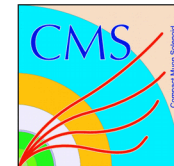




Search for long-lived neutral particles decaying to photons with missing energy using ECAL timing



Approval of EXO-12-035

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University of Minnesota

Sept , 2015



Documentation



- PAS :

<http://cms.cern.ch/iCMS/analysisadmin/versions?analysis=EXO-12-035>

- NOTE :

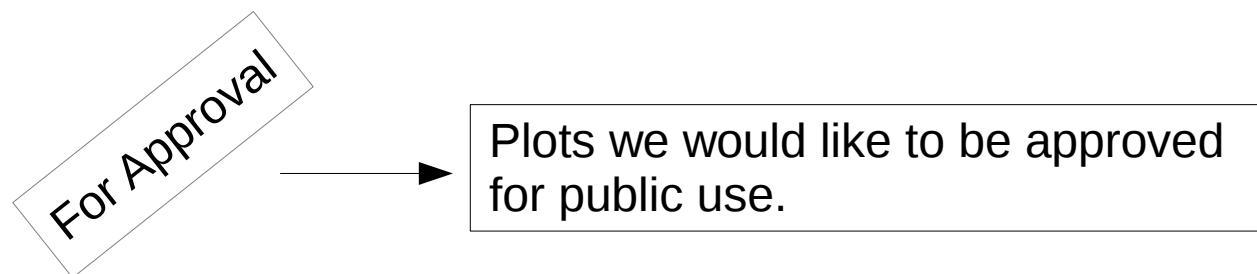
http://cms.cern.ch/iCMS/jsp/openfile.jsp?tp=draft&files=AN2013_001_v8.pdf

- TWIKI :

<https://twiki.cern.ch/twiki/bin/viewauth/CMS/DisplacedPhoton2012>

- CADI :

<http://cms.cern.ch/iCMS/analysisadmin/cadilines?line=EXO-12-035>





Outline



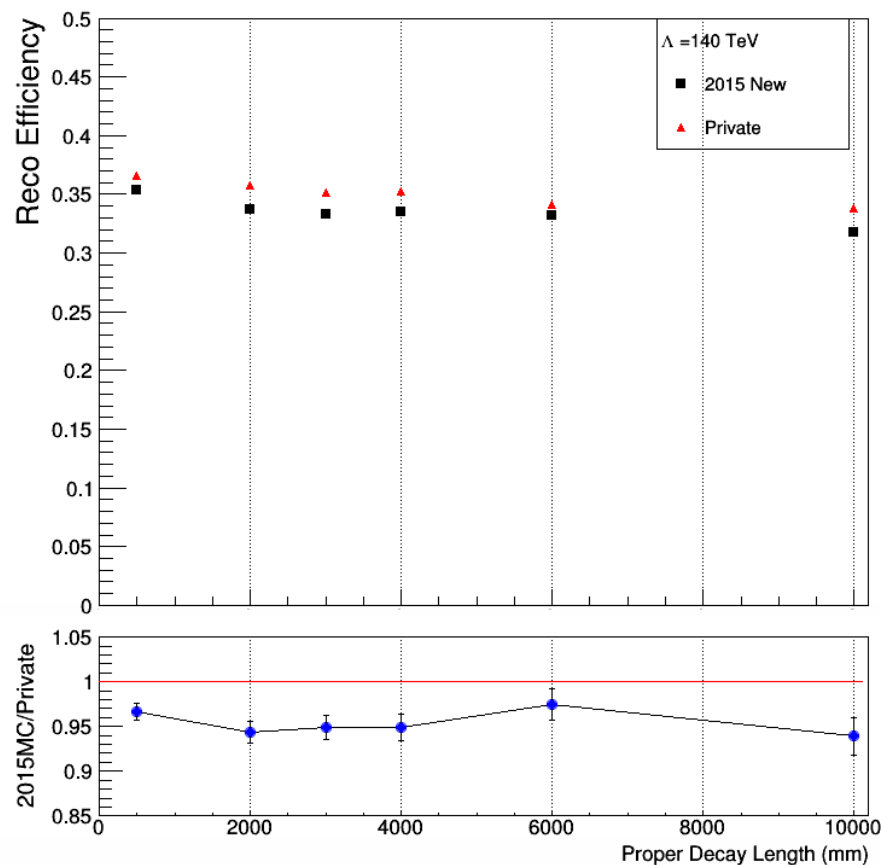
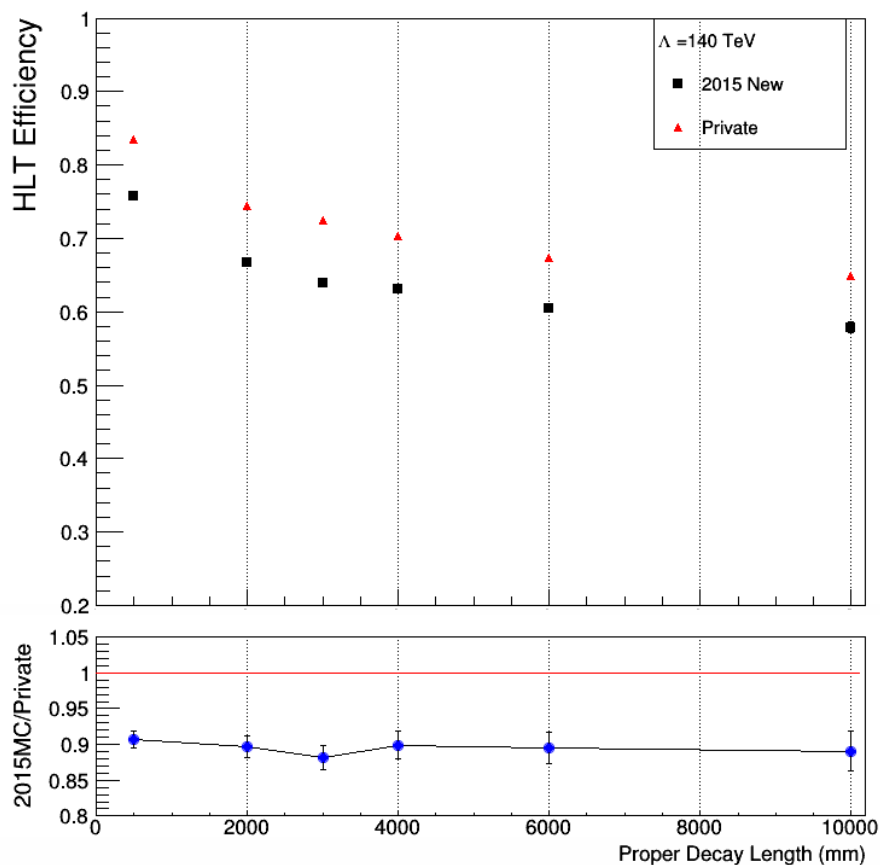
- (1) Introduction
- (2) ECAL Time
- (3) Delay Mechanism
- (4) Efficiency
- (5) MET and MET γ
- (6) Samples
- (7) Event Selection
- (8) Background Estimation
- (9) Uncertainties
- (10) Result and Limit



Updates since pre-approval

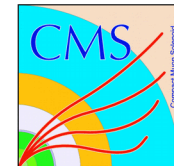


- Results are updated using new generated 8 TeV MC.
 - New samples were produced centrally.
 - Correct lifetime setup and official HLT simulation.
 - 10% drop at HLT stage and 5% at RECO and event selection stage.
 - Overall efficiency difference is about 15% ~ 20% lower.





Updates since pre-approval



- Change collision background estimation from an ABCD method to a $Z \rightarrow ee$ method.
 - Non-Collision background estimation remain the same.
 - Two methods both suggest negligible contribution from collisions.
 - Choose $Z \rightarrow ee$ method since it predicts larger background.

	Estimation in B Region	Estimation in D region
ABCD	$0.14 + 0.11$ $- 0.06$	$0.09 + 0.09$ $- 0.05$
$Z \rightarrow ee$	$0.51 + 0.28$ $- 0.27$	$0.37 + 0.09$ $- 0.07$
ABCD (closure test)	$0.19 + 0.08$ $- 0.06$	$0.05 + 0.05$ $- 0.02$
$Z \rightarrow ee$ (closure test)	$0.64 + 0.35$ $- 0.34$	$0.46 + 0.11$ $- 0.09$



Introduction



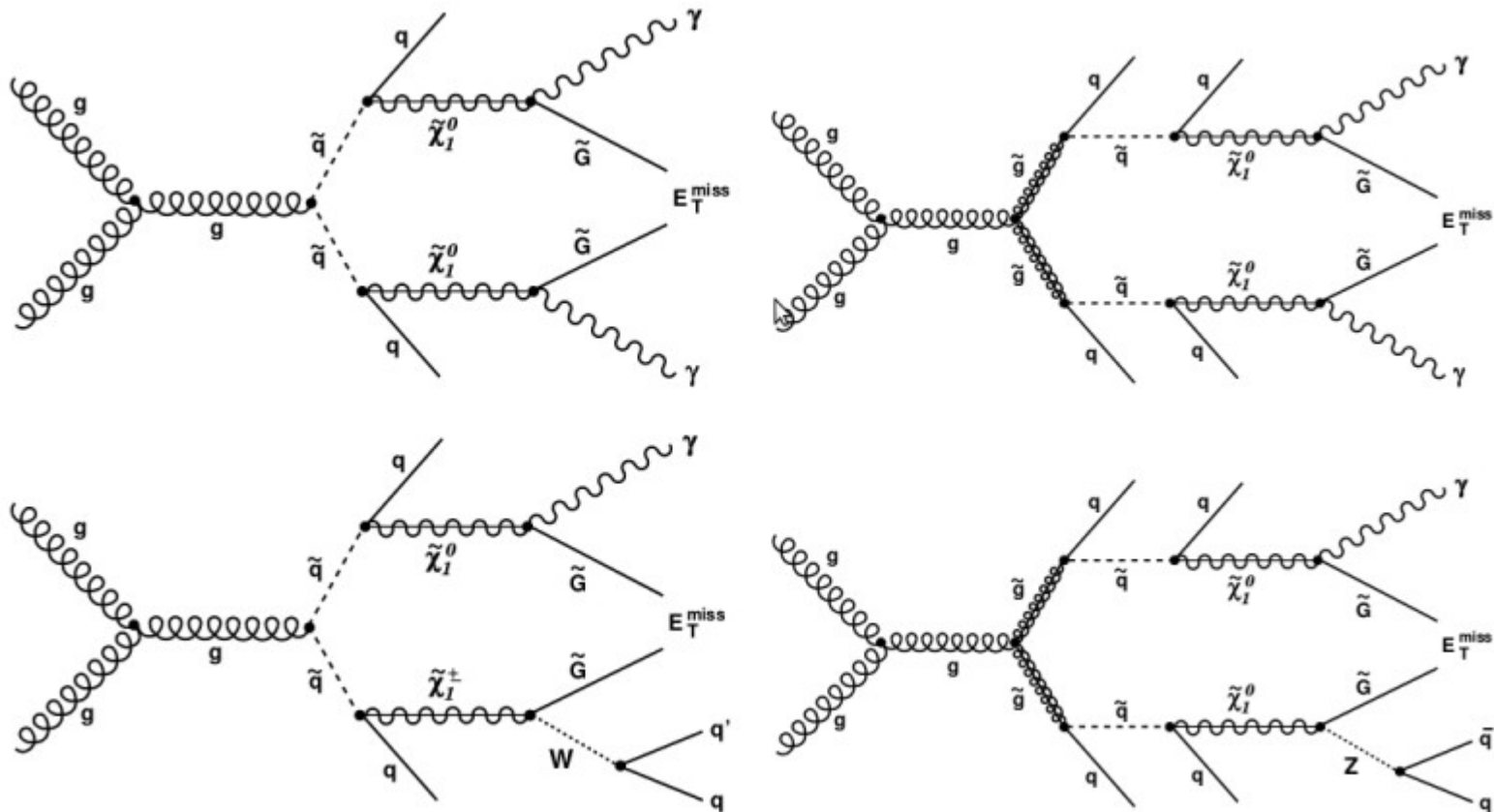
- **Motivation** : Search for long-lived neutral particle decaying to photon with missing energy.
 - **Method** : Delayed ECAL timing
- **Benchmark model** : Snowmass Points and Slopes 8 (**SPS8**)
 - Neutralino ($\tilde{\chi}_1^0$) is NLSP, Gravitino (\tilde{G}) is LSP
 - 83 ~ 94 % BR for $\tilde{\chi}_1^0 \rightarrow \tilde{G} + \gamma$
 - **97 ~ 99 % of events have at least one photon.**
 - SUSY breaking scale Λ changes the mass of $\tilde{\chi}_1^0$ and \tilde{G} .
- Lifetime is proportional to c_{grav}^2 .
 - c_{grav} is Λ dependent. For each lifetime point of a particular Λ , c_{grav} needs to be adjusted accordingly.
- Lifetime of neutralino well constrained by theory, so we look for everywhere we can.



Production Mechanism



- Neutralinos are pair produced from the decay of squarks.
- Signal signature : Photon, MET and Jets





Analysis Method



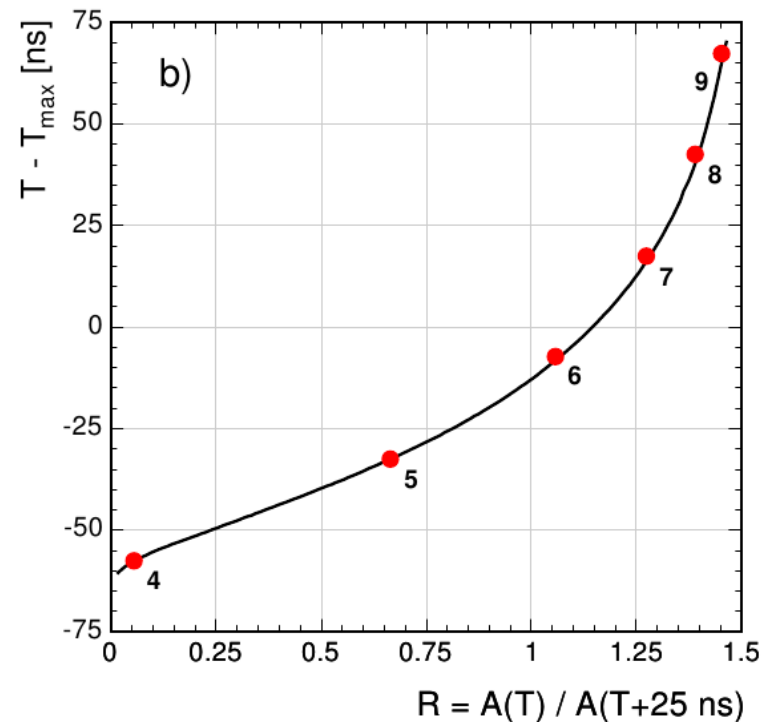
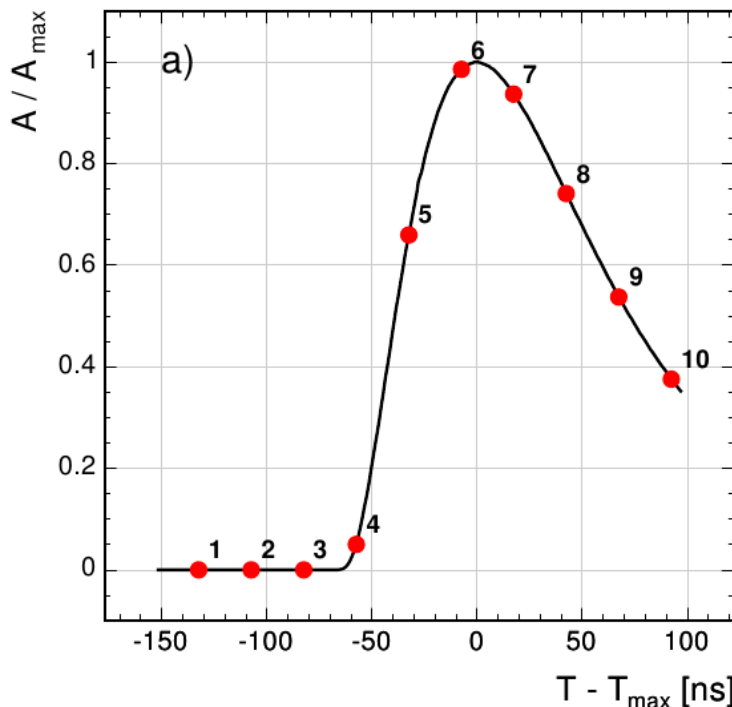
- Simple counting method.
 - Count number of event with at least one late photon (ECAL time > 3 ns).
 - Advantage : Negligible standard model/collision background.
 - Machine induced background and cosmic-rays become the main source.
 - Re-processed photon reconstruction in order to include out-of-time crystals in the reconstruction process.
- Use negative timing as background control region
 - Study and develop background veto methods and background estimation.



ECAL Time (t)

- CMS ECAL Time is determined by 10 pulse sample measurement.
 - Determined by the sample time (T_i) and $T(R_i)$ using the plot (b) and weighted average over samples (4~9).

$$T_{max} = \frac{\sum \frac{T_i - T(R_i)}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$





ECAL Time (t)



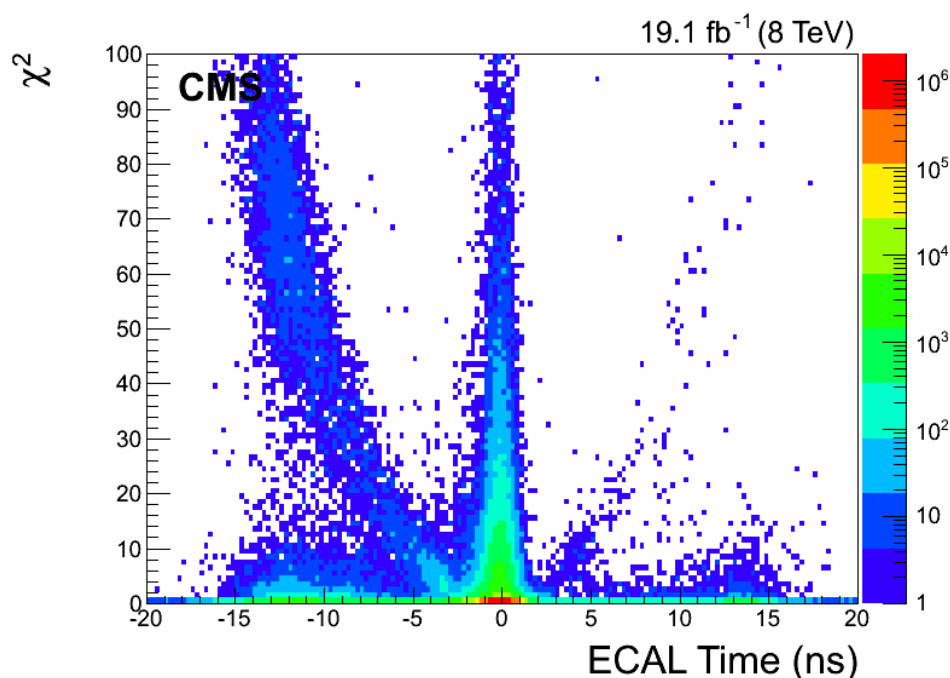
- Using time from seed crystal of the photon cluster.

