Time Series Analysis

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What is Time Series ?

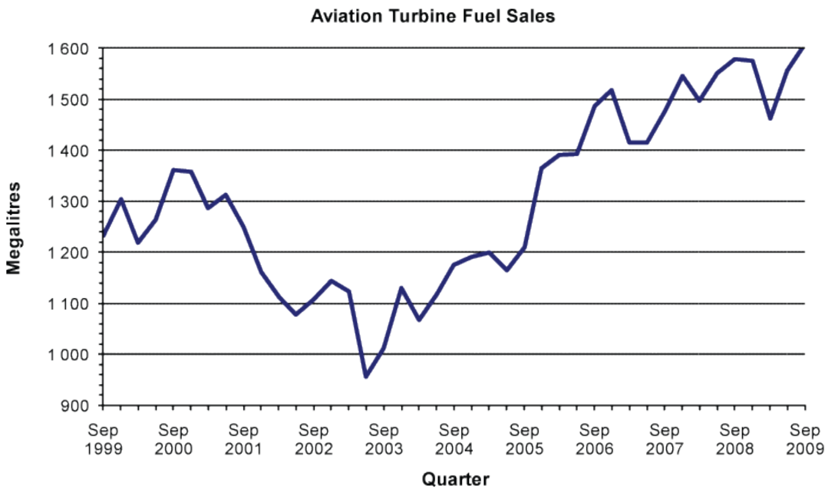
Time Series

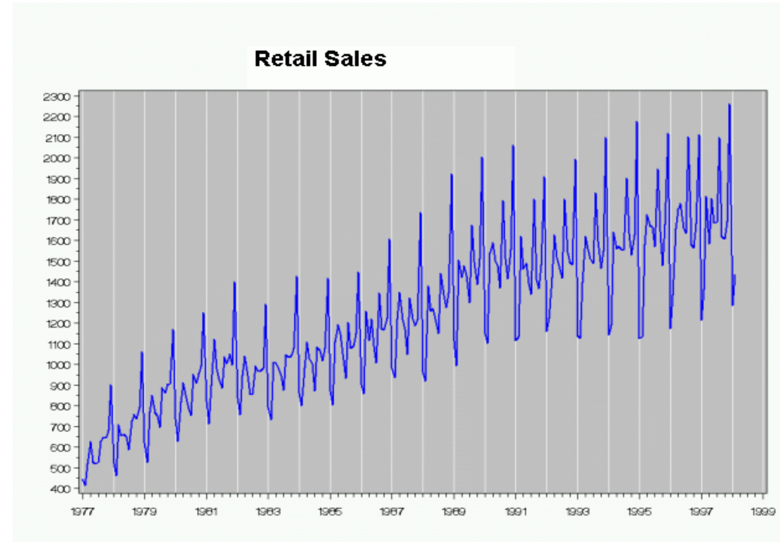
- A time series is a sequence of data points, measured typically at successive times, spaced at uniform time intervals.
- Examples of Time Series : Daily opening/closing values of BSE, Nifty etc, number of defaults a day at a financial institution, number of accidents daily on a particular stretch of road etc.
- **Time series *analysis*** comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data
- **Time series *forecasting*** is the use of a model to forecast future events based on known past events: to predict data points before they are measured.

Objectives of Time Series Analysis

- Identifying the nature of the phenomenon represented by the sequence of observations
- Obtaining an understanding of the underlying forces and structure that produced the observed data
- Fitting a model and proceeding to forecasting   monitoring or even feedback and feed forward control.

Examples of Time Series





As in most other analyses, in time series analysis it is assumed that the data consist of a systematic pattern (usually a set of identifiable components) and random noise (error).

Assumptions of Time Series Analysis

- Pattern of past will continue into the future. (else the series will represent a random walk which cannot be modeled or forecasted)
- Discrete time series data should be equally spaced over time. There should be no missing values in the training data set(or should be handled using appropriate missing value treatment in the data preparation process).
- Time Series Analysis cannot be used to predict effect of random events(Example- Terrorist Attacks or acts of god such as Tsunami disaster etc)

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Identifying Patterns in Time Series Data

Decomposition of Time Series

The decomposition of time series is a statistical method

that deconstructs a time series into notional components.

