(परीक्षार्थी द्वारा भरा जाए)

(To be filled by the Candidate)

Second Periodical Test, January-April/May, 2021

परीक्षा का नाम **(Name of Examination)**........ 2nd periodical 6th sem

अनुक्रमांक अंकों में**(In figures)** 1813031

अनुक्रमांक(शब्दों में)**(Roll No. in Words)** .... Eighteen lakh thirteen thousand thirty one

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ई-मेलआई-डी· **(E-mail ID.)** btbte18076_tejaswini@banasthali.in

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Write NA for questions not attempted		NA		NA		NA

Signature of the Student

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Numerical Methods
MATH - 311.

1 (i) $LHS = \Delta^2 y_8$

$= \nabla y_8 - \nabla y_7$

$\therefore$

$= (y_8 - y_7) - (y_7 - y_6)$

$= y_8 - 2y_7 + y_6 = RHS$

$\therefore \quad \boxed{\Delta^2 y_8 = y_8 - 2y_7 + y_6}$

Hence Proved.

(ii) Given,

let $A = \begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix}$

we know that in Doolittle's Method

$A = LU$ —— ①

where,

$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$

$\therefore$ From eq.^{n} ①

$$\begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ l_{21} U_{11} & l_{21} U_{12} + U_{22} \end{bmatrix}$$

On comparing both sides, we get.

$$U_{11} = 4$$
$$U_{12} = 3$$

$$l_{21} U_{11} = 6 \Rightarrow l_{21} \times 4 = 6$$
$$\Rightarrow l_{21} = \frac{6}{4}$$
$$\boxed{l_{21} = 1.5}$$

$$l_{21} U_{12} + U_{22} = 3 \Rightarrow 1.5 \times 3 + U_{22} = 3$$
$$\Rightarrow U_{22} = 3 - 4.5$$
$$\boxed{U_{22} = 1.5}$$

$$\therefore \quad L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix} \text{ and}$$

$$U = \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & -1.3 \end{bmatrix}$$

L and U are matrix.

(iv) Given,

$$\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$

Now,

$$\cancel{\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}}$$

$$\left(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}\right) y_x = \left(\frac{E-1}{1-E^{-1}} - \frac{1-E^{-1}}{E-1}\right) y_x$$

$$= \left\{ \frac{E-1}{\left(\frac{E-1}{E}\right)} - \frac{\left(\frac{E-1}{E}\right)}{E-1} \right\} y_x$$

$$= \left(E - \frac{1}{E}\right) y_x$$

$$= (E - E^{-1}) y_x$$

$$= \{(1+\Delta) - (1-\nabla)\} y_x$$

$$= \cancel{(\Delta + \Delta)} \; (\Delta + \nabla) y_x$$

Hence,

$$\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$$

Hence Proved.

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(iii) Given,

x :	1	1.25	1.5	1.75	2.0
y :	0	0.223144	0.405465	0.559619	0.693147

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	0				
		0.223144			
1.25	0.223144		-0.040823		
		0.182321		0.012656	
1.5	0.405465		-0.028167		-0.005115
		0.154154		0.007541	
1.75	0.559619		-0.020626		
		0.133528			
2.0	0.693147				

Here,

let, $x = 1.9$

$x_n = 2.0$

$y_n = 0.693147$

$h = x_1 - x_0$

$= 1.25 - 1$

$h = 0.25$

$\because \; p = \dfrac{x - x_n}{h}$

$= \dfrac{1.9 - 2}{0.25}$

$p = -0.4$

$\nabla y_n = 0.133528$

$\nabla^2 y_n = -0.020626$

$\nabla^3 y_n = 0.007541$

$\nabla^4 y_n = -0.005115$

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$\because P_n(x) = y_n + P\nabla y_n + [P(P+1)/2!]\,\nabla^2 y_n + [P(P+1)(P+2)/3!]\,\nabla^3 y_n + [P(P+1)(P+2)(P+3)/4!]\,\nabla^4 y_n$

$= 0.693147 + (-0.4)\times 0.133528 + [(-0.4)(-0.4+1)/2!]\times(-0.020626) + [(-0.4)(-0.4+1)(-0.4+2)/3!]\times(0.007541) + [(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)/4!]\times(-0.005115)$

$= 0.6933147 - 0.0534112 + 0.00247512 - 0.000482624 + 0.000212784$

$\therefore f(1.9) = P_n(x) = 0.64194108$

$\boxed{f(1.9) = 0.64194108}$

(3) Given,

$20x - y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

From the eq^{n}s, we can see that

$|20| > |-1| + |-2|$

$|20| > |3| + |-1|$

$|20| > |2| + |-3|$

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i.e, the given eq.^{n}s are in correct order

$$\therefore \quad x = \frac{1}{20}[17 + y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

Let the initial approximations are

$$x^{(0)} = y^{(0)} = (z)^{(0)} = 0$$

1st Approx.