→ Calculate χ^2

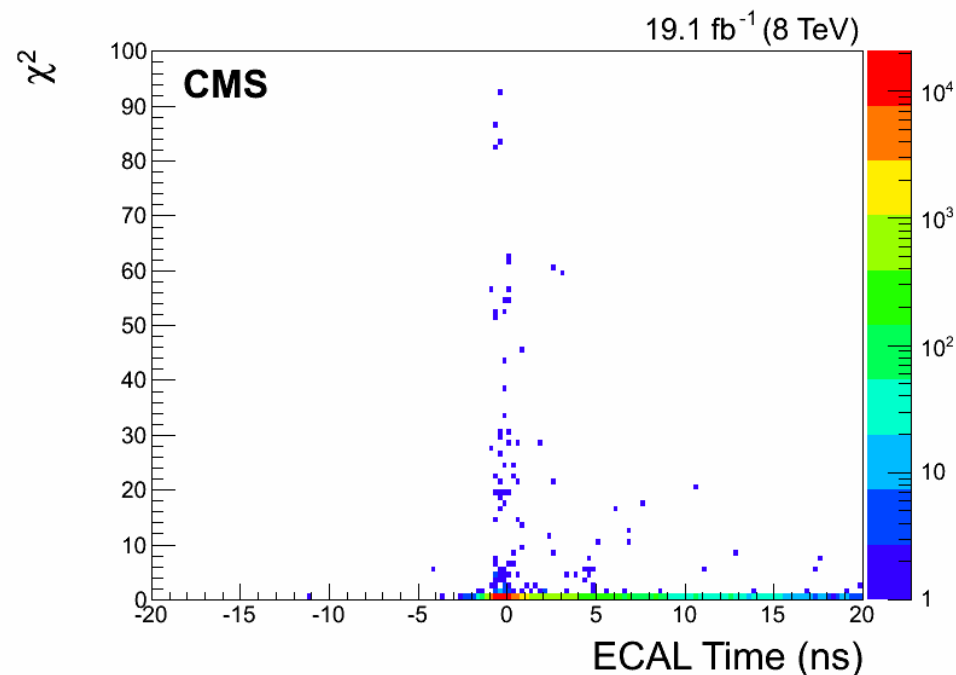
$$\chi^2 = \frac{1}{N-1} \sum \left(\frac{t_i - t_{cluster}}{\sigma_i} \right)^2$$

→ χ^2 cut ($\chi^2 < 4$) reduces spike background.

→ Cluster time ($t_{cluster}$) is the weighted average over the times obtained in all crystals in the seed cluster.



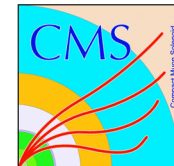
Displaced Photon Skim Dataset
– No selection applied yet.



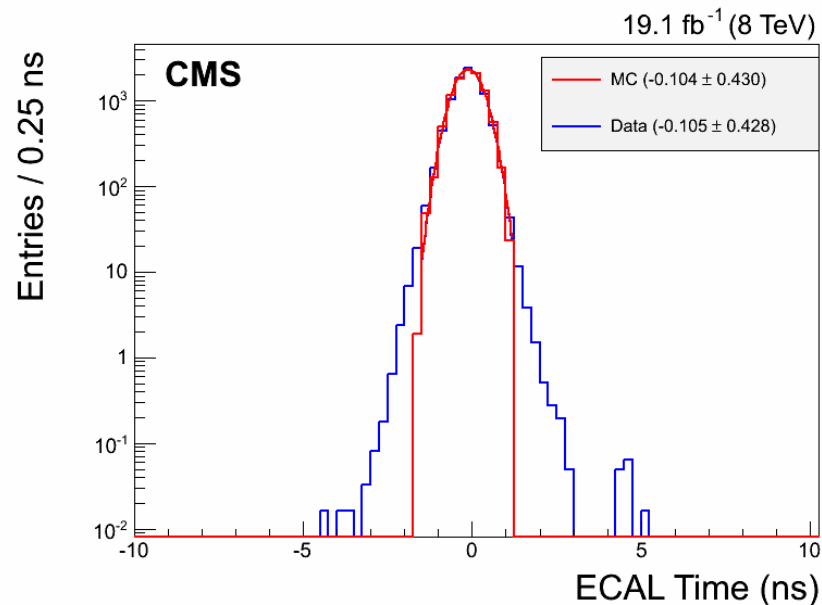
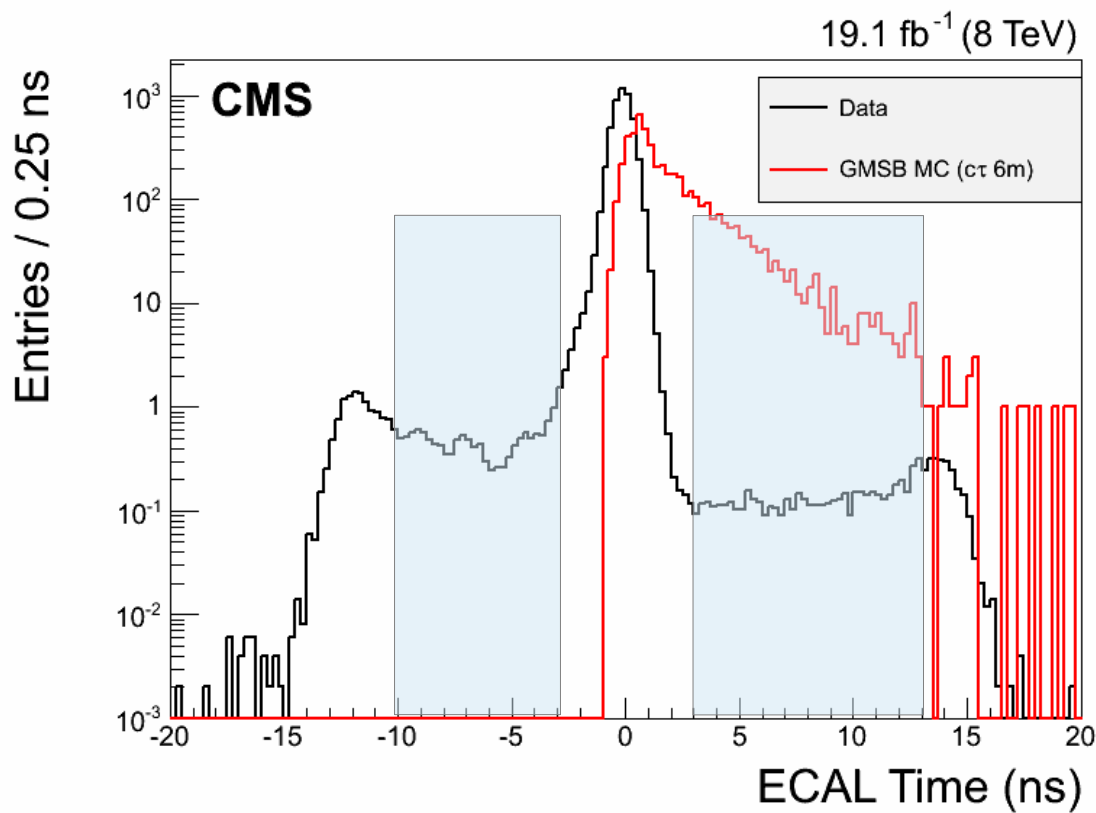
GMSB MC Sample
– No selection applied yet.



ECAL Time (t)



- Resolution : ~ 500 ps
- Observation Scope : 3 \sim 13 ns
 - Linearity is verified up to 14 ns.
- Background control region : -10 \sim -3 ns
 - Avoid -12 ns peak from ECAL spike.



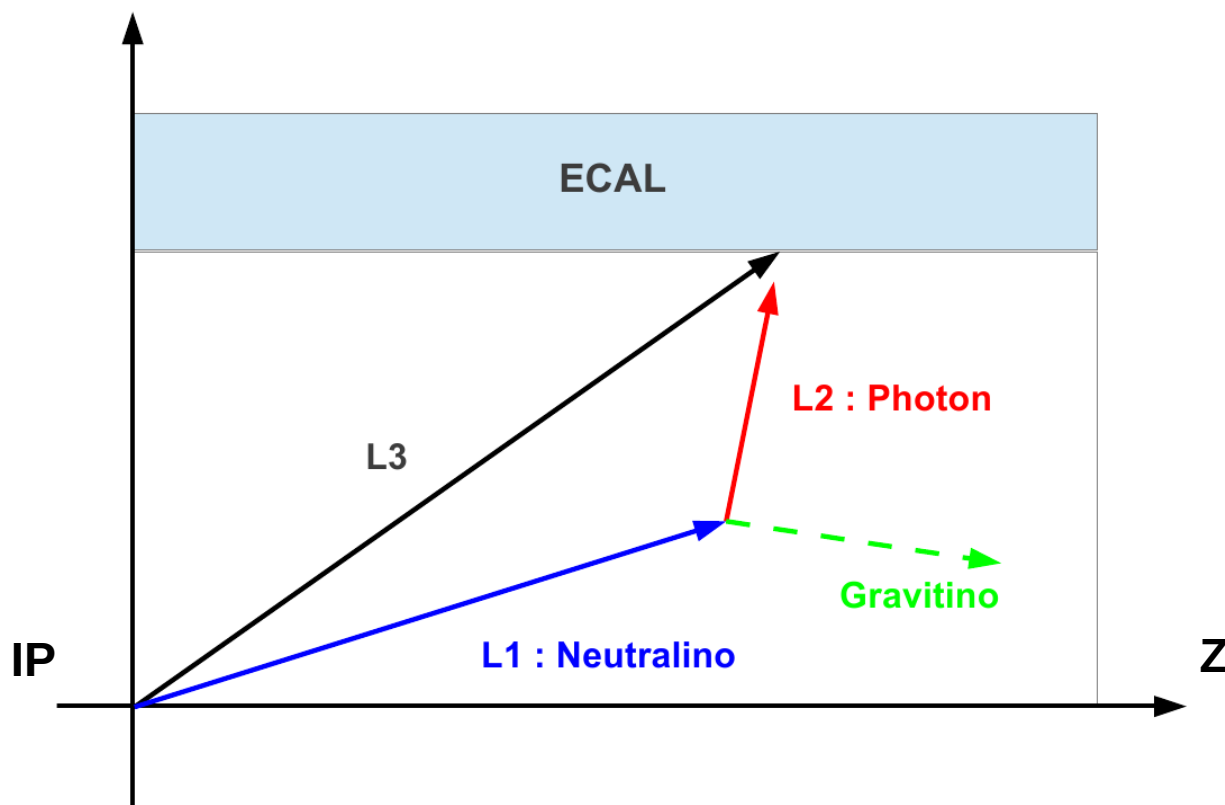
Adding extra smearing to MC samples to correct MC timing resolution.
(Detail in backup slide)



Delay Mechanism

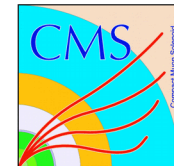


- Delay quantities (Δt_1 and Δt_2) :
 - ECAL Time : $L_1/c\beta + L_2/c - L_3/c$
 - Δt_1 : $L_1/c\beta - L_1/c$; Delay from slow motion of neutralino.
 - Δt_2 : $(L_1 + L_2 - L_3)/c$; Delay from increase in path length.
 - ECAL Time = $\Delta t_1 + \Delta t_2$

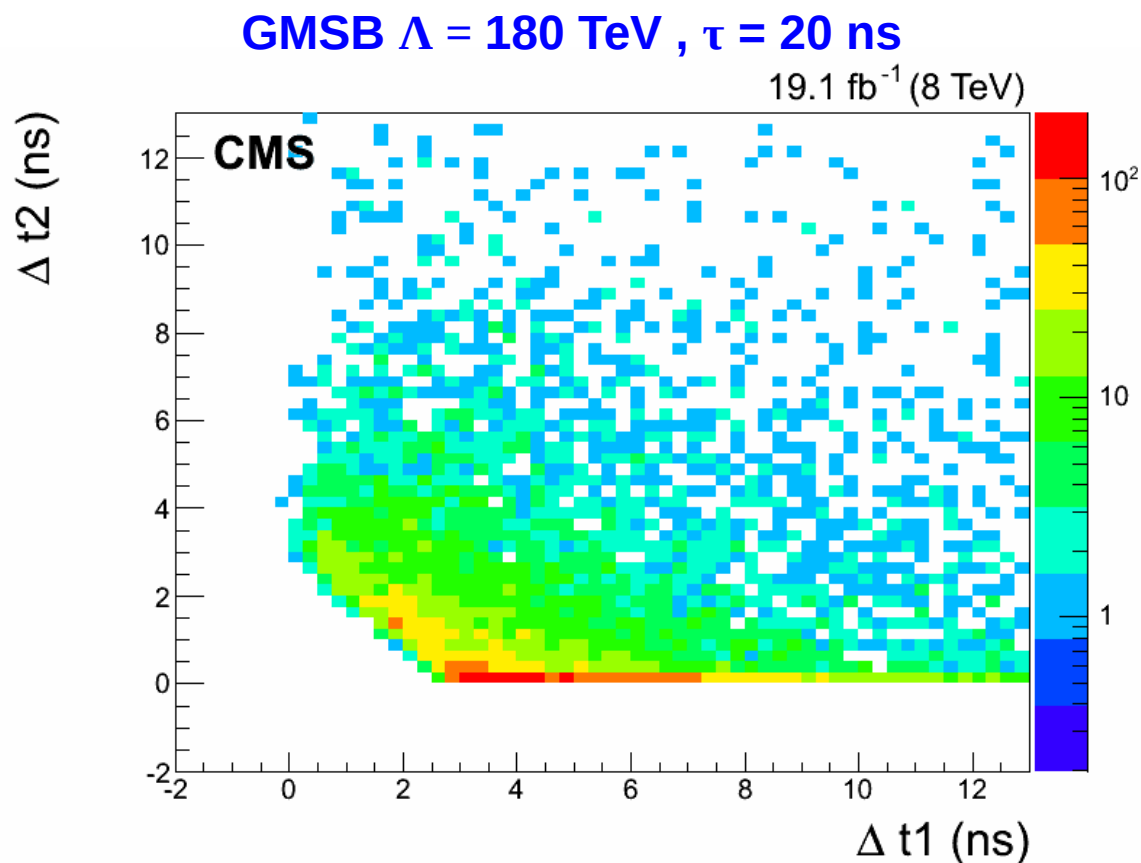




Late Photons



- Check Δt_1 and Δt_2 for late photons ($t > 3\text{ ns}$).
- Delay from slow motion is the main source for delayed signal.
 - 37% of late photons have $\Delta t_2 < 0.5\text{ ns}$.

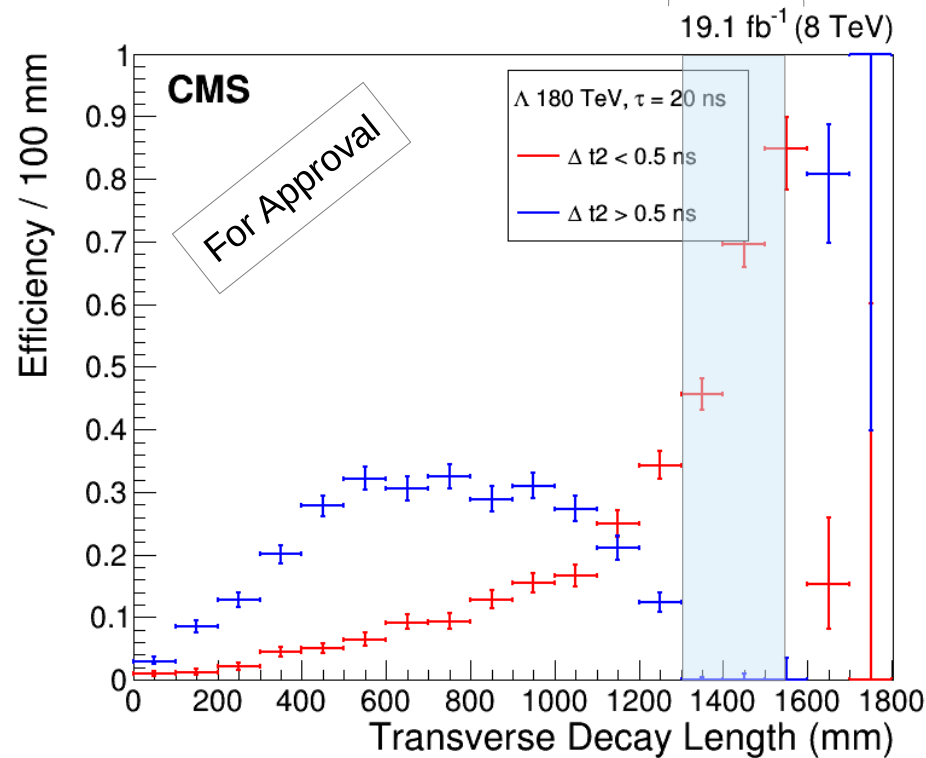
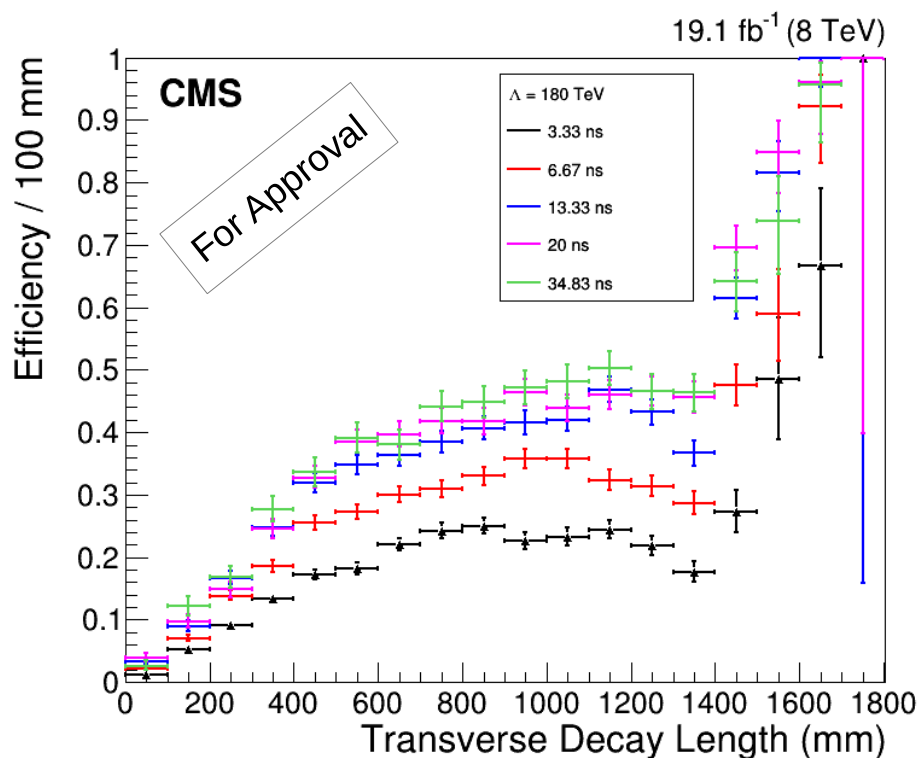




Late Photon Efficiency

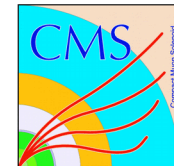


- We evaluated photon efficiency w.r.t. transverse decay length of neutralino.
- The efficiency to detect a late photon ($t > 3\text{ ns}$).
 - Number of late photon / Number of reconstructed photon (EB only)
 - No event selection cut applied yet.
- Separate two kinds of delayed photons using Δt_2 .
- The longer τ is, the higher efficiency (lower beta) would be.

