RELATIONS BETWEEN THE MASSES OF THE SUPERPARTNERS OF LEPTONS AND QUARKS, THE GOLDSTINO COUPLINGS AND THE NEUTRAL CURRENTS ☆

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The mass^2 splittings between leptons and quarks and their spin-0 partners under supersymmetry are related to the goldstino couplings. The bosonic partner of the goldstino cannot be the photon itself. But it should be, in part, a linear combination of the various neutral gauge bosons. As a result, mass relations constrain the neutral current structure of the theory. They require the existence of at least two neutral gauge bosons in addition to the photon and suggest the possibility of a universal mass^2 splitting between leptons and quarks and their spin-0 partners.

Spontaneously broken supersymmetric theories of particles involve new fields [1,2]. The gluons and the photon have the gluinos and the photino as spin-1/2 superpartners, while usual leptons and quarks are associated with heavy spin-0 particles. There is also a (massless neutral spin-1/2) goldstino generated by the spontaneous breaking of the supersymmetry and responsible for the mass splitting between bosons and fermions ‡1. In addition to a very rich structure, these theories have many potential experimental consequences, some of which we mention below.

The neutral currents are, at first sight, similar to those of ordinary gauge theories, although the gauge group used in ref. [1] is SU(2) × U′(1) × U″(1) and not the standard SU(2) × U(1).

The gluinos may lead to the existence of new hadronic states (R-hadrons). Those would be unstable and decay into ordinary hadrons plus a photino or a goldstino [3]. Recent experimental results allow to put upper limits on the pair-production cross section of such hadrons, suggesting that their masses should probably be at least $\sim 1\frac{1}{2}-2\ \mathrm{GeV}/c^2$ [4,5] (see also ref. [6]).

The spin-0 partners of usual leptons and quarks are involved in most of the phenomenological applications of supersymmetry: direct production ‡2; exchanges in R-hadron decays, effective neutral current interactions of the photino and the goldstino with matter, e_+e_- and $q\bar{q}$ annihilation, corrections to the anomalous magnetic moments of fermions, etc. They are presumably heavy, although this is not essential from the standpoint of the theory alone. In the simplest versions of the models described in ref. [1], we get an upper bound $\sim\frac{1}{2}m_W$ for their masses ‡3. It is not our purpose to study here phenomenological lower limits on the masses of various spin-0 particles. Let us mention, however, a relatively stringent constraint for the spin-0 partners of the muon: they should probably have masses $\gtrsim 15\ \mathrm{GeV}/c^2$; otherwise, they would affect the muon anomalous magnetic moment ‡4. In addition, the exchange

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‡1 When gravitation is introduced the goldstino is eliminated while the gravitino (spin 3/2 partner of the graviton) acquires a mass. However if the gravitino is very light it has much the same behavior as a goldstino, so that including explicitly the effects of gravitation does not affect our analysis here.

‡2 The new bosons could be pair-produced if the energy is larger than twice their mass. But they could also appear at lower energy, in association with the corresponding lepton or quark and a photino or a goldstino.

For footnotes 3 and 4 see next page.

of a pair of spin-0 quarks (s or t, with masses m_s, m_t) leads to an effective interaction between ordinary quarks $\sim g^4_{\rm strong}/m_s^2$ or $g^4_{\rm strong}/m_t^2$, which may be of the order of weak interactions (or even larger for small m_s, m_t). Such an interaction preserves parity if $m_s = m_t$ [‡5]. Otherwise we may have to require that s and t are heavy enough (at least some fraction of the W mass) to avoid unacceptable parity-violating effects in nuclear physics.

The attractiveness of supersymmetry is that it relates closely to all the above phenomena, in particular through the question of the masses of the new bosons. We shall learn more about them by studying the goldstino couplings. They will lead to the couplings of the bosonic partner of the goldstino to leptons and quarks and, therefore, to the neutral current structure of the theory.

We have already shown [2] that for a spin-0–spin-1/2 pair, the mass² splitting Δm^2 is proportional to the Yukawa coupling constant of the goldstino to the pair, $e_g\sqrt{2}$:

$$|\Delta m^2| = |e_g d| \,. \tag{1}$$

The parameter d measures the magnitude of the spontaneous breaking of the super-symmetry. We also remind the reader that the interactions of the goldstino, determined by the mass spectrum, are reminiscent of gravitational interactions (and, in fact, are connected with them) [2]. Owing to supersymmetry, the Yukawa couplings of the goldstino can be deduced from the couplings of its bosonic partner to fermions.

