Possible Resonance Effect of Axionic Dark Matter in Josephson Junctions

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We provide theoretical arguments that dark-matter axions from the galactic halo that pass through Earth may generate a small observable signal in resonant S/N/S Josephson junctions. The corresponding interaction process is based on the uniqueness of the gauge-invariant axion Josephson phase angle modulo 2π and is predicted to produce a small Shapiro steplike feature without externally applied microwave radiation when the Josephson frequency resonates with the axion mass. A resonance signal of so far unknown origin observed by C. Hoffmann *et al.* [Phys. Rev. B **70**, 180503(R) (2004)] is consistent with our theory and can be interpreted in terms of an axion mass $m_a c^2 = 0.11$ meV and a local galactic axionic dark-matter density of $0.05\ \mathrm{GeV/cm^3}$. We discuss future experimental checks to confirm the dark-matter nature of the observed signal.

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The existence of dark matter in the Universe is one of the major puzzles of current research in astrophysics, cosmology, and particle physics. While there is clear evidence for the existence of dark matter from astronomical observations, it is still unclear what the physical nature of dark matter is. Important candidate particles for dark matter are weakly interacting massive particles (WIMPS) [1] and axions [2,3]. Many experimental searches to detect WIMPS [4] and axionlike particles [5–11] are currently under way or are being discussed for future implementation. A positive result would be a major breakthrough in our understanding of the matter contents of the Universe.

In this Letter, we propose a new approach to detect QCD-axionic dark matter in the laboratory with high efficiency, exploiting a macroscopic quantum effect. Our proposal is based on S/N/S (superconductor/normal metal/superconductor) Josephson junctions as suitable detectors [12–16]. We will provide theoretical arguments that axions that pass through the weak-link (WL) region of such a Josephson junction in the voltage stage may trigger the transport of additional Cooper pairs if the Josephson frequency ω_J coincides with the axion mass $m_a c^2 = \hbar\omega_J$. The effect is resonantly enhanced.

The basic theoretical idea of our proposal can be regarded as being a kind of a complement of "light shining through wall" experiments [7,8]. In light shining through wall experiments, photons decay into axions in a strong magnetic field, which then pass a "wall" and decay back into photons, which can be detected. Here, we employ the opposite effect where axions convert into photons in a Josephson junction and back into axions when leaving the junction. For brevity, we may call this effect ATJ ("axions tunneling a junction"). The wall for axions in this case is represented by the weak-link region of the biased Josephson junction in the voltage stage.

From an experimental point of view, ATJ predicts a Shapiro steplike feature [17] in the measured *I-V* curve of the S/N/S junction that occurs *without* externally applied microwave radiation [18–20]. The measured differential conductance (see e.g., Refs. [12–15] for typical measurement techniques) is predicted to exhibit a small peak at Josephson frequency $\hbar\omega_J = 2eV = m_a c^2$, whose intensity depends on the velocity of galactic axions hitting the Earth, the size of the weak-link region of the junction, and the local galactic halo density of axions.

Our calculations in this Letter show that the effect of axionic dark matter on the *I-V* curve is small but observable. We will discuss a possible candidate signal of unknown origin that was observed and noted in Ref. [12], which interpreted in terms of ATJ provides a prediction of the axion mass of $m_a c^2 = 0.11$ meV and an estimate of the local halo density of dark-matter axions of $0.05\ \mathrm{GeV/cm^3}$.

Consider an axion field $a = f_a\theta$, where θ is the axion misalignment angle and f_a is the axion coupling constant. If strong external electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present, one has

$$\ddot{\theta} + \Gamma\dot{\theta} + \frac{m_a^2 c^4}{\hbar^2}\sin\theta = -\frac{g_\gamma}{(4\pi^2)}\frac{1}{f_a^2}c^3 e^2 \vec{E}\vec{B} \qquad (1)$$

($g_\gamma = -0.97$ for KSVZ axions [21,22], whereas $g_\gamma = 0.36$ for DFSZ axions [23,24]); Γ is a damping parameter. In the early Universe, $\Gamma = 3H$, where H is the Hubble parameter, but at a later stage of the Universe larger Γ values can be relevant, depending on the interaction processes considered [25]. As shown by Sikivie *et al.* [26], axions at the current stage of the Universe are most likely to form a Bose-Einstein condensate (BEC), which opens up the possibility to exploit macroscopic quantum effects of the axion condensate for detection purposes. The typical parameter ranges that are allowed for QCD dark-matter axions are

$6\times10^{-6}\,\mathrm{eV}\le m_a c^2 \le 2\times10^{-3}\,\mathrm{eV}$ and 3×10^9 GeV $\le f_a \le 10^{12}$ GeV. The product $m_a c^2 f_a$ is expected to be of the order $m_a c^2 f_a \sim 6\times10^{15}$ (eV)^2.

