Next-to-Minimal SOFTSUSY

B.C. Allanach[a], P. Athron[b], Lewis C. Tunstall[b,c], A. Voigt[d], A.G. Williams[b]

[a]*DAMTP, CMS, University of Cambridge, Wilberforce road, Cambridge, CB3 0WA, United Kingdom*
[b]*ARC Centre of Excellence for Particle Physics at the Tera-scale, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005 Australia*
[c]*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland*
[d]*Institut für Kern- und Teilchenphysik, TU Dresden, Zellescher Weg 19, 01069 Dresden, Germany*

Abstract

We describe an extension to the SOFTSUSY program that provides for the calculation of the sparticle spectrum in the *Next-to-Minimal* Supersymmetric Standard Model (NMSSM), where a chiral superfield that is a singlet of the Standard Model gauge group is added to the Minimal Supersymmetric Standard Model (MSSM) fields. Often, a $\mathbb{Z}_3$ symmetry is imposed upon the model. SOFTSUSY can calculate the spectrum in this case as well as the case where general $\mathbb{Z}_3$ violating (denoted as $\mathbb{Z}_3$) terms are added to the soft supersymmetry breaking terms and the superpotential. The user provides a theoretical boundary condition for the couplings and mass terms of the singlet. Radiative electroweak symmetry breaking data along with electroweak and CKM matrix data are used as weak-scale boundary conditions. The renormalisation group equations are solved numerically between the weak scale and a high energy scale using a nested iterative algorithm. This paper serves as a manual to the NMSSM mode of the program, detailing the approximations and conventions used.

Keywords: sparticle, NMSSM, Higgs
PACS: 12.60.Jv
PACS: 14.80.Ly

1. Program Summary

Program title: SOFTSUSY
Program obtainable from: `http://softsusy.hepforge.org/`
Distribution format: tar.gz
Programming language: C++, fortran
Computer: Personal computer.
Operating system: Tested on Linux 4.x
Word size: 32 bits.
External routines: None.
Typical running time: A few seconds per parameter point.
Nature of problem: Calculating supersymmetric particle spectrum and mixing parameters in the next-to-minimal minimal supersymmetric standard model. The solution to the renormalisation group equations must be consistent with boundary conditions on supersymmetry breaking parameters, as well as on the weak-scale boundary condition on gauge couplings, Yukawa couplings and the Higgs potential parameters.
Solution method: Nested iterative algorithm and numerical minimisation of the Higgs potential.
Restrictions: SOFTSUSY will provide a solution only in the perturbative regime and it assumes that all couplings of the model are real (i.e. *CP*−conserving). If the parameter point under investigation is non-physical for some reason (for example because the electroweak potential does not have an acceptable minimum), SOFTSUSY returns an error message.

2. Introduction

While TeV-scale supersymmetric particles have not yet been found[1] at the LHC [1, 2], searches for them continue along with continuing strong theoretical interest in supersymmetric (SUSY) models. This is a testament to the theoretical successes of weak-scale supersymmetry: chiefly the resolution of the technical hierarchy problem, improvement of the apparent unification of Standard Model (SM) gauge couplings and the provision of a potential dark matter candidate. In order to pursue SUSY phenomenology, a long calculational chain is required [3]. Typically, this chain begins with the calculation of the supersymmetric spectrum, including the couplings of the various sparticles and Higgs bosons. Currently, in the Minimal Supersymmetric Standard Model (MSSM), there are several spectrum generators: ISASUSY [4], SOFTSUSY [5], SPheno [6], SUSEFLAV [7] and SUSPECT [8]. Information from these spectrum generators is then passed to other programs (for example those that calculate decays, that simulate collider events, or that calculate the thermal relic density of dark matter) via data in the SUSY Les Houches Accord format [9].