As a result, we are interested in the bosonic partner of the goldstino. Could it be the photon itself (as in some earlier versions of supersymmetric theories [8])? Then the boson–fermion mass² splittings would be proportional to the electrical charge. For the electron and its spin-0 partners s_{e_-}, t_{e_-}, we would have

$$m^2(s_{e_-}) - m^2(e_-) = -ed$$

$$m^2(t_{e_-}) - m^2(e_-) = +ed \,. \tag{2}$$

The mass relation

$$m^2(s_{e_-}) + m^2(t_{e_-}) = 2m^2(e_-) \,, \tag{3}$$

implies that both s_{e_-} and t_{e_-} are very light, which is not acceptable. This prevents us from identifying the goldstino with the photino.

It is instructive to mention that the mass relation (3) has its origin in the vector coupling of the photon [‡6]. Had the photon been axially coupled, we would have obtained equal masses for s_{e_-} and t_{e_-}, making it possible to have both of them heavy. There is no way to make the photon axially coupled but this leads us to consider the other neutral gauge boson(s), i.e., the weak neutral current(s), which must have axial parts. Before deriving constraints imposed on neutral currents by the mass spectrum, we shall illustrate how one can obtain heavy masses for all spin-0 leptons and quarks.

We use, for pedagogical purposes, the simplest version of the models of ref. [1]. Besides the gauge bosons of the standard SU(2) × U′(1) model [9], we have an extra massive gauge boson Z″ (corresponding to a spontaneously broken local U″(1) invariance) axially

‡3 Mass relations connect the supersymmetry breaking in the lepton and quark sectors with the supersymmetry breaking in the gauge boson–Higgs boson sectors. Let W_- and w_- be the charged gauge boson and the associated physical Higgs boson; e_-, s_{e_-} and t_{e_-}, the electron and its spin-0 partners, etc. The relations $m^2(W_-) - m^2(w_-) = 2[m^2(s_{e_-}) + m^2(t_{e_-}) - 2m^2(e_-)] = ...$ yield an upper bound $\sim m_W/\sqrt{2}$ for the masses of spin-0 leptons and quarks (otherwise, the mass of the physical Higgs boson w_- would be too small and might even become negative). If we assume equal masses for the s and t bosons, the upper bound is reduced to $\sim\frac{1}{2}m_W$. However, we should keep in mind that in the absence of the above mass relation (in particular, in those models involving superheavy gauge bosons), the new spin-0 particles might be heavier.

‡4 The two triangle graphs involving the photino and the spin-0 partners of the muon, s_μ and t_μ, give the following contributions to the muon anomaly: $\frac{1}{2}(g_\mu - 2) = -(\alpha/12\pi) \times (m^2(\mu)/m^2(s_\mu) + m^2(\mu)/m^2(t_\mu)) + ...$, in the limit where the partners of the muon are heavy. If s_μ and t_μ are both heavier than 15 GeV/c^2, the above contribution, smaller than 20×10^{-9}, is compatible with the present experimental and theoretical situation [7]. (If one of the spin-0 bosons were much heavier than the other, the latter might be as light as $\sim(15\ \text{GeV}/c^2)/\sqrt{2} \sim 11\ \text{GeV}/c^2$.) But we do not consider that we obtained an absolute lower bound since there might be a partial cancellation with positive contributions (due, for example, to neutral boson exchanges).

‡5 s and t denote the spin-0 partners of left-handed and right-handed spinors, respectively. If they are degenerate $(s + t)/\sqrt{2}$ behaves as a scalar and $(s - t)/\sqrt{2}$ as a pseudoscalar under parity.

