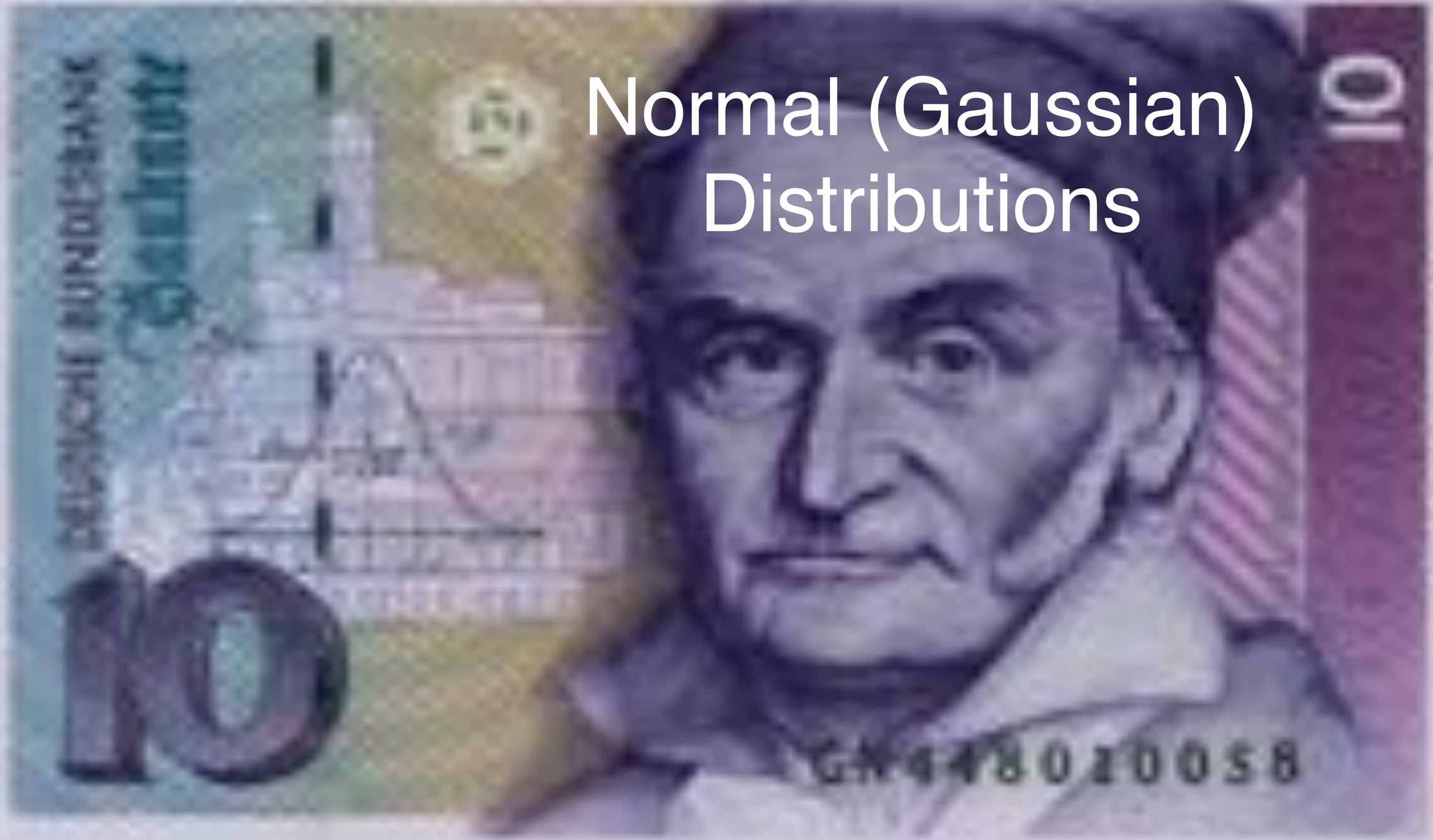
Normal (Gaussian) Distributions



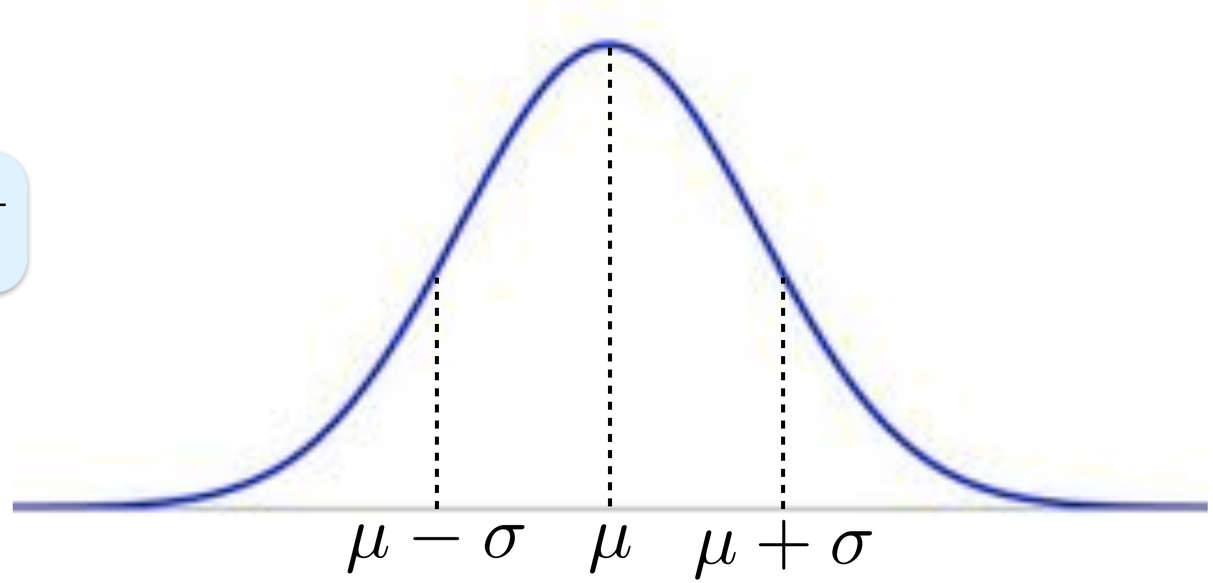
Definition

Mean

Variance

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Bell Curve

Very common

Occurs whenever adding many independent factors

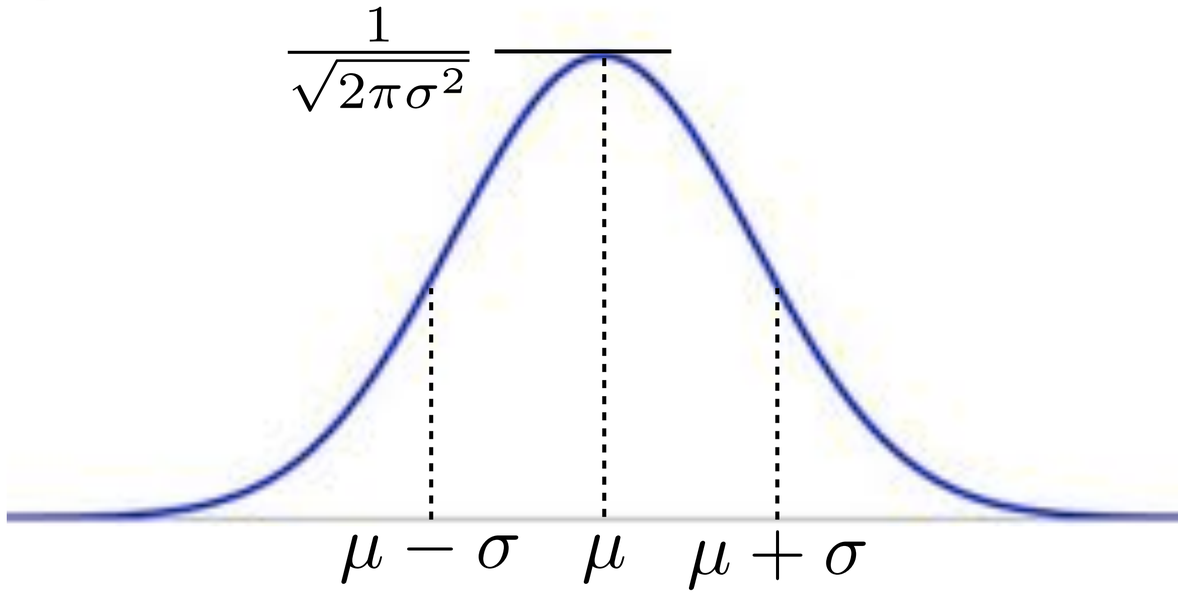
height, weight, rainfall, salaries, ….

Approximates binomial distribution