Recently a boson was discovered in the CMS and ATLAS experiments at over the $5-\sigma$ level [10, 11] with properties consistent with the SM Higgs boson. Using 4.8 fb^{-1} of 7 TeV data and 20.7 fb^{-1} of 8 TeV data, ATLAS measures the mass to be $m_h = 125.5 \pm 0.2^{+0.5}_{-0.6}$ GeV by combining the $H \rightarrow \gamma\gamma$ and $H \rightarrow ZZ$ decay channels [12]. In CMS, these channels give the combined constraint $m_h = 125.3 \pm 0.4 \pm 0.5$ GeV in 5.1 fb^{-1} of 7 TeV data and 5.3 fb^{-1} of 8 TeV data. In the MSSM, one can often obtain a CP even Higgs that couples in a similar way to the Standard Model Higgs boson. At tree-level, its mass is bounded by $m_{h^0} < M_Z$, at odds with the LHC experiments' mass measurements. However, the radiative corrections to the CP even Higgs mass can be sizeable, particularly those from stops. The corrections are larger if the stops are heavy, and if they are heavily mixed. Indeed, the MSSM has enough flexibility [13] such that the experimental values of m_{h^0} are achievable with TeV-scale stops and large mixing. On the other hand, these relatively heavy stops reintroduce the little hierarchy problem, requiring cancellation (at the level of one in several tens) between apparently unrelated parameters in the MSSM Higgs potential. Thus, we have the well known correlation [14] between a higher Higgs mass $m_{h^0} > 106$ GeV and a higher level of apparently unnatural cancellation. In several well-studied simple models of supersymmetry breaking mediation, the problem is much exacerbated [15].

In order to reduce the unnatural cancellations implied by the Higgs mass measurement [16, 17, 18, 19, 20], one can augment the MSSM by a gauge singlet chiral superfield [21, 22, 23]. This model is referred to as the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [24]. We shall distinguish between a version where an extra symmetry is assumed (often a $\mathbb{Z}_3$ symmetry) and a version where it is not ($\cancel{\mathbb{Z}}_3$) [16, 25, 26, 27]. In the MSSM, (based on a two Higgs doublet version of the SM with softly broken $N = 1$ global supersymmetry) the tree-level bound upon the Higgs mass comes from the fact that the quartic Higgs couplings are related to the electroweak gauge couplings by supersymmetry. The Higgs potential is modified by the addition of a gauge singlet, and the resulting lightest CP even Higgs boson can receive additional positive corrections to its mass at tree-level. In addition, the neutral Higgs potential (now a function of three fields rather than two in the MSSM) is heavily modified, with associated potential reductions in the unnatural cancellations. Along with other factors this had lead to considerable interest in the NMSSM in the recent literature and benchmarks points with a 125 GeV Higgs have already been proposed [28]. It is therefore essential for the research community to have access to a variety of reliable computational tools to calculate the relevant NMSSM observables.

As mentioned above in the MSSM case, the initial step in a calculational chain is typically spectrum and couplings calculation. Currently, there is one out-of-the-box package NMSPEC [29] which calculates the spectrum of the NMSSM, matching weak-scale data with theoretical boundary conditions on supersymmetry breaking and Higgs potential parameters. However, one can also marry SARAH [30, 31, 32, 33] with SPheno [6] in order to be able to calculate the spectrum after setting up the model. The NMSSM was included in an extended version of the SUSY Les Houches Accord [34] so that this information may be passed to programs performing other calculations. For instance, NMHDECAY [35] is capable of calculating the NMSSM Higgs decays, and NMSDECAY [36, 37] calculates sparticle decays. PYTHIA [38] is then capable of simulating particle collisions in the NMSSM and, in addition, micrOMEGAs [39] can calculate the thermal dark matter relic density.

Having several public spectrum generators for the MSSM has proved fruitful for the community. As well as comparisons and bug-finding, the various generators have different levels of approximations and are able to calculate

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[1]In some cases, lower bounds of 1 TeV or more have been placed upon gluinos and squarks by LHC experiments.

in different generalisations of the MSSM. For example, some are easier (or harder) to use for certain assumptions about supersymmetry breaking mediation. The advantages of having several supported, publicly available spectrum generators naturally also extends to the case of the increasingly popular NMSSM. The extension of SOFTSUSY to include the NMSSM will hopefully aid the accuracy and feasibility of a variety of NMSSM studies.