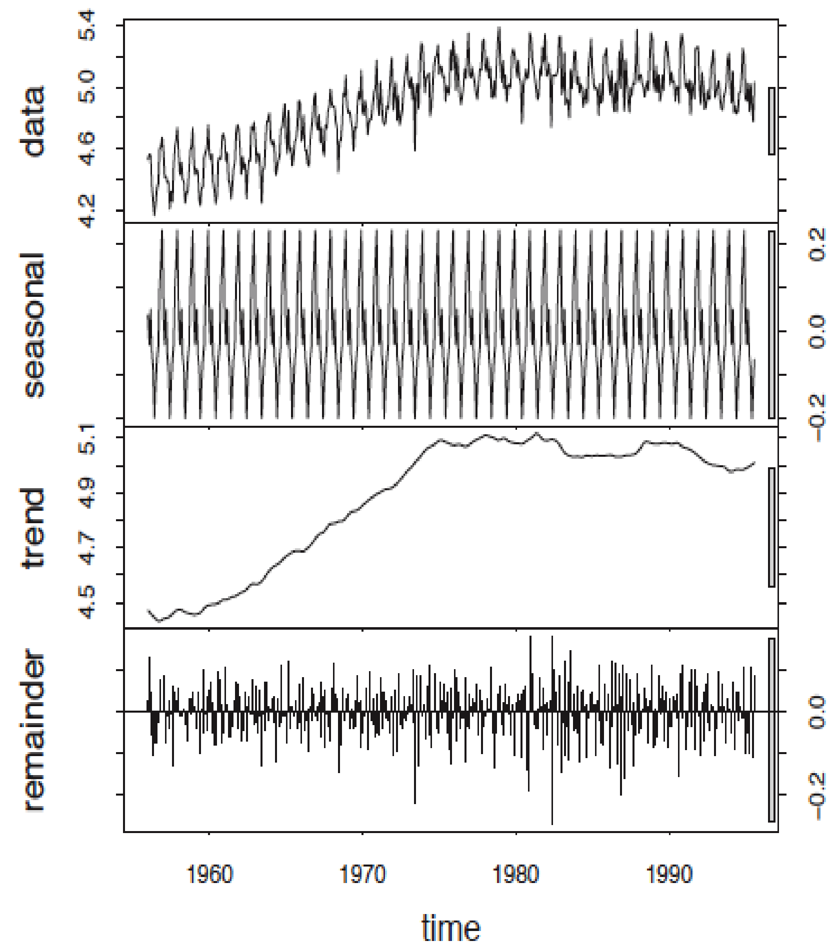
Decomposition based on rates of change:

A time series can be represented as the combination of the following components

- **Trend** Component that reflects the long term progression of the series
- **Cyclical** Component that describes regular fluctuations caused by the economic cycle
- **Seasonal** Component reflecting seasonality (seasonal fluctuations)
- **Irregular** Component (or "noise") that describes random, irregular influences. Compared to the other components it represents the residuals of the time series.

Hence, **Y = f (T,S,C,I)**

Where Y denotes the result of the four elements; T = Trend ; S = Seasonal component; C = cyclical components; I = Irregular component