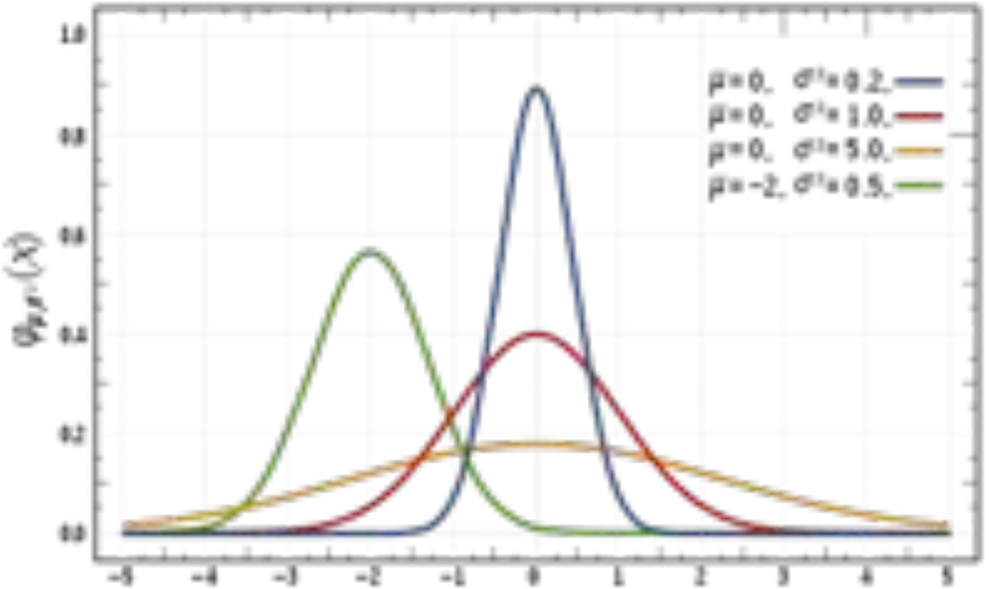
Observations

Symmetric around mean μ

μ most likely $f(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}}$



As σ grows, distribution gets more flat



Linear Transformations

Linear transformations of normal distributions are normal

$$X \sim N(\mu, \sigma)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$Y = aX + b$$