In the present paper, we focus on the recent components that have been added to SOFTSUSY in order to include the effects of the gauge singlet superfield. Up-to-date versions of this manual (along with other SOFTSUSY manuals) will be released along with the code in the doc/ subdirectory. The other manuals in this subdirectory detail the standard R−parity conserving MSSM [5], the R−parity violating MSSM [40] and the loop-level neutrino mass computation in the R−parity violating MSSM [41]. The remainder of the paper proceeds as follows: in section 3, we introduce the NMSSM supersymmetric parameters and the soft supersymmetry breaking parameters using our conventions. In section 4, we describe the algorithm employed to calculate the spectrum of masses and couplings of NMSSM particles, detailing our level of approximation for various parts of the calculation. More technical information is relegated to the appendices. In section Appendix A, we explain how to run the program. The class structure, along with the data contained within each class, is shown in section Appendix C. Finally, in section Appendix D, we reproduce the renormalisation group equations of the NMSSM to two-loops including the full 3 by 3 flavour structure.

3. NMSSM Parameters

In this section, we introduce the NMSSM parameters in the SOFTSUSY conventions. The translations to the variable names used in the program code are shown explicitly in section Appendix C.

3.1. Supersymmetric parameters

The chiral superfield particle content of the NMSSM has the following $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers

$$
\begin{aligned}
&L : (1, 2, -\tfrac{1}{2}), &&\bar{E} : (1, 1, 1), &&Q : (3, 2, \tfrac{1}{6}), &&\bar{U} : (\bar{3}, 1, -\tfrac{2}{3}), \\
&\bar{D} : (\bar{3}, 1, \tfrac{1}{3}), &&H_1 : (1, 2, -\tfrac{1}{2}), &&H_2 : (1, 2, \tfrac{1}{2}), &&S : (1, 1, 0).
\end{aligned}
\tag{1}
$$

S is the gauge singlet chiral superfield that is particular to the NMSSM. L, Q, H_1, and H_2 are the left-handed doublet lepton and quark superfields and the two Higgs doublets. $\bar{E}$, $\bar{U}$, and $\bar{D}$ are the lepton, up-type quark and down-type quark right-handed superfield singlets, respectively. Note that the lepton doublet superfields L_i^a and the Higgs doublet superfield coupling to the down-type quarks H have the same SM gauge quantum numbers. We denote an $SU(3)$ colour index of the fundamental representation by $\{x, y, z\} \in \{1, 2, 3\}$. The $SU(2)_L$ fundamental representation indices are denoted by $\{a, b, c\} \in \{1, 2\}$ and the generation indices by $\{i, j, k\} \in \{1, 2, 3\}$. $\epsilon_{xyz} = \epsilon^{xyz}$ and $\epsilon_{ab} = \epsilon^{ab}$ are totally antisymmetric tensors, with $\epsilon_{123} = 1$ and $\epsilon_{12} = 1$, respectively. Currently, only real couplings in the superpotential and Lagrangian are included.

The full renormalisable, R−parity conserving superpotential is given by

$$
W_{\mathbb{Z}_3} = \epsilon_{ab} \left[(Y_E)_{ij} L_i^b H_1^a \bar{E}_j + (Y_D)_{ij} Q_i^{bx} H_1^a \bar{D}_{jx} + (Y_U)_{ij} Q_i^{ax} H_2^b \bar{U}_{jx} + (\lambda S + \mu)(H_2^a H_1^b) \right] + \xi_F S + \frac{\mu'}{2} S^2 + \frac{\kappa}{3} S^3
\tag{2}
$$

$$
= W_{\text{MSSM}}^{\mu=0} + \epsilon_{ab} \left[(\lambda S + \mu)(H_2^a H_1^b) \right] + \xi_F S + \frac{\mu'}{2} S^2 + \frac{\kappa}{3} \kappa S^3
\tag{3}
$$

where the subscript $\mathbb{Z}_3$ reflects the fact that this superpotential contains terms which violate the $\mathbb{Z}_3$ symmetry that is commonly imposed on the NMSSM. Imposing the $\mathbb{Z}_3$ symmetry restricts the superpotential to

$$
W_{\mathbb{Z}_3} = \epsilon_{ab} \left[(Y_E)_{ij} L_i^b H_1^a \bar{E}_j + (Y_D)_{ij} Q_i^{bx} H_1^a \bar{D}_{jx} + (Y_U)_{ij} Q_i^{ax} H_2^b \bar{U}_{jx} + \lambda S (H_2^a H_1^b) \right] + \frac{\kappa}{3} S^3
\tag{4}
$$

$$
= W_{\text{MSSM}}^{\mu=0} + \epsilon_{ab} \lambda S (H_2^a H_1^b) + \frac{\kappa}{3} S^3.
\tag{5}
$$