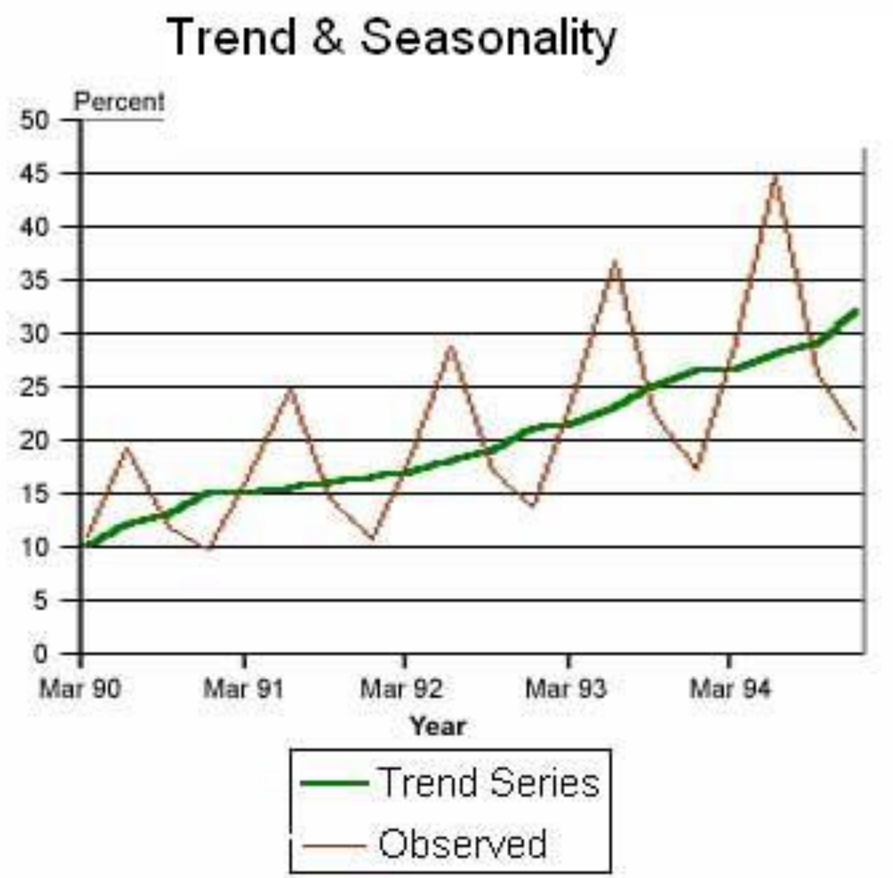
General Aspect of Analysis

Most time series patterns are described in terms of two basic components:

- **Trend**
- **Seasonality**

Trend Analysis : Smoothing is commonly used for extracting the trend. Various smoothing techniques are:

- Simple Moving Average
- Weighted Moving Average
- Exponential Smoothing (Single, double etc)



Seasonality: Refers to the regular repeating component of the series, formally defined as correlational dependency of order *k* between each *i*'th element of the series and the (*i*-*k*)'th element and measured by autocorrelation; *k* is usually called the *lag*.

Methods used for capturing seasonality are:

- Autoregression
- Moving Averages

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Smoothing Techniques

Smoothing techniques are used to reduce irregularities (random fluctuations) in time series data. They provide a clearer view of the true underlying behavior of the series. Methods for smoothing are:

- **Simple Moving Averages:** Involves simply taking a certain number of past periods adding together; then dividing by the number of periods. Simple Moving Averages (MA) is effective and efficient approach provided the time series is stationary in both mean and variance. The following formula is used in finding the moving average of order n, MA(n) for a period t+1,

$$MA_{t+1} = [D_t + D_{t-1} + \dots + D_{t-n+1}] / n \qquad \hat{y}$$

- **Weighted Moving:** A slightly more intricate method for smoothing a raw time series $\{x_t\}$ is to calculate a weighted moving average by first choosing a set of weighting factors $\{w_1, w_2, \dots, w_k\}$ such that $\sum_{n=1}^{k} w_n = 1$

  and then using these weights to calculate the smoothed statistics $\{s_t\}$:

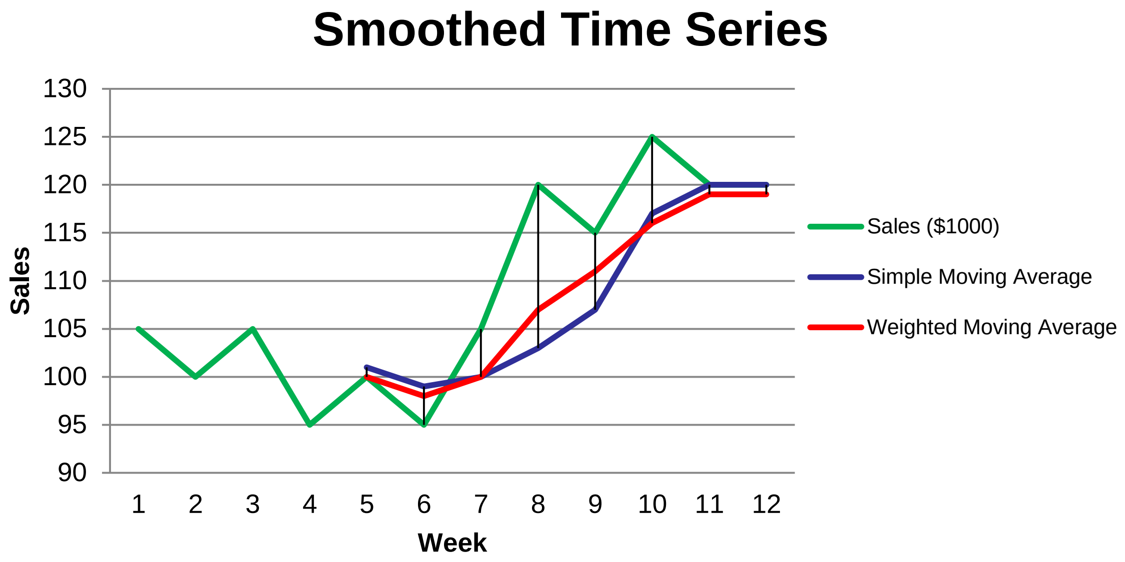
$$s_t = \sum_{n=1}^{k} w_n x_{t+1-n} = w_1 x_t + w_2 x_{t-1} + \cdots + w_k x_{t-k+1}.$$

In practice this is used to give more weight to recent terms as compared to older terms, whereas in SMA; all terms have equal weight.

For e.g. Weighted MA(3) = w1.Dt + w2.Dt-1 + w3.Dt-2 where the weights are any positive numbers such that: w1 + w2 + w3 = 1. A typical weights for this example is, w1 = 3/(1 + 2 + 3) = 3/6, w2 = 2/6, and w3 = 1/6.

Smoothing Techniques

Week	Sales ($1000)	MA(5)	WMA(5)
1	105	-	-
2	100	-	-
3	105	-	-
4	95	-	-
5	100	101	100
6	95	99	98
7	105	100	100
8	120	103	107
9	115	107	111
10	125	117	116
11	120	120	119
12	120	120	119



The moving average and weighted moving average of order five are calculated in the following table.

A window of 5 has been taken for calculation the smoothed value.

Smoothing Techniques

EXPONENTIAL SMOOTHING

The simplest form of exponential smoothing is given by the formulae

$$s_0 = x_0$$

$$s_t = \alpha x_t + (1-\alpha)s_{t-1} = s_{t-1} + \alpha(x_t - s_{t-1})$$

– where α is the *smoothing factor*, and 0 < α < 1.

– X(t) is the actual value

– S(t) is the forecast value

– t is the current time period (smoothed value also becomes forecast for the period t+1)

- **A small α** provides a detectable and visible smoothing. While **a large α** provides a fast response to the recent changes in the time series but provides a smaller amount of smoothing.
- An exponential smoothing over an already smoothed time series is called **double-exponential smoothing**. In some cases, it might be necessary to extend it even to a **triple-exponential smoothing**.
- While simple exponential smoothing requires stationary condition, the double-exponential smoothing can capture linear trends, and triple-exponential smoothing can handle almost all other business time series (parabolic etc).

Smoothing Techniques

Holt's Linear Exponential Smoothing

- For non seasonal series with trend. Trend(T_t) at time t defined as the difference between current and previous level.
- Therefore Smoothing takes the form of Level Trend, where level is determined as similar to Exponential smoothing.
  - Level (L_t) → $a\, y_t + (1 - a)\, F_t$
  - Trend(T_t) → $b\, (L_t - L_{t-1}) + (1 - b)\, T_{t-1}$
  - Two smoothing parameters (a,b) are used, both positive and less than 1