As for all r.v.

$$\mu_Y = a\mu_X + b$$

$$\sigma_Y = a\sigma_X$$

Show normal

Variable transformation

$$f_Y(y) = \frac{1}{(ax+b)'} f_X(x)\Big|_{y=ax+b} \leftrightarrow x = \frac{y-b}{a}$$

$$= \frac{1}{a} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\frac{y-b}{a}-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi\underbrace{(a\sigma)^2}_{\sigma_Y}}} e^{-\frac{(y-\overbrace{(a\mu+b)}^{\mu_Y})^2}{\underbrace{2(a\sigma)^2}_{\sigma_Y}}}$$

$$Y \sim N(a\mu + b, (a\sigma)^2)$$

Standard Normal Distribution

Without loss of generality consider X ~ N(0,1)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

Helpful integral $\int x e^{-\frac{x^2}{2}} dx = -e^{-\frac{x^2}{2}}$

Σ WILL IT ADD?

$$I = \int_{-\infty}^{\infty} e^{\frac{-x^2}{2}}\, dx$$

$$I^2 = \left(\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}}\, dx\right)\left(\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}}\, dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-(x^2+y^2)}{2}}\, dx\ dy$$

$$= \int_{0}^{\infty}\int_{0}^{2\pi} e^{\frac{-r^2}{2}}\, r\ d\theta\ dr$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$dx\ dy = r\ dr\ d\theta$$

$$= \int_{0}^{\infty} r\ e^{\frac{-r^2}{2}} \int_{0}^{2\pi} d\theta\ dr$$

$$= 2\pi \int_{0}^{\infty} r\ e^{\frac{-r^2}{2}}\, dr$$

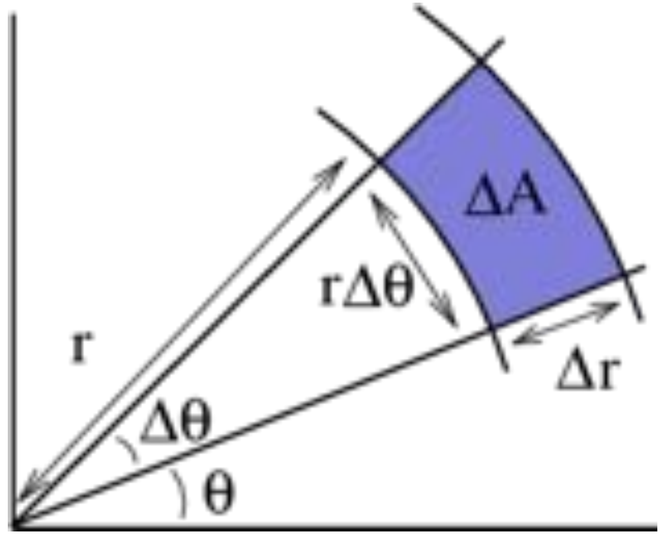
$$\int r e^{\frac{-r^2}{2}}\, dr = -e^{\frac{-r^2}{2}}$$

$$= -2\pi e^{\frac{-r^2}{2}}\Big|_0^{\infty} = 2\pi$$

$$I = \sqrt{2\pi}$$

$$\int \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx = 1$$

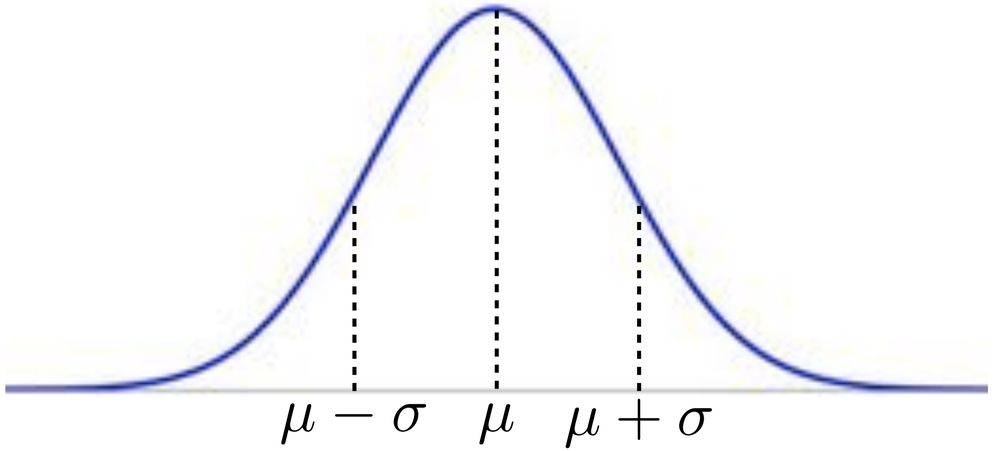
YES IT ADDS!