The $\mathbb{Z}_3$-NMSSM superpotential Eq. (5) contains no explicit mass parameter, thereby allowing a solution to the μ-problem when the singlet field acquires a Vacuum Expectation Value (VEV) and generates an effective μ term of the right size. As such, it is sometimes referred to as the scale invariant NMSSM in the literature. In this paper, we will always write $\mathbb{Z}_3$-NMSSM for the $\mathbb{Z}_3$ conserving case Eq. (5) and $\mathbb{Z}_3$-NMSSM for the general $\mathbb{Z}_3$ violating one Eq. (3).

For parameters common to both the MSSM and either the $\mathbb{Z}_3$-NMSSM or $\mathbb{Z}_3$-NMSSM, a comparison of the SOFTSUSY conventions and the literature can be found in Table 1 of the MSSM SOFTSUSY manual [5]. Elsewhere, our conventions are those of the SUSY Les Houches Accord [34] and thus consistent with the review of Ellwanger, Hugonie and Teixeira (EHT) [22] and also Ref. [42]. (Note however that our definitions of the neutral Higgs VEVs (section 3.3) differ by a factor of $\sqrt{2}$ compared to Refs. [22, 42].)

3.2. Next-to-minimal SUSY breaking parameters

The soft breaking scalar potential is given by

$$V_{\text{soft}} = V_3 + V_2\big|_{m_3^2=0} + m_S^2|S|^2 + \epsilon_{ab}\lambda A_\lambda S H_2^a H_1^b + \frac{\kappa A_\kappa}{3}S^3 + V_{\mathbb{Z}_3}, \tag{6}$$

where all $\mathbb{Z}_3$ terms are included in

$$V_{\mathbb{Z}_3} = \xi_S S + \frac{m_S'^2}{2}S^2 + \epsilon_{ab}m_3^2 H_2^a H_1^b + \text{h.c.}. \tag{7}$$

Expressions for the trilinear scalar interaction potential V_3 and scalar bilinear SUSY breaking potential V_2 of the MSSM are given in Sect. 2.2 of the SOFTSUSY manual [5] for the R-parity conserving MSSM. The notation $V_2\big|_{m_3^2=0}$ indicates that the $\mathbb{Z}_3$ soft bilinear mass m_3^2 present in V_2 is set to zero to avoid double counting with the third term in Eq. (7).

3.3. Higgs potential and electroweak symmetry breaking

At tree-level, the Higgs potential is given by

$$V_{\text{Higgs}} = V_F^H + V_D^H + V_{\text{soft}}^H \tag{8}$$

$$= V_{\text{MSSM}}^{\mu=0} + V_F^{HN} + V_{\text{soft}}^{HN}, \tag{9}$$

where

$$V_F^{HN} = |\lambda S + \mu|^2(|H_2|^2 + |H_1|^2) + |\lambda H_2 H_1 + \kappa S^2 + \mu' S + \xi_S|^2, \tag{10}$$

$$V_{\text{soft}}^{HN} = m_S^2|S|^2 + \left(\lambda A_\lambda S H_2 H_1 + \frac{\kappa}{3}A_\kappa S^3 + \frac{m_S'^2}{2}S^2 + \xi_S S + \text{h.c.}\right). \tag{11}$$

The three neutral Higgs fields then pick up VEVs

$$\langle H_1^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle H_2^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad \langle S \rangle = \frac{1}{\sqrt{2}}s, \tag{12}$$

which are related to the soft masses via the minimization conditions:

$$m_{H_1}^2 = -\frac{M_Z^2}{2}\cos(2\beta) - \frac{\lambda^2}{2}v_2^2 + (m_3^2)_{\text{eff}}\tan\beta - |\mu_{\text{eff}}|^2, \tag{13}$$

$$m_{H_2}^2 = \frac{M_Z^2}{2}\cos(2\beta) - \frac{\lambda^2}{2}v_1^2 + \frac{(m_3^2)_{\text{eff}}}{\tan\beta} - |\mu_{\text{eff}}|^2, \tag{14}$$

$$m_S^2 = -\kappa^2 s^2 - \frac{\lambda^2}{2}v^2 + \kappa\lambda v_2 v_1 + \lambda A_\lambda \frac{v_2 v_1}{\sqrt{2}s} - \kappa A_\kappa s - m_S'^2 - \mu'^2 + 2\kappa\xi_F - 3\kappa s\mu'. \tag{15}$$