Let us consider as a suitable detector a Josephson junction (JJ) [27], which we initially treat in the approximation of the resistively shunted junction (RSJ) model [16] (later we will come to more specific physics for S/N/S junctions). In the RSJ model the phase difference δ of a JJ driven by a bias current I satisfies

$$\ddot{\delta} + \frac{1}{RC}\dot{\delta} + \frac{2eI_c}{\hbar C}\sin\delta = \frac{2e}{\hbar C}I \tag{2}$$

where I_c is the critical current of the junction and R and C are the normal resistance and capacity of the junction, respectively.

As pointed out in Refs. [19,20], the equations of motions for axions (1) and JJs (2) are formally identical, and also the numerical values of the coefficients in the equations are of similar order of magnitude (see Ref. [19] for some numerical examples). In this formal analogy the axion mass parameter squared essentially corresponds to the critical current I_c, the product $\vec{E}\cdot\vec{B}$ corresponds to the bias current I, and the damping Γ corresponds to $(RC)^{-1}$.

Now let us consider an axion with misalignment angle θ that enters the WL region of a JJ with phase difference δ [Fig. 1(a)]. Both the axion (as a BEC with an equation of motion identical to a JJ) and the Josephson junction [as a coherent state of two superconductors (SCs) separated by a WL] are macroscopic quantum systems and can be described by a joint macroscopic wave function Ψ in the vicinity of WL. Similar to the case where two JJs are put together in a superconducting quantum interference device configuration [16] the phase variable φ of this wave function $\Psi = |\Psi|e^{i\varphi}$ must be single valued. This means that for a given closed integration curve covering the interior of both SCs and the WL region one has

$$\int_{\mathrm{SC}} \nabla\varphi\cdot d\vec{s} + \delta + \theta = 0 \bmod 2\pi \tag{3}$$

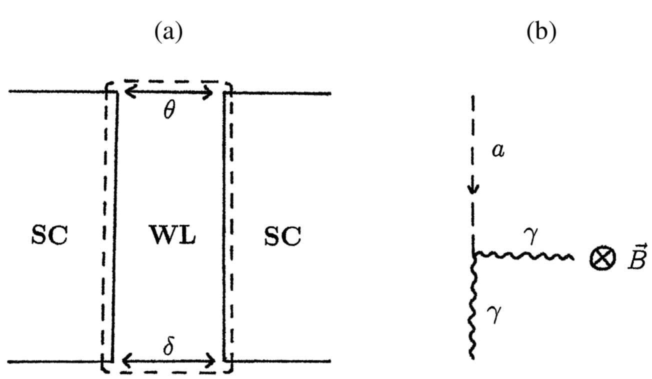

FIG. 1. (a) Closed integration curve (dashed line) over axion and Josephson phase angles in the WL region of a JJ. (b) Feynman graph underlying axion-photon decay in a JJ.

[see Fig. 1(a)]. Here, we assume that θ (thought to be of order 10^{-19}) enters the loop as a tiny phase difference over WL, just as a second JJ does. The condition of Eq. (3) implies that δ and θ are no longer independent of each other but influence each other. Our physical interpretation is that the incoming axion produces a small perturbation θ in the *CP* symmetry status of the two SCs separated by WL, to which the JJ reacts by building up a small response phase δ so that *CP* symmetry is restored.

In the presence of a vector potential $\vec{A}$ we may define gauge-invariant phase differences γ_i ($i = 1, 2$) by

$$\gamma_1 := \delta - \frac{2\pi}{\Phi_0}\int_{\text{weak link 1}} \vec{A}\cdot d\vec{s}, \tag{4}$$

$$\gamma_2 := \theta - \frac{2\pi}{\Phi_0}\int_{\text{weak link 2}} \vec{A}\cdot d\vec{s}. \tag{5}$$

Here, $\Phi_0 = (h/2e)$ denotes the flux quantum and "weak link 1" denotes the link from the right to the left SC at the bottom of Fig. 1(a), whereas "weak link 2" denotes the link from the left to the right SC at the top of Fig. 1(a). For practical purposes, it is more convenient to define all links in the same direction (say from the left SC to the right SC), and the standard formalism employing uniqueness of the phase φ modulo 2π (see e.g., Ref. [16]) then yields the relation

$$\gamma_1 - \gamma_2 = 2\pi\frac{\Phi}{\Phi_0} \mod 2\pi, \tag{6}$$

where Φ is the magnetic flux through the area enclosed by the chosen closed line of integration. Equation (6) physically means that γ_1 and γ_2 synchronize.