Expectation

Symmetry

$$E(X) = 0$$



Calculation

$$E(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{\frac{-x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[-e^{\frac{-x^2}{2}} \right|_{-\infty}^{\infty} = 0$$

Variance

$$E(X^2) = \frac{1}{\sqrt{2\pi}} \int x^2 e^{\frac{-x^2}{2}} dx$$

$$u = x \qquad dv = x\ e^{\frac{-x^2}{2}} dx$$

$$du = dx \qquad v = -e^{\frac{-x^2}{2}}$$

$$\int u\ dv = uv - \int v\ du$$

$$= \frac{-1}{\sqrt{2\pi}} x e^{\frac{-x^2}{2}} \Big|_{-\infty}^{\infty} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx = 0 + 1 = 1$$

$$V(X) = E(X^2) - (EX)^2 = 1 - 0 = 1$$

Normal (Gaussian) Distributions

$$\mathbb{N}(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

$$EX = \mu$$

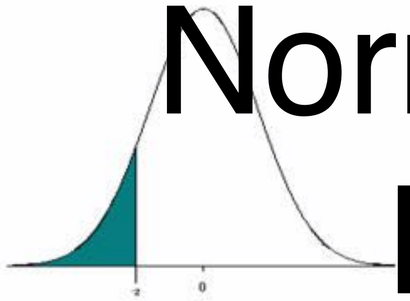
$$V = \sigma^2$$

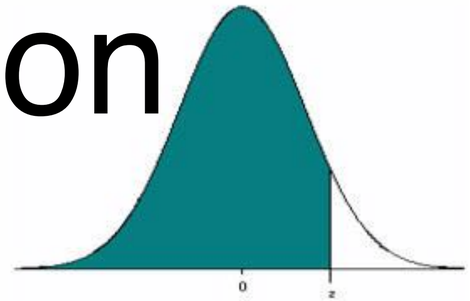
$$\sigma = \sigma$$

Very common in nature

Next Probabilities

Normal Distribution Probabilities





Interval probabilities

Z table and scores

Standard deviation

Approximating Bernoulli

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3						11	0.0011	0.0011	0.0010	0.0010
-2						16	0.0015	0.0015	0.0014	0.0014
-2						22	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2						0.0094	0.0091	0.0089	0.0087	0.0084
-2						0.0122	0.0119	0.0116	0.0113	0.0110
-2						0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1						0606	0.0594	0.0582	0.0571	0.0559
-1						0735	0.0721	0.0708	0.0694	0.0681
-1						0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0						5		0.2206	0.2177	0.2148
-0						5		0.2514	0.2483	0.2451
-0.						7		0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

CDF

$$X \sim N(0,1)$$

$$\Phi(x) \triangleq F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-y^2}{2}} dy$$

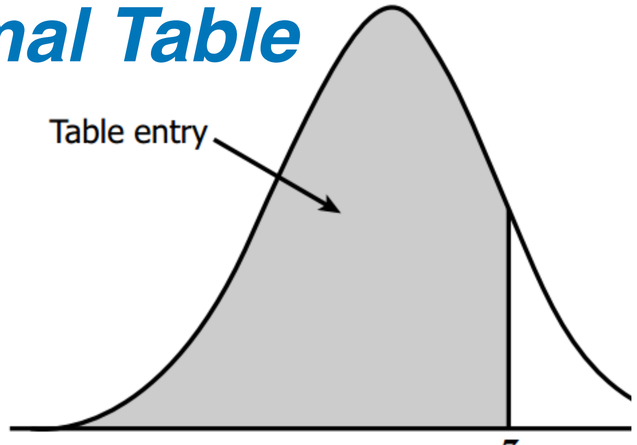
No known formula

Instead use table or computer

Table for each μ, σ? 1 suffices!