We have $\tan\beta = v_2/v_1$, and for simplicity we have introduced

$$(m_3^2)_{\text{eff}} \equiv \frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m_3^2}, \tag{16}$$

and

$$\mu_{\text{eff}} \equiv \mu + \frac{\lambda s}{\sqrt{2}}, \quad B_{\text{eff}} \equiv A_\lambda + \frac{\kappa s}{\sqrt{2}}, \quad \widehat{m_3^2} \equiv m_3^2 + \lambda\left(\frac{\mu' s}{\sqrt{2}} + \xi_F\right). \tag{17}$$

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3.4. Tree-level masses

The chargino and sfermion masses are obtained by substituting $\mu \rightarrow \mu_{\text{eff}}$ into the MSSM expressions. The neutralino mass matrix is contained in the Lagrangian term $-\frac{1}{2}\tilde{\psi}^{0T}\mathcal{M}_{\tilde{\psi}^0}\tilde{\psi}^0 + \text{h.c.}$, where $\tilde{\psi}^0 = (-i\tilde{b}, -i\tilde{w}^3, \tilde{h}_1, \tilde{h}_2, \tilde{s})^T$ and

$$\mathcal{M}_{\tilde{\psi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W & 0 \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W & 0 \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu & -\lambda v_2 \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 & -\lambda v_1 \\ 0 & 0 & 0 & 0 & 2\kappa s + \mu' \end{pmatrix}. \tag{18}$$

We use s and c for sine and cosine, so that $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$ and $s_W(c_W)$ is the sine (cosine) of the weak mixing angle. The 5 by 5 neutralino mixing matrix is an orthogonal matrix O with real entries, such that $O^T \mathcal{M}_{\tilde{\psi}^0} O$ is diagonal. The neutralinos χ_i^0 are defined such that their absolute masses increase with increasing i. Note that some of their mass values can be negative.

The CP-even gauge eigenstates $(H^0)^T = (H_1^0, H_2^0, S)$ are rotated into mass eigenstates $(h^0)^T = (h_1, h_2, h_3)$ by a mixing matrix R,

$$h^0 = RH^0 \,. \tag{19}$$

The mass matrix arises from the Lagrangian term $-(H^0)^T M_{H^0}^2 H^0$, where

$$M_{H^0}^2 = \begin{pmatrix} \dfrac{\partial^2 V}{\partial^2 v_1} - \dfrac{1}{v_1}\dfrac{\partial V}{\partial v_1} & \dfrac{\partial^2 V}{\partial v_1 \partial v_2} & \dfrac{\partial^2 V}{\partial v_1 \partial s} \\ \dfrac{\partial^2 V}{\partial v_2 \partial v_1} & \dfrac{\partial^2 V}{\partial^2 v_2} - \dfrac{1}{v_2}\dfrac{\partial V}{\partial v_2} & \dfrac{\partial^2 V}{\partial v_2 \partial s} \\ \dfrac{\partial^2 V}{\partial s \partial v_1} & \dfrac{\partial^2 V}{\partial s \partial v_2} & \dfrac{\partial^2 V}{\partial^2 s} - \dfrac{1}{s}\dfrac{\partial V}{\partial s} \end{pmatrix}, \tag{20}$$

and we have rewritten the elements of $M_{H^0}^2$ by imposing tree-level electroweak symmetry breaking (EWSB) substitution on the the double derivatives of the effective potential. This is equivalent to defining

$$(M_{H^0}^2)_{ij} \equiv \frac{\partial^2 V}{\partial v_i \partial v_j} - \frac{\delta_{ij}}{v_i}\frac{\partial V}{\partial v_i} \tag{21}$$

with $v_3 \equiv s$. The explicit elements of the matrix are

$$(M_{H^0}^2)_{11} = M_Z^2 c_\beta^2 + \left(\frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2\right)\tan\beta \,, \tag{22}$$

$$(M_{H^0}^2)_{12} = (4\lambda^2 - \bar{g}^2)\frac{v_2 v_1}{4} - \frac{\lambda s}{\sqrt{2}}B_{\text{eff}} - \widehat{m}_3^2 \,, \tag{23}$$