Now consider a JJ in the voltage stage, where $\delta(t) = \delta(0) + [(2eV)/\hbar]t$. There is an oscillating supercurrent $I_c\sin\delta(t)$, and the junction emits Josephson radiation with frequency $\hbar\omega_J = 2eV$. Equations (4)–(6) imply that $\theta(t) = \delta(t) + \mathrm{const}$ (where the constant depends on the magnetic flux included in the loop), meaning that (classically, in WL) the axion misalignment angle evolves in the same way as the Josephson phase difference, up to a constant. In particular, if δ increases linearly in time, then also θ is forced to increase linearly in time with the same rate as that of δ. We thus get

$$\dot{\theta} = \dot{\delta} = \frac{2eV}{\hbar} \tag{7}$$

from the phase synchronization condition (6) and

$$\dot{\theta} = -\frac{g_\gamma}{(4\pi^2)}\frac{1}{\Gamma f_a^2}c^3 e^2 \vec{E}\vec{B} \tag{8}$$

from the original equation of motion (1) of the axion, neglecting the first and third term. The joint validity of Eqs. (7) and (8) implies that the JJ environment effectively simulates to the axion the existence of a large nonzero product $\vec{E}\cdot\vec{B}$ in the weak-link region. Using $|\vec{E}| = V/d$, where d is the distance between the superconducting electrodes of the JJ, we get from Eqs. (7) and (8) a *formal* magnetic field given by

$$B = \frac{8\pi^2 \Gamma f_a^2 d}{g_\gamma \hbar c^3 e}. \tag{9}$$

Note that the result of Eq. (9) is independent of the applied voltage V. The direction of this effective $\vec{B}$ field is in the same direction as that of $\vec{E}$; i.e., it is orthogonal to the superconducting plates of the junction.

Putting in typical values for the QCD axion coupling f_a and the distance d, one gets huge numerical values for B, many orders of magnitude higher than what can be achieved by externally producing a B field in the lab. As a numerical example, for a typical tunnel junction, $d = 10^{-9}$ m and $RC \sim 10^{-12}$ s [28]. Assuming $f_a \sim 5 \times 10^{19}$ eV $= 8J$ and $\Gamma \sim (RC)^{-1}$ one gets $B \sim 10^{34}$ T, an incredibly large value. Much smaller choices of Γ, of the order of the current Hubble parameter $H \sim 10^{-18}$ s^{-1}, still yield a big $B \sim 10^3$ T.

Our conclusion is that a phase-synchronized axion cannot exist in the junction but decays into microwave photons [Fig. 1(b)]. To roughly estimate the probability $P_{a\to\gamma}$ of this happening, we may use the well-known formula from the Primakoff effect [29]

$$P_{a\to\gamma} = \frac{1}{4\beta_a}(gBecL)^2 \frac{1}{4\pi\alpha}\left(\frac{\sin\frac{qL}{2\hbar}}{\frac{qL}{2\hbar}}\right)^2 \tag{10}$$

where q is the axion-photon momentum transfer, $\beta_a = v/c$ is the axion velocity, L is the length of the detector, and $g := (g_\gamma \alpha)/(\pi f_a)$, where α is the fine structure constant. Inserting the formal value of the B field (9), one obtains for $qL \ll 2\hbar$

$$P_{a\to\gamma} = \frac{1}{\beta_a \hbar^2 c^4}(f_a \Gamma d L)^2 4\pi\alpha. \tag{11}$$

In particular, $P_{a\to\gamma} = 1$ corresponds to a very short length scale, namely,

$$L = \frac{\hbar c^2}{\sqrt{4\pi\alpha}}\sqrt{\beta_a}\frac{1}{f_a \Gamma d}. \tag{12}$$

For our previous numerical example and an axion velocity of $v = 2.3 \times 10^5$ m/s, one gets $L \sim 10^{-22}$ m, which means that the axion immediately decays at the surface of the weak-link region. Moreover, since $P_{a\to\gamma} = P_{\gamma\to a}$, it can equally likely recombine back into an axion when leaving the weak-link region, thus producing ATJ. The total probability of axion decay in the junction is given by $pP_{a\to\gamma}$, where p is the probability that the axion phase synchronizes. There are no astrophysical or cosmological constraints on p since almost all matter of the Universe is not in the form of JJs; hence, $p \sim O(1)$ is compatible with the astrophysical dark-matter status of the axion.