Standard Normal Table

Z Table

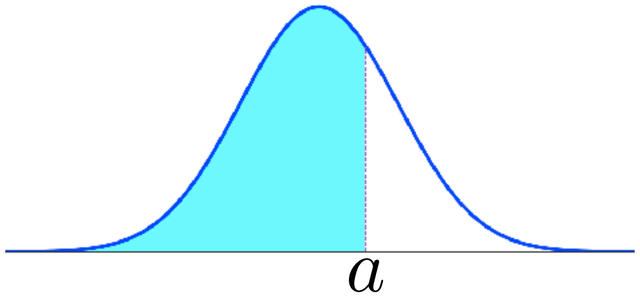


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

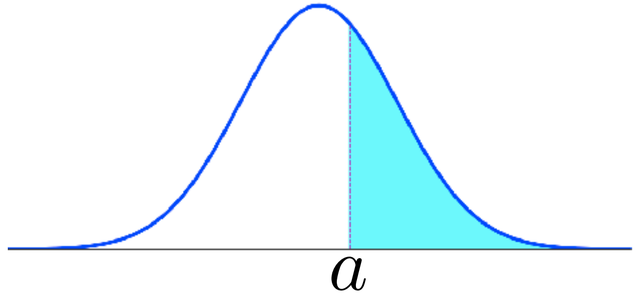
Using the Z Table

$X \sim N(0,1)$

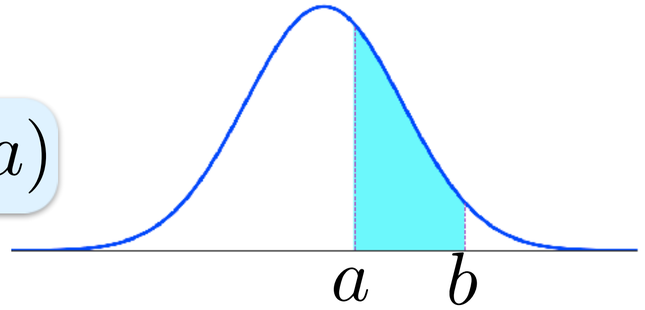
$P(X \leq a) = \Phi(a)$



$P(X \geq a) = 1 - \Phi(a)$



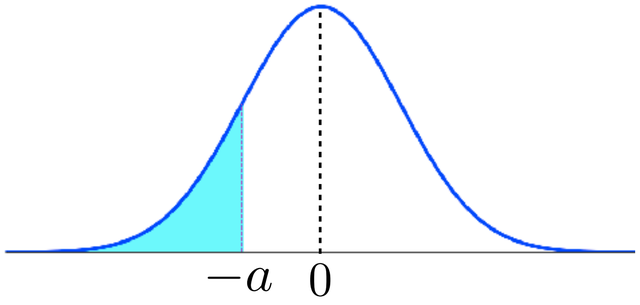
$P(a \leq X \leq b) = \Phi(b) - \Phi(a)$