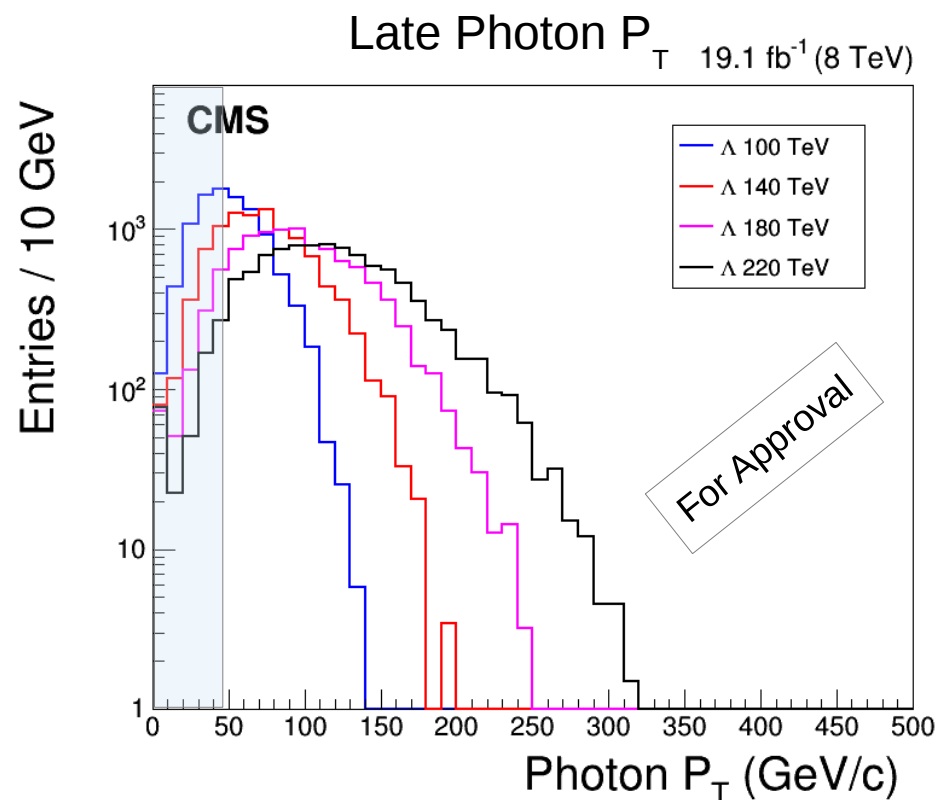
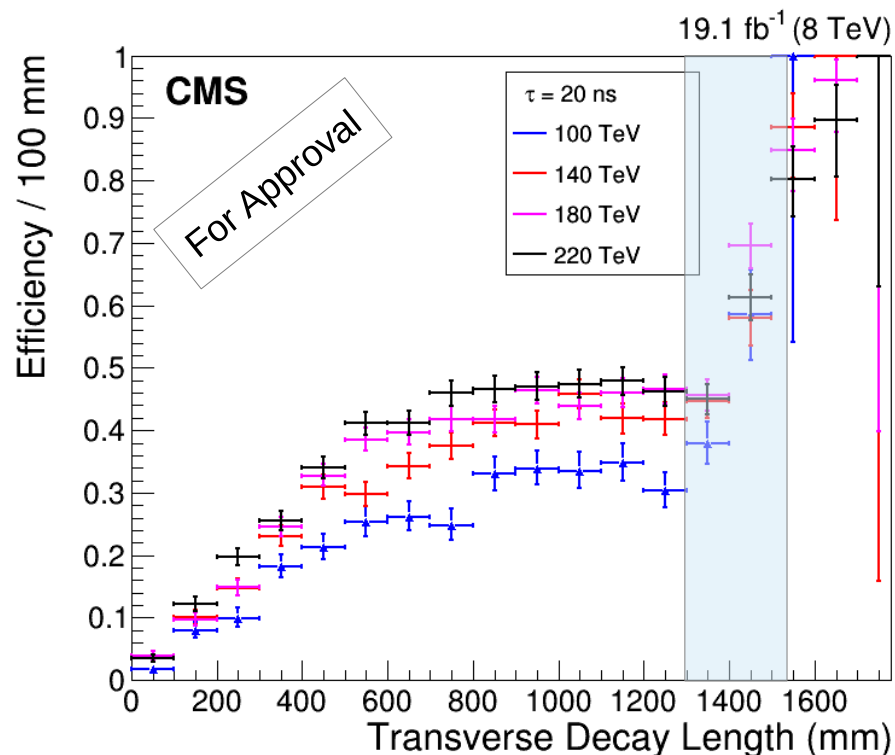




Late Photon Efficiency



- Efficiency among different Λ (with a fixed τ)
 - The larger Λ (heavier neutralino) is, the higher efficiency will be due to harder photon pT spectrum!



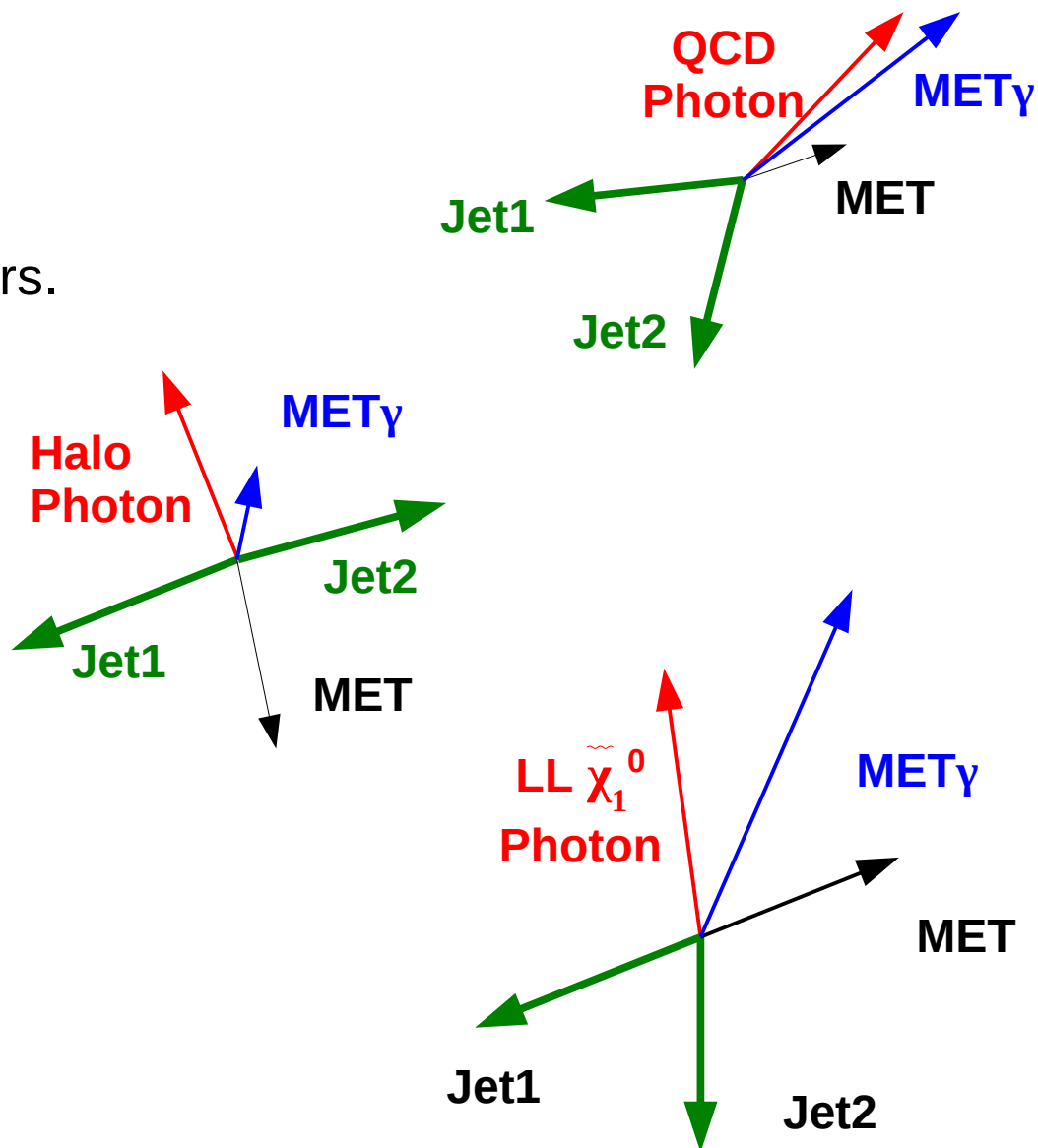


MET and MET γ



- Define new variable - **MET γ**
 - PFMET + photon Et (MET γ)
- Set 60 GeV as threshold both MET quantities.
 - MET can separate QCD from others.
 - MET γ separates Halo from others

	MET	MET γ
QCD	Small	Large
Halo	Large	Small
LL $\tilde{\chi}_1^0$	Large	Large





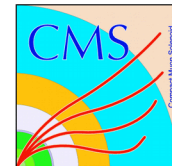
Samples



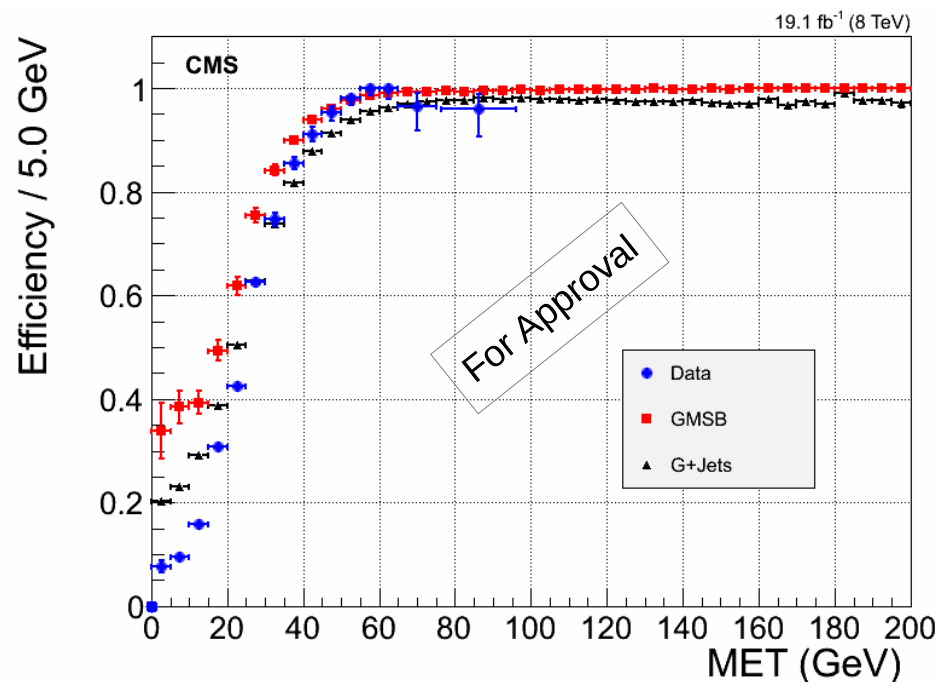
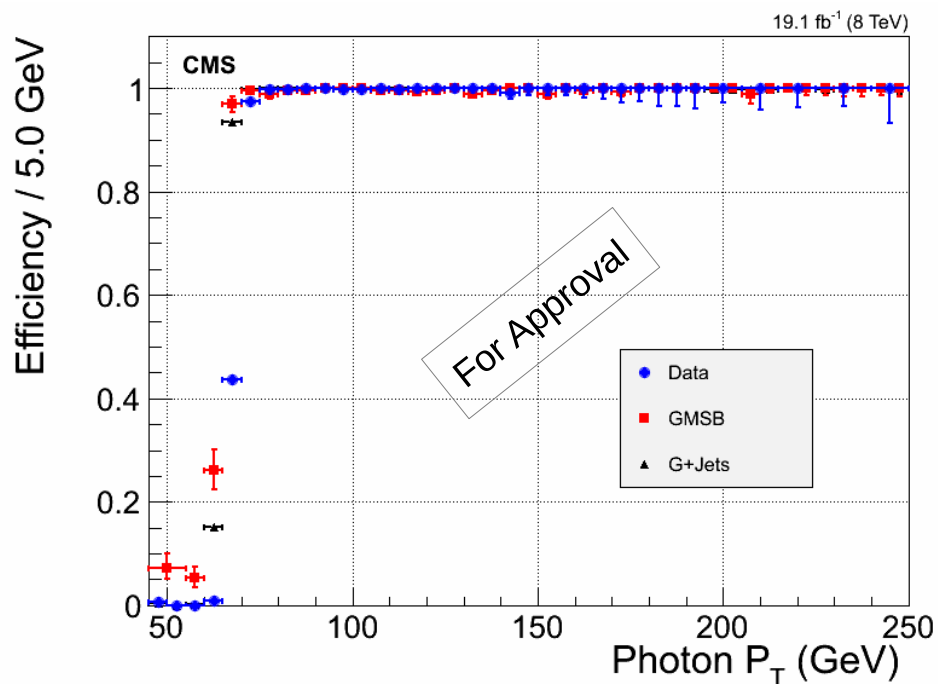
- Data :
 - Displaced Photon Skim (Events fired DiplacedPhoton Trigger, HLT_DisplacedPhoton65_CaloldVL_IsoL_PFMET25)
 - Total 19.1 /fb, 2012A is not included because the trigger was not available.
- Cosmic Dataset
 - For studying background
- IsoPhoton50 Data (HLT_Photon50_CaloldVL_IsoL)
 - For trigger efficiency study
- GMSB MC
 - Signal MC (48000 events for each τ and Λ point)
 - (6 Λ values from 100 TeV to 220 TeV, 7 $c\tau$ points from $\sim 0.2\text{m}$ to 10m)
- Gamma+Jets MC
 - Background MC, only used for sanity check and timing resolution study



Trigger

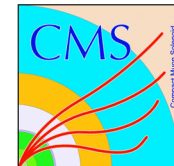


- Displaced Photon Trigger
 - IsoPhoton(65 GeV) + Track rejection ($dR > 0.5$) + sMinor constrain(0.1 ~ 0.4) + PFMET(25 GeV)
 - 100% efficiency for photon > 80 GeV/c and PFMET > 60 GeV. (used in off-line selection)

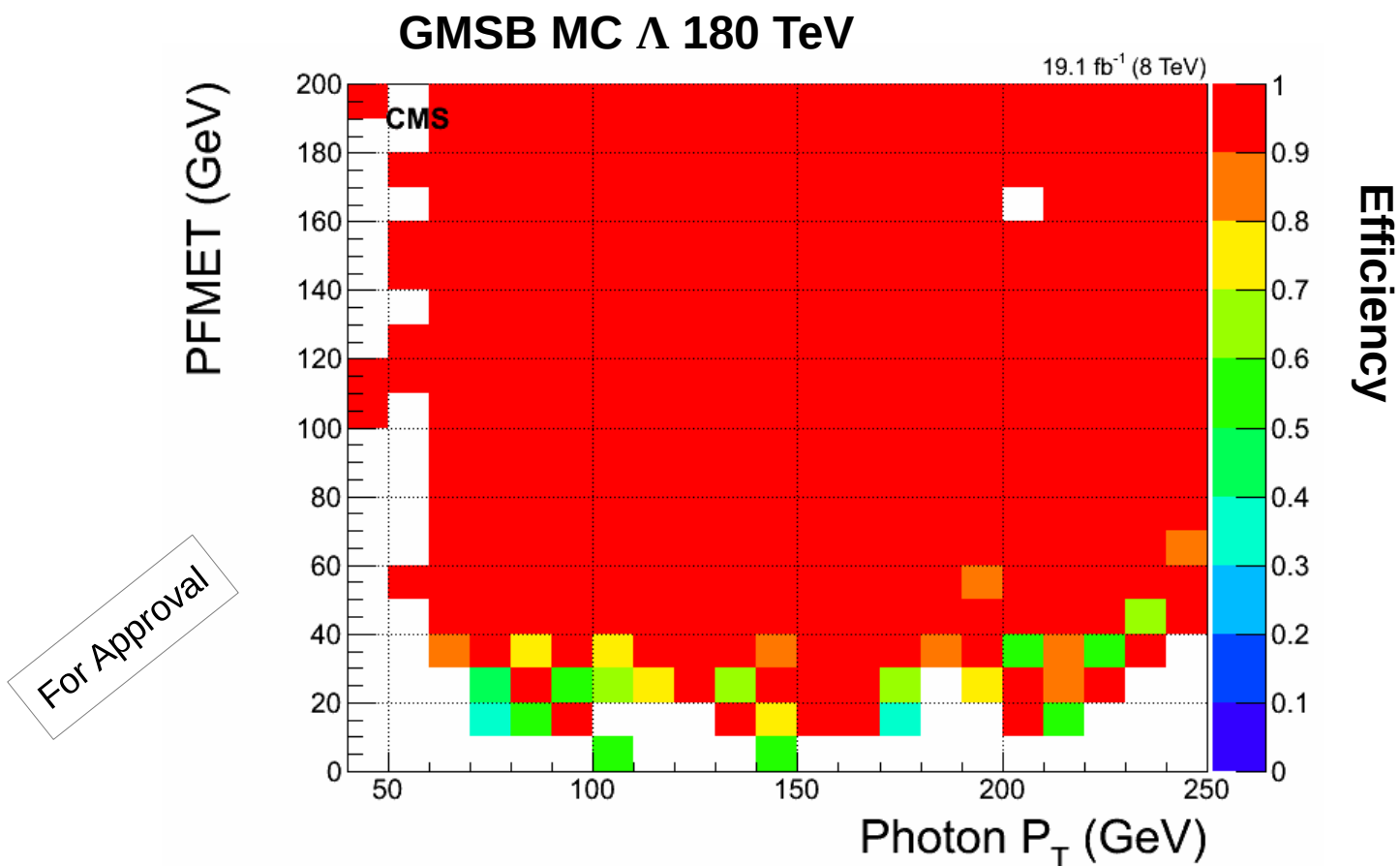




Trigger



- No correlation is found between photon object and PFMET.