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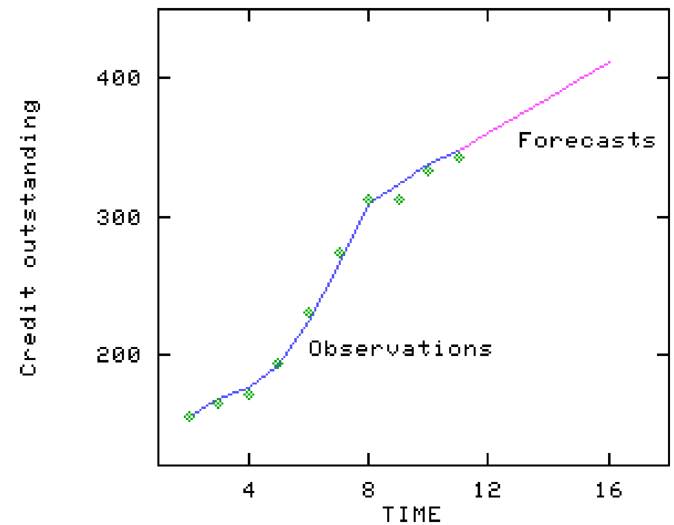
- Forecasting for k periods into the future is done as:
  - $F_{n+k} = L_n + k.\, T_n$

Applying the Holt's techniques with smoothing with parameters a = 0.7 and b = 0.6, a graphical representation of the time series, its forecasts, together with a k step ahead forecasts, are depicted sideways

Year-end Past credit	
Year	credit (in millions)
1	133
2	155
3	165
4	171
5	194
6	231
7	274
8	312
9	313
10	333
11	343
K-Period Ahead Forecast	
K	Forecast (in millions)
1	359.7
2	372.6
3	385.4
4	398.3



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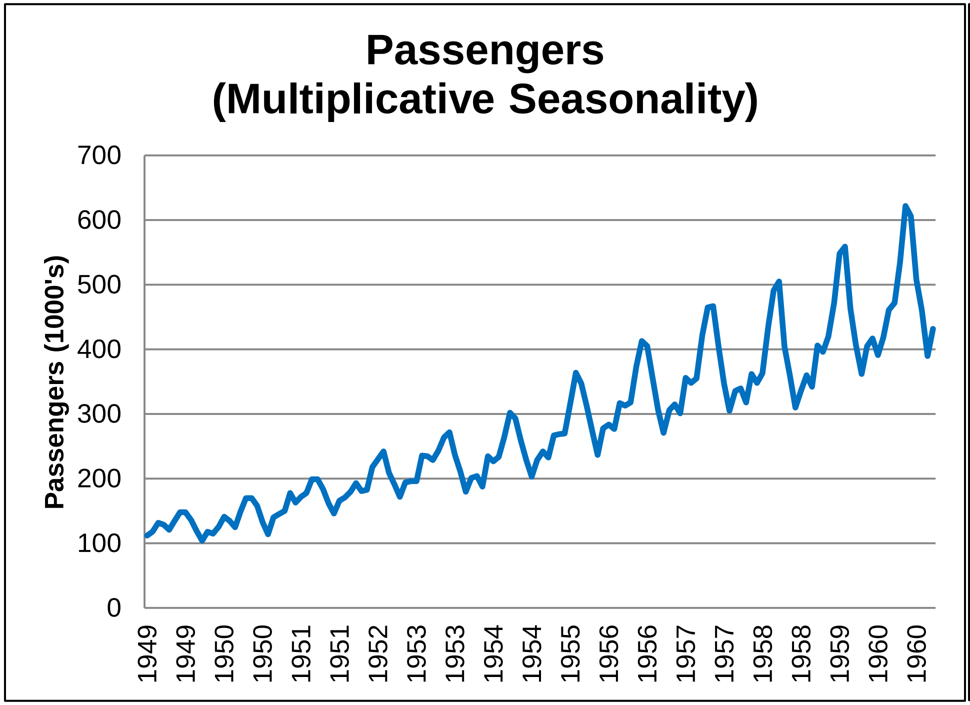
Seasonality