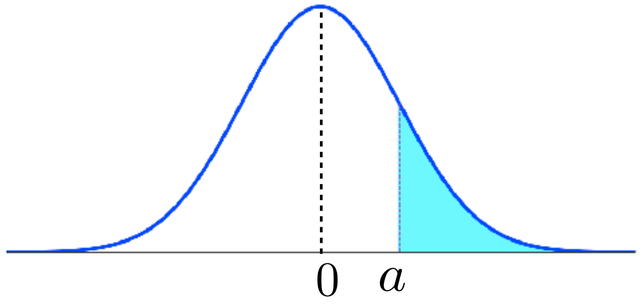


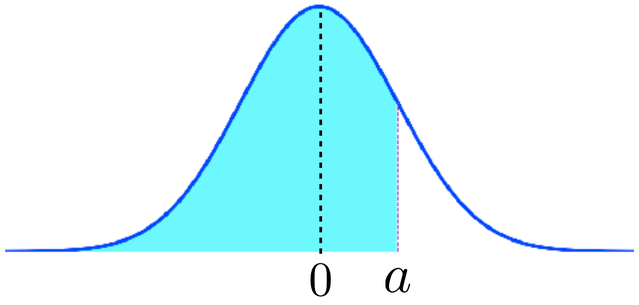
Negative Values

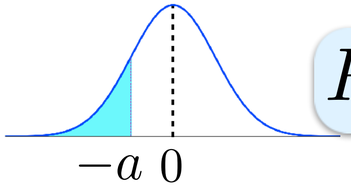
a > 0

$$\Phi(-a) = P(X \leq -a) = P(X \geq a) = 1 - P(X \leq a) = 1 - \Phi(a)$$

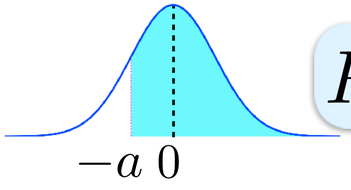




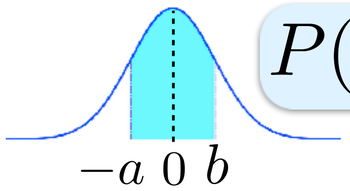




$$P(X \leq -a) = \Phi(-a) = 1 - \Phi(a)$$



$$P(X \geq -a) = 1 - \Phi(-a) = \Phi(a)$$



$$P(-a \leq X \leq b) = \Phi(b) - \Phi(-a) = \Phi(b) - (1 - \Phi(a))$$

$$= \Phi(a) + \Phi(b) - 1$$

General Normal

$$X \sim N(\mu, \sigma^2)$$

Variable	E(X)	V(X)
X	μ	σ^2
$X - \mu$	0	σ^2
$\frac{X-\mu}{\sigma}$	0	1

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

Standardized Version of X

$$P(a \leq X \leq b)$$

$$= P\left(\frac{a-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right)$$

Example

$X \sim N(15, 4)$ $\mu = 15$ $\sigma = 2$

Z score $Z = \frac{X-15}{2}$

$$P(11 \leq X \leq 17) = P(\tfrac{11-15}{2} \leq Z \leq \tfrac{17-15}{2})$$

$$= P(-2 \leq Z \leq 1)$$

$$= \Phi(1) + \Phi(2) - 1$$

$$\approx 0.8413 + 0.9772 - 1$$

$$= 0.8185$$

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700
0.5	.6915	.6950	.6985	.7019	.7054
0.6	.7257	.7291	.7324	.7357	.7389
0.7	.7580	.7611	.7642	.7673	.7704
0.8	.7881	.7910	.7939	.7967	.7995
0.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925
1.3	.9032	.9049	.9066	.9082	.9099
1.4	.9192	.9207	.9222	.9236	.9251
1.5	.9332	.9345	.9357	.9370	.9382
1.6	.9452	.9463	.9474	.9484	.9495
1.7	.9554	.9564	.9573	.9582	.9591
1.8	.9641	.9649	.9656	.9664	.9671
1.9	.9713	.9719	.9726	.9732	.9738
2.0	.9772	.9778	.9783	.9788	.9793
2.1	.9821	.9826	.9830	.9834	.9838
2.2	.9861	.9864	.9868	.9871	.9875
2.3	.9893	.9896	.9898	.9901	.9904
2.4	.9918	.9920	.9922	.9925	.9927
2.5	.9938	.9940	.9941	.9943	.9945
2.6	.9953	.9955	.9956	.9957	.9959

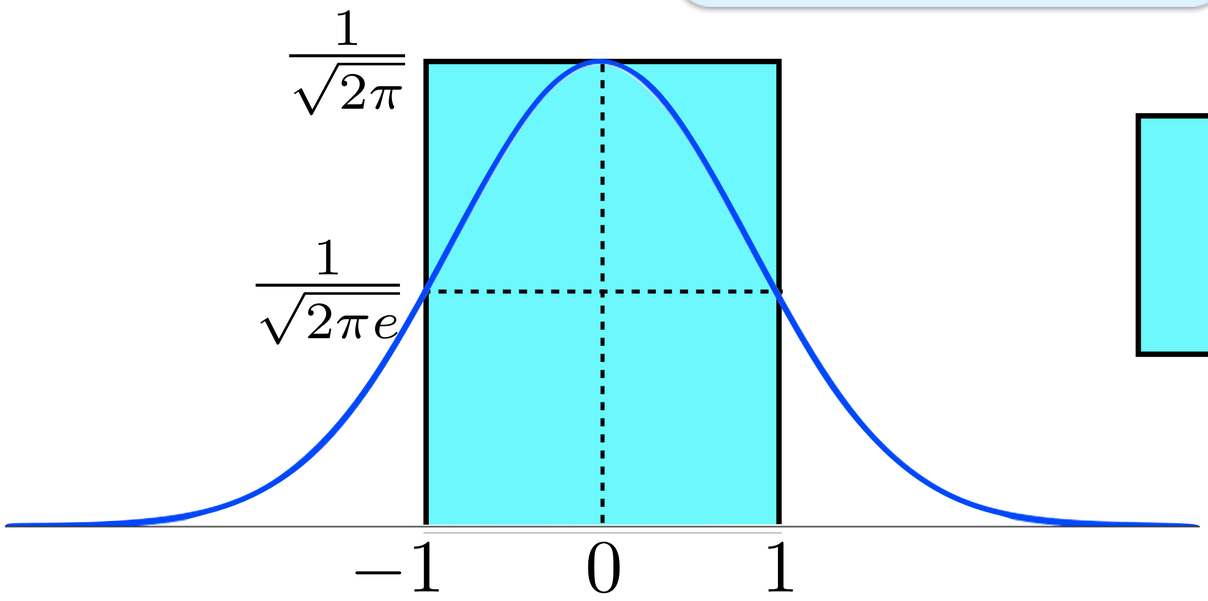
Standard Deviation and Probability

Probability of within σ

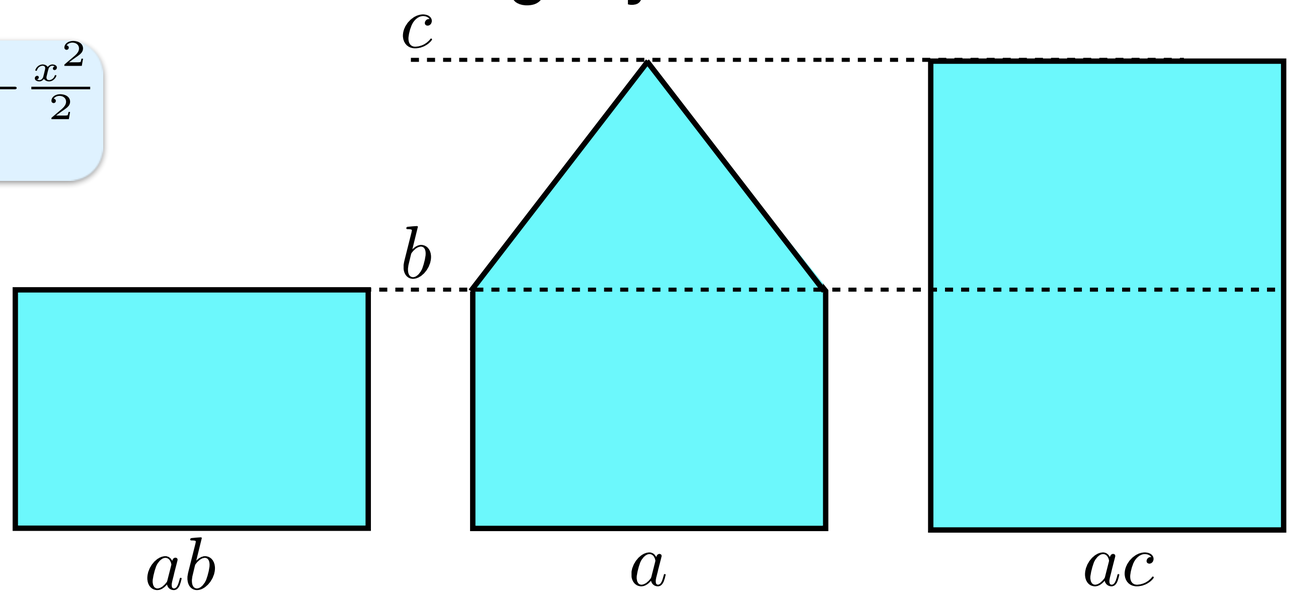
$P(|X-\mu| \le \sigma)$?