Object and Event Selection



- Photon
 - No PF isolation required , $dR(\gamma, \text{track}) > 0.6$, Standard Photon ID
 - Leading photon $P_t > 80 \text{ GeV}/c$, other photons $P_t > 45 \text{ GeV}/c$, EB only
- Jet
 - PFJet , $p_t > 35 \text{ GeV}$, $|\eta| < 2.4$ (Standard PFJet ID), $dR(\gamma, \text{jet}) > 0.3$
- MET
 - PFMET (MET)
 - PFMET + photon E_t (MET_γ)
- Signal Selection :
 - ≥ 1 photon, ≥ 2 jets , $\text{MET} > 60 \text{ GeV}$, $\text{MET}_\gamma > 60 \text{ GeV}$.
- Closure Test :
 - ≥ 1 photon, 0 and 1-jet events, $\text{MET} > 60 \text{ GeV}$, $\text{MET}_\gamma > 60 \text{ GeV}$.



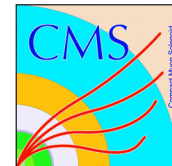
Background Estimation



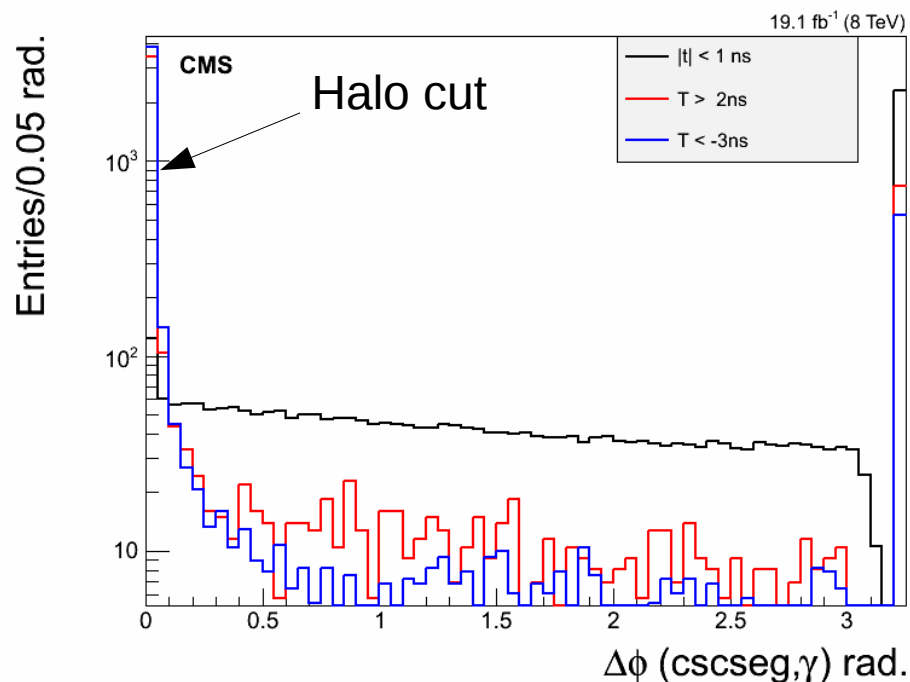
- Comparing with in-time and off-time photon behavior.
 - Use 0 and 1-jet events as control region to study background particularly from non-collision sources.
 - $t < -3 \text{ ns}$, $t > 2 \text{ ns}$
 - Three non-collision sources : Halo, Cosmic-ray, Spikes
 - Find tagging criteria and veto the tagged photon objects.
- Residual non-collision backgrounds survived from veto
 - ABCD method.
- Collision background. (Non-Gaussian tail of ECAL time)
 - $Z \rightarrow e^+e^-$ method.
 - No discrimination to event type (e.g. QCD, W, Top ..).
 - Small contribution (cross-checked using an ABCD method)



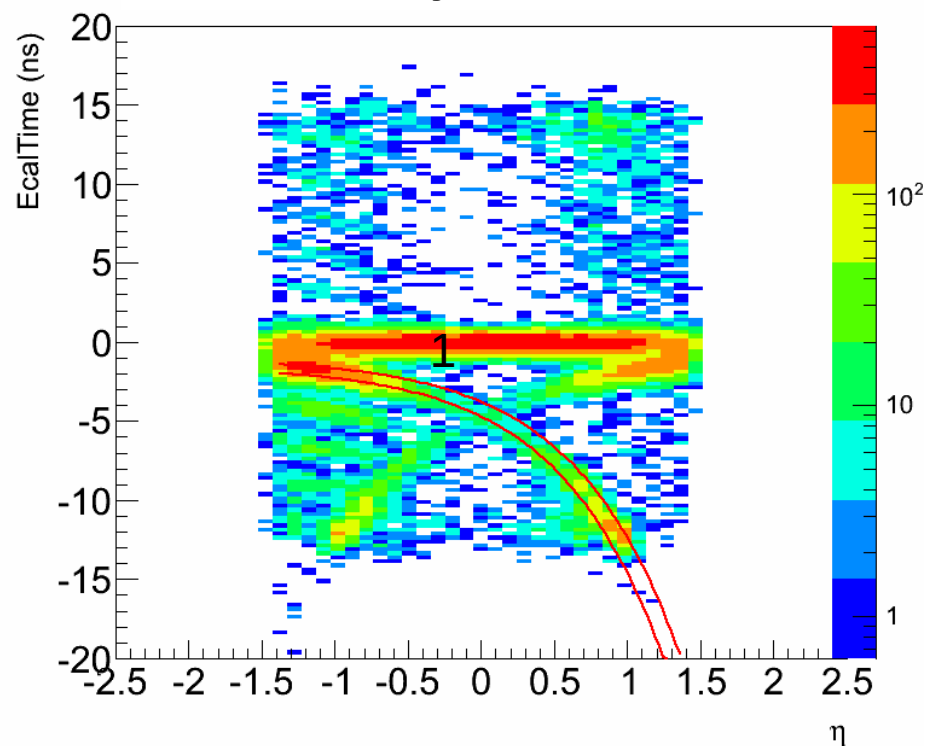
Beam Halo



- The timing of beam halo is η dependent.
 - Its timing is earlier than the associated beam crossing event.
 - Beam halo from satellite bunches give possible late ECAL time.
 - Veto photon objects if its $\Delta\Phi(\gamma, \text{cscSegment}) < 0.05$.

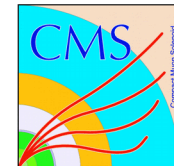


0 & 1-jet events

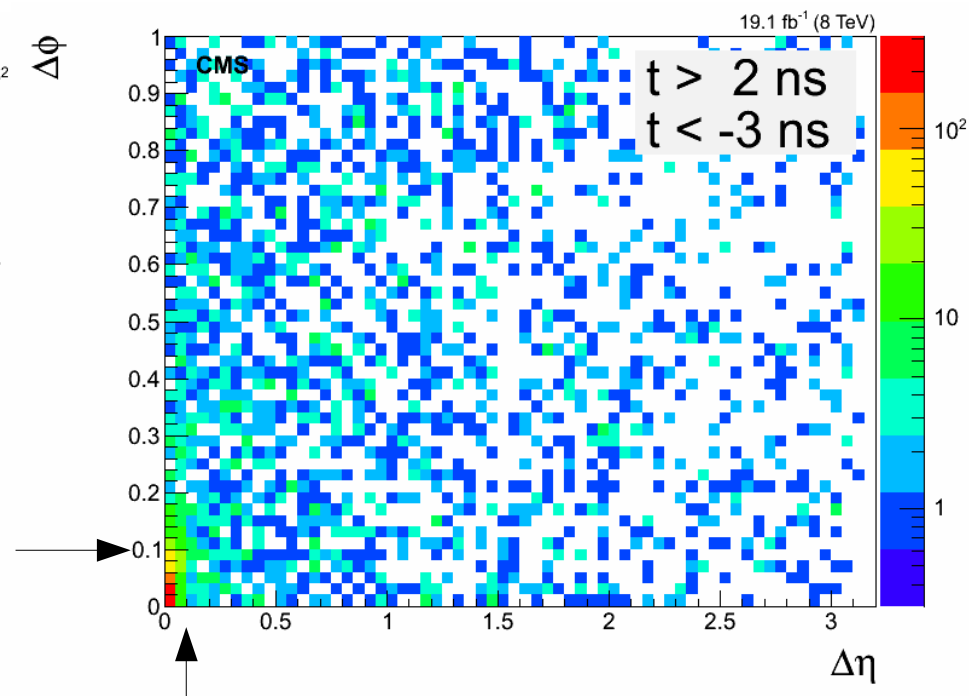
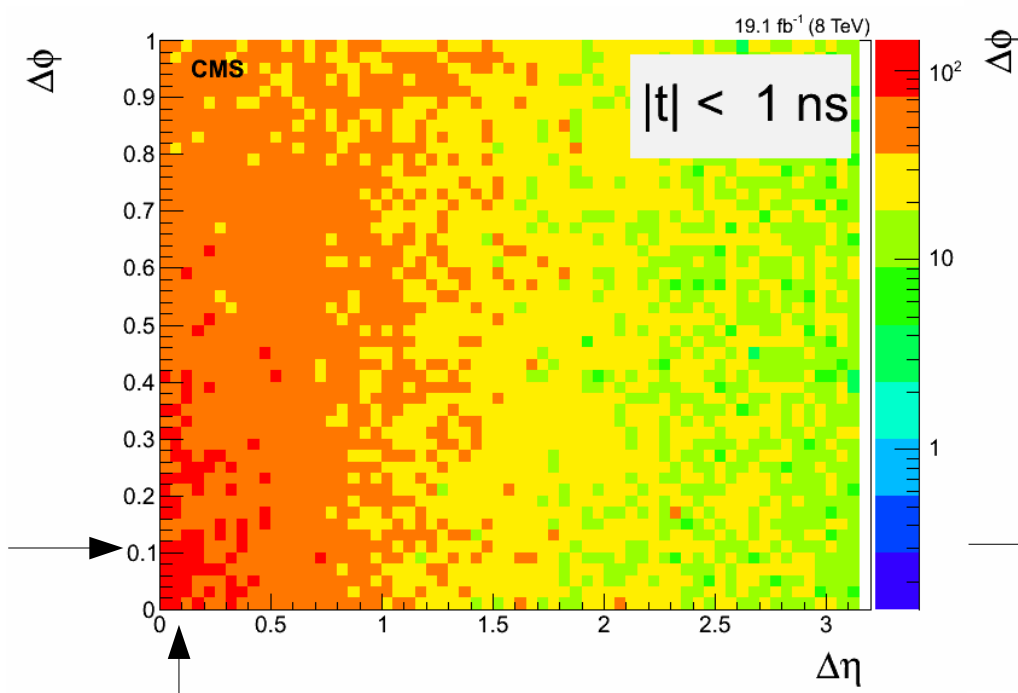




Cosmic-rays



- Cosmic-ray Rejection :
 - $\Delta\eta(\gamma, \text{DTCosmicRaySegment}) < 0.1$ & $\Phi(\gamma, \text{DTCosmicRaySegment}) < 0.1$
 - Using cosmic-rays dataset to verify this method.



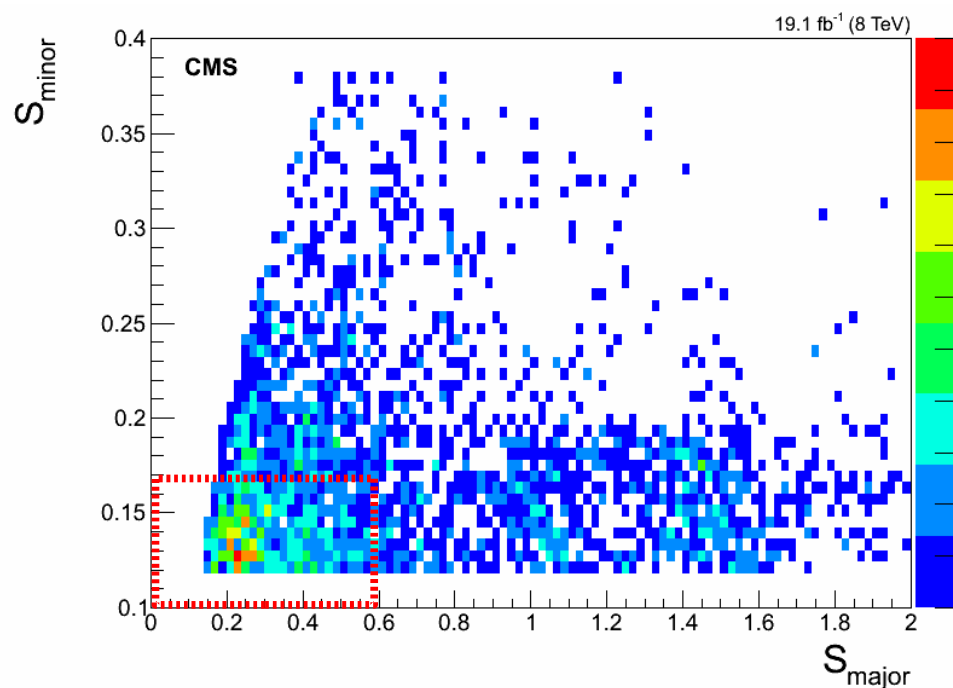


ECAL Spikes

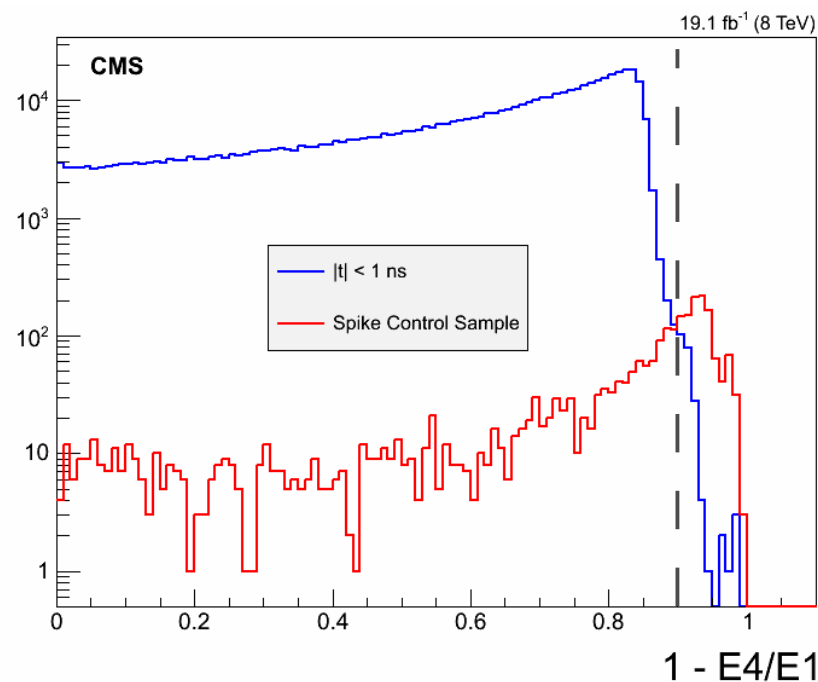


- Direct energy deposit by particle in APD (Avalanche Photo-Diode)
- Most of them are negative timing.
- Remains from halo and cosmic-rays veto.
- Veto criteria : $1 - E4/E1 > 0.9$ or $sMajor < 0.6$ & $sMinor < 0.17$

	E4	
E4	E1	E4
	E4	

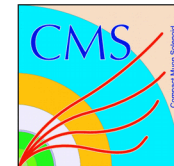


Entries/0.01



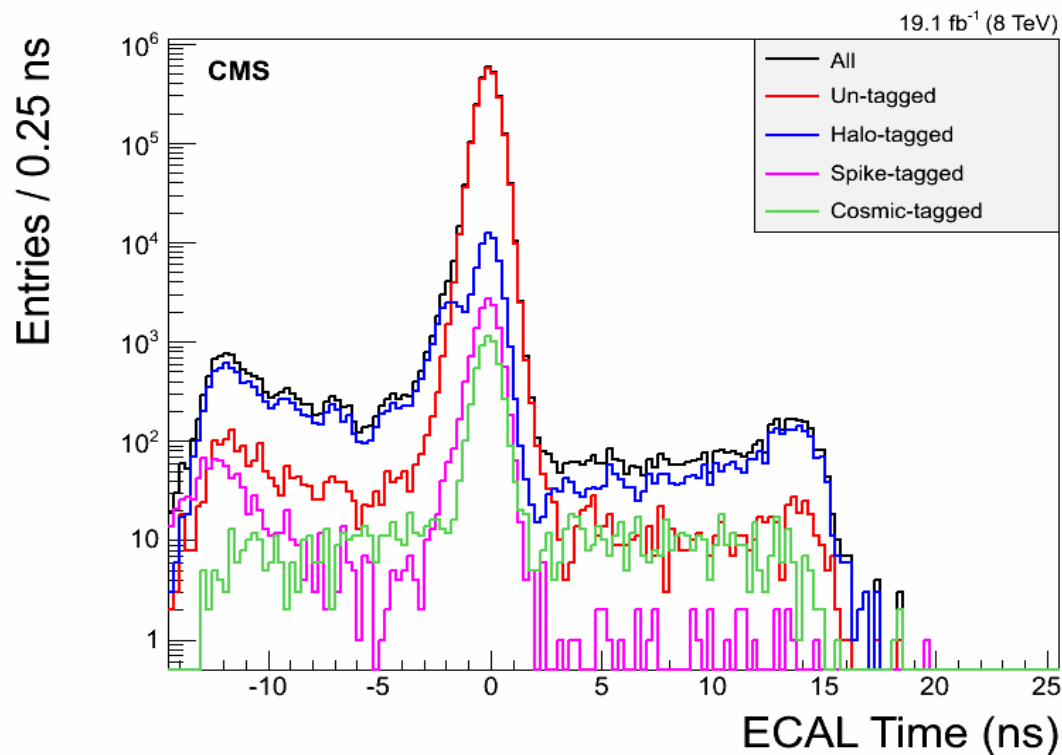


Veto Performance and Fake Rate



- The efficiency for each veto method is difficult to estimate.
 - It's hard to define a pure control sample for each background source without contamination or bias.
- Fake rates of the veto are measured from in-time photon sample.
- Since the efficiency is not 100%, the residual non-collision background is not trivial.
 - Develop an ABCD method to estimate the leftover photons.