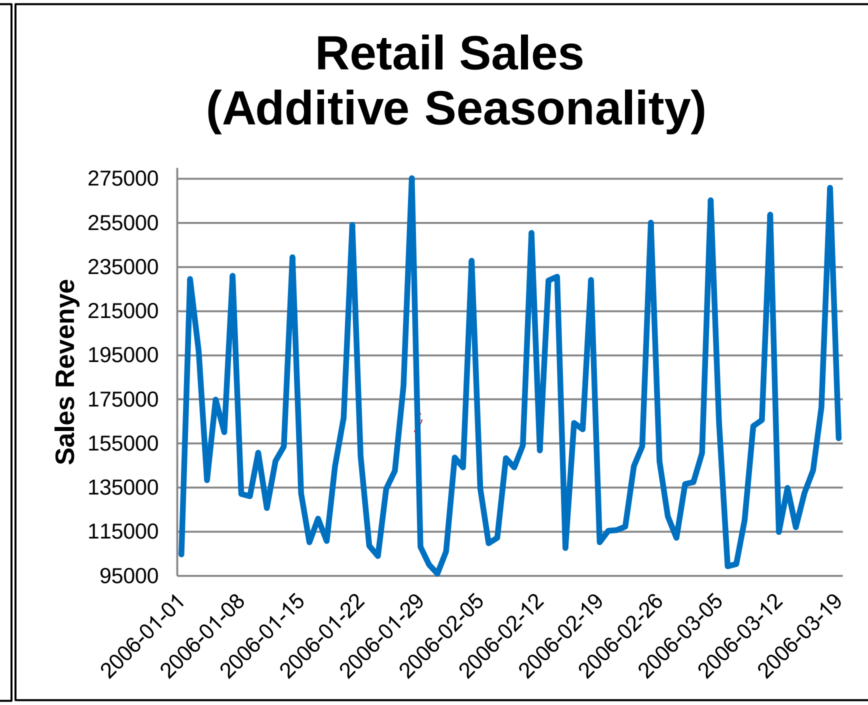
Many time series data follow recurring seasonal patterns. For example sales may peak around Diwali or Christmas year after year. Movie ticket sales may increase noticeably on weekends. Thus, it may be useful to smooth the seasonal component independently with an extra parameter.

- Seasonality can be **Additive** or **Multiplicative** in nature.
  - Forecast = $S_t + I_{t-p}$ (Additive)
  - Forecast = $S_t * I_{t-p}$ (Multiplicative) $\hat{y}$
  - S_t → Exponentially smoothed value at time t
  - I_{t-p} → Smoothed seasonal factor at time t-p, where p is the length of the season (Seasonal lag)

- We can extend the previous example to illustrate the additive and multiplicative trend-cycle components. In terms of our toy example, a "fashion" *trend* may produce a steady increase in sales (e.g., a trend towards more educational toys in general); as with the seasonal component, this trend may be additive (sales increase by 3 million dollars per year) or multiplicative (sales increase by 30%, or by a factor of 1.3, annually) in nature.
- **Detection:** Detection of seasonality can involve plotting and visually inspecting the series, and also by analyzing the autocorrelogram. (A repeating rise and fall pattern in the ACF indicates presence of a seasonal component with)

Seasonality





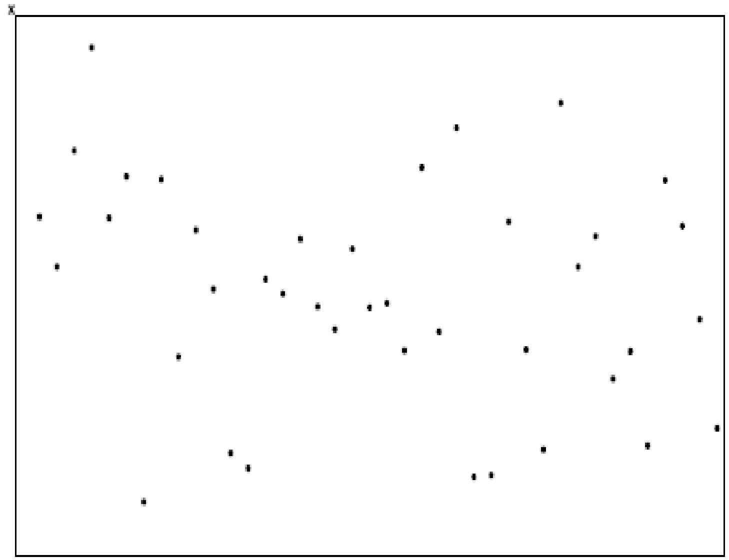
- Fig 1 (Passengers) displays a multiplicative seasonality with a exponentially rising trend.
- Fig 2 (Retail Sales) displays an additive seasonality with constant mean (no linear trend)

FORECASTING

TIME TREND MODELS

Time trend models assume that there is some permanent deterministic pattern across time. These models are best suited to data that are not dominated by random fluctuations.