Standard normal

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$



How big's your house?



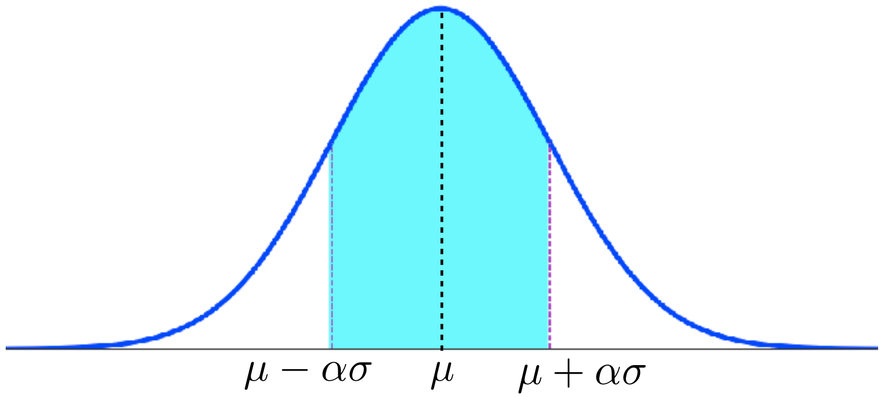
$ab + \frac{a(c-b)}{2} = \frac{ab+ac}{2}$

$0.48 \approx \sqrt{\frac{2}{\pi e}} \le P(|X-\mu| \le \sigma) \le \sqrt{\frac{2}{\pi}} \approx 0.8$

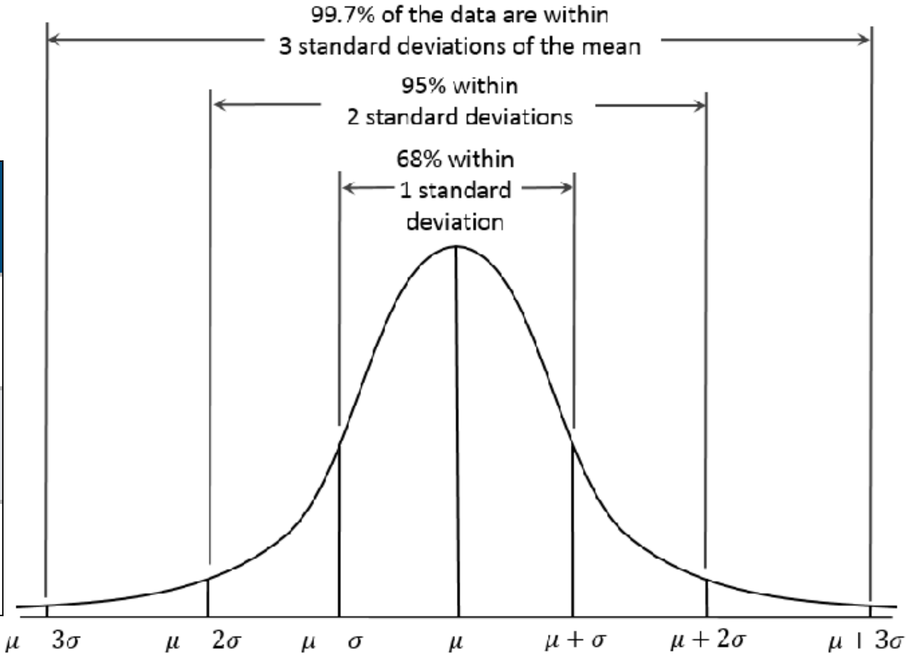
$\frac{\sqrt{\frac{2}{\pi e}}+\sqrt{\frac{2}{\pi}}}{2} \approx 0.64$

68 - 95 - 99.7 Rule

$$P(\mu - \alpha\sigma \leq X \leq \mu + \alpha\sigma) = P(-\alpha \leq Z \leq \alpha) = 2\Phi(\alpha) - 1$$



α	P(|X-μ| ≤ $\alpha\sigma$)
1	2(0.8413) - 1 = 0.682
2	2(0.9772) - 1 = 0.9544
3	2(0.9987) - 1 = 0.9974