	Fake Rate
Halo	~3%
Cosmic-rays	1.4%
ECAL Spike	0.4%





ABCD – Non Collision Background



$\text{MET} > 60 \text{ GeV}$	$\text{MET}_\gamma < 60 \text{ GeV}$	$\text{MET}_\gamma > 60 \text{ GeV}$
$3 \text{ ns} < t < 13 \text{ ns}$	C	D
$-10 \text{ ns} < t < -3 \text{ ns}$	A	B

- Estimate the residual from halo/cosmic-rays/spike veto
- $\text{MET} > 60 \text{ GeV}$:
 - QCD is suppressed, Non-collision backgrounds has small MET_γ .
 - A and C are nearly QCD free.
- $D = B/A \times C$



$Z \rightarrow e^+e^-$ Method

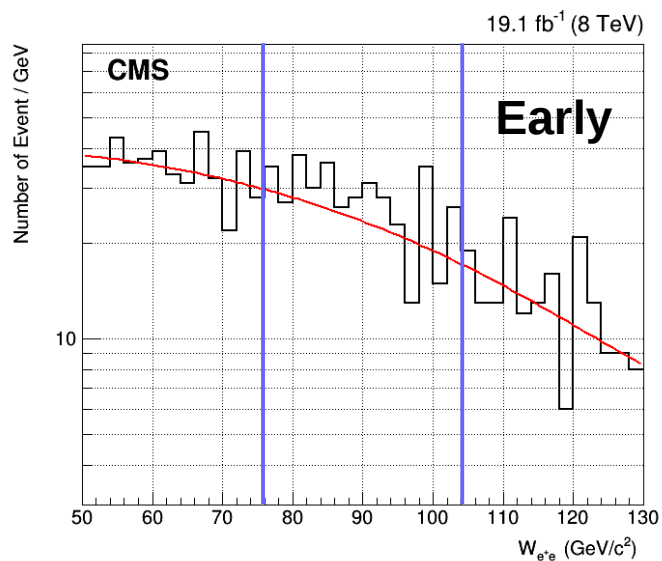


- Using Z events as a representative for collision background
 - Count Z events in three different time zone, in-time ($|t| < 2 \text{ ns}$), early ($t < -3 \text{ ns}$), late ($t > 3 \text{ ns}$), within Z mass window ($76 \sim 104 \text{ GeV}$)
 - Polynomial fit sideband ($50 \sim 76 \text{ GeV}$ and $104 \sim 130 \text{ GeV}$) to determine background of Z events.
 - Take ratio of out-of-time and in-time electrons as the probability to get out-of-time measurement and apply it to our analysis selection.
 - Observed 3 events with both electron late ($t > 3 \text{ ns}$).
 - Only one late electron : 1.09×10^{-5}
 - Two late electrons : 0.22×10^{-5}

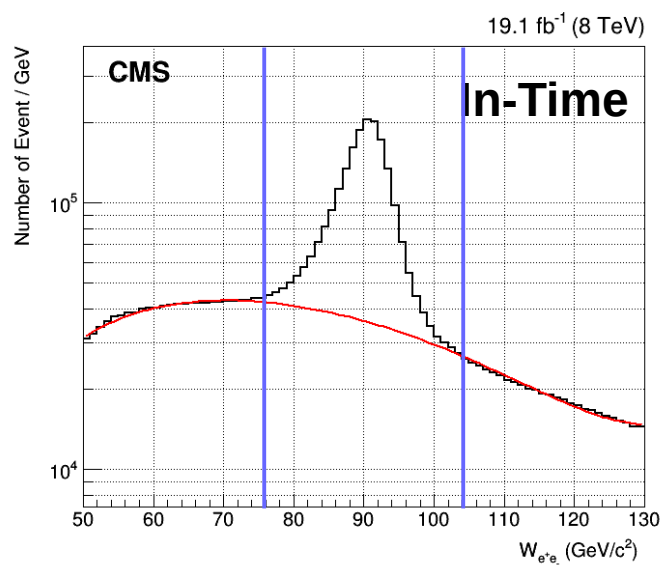
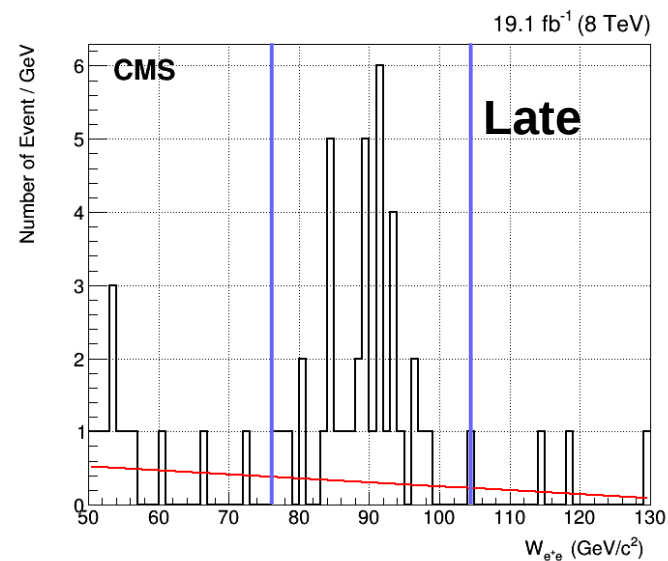
	$T < -3 \text{ ns}$	$ T < 2 \text{ ns}$	$T > 3 \text{ ns}$
Total	378	2349187.0	41.0
DY Background	329	996803.6	8.6
Estimated Z	49	1352383.4	32.4



Z Mass Distribution



OR





$Z \rightarrow e^+e^-$ Method

- $N = n_1 \times p_1 + n_2 \times (2 p_1 (1 - p_1) + p_1^2) + n_1 \times p_2 + n_2 \times p_2$

n_1 : in-time 1 photon events , n_2 : in-time 2 photon events.

p_1 : Probability to get a late/early timing measurement.

p_2 : Probability to get a late/early collision event (satellite bunch collisions).

→ **N = Estimated out-of-time background events.**

- Calculate p_1 and p_2 using values from previous table.

→ The observed 3 events (with two late electrons) are considered as events from satellite bunch collisions and are used for p_2 .

	p_1	p_2
Late	1.09×10^{-5}	0.22×10^{-5}
Early	1.81×10^{-5}	0



Combined Background Estimation and Closure Test



- Combined ABCD from non-collision and $Z \rightarrow ee$ collision background

$$\rightarrow D = (B - Q_b) \times C / A + Q_d$$

- Using 0 and 1-jet events for closure test

$$\rightarrow \text{Estimate : } (38 - 0.64) \times 359 / 851 + 0.46 = \mathbf{16.41^{+3.00}_{-2.59}} \leftrightarrow \text{Observed : } \mathbf{10}$$

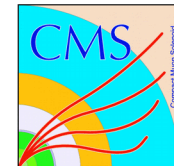
$$\rightarrow Q_d : \mathbf{0.46^{+0.11}_{-0.09}}, \quad Q_b : \mathbf{0.64^{+0.35}_{-0.33}}$$

For Approval

Non-Collision	$\text{MET}_\gamma < 60 \text{ GeV}$	$\text{MET}_\gamma > 60 \text{ GeV}$
$3 \text{ ns} < t < 13 \text{ ns}$	(C) 359	(D) 10 (16.41)
$ t < 2 \text{ ns}$		35097 (1 γ) + 174 (2 γ)
$-10 \text{ ns} < t < -3 \text{ ns}$	(A) 851	(B) 38



Result



- Background estimation for signal sample :

→ Predict background : $((1 - 0.51) \times 0/3) + 0.37 = 0.37^{+0.39}_{-0.07}$

→ Observed 1

→ $Q_d : 0.37^{+0.09}_{-0.07}$, $Q_b : 0.51^{+0.28}_{-0.27}$

For Approval

Non-Collision	$MET_\gamma < 60 \text{ GeV}$	$MET_\gamma > 60 \text{ GeV}$
$3 \text{ ns} < t < 13 \text{ ns}$	(C) 0	(D) 1
$ t < 2 \text{ ns}$	35097 (1γ) + 174 (2γ)	
$-10 \text{ ns} < t < -3 \text{ ns}$	(A) 3	(B) 1



Uncertainties



- The main impact of the systematic is to signal efficiency.
 - Two factors, time bias (T0) and unclustered energy (MET), are the main sources of systematic uncertainty.
 - Reflect the method, ECAL time cut and MET cut.
 - Vary each systematic (1-sigma upward and downward) individually to get the percentage of the yield change.

- Smaller Δ corresponds to larger systematic.
 - Efficiency is lower. Statistic uncertainty is dominated
- Uncertainty from background estimation is quoted from its statistical uncertainty
 - $0.37 + 0.39$
- 0.07

For Approval

Source	Uncertainty (%)
Time Bias (T0)	6 ~ 10
Unclustered Energy (MET)	4 ~ 10
ECAL Time Resolution	2 ~ 5
Jet Energy Scale	3 ~ 9
Jet Energy Resolution	2 ~ 9
Photon Energy Scale	2 ~ 4
Luminosity	2.6
PDF	< 1



One Observed Event



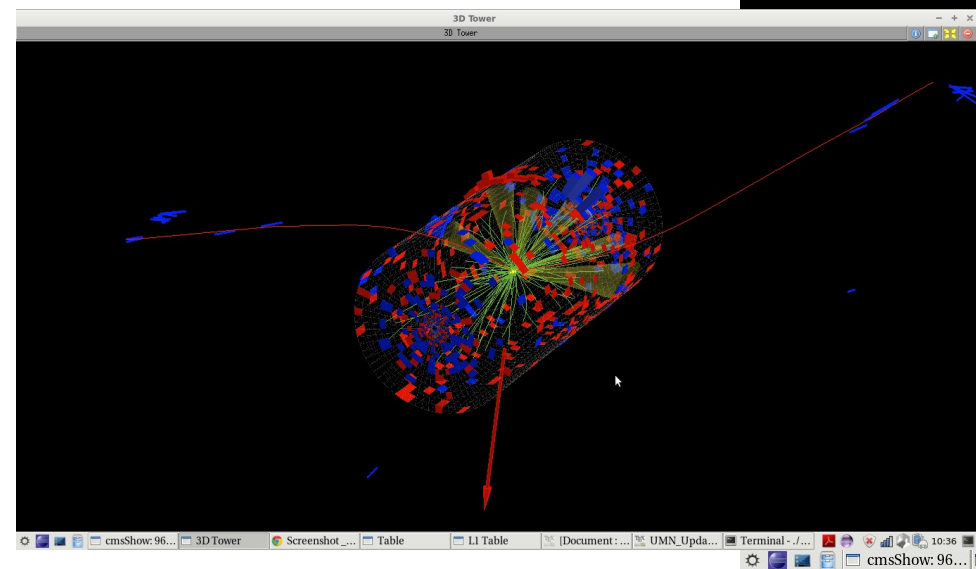
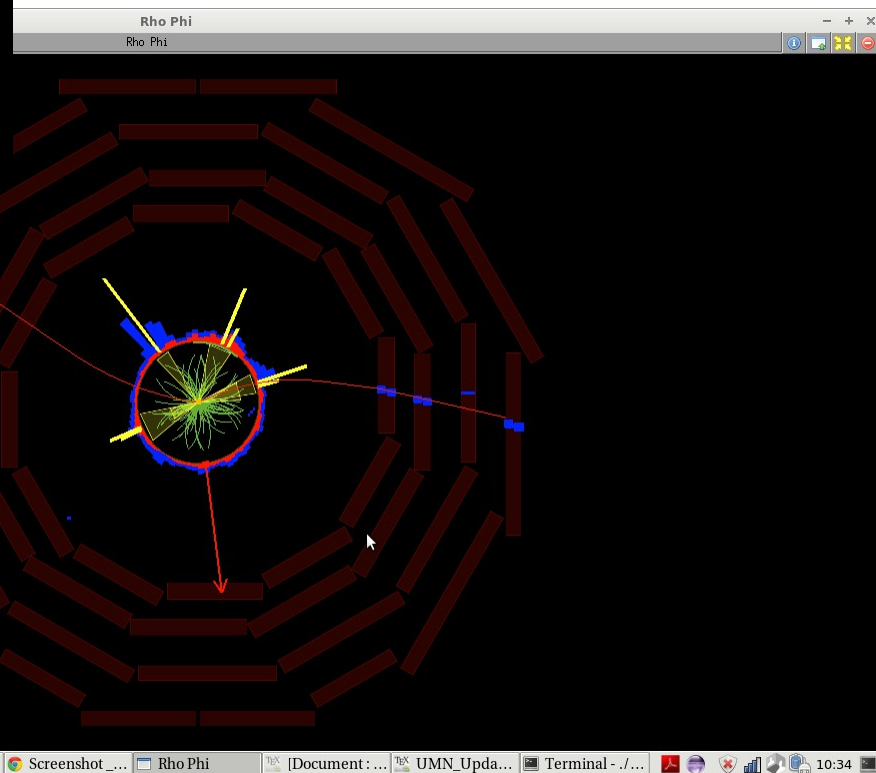
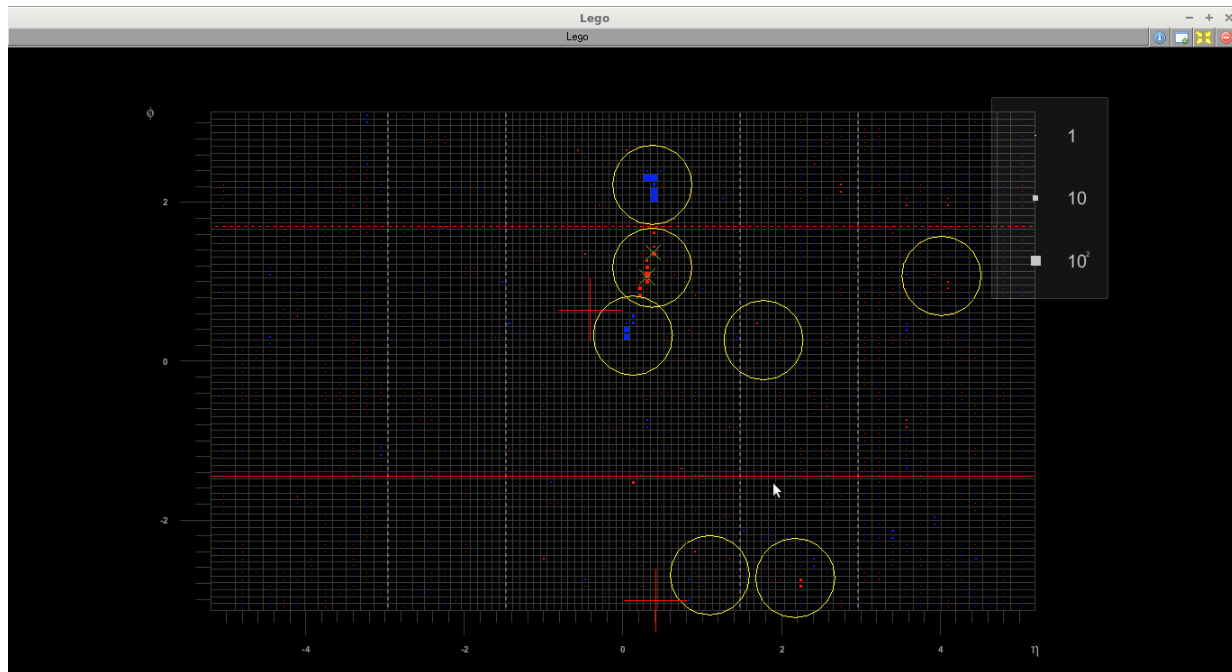
- Run Number: 206484 LumiSection : 620 Event Number: 871295869
- Photon :
 - Pt : 225 GeV, Ecaltime : 12 ns , η : 0.32, Φ : 1.13 , sMajor: 2.82, sMin: 0.16
- MET :
 - MET $_{\gamma}$: 125 GeV
 - MET : 333 GeV
 - Larger MET and smaller MET $_{\gamma}$ indicate Halo/Cosmic-rays like behavior.
- Jet :
 - 2 jets (86 and 36 GeV)



Event Display



- Run Number: 206484
- LumiSection : 620
- Event Number: 871295869



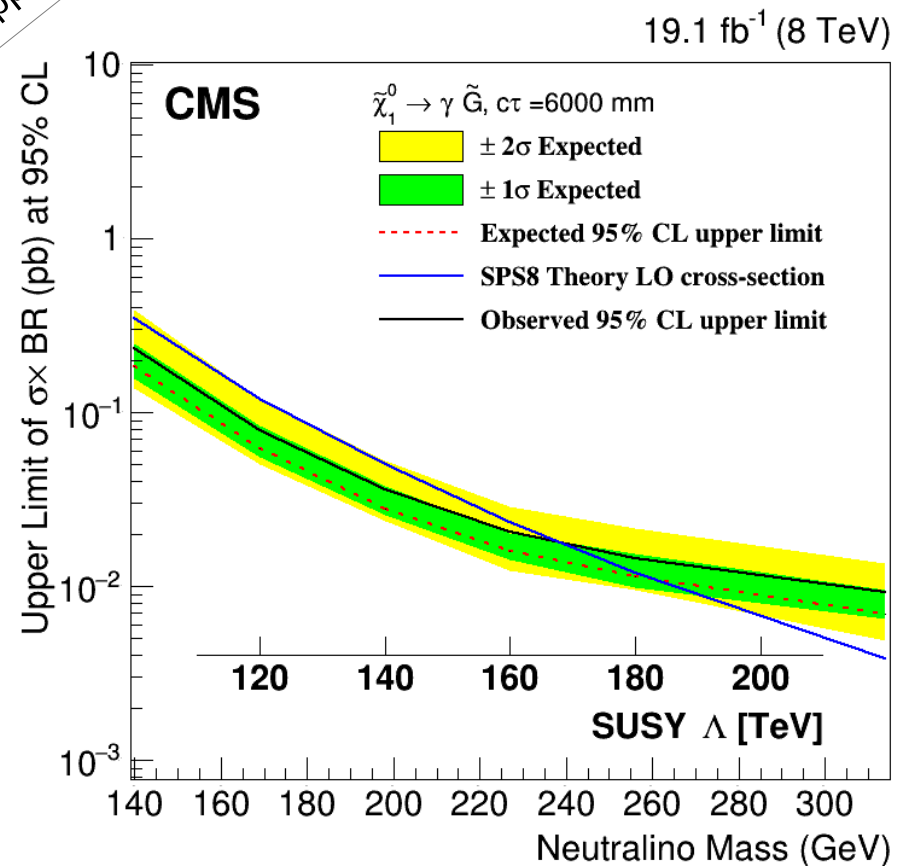
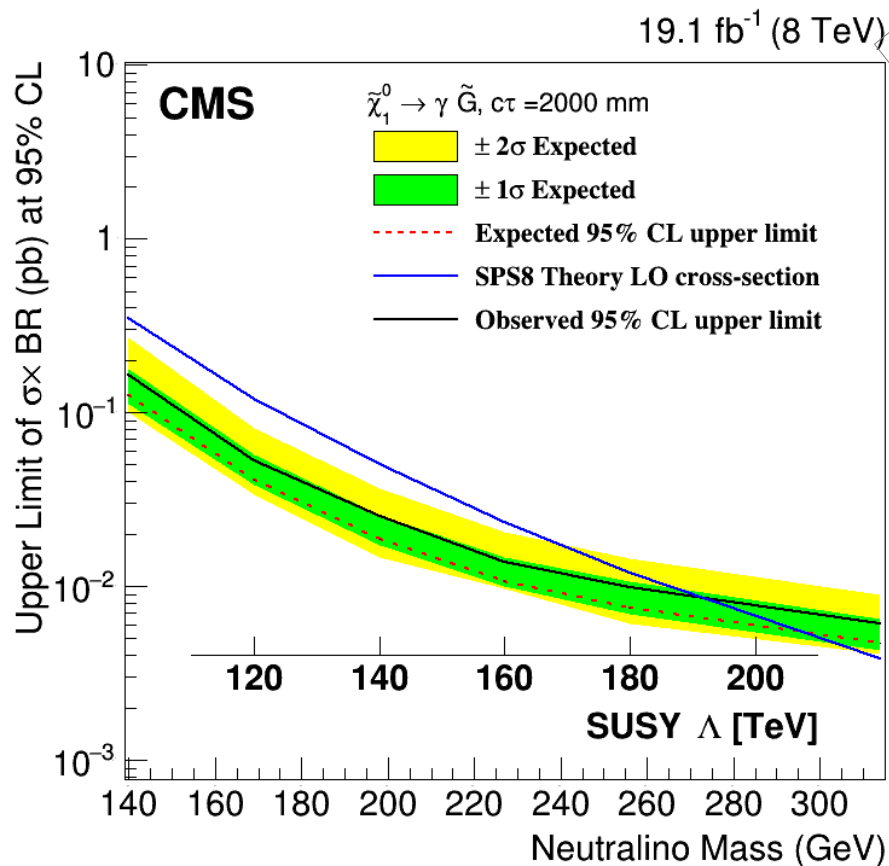


Limit – Neutralino Mass



- Set limits using CLs method.
- Under current background expectation (0.37) and observation (1), the upper limit of 4.07 events is set at 95% C.L.