Let us now come to measurable effects to test this theoretical idea. It is well known that external microwave radiation applied to a Josephson junction leads to distortions in the I-V curve, the well-known Shapiro steps [17]. Within the RSJ model, one can calculate (see e.g., Ref. [30]) that external monochromatic microwave radiation of signal frequency $\hbar\omega_s = 2eV_s$ leads to a distortion I_s in the measured current-voltage curve $I(V)$ given by

$$I_s(V) = \frac{P_s}{4}(RI_c)^2 \frac{1}{V^2}\left[\frac{V+V_s}{(V+V_s)^2 + (\frac{\delta V}{2})^2} + \frac{V-V_s}{(V-V_s)^2 + (\frac{\delta V}{2})^2}\right]. \tag{13}$$

Here, P_s is the signal power, and δV is determined by the line width of the Josephson frequency.

When the Josephson frequency $\hbar\omega_J = 2eV$ resonates with the axion mass $m_a c^2$ we may assume that all axions hitting the weak-link region synchronize and decay ($p = 1$); hence, the expected signal power P_s is given by

$$P_s = \rho_a v A. \tag{14}$$

Here, ρ_a is the halo axionic dark-matter energy density close to Earth, v is the velocity of Earth relative to the galactic center, and A is the area of the weak-link region perpendicular to the axion flow. Neglecting fluctuations, the velocity v is known to be approximately 2.3×10^5 m/s, with a yearly modulation of about 10% around its mean value [4,31].

We propose as a specific microscopic model underlying the interaction of axions with resonant S/N/S junctions the process sketched in Fig. 2. An axion entering the weak-link

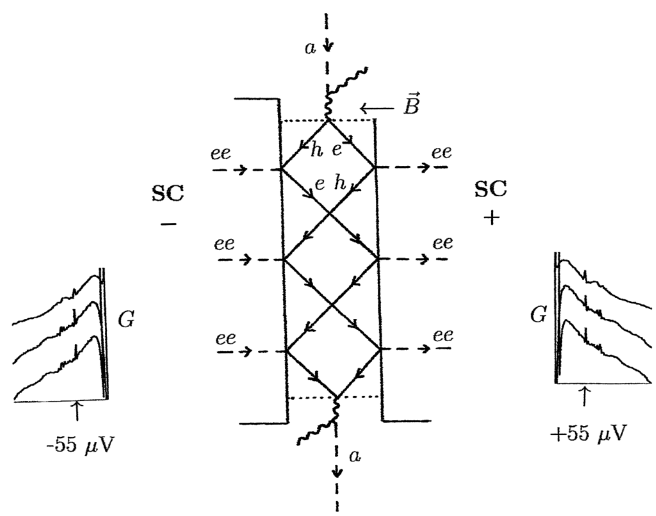

FIG. 2. Axion triggering the transport of n Cooper pairs ee in an S/N/S junction by multiple Andreev reflections, here shown for the example $n = 3$. The dotted line corresponds to the normal metal surface. The left and right insets show the shape of the differential conductance curve $G(V)$ as measured by Hoffmann *et al.* [12] for $T = 0.9$, 0.5, 0.1 K (top to bottom), with a peak occurring at ± 0.055 mV.

region with a transversal velocity component decays close to the normal metal surface into two microwave photons, one with $q \approx 0$ and the other one with frequency $\hbar\omega_s \approx m_a c^2 = 2eV_s$. The $q \approx 0$ photon transfers its small momentum to a hole-electron pair in the weak-link region (holes h and electrons e can enter from the surrounding superconductors with energy $-\Delta$, where Δ is the gap energy). The hole and the electron perform multiple Andreev reflections in the usual way [12,13], meaning the hole is reflected at the S/N interface as an electron and annihilates in this process a Cooper pair, whereas the electron is reflected at the other S/N interface as a hole, creating in this process a Cooper pair. In total, there are n Andreev reflections, with

$$n \approx \frac{2\Delta}{eV} + 1 \tag{15}$$

[12]. At the end of this process, when both the electron and hole energy exceed the gap energy Δ they either just leave the weak-link region or annihilate back into a low-energy photon, which together with another photon of Josephson frequency $\hbar\omega_J = 2eV_s = m_a c^2$ can recombine back into an axion, which leaves the detector unharmed. In total, there is an ATJ process and each incident axion triggers the transport of n Cooper pairs. These additional Cooper pairs produce a signal $G_s = dI_s/dV$ in the measured differential conductance $G(V)$ of the junction at the signal voltage $V_s = m_a c^2/(2e)$. The total signal current produced by axions is given by

$$I_s = \int G_s dV = \frac{N_a}{\tau} n 2e = \frac{\rho_a}{m_a c^2} vAn2e \tag{16}$$

where N_a/τ is the number of axions hitting the normal metal region per time unit τ. Since $2eV_s = m_a c^2$, we get

$$\rho_a = \frac{I_s V_s}{vAn}. \tag{17}$$

This can be used to experimentally estimate the axion dark-matter density ρ_a from an experimental measurement of V_s and I_s.