For time series without trend → $x(t) = b_0 + e_t$



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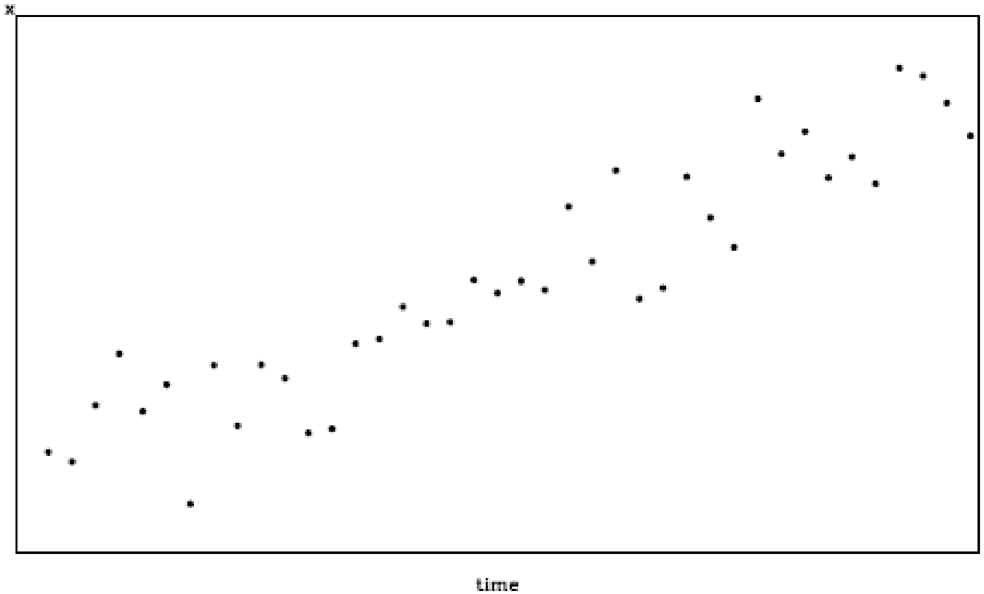
- e_t is an independent zero-mean random error element
- B_0 is the series mean

FORECASTING

- For time series with trend → $x(t) = b_0 + b_1 t + e_t$



- This can be extended to generate polynomial models (i.e. quadratic for parabola etc)

$$x(t) = b_0 + b_1 t + b_2 t^2 + e_t$$

- Exponential Smoothing can fit three types of time trend models → **Constant, Linear and Quadratic.**
- Additionally **Winters method** can be used to incorporate the seasonal factor in exponential smoothing.

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Autoregressive and Moving Averages

Also known as **Box and Jenkins Methodology,** applies autoregressive moving average ARMA or ARIMA models to find the best fit of a time series to past values of this time series, in order to make forecasts.

Autoregressive process. Most time series consist of elements that are serially dependent in the sense that one can estimate a coefficient or a set of coefficients that describe consecutive elements of the series from specific, time-lagged (previous) elements. This can be summarized in the equation:

$$X(t) = F_0 + F_1X(t\text{-}1) + F_2X(t\text{-}2) + F_2X(t\text{-}3) + \ldots + F_pX(t\text{-}p) + e_t,$$

Where:

F_0 is a constant (intercept), and

$F_{1,\ 2,\ 3 \ldots p}$ are the autoregressive model parameters.

Stationarity requirement. Note that an autoregressive process will only be stable if the parameters are within a certain range; for example, if there is only one autoregressive parameter then is must fall within the interval of -1 << 1. Otherwise, past effects would accumulate and the values of successive x_t' s would move towards infinity, that is, the series would not be stationary. If there is more than one autoregressive parameter, similar restrictions on the parameter values can be defined

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Autoregressive and Moving Averages

Moving average process Independent from the autoregressive process, each element in the series can also be affected by the past error (or random shock) that cannot be accounted for by the autoregressive component, that is:

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

Where,

- μ is the mean of the series
- $\theta_1, \ldots, \theta_q$ are the parameters of the model
- $\varepsilon_t, \varepsilon_{t-1}, \ldots$ are white noise error terms
- The value of *q* is called the order of the MA model

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Put in words, each observation is made up of a random error component (random shock,) and a linear combination of prior random shocks.