For Approval

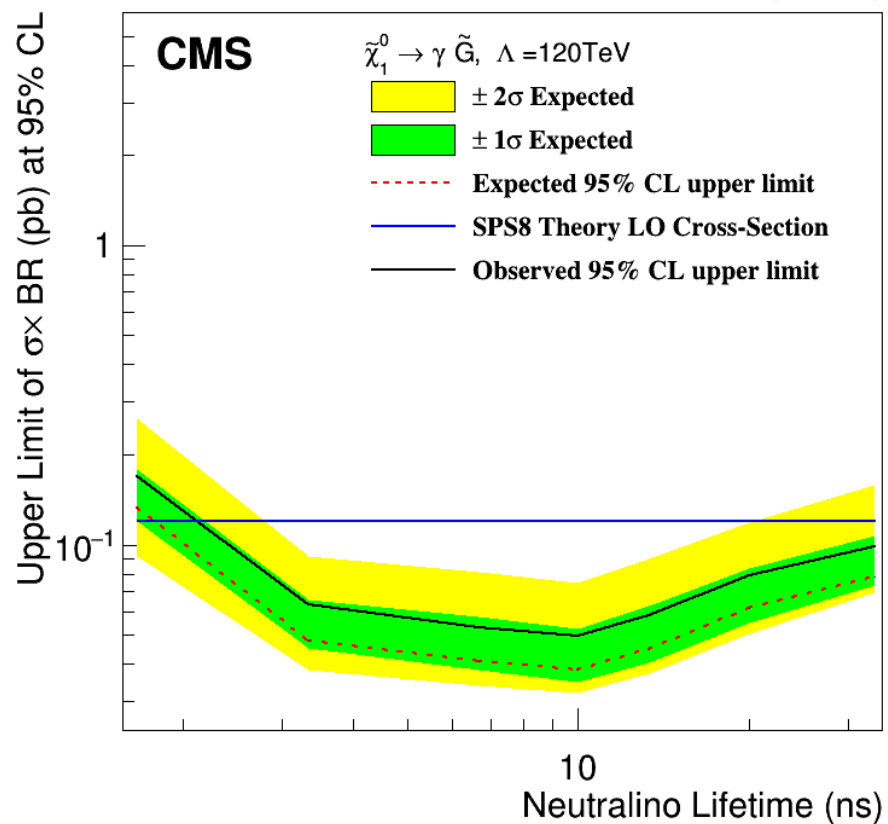




Limit - Lifetime

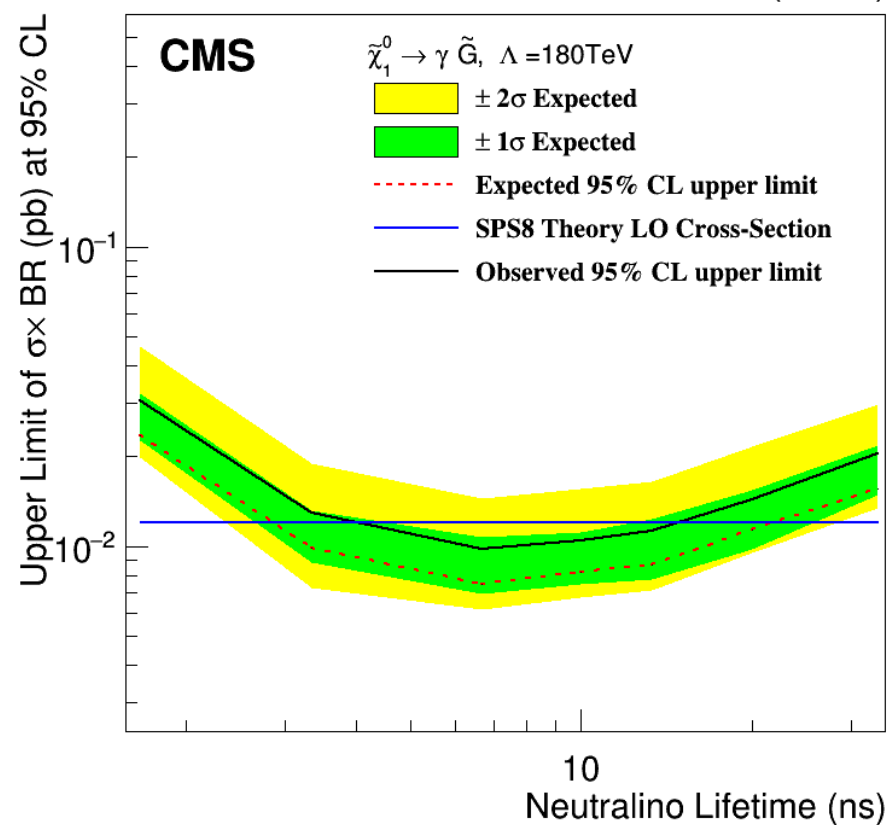


19.1 fb⁻¹ (8 TeV)



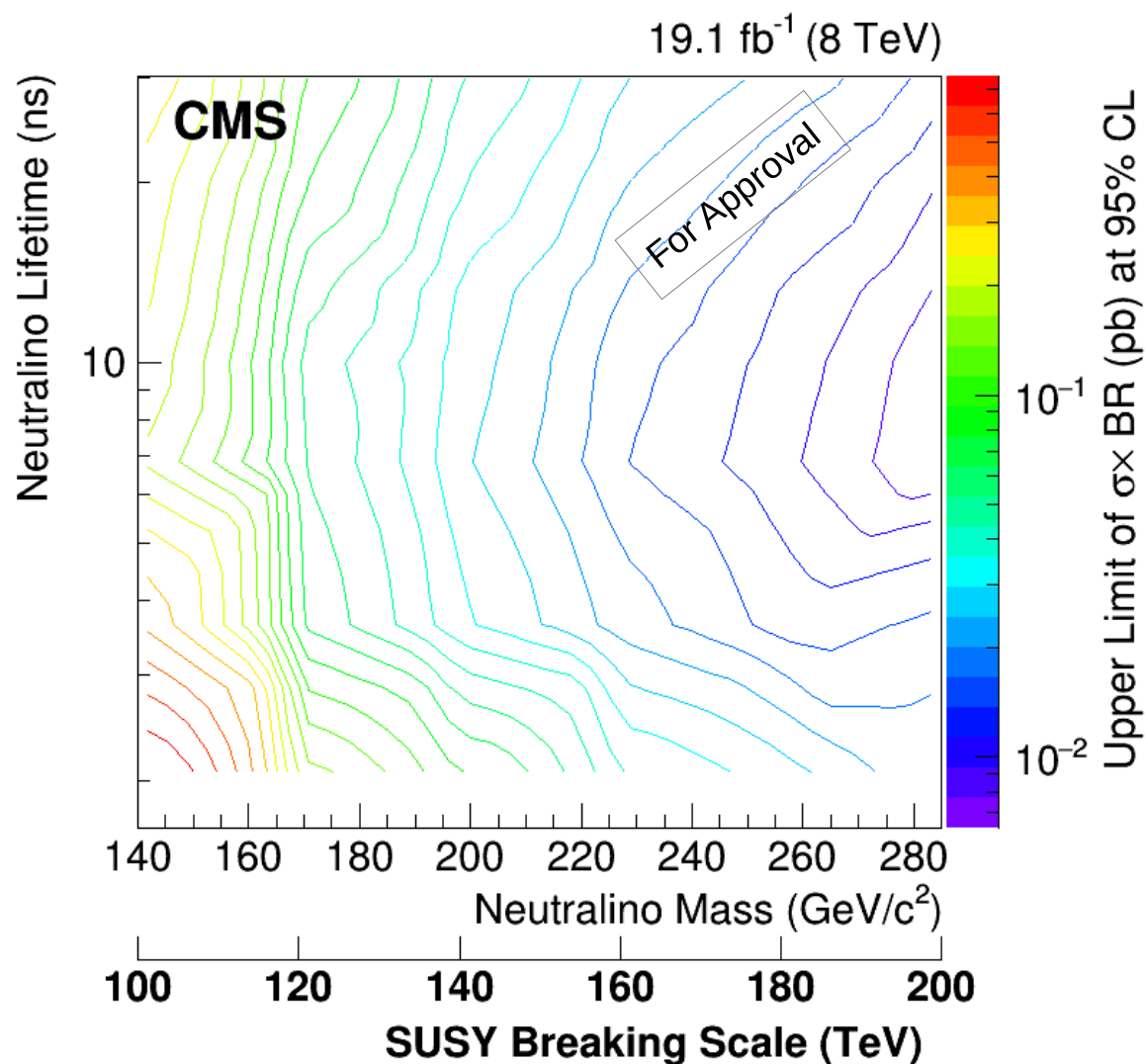
For Approval

19.1 fb⁻¹ (8 TeV)





- An upper-limit of cross-section times branching ratio is provided in terms of neutralino's lifetime and mass.

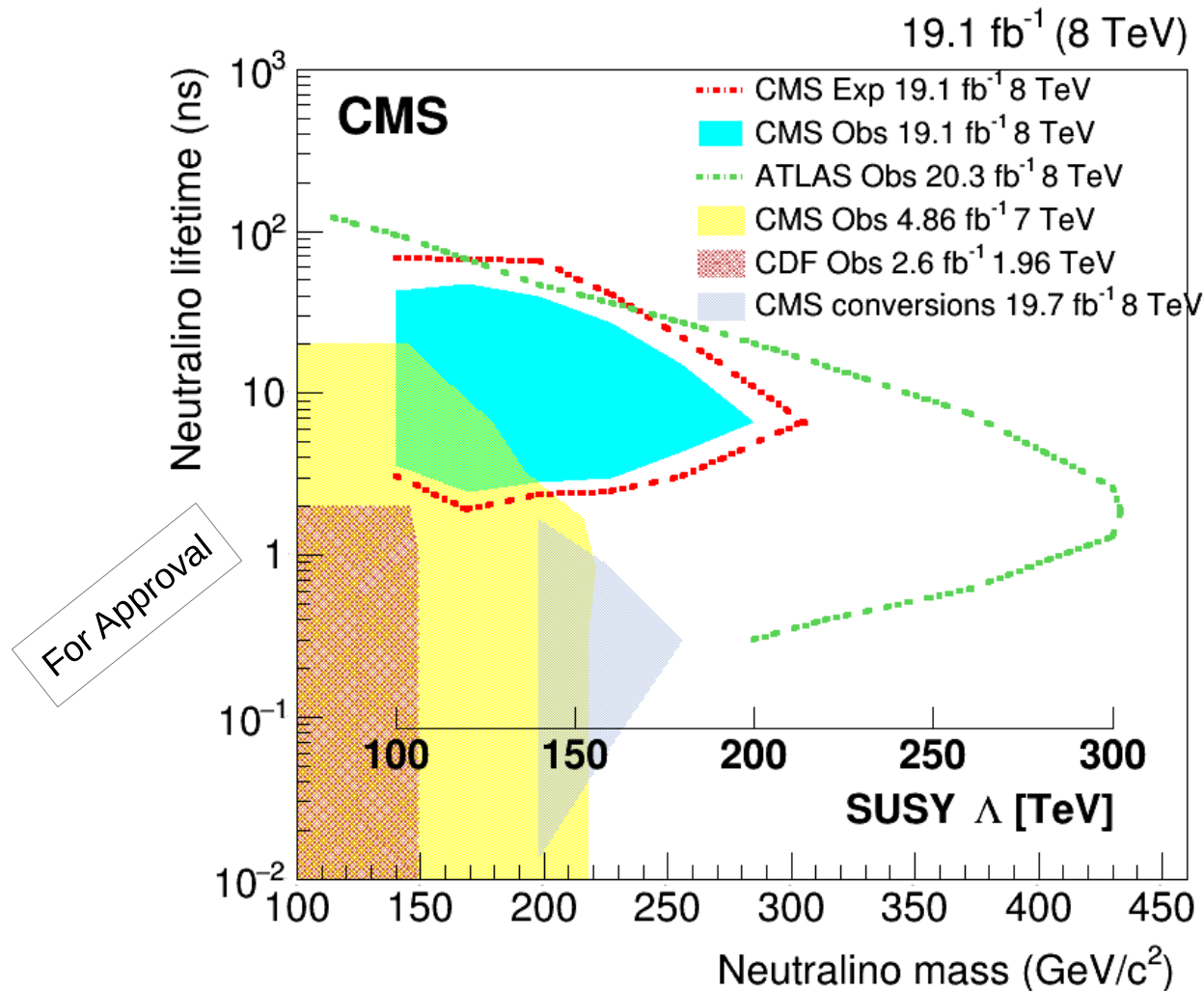




Conclusion



- An exclusion region of neutralino mass and lifetime at 95% C.L. is set.
 - Lifetime : 3ns ~ 30 ns , mass: 140 ~ 260 GeV





Back Up

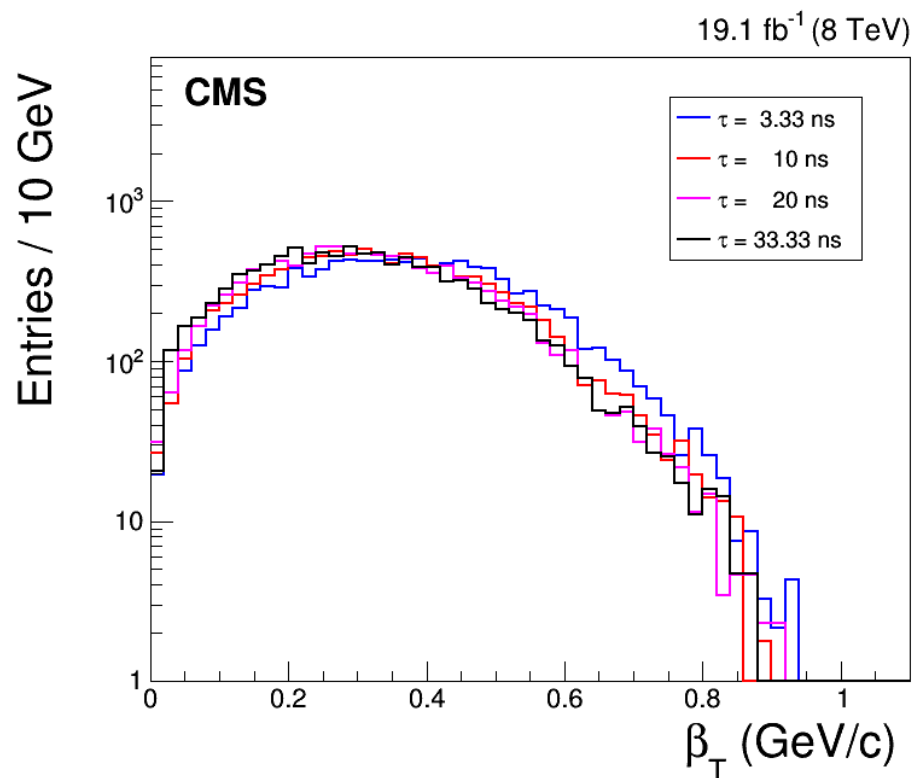




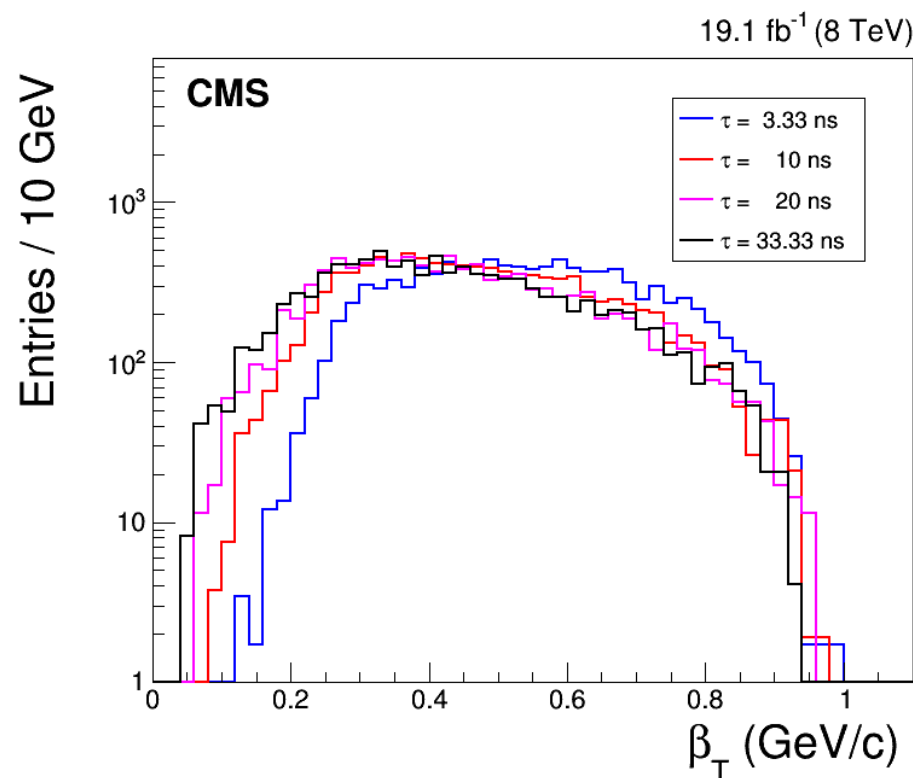
Beta of Neutralino



- Delayed photons ($T > 3\text{ ns}$) are mostly from slow neutralino.
- Shorter lifetime model need harder boost in order to reach the same transverse decay length ($900 \sim 1200\text{ mm}$),



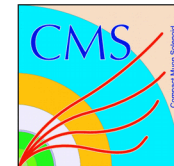
β_T distribution of neutralino
which produces a late photon.