Rare Events

α	$P(\lvert X - \mu \rvert \leq \alpha\sigma)$	$P(\lvert X - \mu \rvert > \alpha\sigma)$	Daily event ~
1	68.2%	31.8%	3 days
2	95.4%	4.6%	3 weeks
3	99.7%	0.3%	year
4	99.99%	0.01%	43 years
5	99.9999%	10^{-6}	4776 years
6	99.9999998%	$2 \cdot 10^{-9}$	1.3 millions years

6 σ philosophy

ab-

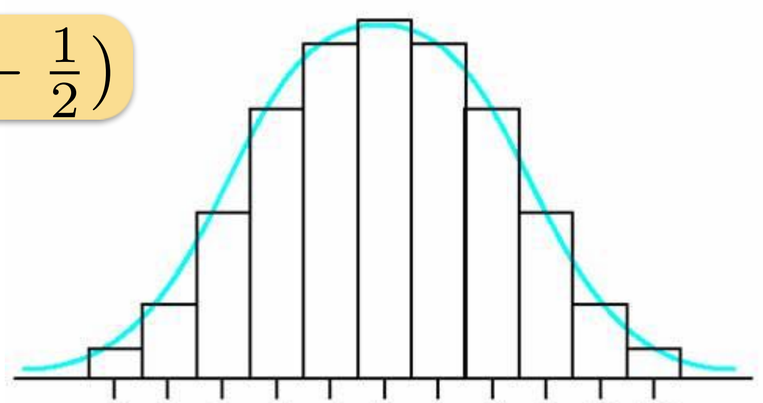
Life is normal

Normal Approximation of Binomial Distribution

Normal Approximation of Binomial

$X \sim B_{n,p}$ $\mu = np$ $\sigma = \sqrt{npq}$

$Y \sim N(np, npq)$ $P(X = k) \approx P(k - \frac{1}{2} \leq Y \leq k + \frac{1}{2})$



$X \sim B_{100,0.5}$ $np = 50$ $\sqrt{npq} = 5$

$Y \sim N(50, 25)$

$Z = \frac{Y-50}{5}$

$P(X = 60) \approx P(59.5 \leq Y \leq 60.5)$

$= P(1.9 = \frac{59.5-50}{5} \leq Z \leq \frac{60.5-50}{5} = 2.1)$

$= \Phi(2.1) - \Phi(1.9) \approx 0.9821 - 0.9713 = 0.0108$

$P(X = 60) = \binom{100}{60} 0.5^{100} \approx 0.0108$

Interval Probabilities

$X \sim B_{100,0.5}$ $np = 50$ $\sqrt{npq} = 5$

$Y \sim N(50, 25)$

$$P(42 \leq X \leq 53) = P(41.5 \leq Y \leq 53.5)$$

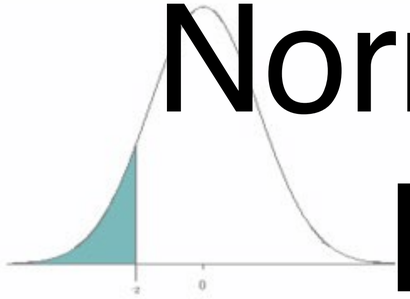
$$= P(-1.7 = \tfrac{41.5-50}{5} \leq Z \leq \tfrac{53.5-50}{5} \leq 0.7)$$

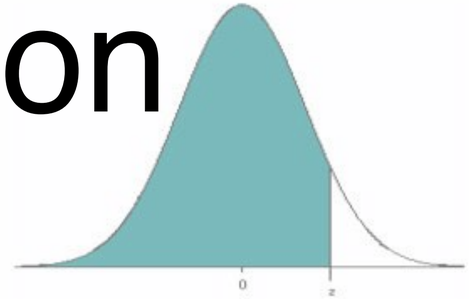
$$= \Phi(0.7) + \Phi(1.7) - 1$$

$$\approx 0.7580 + 0.9554 - 1 = 0.7134$$

$$\text{Actual} = \sum_{k=42}^{53} \binom{100}{k} (\tfrac{1}{2})^{100} \approx 0.7136$$

Normal Distribution Probabilities





Interval probabilities

Z table and scores

Standard deviation

Approximating Bernoulli

Next

Inequalities and limits

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3							0.0004	0.0004	0.0004	0.0003
-3.							0.0006	0.0005	0.0005	0.0005
-3.							0.0008	0.0008	0.0007	0.0007
-3.0							0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7						0.0030	0.0029	0.0028	0.0027	0.0026
-2.						0.0040	0.0039	0.0038	0.0037	0.0036
-2.						0.0054	0.0052	0.0051	0.0049	0.0048
-2.						0.0071	0.0069	0.0068	0.0066	0.0064
-2.3						0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.						0.0256	0.0250	0.0244	0.0239	0.0233
-1						0.0322	0.0314	0.0307	0.0301	0.0294
-1						0.0401	0.0392	0.0384	0.0375	0.0367
-1.						0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3								0.0853	0.0838	0.0823
-1								0.1020	0.1003	0.0985
-1								0.1210	0.1190	0.1170
-1.								0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.		0.3050								0.2776
-0		0.3409								0.3121
-0		0.3783								0.3483
-0.		0.4168								0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998