In Ref. [12], Hoffmann *et al.* observed a signal of unknown origin that is consistent with our theoretical expectations. Independent of the temperature (which is varied from 0.1 to 0.9 K), they consistently observe a small peak in their measured differential conductance $G(V)$ at the voltage $V_s = \pm 0.055$ mV (see insets of Fig. 2, data from Ref. [12]). Their measurements provide evidence for a signal current feature of size $I_s = (8.1 \pm 1.0) \times 10^{-8}$ A, which is obtained by integrating the area under the observed signal peak of the differential conductance. Their noise measurements also indicate that every quasiparticle performs $n = 7$ Andreev reflections [12]. The area of the metal plate of their junction is $A = 0.85\ \mu\text{m} \times 0.4\ \mu\text{m} = 3.4 \times 10^{-13}\ \text{m}^2$. From $2eV_s = m_a c^2$, we thus obtain an axion mass prediction of $m_a c^2 = 110\ \mu$eV (equivalent to $f_a \sim 5.5 \times 10^{10}$ GeV), and Eq. (17) yields the prediction $\rho_a = (0.051 \pm 0.006)\ \text{GeV/cm}^3$.

Astrophysical observations suggest that the galactic dark-matter density ρ_d near Earth is about $\rho_d = (0.3 \pm 0.1)\ \text{GeV/cm}^3$ [32]. But this includes all kinds of dark-matter particles, including WIMPS. Generally, axions of high mass will make up only a fraction of the total dark-matter density of the Universe, which can be estimated from cosmological considerations to be about $\rho_a/\rho_d \approx (24\ \mu\text{eV}/m_a c^2)^{7/6}$ [3]. For $m_a c^2 = 110\ \mu$eV, we thus expect an axionic dark-matter density that is a fraction $(24/110)^{7/6} \approx 0.17$ of the total dark-matter density, giving $\rho_a \approx 0.17 \times \rho_d = (0.051 \pm 0.017)\ \text{GeV/cm}^3$. The experimental results of Hoffmann *et al.* together with our theoretical prediction (17) are thus in perfect agreement with what is expected from astrophysical observations.

To either refute or confirm our hypothesis that the signal seen in Ref. [12] is produced by dark-matter axions, further measurements are needed. Clearly, one should test if the signal survives careful shielding of the junction from any external microwave radiation. A signal produced by axions cannot be shielded. Moreover, one might look for a possible dependence of the measured signal intensity on the spatial orientation of the metal plate relative to the galactic axion flow (a precise directional measurement would be extremely helpful). Finally and most importantly, the velocity v by which Earth moves through the axionic BEC of the galactic halo exhibits a yearly modulation of about 10%, with a maximum in June and a minimum in December (as used in searches for WIMPS [31]). If the JJ signal is produced by axions, then its intensity should show the same 10% modulation effect over a period of a year. This can be tested in future experiments. In Ref. [12] also some fine structure of $G(V)$ near 0.08 mV is observed, which might be a hint for further axionlike particles with different mass.

To conclude, in this Letter we have described a macroscopic quantum effect in Josephson junctions that may help to prove the existence of axionic dark matter in future measurements. Phase-synchronized axions cannot exist in the weak-link region of JJs due to a (formal) huge magnetic field that is simulated to them by the driven JJ environment in the voltage stage. Axions are expected to decay when entering the weak-link region of the junction and trigger the transport of additional Cooper pairs. This leads to a small measurable signal for the differential conductance (a Shapiro steplike signal without externally applied microwave radiation) if the axion mass resonates with the Josephson frequency. The effect is particularly strong in S/N/S junctions that have a much larger weak-link region than tunnel junctions and where the Cooper pair transport is amplified by multiple Andreev reflections. A candidate signal of unknown origin has been observed in measurements of Hoffmann *et al.* [12], which interpreted in this way points to an axion mass of 0.11 meV and a local axionic energy density of 0.05 GeV/cm^3.

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