β_T distribution for neutralino with
transverse decay length
between 900 and 1200 mm.

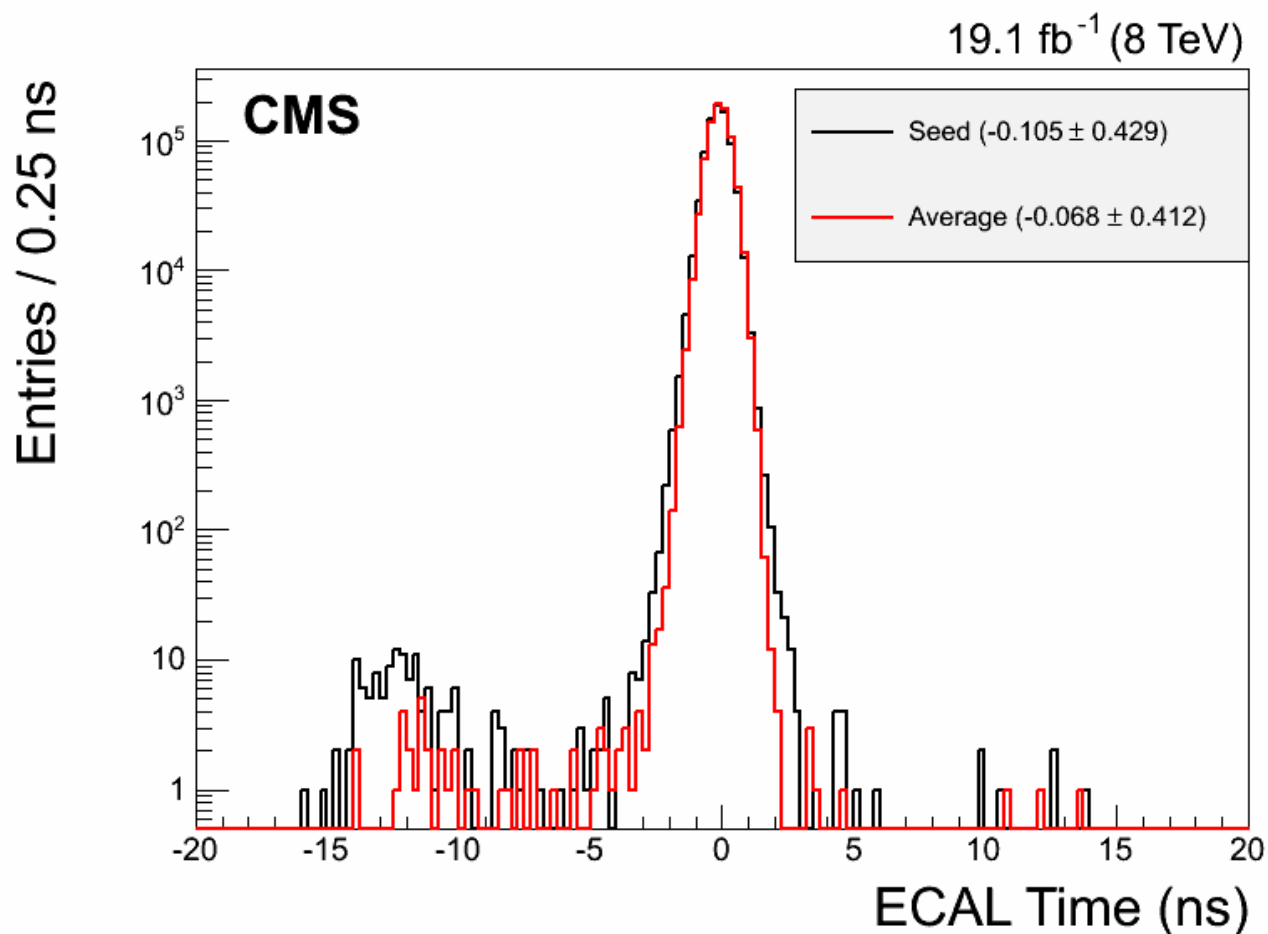


Cluster Time and Seed Time



- Cluster time is weighted average over times from all crystals in seed cluster.
 - Both timing have similar performance.

$$t_{cluster} = \frac{\sum \frac{t_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$





Dataset



- Displaced Photon Skim

/SinglePhoton/Run2012B-EXODisplacedPhoton-22Jan2013-v1/RECO

/SinglePhoton/Run2012C-EXODisplacedPhoton-19Dec2012-v1/RECO

/SinglePhoton/Run2012D-EXODisplacedPhoton-19Dec2012-v1/RECO

- IsoPhoton50

/SinglePhoton/Run2012B-22Jan2013-v1/RECO

/SinglePhoton/Run2012C-22Jan2013-v1/RECO

- GJets MC ($P_{t, \text{hat}}$: 50 ~ 800 GeV)

/G_Pt-50to80_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO

/G_Pt-80to120_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO

/G_Pt-120to170_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO

/G_Pt-170to300_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO

/G_Pt-300to470_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO

/G_Pt-470to800_TuneZ2star_8TeV_pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/GEN-SIM-RECO



GMSB MC Samples



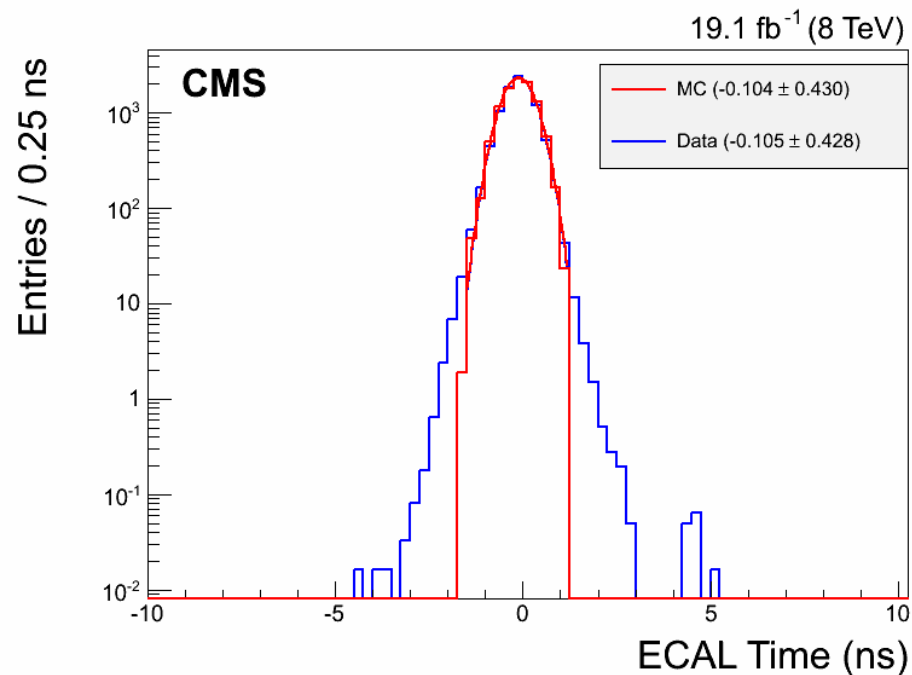
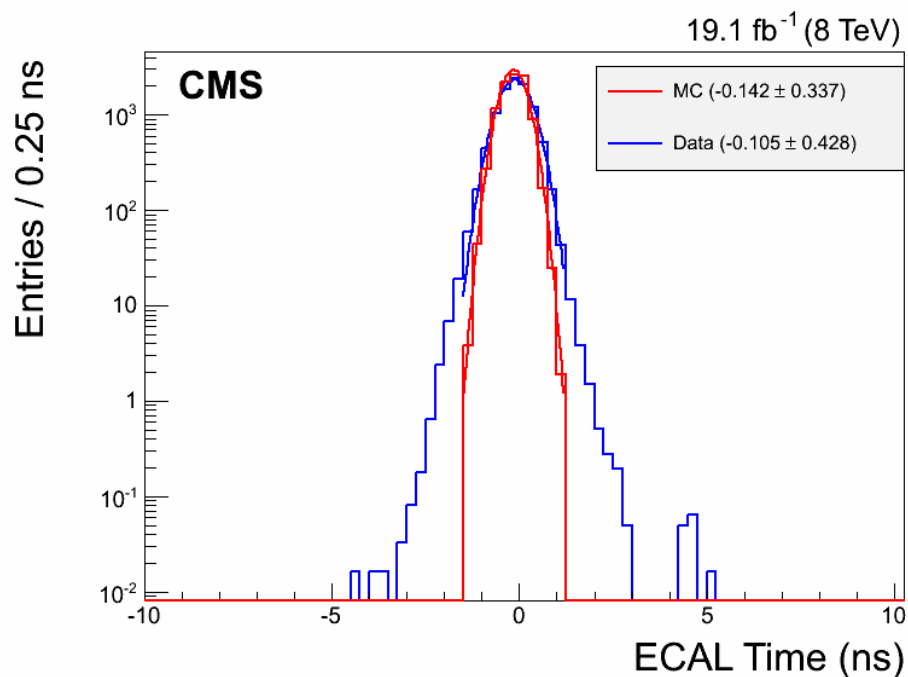
- /GMSB_Lambda-xxx_CTau-yyyy_TuneZ2star_8TeV-pythia6/Summer12DR53X-PU_S10_START53_V19-v2/GEN-SIM-RECO
→ xxx is the value of Λ and yyyy is the value of $c\tau$ (in mm).
- Range of SUSY breaking scale (Λ)
→ 100, 120, 140, 160, 180, 220 TeV
- Range of neutralino lifetime ($c\tau$)
→ 500, 1000, 2000, 3000, 4000, 6000, 10000 mm



MC ECAL Time Calibration

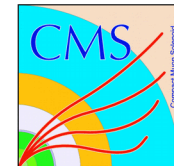


- Events Pass signal selection.
 - One PF-Isolated photon with $P_t > 80$ GeV , At least 1 or 2 jet in the events, $MET < 30$ GeV.
 - Comparing data and Gamma+Jets MC.
- Apply T0 shift and smear time resolution on each photon time to signal MC samples.

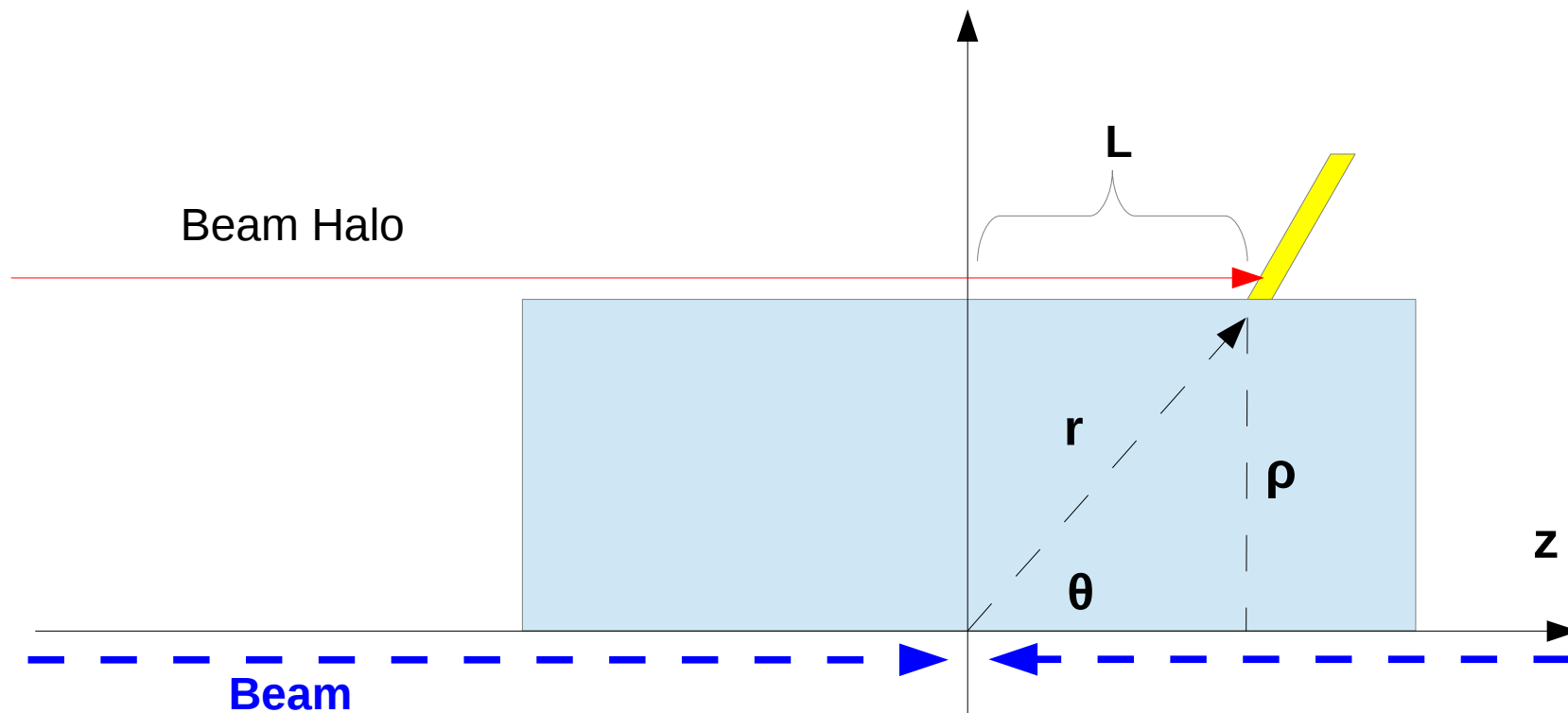




Halo Timing Approximation



- Approximation of halo behavior :
 - $t_0 = r/c = \rho / c \sin\theta$
 - $t_{\text{halo}} = L / c = \rho / c \tan\theta$
 - $t_{\text{ECAL}} = \rho / c \tan\theta - \rho / c \sin\theta = \rho/2c (\tan\theta/2) = - \rho/2c \exp(-\eta)$
 - Halo timing would be earlier than the normal collision.

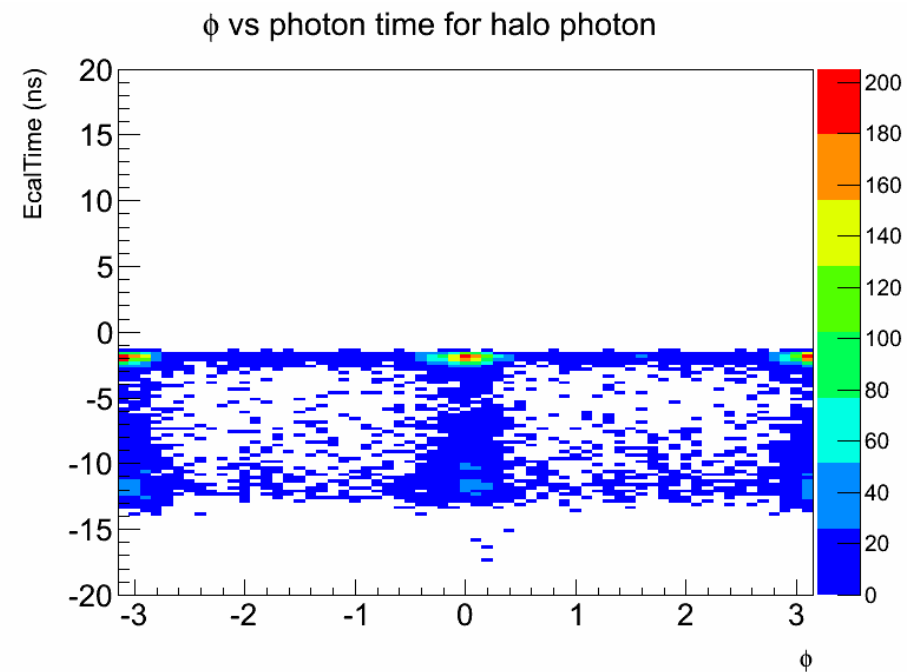
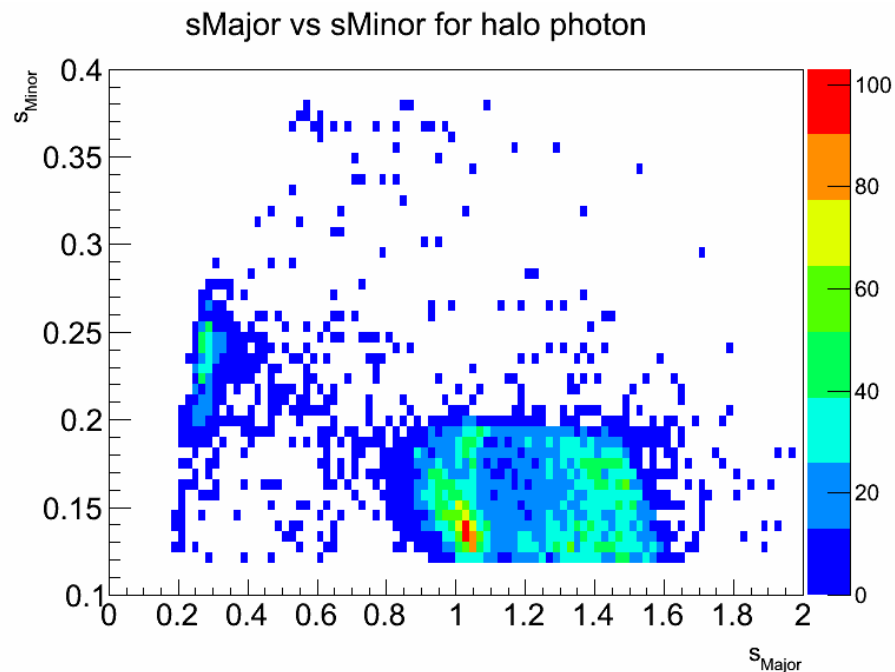