Autoregressive moving average model

The notation ARMA(*p*, *q*) refers to the model with *p* autoregressive terms and *q* moving average terms. This model contains the AR(*p*) and MA(*q*) models

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}.$$

Autoregressive and Moving Averages

ARIMA or Autoregressive and **Integrated** Moving Averages is a special case of ARMA in which the input time series has undergone '**d**' number of differencing passes to make the series stationary.

- In ARIMA the three types of parameters in the model are:
  - The autoregressive parameters (p)
  - The number of differencing passes (d) $\hat{y}$
  - Moving average parameters (q)
- In the notation introduced by Box and Jenkins, models are summarized as ARIMA (*p, d, q*)
- A model described as (0, 1, 2) means that it contains 0 (zero) autoregressive (*p*) parameters and 2 moving average (*q*) parameters which were computed for the series after it was differenced once.

Autoregressive and Moving Averages

Seasonality in ARIMA

If the data is thought to contain seasonal effects, it may be modeled by a SARIMA (seasonal ARIMA) or a periodic ARMA model.

Analogous to the simple ARIMA parameters, seasonal ARIMA also has similar parameters:

- Seasonal autoregressive (*ps*)
- Seasonal differencing (*ds*)
- Seasonal moving average parameters (*qs*) $\hat{y}$

For example, the model (*0,1,2*)(*0,1,1*) describes a model that includes no autoregressive parameters, 2 regular moving average parameters and 1 seasonal moving average parameter, and these parameters were computed for the series after it was differenced once with lag 1, and once seasonally differenced.

Seasonal lag can usually be identified by the ACF and PACF, they will display sizable coefficients at multiples of the seasonal lag. Order of the seasonal parameters can be determined by applying general recommendations of normal ARIMA, although at seasonal lags.

Autoregressive and Moving Averages

BUILDING AN ARIMA MODEL

There are three basic stages in performing an ARIMA analysis:

- Identifying the time series
- Estimating the time series
- Forecasting future values of the time series

IDENTIFICATION

- Detecting Stationarity
  - The input series must be stationary (i.e. constant mean, variance and ACF through time)
  - The number of times the series needs to be differenced to achieve stationarity is reflected in the *d* parameter.
  - In order to determine the necessary level of differencing, one should examine the plot of the data and ***autocorrelogram***.
- Detecting Seasonality
  - Significant changes in level (strong upward or downward changes) usually require first order non seasonal (lag=1) differencing; strong changes of slope usually require second order non seasonal differencing.
  - Seasonal patterns require respective seasonal differencing . If the estimated autocorrelation coefficients decline slowly at longer lags, first order differencing is usually needed.

Autoregressive and Moving Averages

- **Detecting order of AR and MA**
  - Once series is stationary (after normal and seasonal differencing), the order of AR(p) and MA(q) needs to be determined for normal as well as seasonal parameters.
  - The order can be determined by examining the ACF and PACF of the stationary series. The following table summarizes interpretation of ACF to determine order of series

Shape	Indicated Model
Exponential, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to identify the order of the autoregressive model.
Alternating positive and negative, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to help identify the order.
One or more spikes, rest are essentially zero	Moving average model, order identified by where plot becomes zero.
Decay, starting after a few lags	Mixed autoregressive and moving average (ARMA) model.
All zero or close to zero	Data is essentially random.
High values at fixed intervals	Include seasonal autoregressive term.
No decay to zero	Series is not stationary.

- **Seasonal Models** : Analogous to the simple ARIMA parameters, seasonal parameters(*ps*), (*ds*), and (*qs*) need to be identified. The general recommendations concerning the selection of parameters to be estimated (based on ACF and PACF) also apply to seasonal models.(ACF and PACF interpreted at the seasonal lag)

- Also, note that since the number of parameters (to be estimated) of each kind is almost never greater than 2, it is often practical to try alternative models on the same data and select the best model based on the information criterion (AIC or BIC etc)

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THANK YOU