‡6 Since the photon has opposite couplings with e_{-L} and $\overline{e_{+L}}$, the mass² splittings between (i) s_{e_-} and e_-, (ii) $\overline{t_{e_-}}$ and e_+ (i.e., t_{e_-} and e_-) have opposite signs, hence the relative minus sign in formula (2).

coupled to ordinary leptons and quarks. The mass of the latter is generated by the spontaneous breaking of the gauge invariance. The goldstino field λ_g is a mixing of two Majorana spinors λ'' and ζ, associated, respectively, with the spin-1 boson Z″ and a complex spin-0 boson $n = (a - ib)/\sqrt{2}$:

$$\lambda_g = \cos\beta\,\lambda'' + \sin\beta\,\gamma_5\zeta\,. \tag{4}$$

Since Z″ couples to usual leptons and quarks but n does not [‡7], the mass^2 splittings are proportional to the U″(1) coupling constants. Z″ being axially coupled with equal strength to all usual leptons and quarks, we have in these sectors a universal boson–fermion mass^2 splitting

$$\Delta m^2 = m^2(s_i) - m^2(\psi_i) = m^2(t_i) - m^2(\psi_i)\,, \tag{5}$$

which has $\frac{1}{4}m_W^2$ as an upper bound.

The actual neutral current structure (and, therefore, the mass spectrum of spin-0 leptons and quarks) may well not be as simple as presented above for illustration. Moreover, in view of the large amount of experimental information now available, which seems in good agreement with the standard model, we would like to know whether SU(2) × U(1) itself could be chosen as the gauge group.

The goldstino can always be expressed as a linear combination of the superpartners of a spin-1 and a spin-0 boson, as in formula (4). It is often possible to define a conserved quantum number, R, one unit of which is carried by the supersymmetry generator. In that case, the above spin-0 boson has $R = 2$ and cannot couple directly to usual leptons and quarks which have $R = 0$. As a result, the mass^2 splittings in these sectors can be associated entirely with the neutral currents (including, in particular, the electromagnetic current).

This might not be true in the absence of a continuous R-invariance. In that case, the s_i and t_j sectors may be mixed by mass terms in the lagrangian density [‡8]

$$\sim t_j^\dagger s_i + \text{h.c.} \tag{6}$$

[‡7] This follows, in particular, from R-invariance: ζ_L, which should transform as λ''_L under R, must have $R = 1$. Therefore, n has $R = 2$ and cannot couple directly to ordinary fermions.

[‡8] Such terms would be induced by non-vanishing vacuum expectation values for the auxiliary F- or G-components associated with ζ, i.e., they would correspond to the existence of a direct Yukawa coupling $\bar\psi_{jR}\psi_{iL}n$.

The effect of such terms would be to raise some mass^2 and lower others, including the lowest one in each sector. The average boson mass^2 is not modified and the mass relations we shall derive later are not affected.

Let us return to the boson–fermion mass^2 splitting associated with neutral currents (i.e., in technical terms, induced by non-vanishing vacuum expectation values for the auxiliary D-components of neutral gauge superfields). Instead of formula (2), we find a linear combination of the Yang–Mills and abelian generators, represented, respectively, by T_ℓ^a and F'_k:

$$\Delta m^2 = \mp \sum \left(g_\ell T_\ell^a \langle D_\ell^a\rangle + \tfrac{1}{2} g'_k F'_k \langle D'_k\rangle\right). \tag{7}$$

We still have a double sign (− for the s_i sector, + for the t_j sector) due to the fact that s_i behaves as ψ_{iL} and t_j as ψ_{jR} under gauge transformations.

A light fermion such as the electron, ..., must have both its spin-0 partners heavier than itself. Therefore the left-handed and right-handed fields which describe it cannot have the same gauge transformation properties. As a result the masses of the light leptons and quarks must be generated by the translation of Higgs fields which leads to the spontaneous breaking of gauge invariance.

If SU(2) × U(1) were the flavor gauge group formula (7) would give for right-handed singlets a boson–fermion mass^2 splitting proportional to the electrical charge. Having u_R and d_R as SU(2) singlets would lead to the unacceptable consequence that their spin-0 partners must be very light to avoid negative mass^2 (and this situation cannot be improved by mixing them with spin-0 bosons in the s-sector through $t_j^\dagger s_i$ terms).