More Halo Signatures

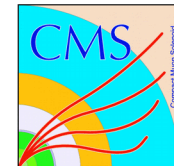


- Halo sample can be extracted using the approximate formula. Other signature can be summarized
 - Mostly around 0 and π .
 - Large s_{Major} and small s_{Minor}

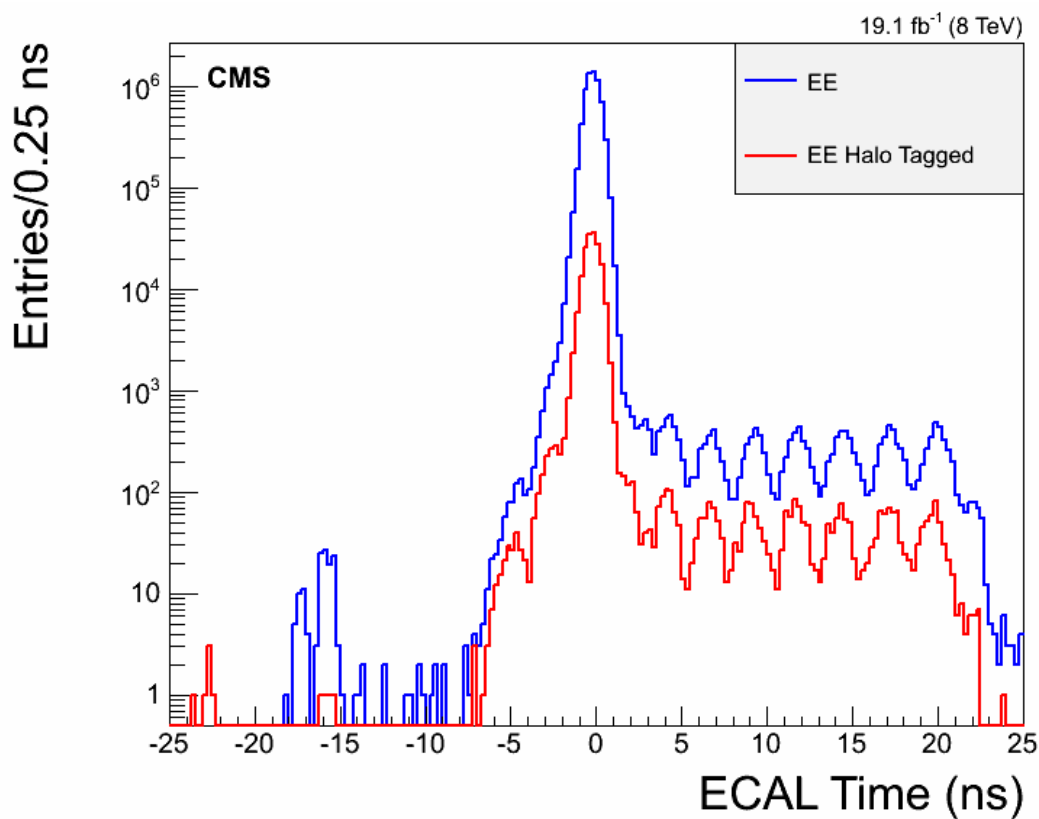




Satellite Bunches



- Satellite bunches can be found from EE time distribution.
- Halo from satellite bunches gives late photon signal.

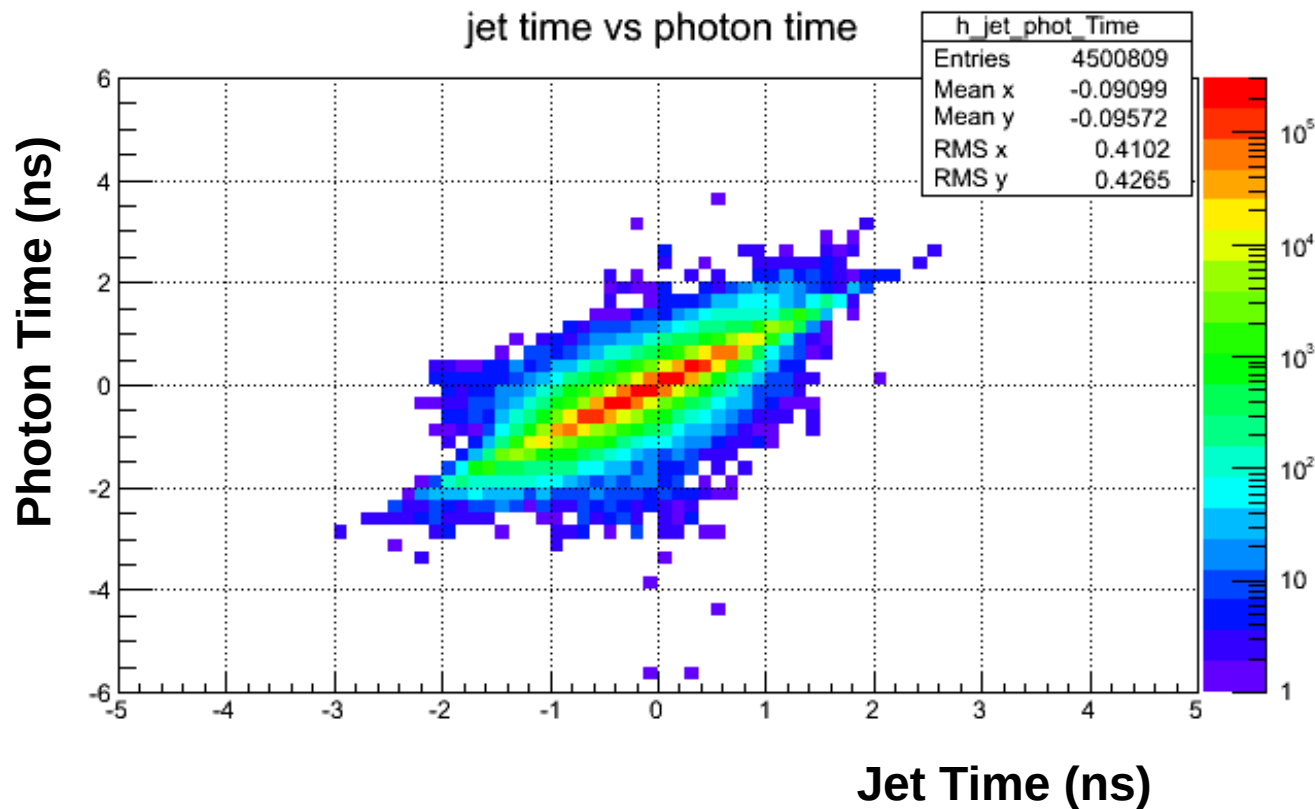




Jet and Photon Time



- Events with at least 1 photon and at least 1 jet
- Use cluster time for jets and seed time for photon
- If a jet is mis-identified as a photon, its ECAL timing is still within normal time window ($|t| < 3$ ns)

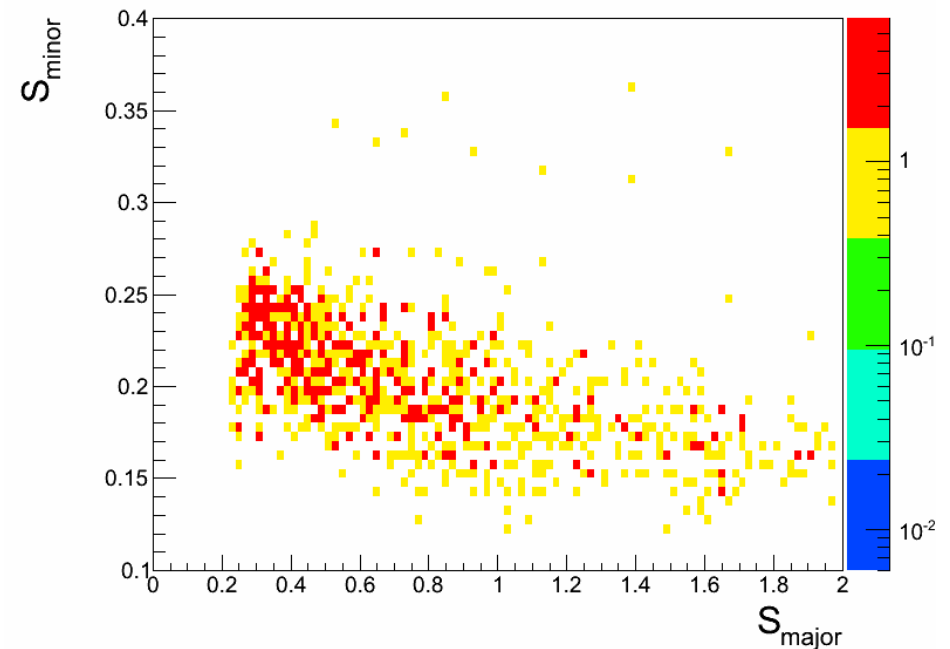
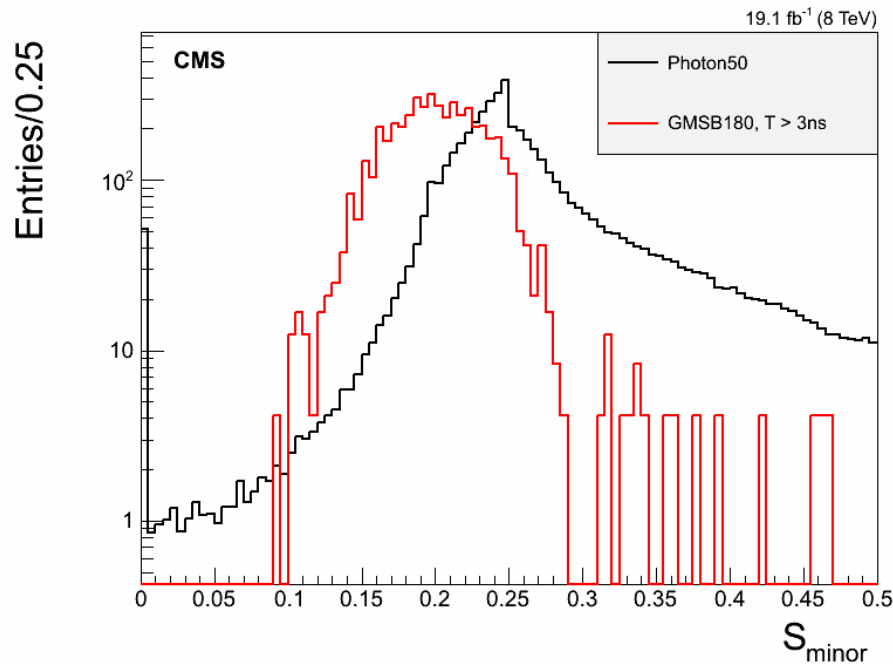


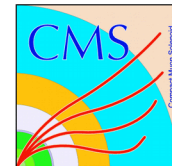


Sminor Constraint



- The variable, sMinor, can be used to enhance signal from late photons.
 - For late photon signal (for those off-pointing), the energy deposit produce elliptical cluster shape. Normal photons usually are round shape(with similar sMajor and sMinor value ~ 0.25)
 - ECAL Spikes usually have small or zero sMinor.



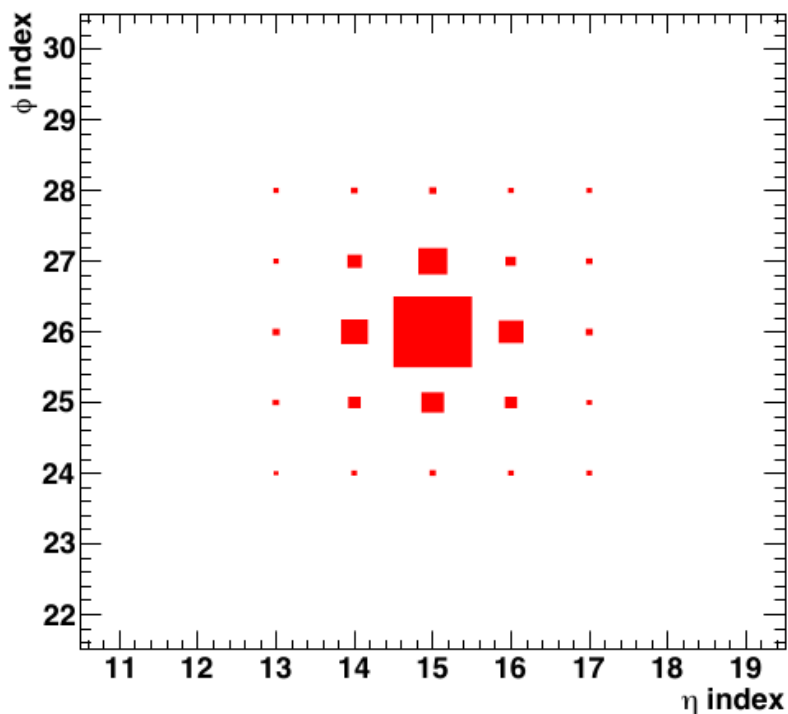


Smajor and Sminor

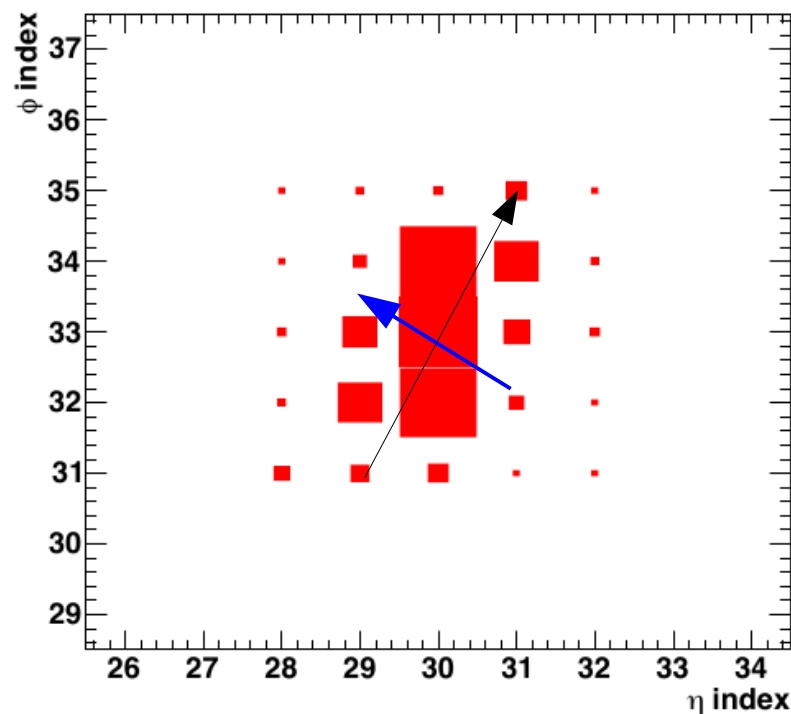
- Smajor and Sminor are the major and minor axes of the ellipse of ECAL cluster. They can be obtained by diagonalized the covariance matrix of

$$COV_{\eta\phi} = \begin{bmatrix} \sigma_{\eta\eta} & \sigma_{\eta\phi} \\ \sigma_{\phi\eta} & \sigma_{\phi\phi} \end{bmatrix} \rightarrow \begin{bmatrix} S_{major} & 0 \\ 0 & S_{minor} \end{bmatrix}$$

ECAL cluster from a pointing photon

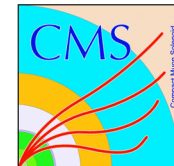


ECAL cluster from an off-pointing photon





Cosmic-Rays



- Use cosmic-rays dataset to verify cosmic-rays veto method.

- Dataset :

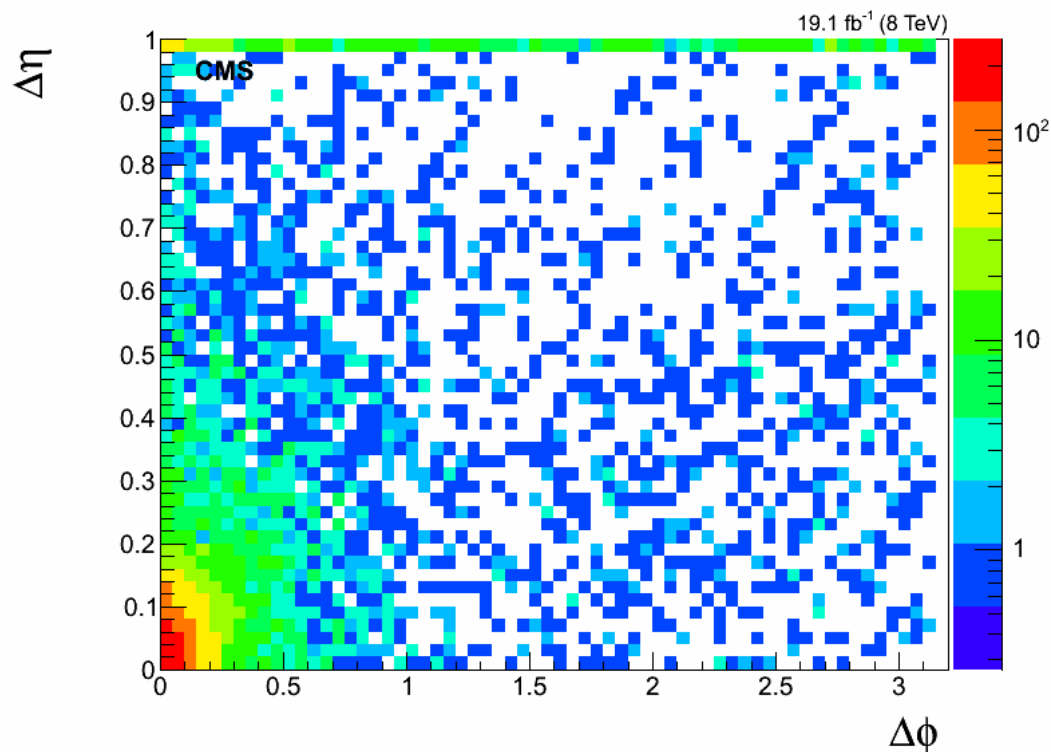
/Cosmics/Run2012C-22Jan2013-v1/RECO

/Cosmics/Run2012D-22Jan2013-v1/RECO

- No Photon object available in cosmic dataset.

→ Use supercluster (> 10 GeV) instead

→ Efficiency : 75%





ABCD – Collision Background



$MET_{\gamma} > 60 \text{ GeV}$	$MET < 60 \text{ GeV}$	$MET > 60 \text{ GeV}$
$3 \text{ ns} < t < 13 \text{ ns}$	D'	D
$-2 \text{ ns} < t < 2 \text{ ns}$	F'	F
$-10 \text{ ns} < t < -3 \text{ ns}$	B'	B

- $MET_{\gamma} > 60 \text{ GeV}$:
 - Halo/Cosmic-rays/Spikes are suppressed.
- F' and F are dominated by collision events.
- Collision events in D and B region can be estimated by
 - Collision in D : $Q_d = D' \times F/F'$, Collision in B : $Q_b = B' \times F/F'$