PROBLEM 4:

A bag contains 5 green balls and 3 red balls. If two balls are drawn from the bag randomly with replacement, find the probability distribution of the number of green balls drawn.

SOLUTION:

$$p = \frac{5}{5+3} = \frac{5}{8} \qquad q = 1 - \frac{5}{8} = \frac{3}{8}$$

$$P(X=0) = {}^{2}C_{0}\, p^{0} q^{2-0} = 1 \times 1 \times \left(\frac{3}{8}\right)^{2} = \frac{9}{64}$$

$$P(X=1) = {}^{2}C_{1}\, p^{1} q^{2-1} = 2 \times \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$$

$$P(X=2) = {}^{2}C_{2}\, p^{2} q^{2-2} = 1 \times \left(\frac{5}{8}\right)^{2} \times \left(\frac{3}{8}\right)^{0} = \frac{25}{64}$$

POISSON'S DISTRIBUTION

PROBLEM 1:

As only 3 students came to attend the class today, find the probability for exactly 4 students to attend the classes tomorrow.

SOLUTION:

Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

1. $\lambda = 3$

$x = 4$

$$P(X=4) = \frac{e^{-3}\, 3^{4}}{4!} = 0.16803$$

PROBLEM 2:

Traffic accidents at a particular intersection follow Poisson distribution with an average rate of 1.4 per week.

a) What is the probability that the next week is accident-free?

b) What is the probability that there will be exactly 3 accidents next week?

c) What is the probability that there will be at most 2 accidents next week?

SOLUTION:

2. a) $P(X=0) = \frac{1.4^0 \cdot e^{-1.4}}{0!} \approx 0.2466$.

b) 1 week $\rightarrow \lambda = 1.4$

$P(X=3) = \frac{1.4^3 \cdot e^{-1.4}}{3!} = 0.1128$

c) 1 week $\longrightarrow \lambda = 1.4$

$P(x \leq 2) = P(X=0) + P(X=1) + P(X=2$

$= 0.2466 + \frac{1.4^1 \cdot e^{-1.4}}{1!} + \frac{1.4^2 \cdot e^{-1.4}}{2!}$

$= 0.2466 + 0.3452 + 0.2417$

$= 0.8335$

PROBLEM 3:

Example 1: Aclinic deals only with flu vaccinations. The number of patients arriving every 15 minutes is modelled by random variable X with distribution $Po(4.2)$

(i) State two assumptions required for the Poisson model to be valid. -- [2]

(ii) Find the probability that at least 1 patent will arrive in a 15-minute period. -- [2]

[S-10/73/Q10]

SOLUTION:

(i) Patient ~~at~~ arrive at constant mean rate.
Patient arrive at random
Patient arrive independently.
Patient arrive singly.

(ii) $P(x \geq 1) = 1 - P(0)$

$= 1 - \dfrac{e^{-\lambda} \cdot \lambda^{0}}{0!}$

$= 1 - e^{-8}$

$= 1 - 0.0145 = 0.985$

PROBLEM 4:

Example 2: The number of goals scored per match by Everly Rovers is represented by the random variable X, which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. ---[2]

Assume now that $X \sim Po(1.8)$

(ii) Find $P(2 < X < 6)$ ---[2]

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [S-11/72/Q3] ---[3]

SOLUTION:

(i) constant average rate of goals scored.
Goals at random
goal independent

(ii) $x \sim P_o(1.8) \Rightarrow \lambda = 1.8$

$$P(2 < x < 6) = P(3) + P(4) + P(5)$$

$$= \frac{e^{-1.8} 1.8^3}{3!} + \frac{e^{-1.8} \cdot 1.8^4}{4!} + \frac{e^{-1.8} \cdot 1.8^5}{5!}$$

$$= 0.259.$$

(iii) In each match $P(\text{atleast } 1 \text{ goals}) = P(X \geq 1)$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{e^{-\lambda} \lambda^0}{0!}$$

$$= 1 - e^{-1.18}$$

Probability that in 10 matches they score at least on goal

$$P(\text{win bonus}) = (1 - e^{-1.18})^{10}$$

$$= 0.164$$

UNIFORM DISTRIBUTION

PROBLEM 1:

The average weight gained by a person over the winter months is uniformly distributed and ranges from 0 to 30 lbs. Find the probability of a person that he will gain between 10 and 15lbs in the winter months.

SOLUTION:

1. First find height of distributing.
area under propability = 1
Since there are 30 units so height $= \frac{1}{30}$
width $= 15 - 10 = 5$

$$\text{Probability} = 5 \times \frac{1}{30} = \frac{1}{6}$$

PROBLEM 2:

The data that follow are the number of passengers on 35 different charter fishing boats. The sample mean = 7.9 and the sample standard deviation = 4.33. The data follow a uniform distribution where all values between and including zero and 14 are equally likely. State the values of *a* and *b*. Write the distribution in proper notation, and calculate the theoretical mean and standard deviation.

Table 5.3.2

1	12	4	10	4	14	11
7	11	4	13	2	4	6
3	10	0	12	6	9	10
5	13	4	10	14	12	11
6	10	11	0	11	13	2

SOLUTION:

2. $a = 0$

$b = 14$

$X \sim U(0, 14)$

$\mu = \frac{14-0}{2} = 7$

$\sigma = \sqrt{\frac{(14-0)^2}{12}} = 4.04$

PROBLEM 3:

The data in Table 5.3.1 are 55 smiling times, in seconds, of an eight-week-old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

a) **What is the probability that a randomly chosen eight-week-old baby smiles between two and 18 seconds?**

SOLUTION:

$P(2 < x < 18) = (18-2)\left(\frac{1}{23}\right) = \frac{16}{23}$

b) **Find the 90th percentile for an eight-week-old baby's smiling time.**

SOLUTION:

Ninety per cent of smiling times fall below 90th percentile, k

$P(x<k) = 0.90$

$P(x<k) = 0.9$

base × height = 0.9

$(k-0)\left(\frac{1}{23}\right) = 0.9$

$k = 20.7$

PROBLEM 4:

The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.

a) What is the probability that a person waits fewer than 12.5 minutes?

SOLUTION:

a)

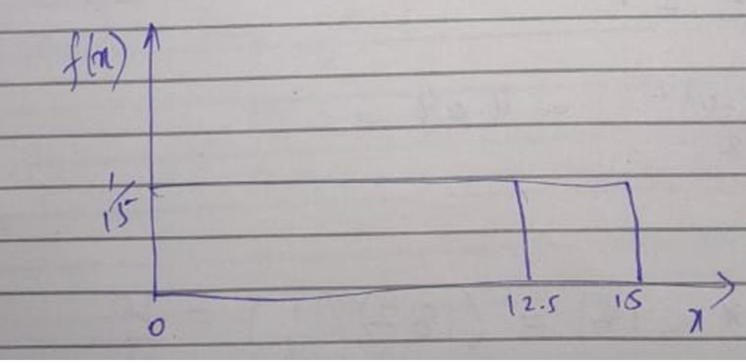
$a = 0$

$b = 15$

$X \sim U(0,15)$

$f(x) = \frac{1}{15-0} = \frac{1}{15} \quad 0 \le x \le 15$

$P(x<k)$ = base×height = $(12.5)\left(\frac{1}{15}\right) = 0.8333$



b) On the average, how long must a person wait? Find the mean, μ and the standard deviation, σ

SOLUTION:

b) $\mu = \frac{15+0}{2} = 7.5$

$\sigma = \sqrt{\frac{(b-a)^2}{12}}$

$= \sqrt{\frac{12^2}{12}} = 4.3$