We can replace the hypothesis of right-handed singlets by the experimental information available on the quark couplings in the eventuality of a single weak neutral current. Then formula (7) would involve a linear combination of the electrical charge Q_γ and the weak neutral charge Q_Z. But their values [10] are such that no combination

$$xQ_\gamma + yQ_Z\,, \tag{8}$$

is positive for u_L, d_L, $\bar u_L$, $\bar d_L$ (i.e., positive for u_L, d_L, negative for u_R, d_R) so that we cannot avoid spin-0 quarks with negative or at best very small mass^2. As a result, in the present framework of spontaneously broken supersymmetry, a gauge group as small as

SU(2) × U(1) is excluded and the theory should include at least two neutral gauge bosons in addition to the photon and the charged W. However, this does not imply that we necessarily have to abandon the successful predictions of the standard model ‡9.

Before concluding, we shall derive a property of the spectrum which may be useful when searching for other gauge groups and further extensions of supersymmetric theories. The supersymmetry breaking contribution to the boson mass^2 matrix, Δm^2, does not necessarily commute with the supersymmetric contribution (in that case, the boson and fermion masses are diagonalized differently). But we can relate the sum of the eigenvalues by taking traces and find:

$$\sum_{\substack{\text{Complex spin-0 fields}\\ \text{constructed from } s_i,\, t_j}} m^2 = 2 \sum_{\substack{\text{Dirac spinors constructed}\\ \text{from } \psi_{iL},\, \psi_{jR}}} m^2 + \mathrm{Tr}\, \Delta m^2 \,. \tag{9}$$

The summation runs over all fields with the same quantum numbers, and Δm^2 is given by eq. (7), since (6) cannot contribute to the trace.

Let us now sum over all particles in a given sector (electronic, muonic, ...). If the gauge group is semisimple, we find immediately from eq. (7) that Σ Tr Δm^2 vanishes. In general, if for every abelian gauge boson related with the goldstino we have ‡10

$$\sum_{\text{left-handed spinors}} \text{Weak hypercharge} = \sum_{\text{right-handed spinors}} \text{Weak hypercharge} \,, \tag{10}$$

then again

$$\sum \mathrm{Tr}\, \Delta m^2 = 0 \,, \tag{11}$$

so that we get the relation

$$\sum_{\substack{\text{Complex spin-0 fields}\\ \text{constructed from } s_i,\, t_j}} m^2 = 2 \sum_{\substack{\text{Dirac spinors constructed}\\ \text{from } \psi_{iL},\, \psi_{jR}}} m^2 \,. \tag{12}$$

The summation runs on all fields with the same quantum numbers. The average value of the mass^2 in a given sector (electronic, muonic, ...) is the same for bosons and fermions, independently of the gauge assignments of leptons and quarks, and of the presence or absence of a continuous R-invariance ‡11.

Therefore, the use of a semi-simple gauge group (if it is possible ‡12) or more generally of any gauge group satisfying eq. (10) requires the existence of heavy fermions in each sector (a simple look at the sign of Δm^2 tells us immediately which fermions should be heavy).

In summary, we have shown how, in the present framework of supersymmetric theories, the partial association of the goldstino with a new neutral gauge boson connects the mass spectrum to the neutral currents, and requires the masses of light leptons and quarks to be generated spontaneously. In particular, a universal axial coupling of the new gauge boson to usual leptons and quarks ‡13 would lead to a universal mass^2 splitting between them and their spin-0 partners. In that case, models such as those described in ref. [1] suggest $\sim 15\ \mathrm{GeV}/c^2 \to \sim \frac{1}{2} m_W (\sim 40\ \mathrm{GeV}/c^2)$ as a plausible mass interval for spin-0 leptons and quarks.

‡9 The new neutral gauge boson Z″ may be heavier than the ordinary one or more weakly coupled. But we find especially attractive the possibility that Z″ may be both very light and very weakly coupled. Such a situation, in which the new gauge boson does not contribute to the ordinary neutral current phenomenology, will be discussed elsewhere.

‡10 This is true, in particular, when the gauge group contains only one invariant abelian subgroup, the corresponding weak hypercharge being a linear combination of the electrical charge and Yang–Mills generators, as in the standard model.

‡11 A similar formula relating the mass^2 of bosons and fermions in supergravity theories has been considered by S. Ferrara et al.

‡12 The embedding of SU(3) × SU(2) × U′(1) × U″(1) into a semi-simple gauge group may, conceivably, lead to the consideration of a larger framework in which some of the gauge bosons are associated with gravitinos.

‡13 Axial couplings (preferred although probably not necessary) lead to equal masses for s and t so that the question of parity violation in strong interactions does not arise.

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