$$x^{(1)} = \frac{1}{20}[17 + y^{(0)} + 2z^{(0)}] = \frac{17}{20} = 0.85$$

$$y^{(1)} = \frac{1}{20}[-18 - 3x^{(1)} + z^{(0)}] = \frac{1}{20}[-18 - 3\times0.85 + 0]$$

$$= -1.0275$$

$$z^{(1)} = \frac{1}{20}[25 - 2x^{(1)} + 3y^{(1)}] = \frac{1}{20}[25 - 2\times0.85 + 3(-1.0275)]$$

$$= 1.0109$$

2nd Approx.

$$x^{(2)} = \frac{1}{20}[17 + y' + 2z'] = \frac{1}{20}[17 - 1.0275 + 2\times1.0109]$$

$$= 0.8997$$

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$$y^{(2)} = \frac{1}{20}[-18-3x^{(2)}+z^{(1)}] = \frac{1}{20}[-18-3\times 0.8997+1.0109]$$

$$= -0.9844$$

$$z^{(2)} = \frac{1}{20}[25-2x^{(2)}+3y^{(2)}] = \frac{1}{20}[25-2\times 0.8997+3(-0.9844)]$$

$$= 1.0124.$$

3rd Approx.

$$x^{(3)} = \frac{1}{20}[17+y^{(2)}+2z^{(2)}] = \frac{1}{20}[17-0.9844+2\times 1.0124]$$

$$= 0.9020$$

$$y^{(3)} = \frac{1}{20}[-18-3x^{(3)}+z^{(2)}] = \frac{1}{20}[-18-3\times 0.9020+1.0124]$$

$$= -0.9841$$

$$z^{(3)} = \frac{1}{20}[25-2x^{(3)}+3y^{(3)}] = \frac{1}{20}[25-2\times 0.9020+3\times(-0.98468)]$$

$$= 1.0122.$$

Hence,

$x = 0.9020$
$y = -0.9847$
$z = 1.0122$

Sol.^n of the given eq.^ns.

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⑤ Given,

$4x + 2y + 6z = 16$

$2x + 82y + 39z = 206$

$6x + 39y + 26z = 1\cdot 13.$

The given system of linear eq^n can be written as -

$$AX = B \quad \text{—— } ①$$

where,

$$A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 16 \\ 206 \\ 113 \end{bmatrix}$$

Now,

$$A^T = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix} = A.$$

$\because A^T = A$, the matrix A is symmetric. Thus, we can apply Cholesky's Method.

Let,

$$A = LL^T \quad \text{—— } ②$$

Where,

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

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from eq.^{n} 1 & 2

$$(LL^T)X = B$$
$$L(L^TX) = B \quad \text{—— } (3)$$

let $L^TX = V$ —— (4) where $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$LV = B$ —— (5)

from eq.^{n} 3

$$\begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{11}l_{21} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{11}l_{31} & l_{21}l_{31} + l_{22}l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

On comparing both sides,

$l_{11}^2 = 4 \Rightarrow l_{11} = 2$

$l_{11}l_{21} = 2 \Rightarrow l_{21} = 1$

$l_{11}l_{31} = 6 \Rightarrow l_{31} = 3$

$l_{11}l_{21} = 2 \Rightarrow l_{21} = 1$

$l_{21}^2 + l_{22}^2 = 82 \Rightarrow l_{22}^2 = 81 \Rightarrow l_{22} = 9$

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$l_{21} l_{31} + l_{22} l_{32} = 39 \Rightarrow 1 \times 3 + 9 \times l_{32} = 39$

$\Rightarrow l_{32} = \frac{36}{9}$

$l_{32} = 4$

$l_{31}^2 + l_{33}^2 + l_{33}^2 = 26 \Rightarrow (3)^2 + 4^2 + l_{33}^2 = 26$

$l_{33} = 1.$

$\therefore \quad L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 9 & 0 \\ 3 & 4 & 1 \end{bmatrix}$

from eq.^{n} (3)

$$\begin{bmatrix} 2 & 0 & 3 \\ 1 & 9 & 4 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 206 \\ 113 \end{bmatrix}$$

$\therefore \quad 2V_1 = 16 \Rightarrow V_1 = 8$

$V_1 + 9V_2 = 206 \Rightarrow 9V_2 = 206 - 8 \Rightarrow V_2 = \frac{198}{9}$

$\Rightarrow V_2 = 22$

$3V_1 + 4V_2 + V_3 = 113 \Rightarrow 3 \times 8 + 4 \times 22 + V_3 = 113$

$\Rightarrow V_3 = 113 - 112$

$V_3 = 1$

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from eq.^{n} 4

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 9 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 22 \\ 1 \end{bmatrix}$$

$\therefore \quad z = 1$

$9y + 4z = 22 \Rightarrow 9y + 4 = 22 \Rightarrow y = 2$

$2x + y + 3z = 8 \Rightarrow 2x + 2 + 3 = 8 \Rightarrow x = 1.5$

Hence,

$x = 1.5$
$y = 2$
$z = 1$

sol.^{n} of the given system.