$$(M_{H^0}^2)_{13} = \lambda\left[2\mu_{\text{eff}}\frac{v_1}{\sqrt{2}} - (B_{\text{eff}} + \kappa s + \mu')\frac{v_2}{\sqrt{2}}\right], \tag{24}$$

$$(M_{H^0}^2)_{22} = M_Z^2 s_\beta^2 + \left(\frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2\right)/\tan\beta \tag{25}$$

$$(M_{H^0}^2)_{23} = \lambda\left[2\mu_{\text{eff}}\frac{v_2}{\sqrt{2}} - (B_{\text{eff}} + \frac{\kappa s}{\sqrt{2}} + \mu')\frac{v_1}{\sqrt{2}}\right], \tag{26}$$

$$(M_{H^0}^2)_{33} = \frac{\lambda}{\sqrt{2}}(A_\lambda + \mu')\frac{v_2 v_1}{s} + \frac{\kappa s}{\sqrt{2}}(A_\kappa + 4\frac{\kappa s}{\sqrt{2}} + 3\mu') - \sqrt{2}(\xi_S + \xi_F \mu')/s \,. \tag{27}$$

The three imaginary components of the neutral Higgs fields $(H^I)^T = (H_1^I, H_2^I, S^I)$ mix to give the two physical CP odd bosons $A_{1,2}$ and the Goldstone boson G^0. A mixing matrix P relates the two bases

$$a = PH^I \,, \tag{28}$$

where $a^T = (G^0, A_1, A_2)$. Here, P matches the conventions of [42], while deleting the first row from P produces the 2 by 3 mixing matrix for the physical CP-odd Higgs bosons in SLHA2 conventions [34]. Following EHT [22], the entries of the 3 by 3 mass matrix $M_P'^2$ in the H^I basis read

$$(M_P'^2)_{11} = \left(\frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2\right)\tan\beta, \tag{29}$$

$$(M_P'^2)_{12} = \frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2, \tag{30}$$

$$(M_P'^2)_{13} = \lambda v_u(A_\lambda - 2\kappa s - \mu'), \tag{31}$$

$$(M_P'^2)_{22} = \left(\frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2\right)/\tan\beta, \tag{32}$$

$$(M_P'^2)_{23} = \lambda v_d(A_\lambda - 2\kappa s - \mu') \tag{33}$$

$$(M_P'^2)_{33} = \lambda(B_{\text{eff}} + 3\kappa s + \mu')\frac{v_u v_d}{s} - 3\kappa A_\kappa s - 2m_S'^2 - \kappa\mu's - \xi_F\left(4\kappa + \frac{\mu'}{s}\right) - \frac{\xi_S}{s}. \tag{34}$$

where tree-level EWSB has been imposed.

Note that — as in the MSSM — the mixing of the Goldstone boson G^0 depends only on $\tan\beta$. As shown in EHT [22], this can be seen by first performing a rotation by β, which converts $M_P'^2$ to be block diagonal. The resulting 2 by 2 submatrix may then be diagonalised. Therefore the CP-odd mixing can be stored as a single mixing angle.[2]

Finally, the charged Higgs fields in the mass basis contain one massless charged Goldstone boson G^+ and a charged Higgs, $H^\pm$ with mass

$$m_{H^\pm}^2 = \left(\frac{\lambda s}{\sqrt{2}}B_{\text{eff}} + \widehat{m}_3^2\right)(\tan\beta + \cot\beta) + M_W^2 - \frac{\lambda^2 v^2}{2}. \tag{35}$$

4. Calculation Algorithm

We now describe the algorithm used to perform the calculation. The full iterative algorithm to determine the mass spectrum is shown schematically in Fig. 1. Here we will provide a detailed description of this procedure and specify all contributions that are included in the calculation.

As in MSSM SOFTSUSY, the SM fermion and gauge boson masses, and the couplings $\alpha(M_Z)$, G_F^μ, and $\alpha_s(M_z)$ act as low energy constraints. Below M_Z, the evolution of these input parameters proceeds in the manner described in Sect. 3.1 of the MSSM SOFTSUSY manual [5]. Similarly, the initial guess for the SUSY preserving $\overline{DR}$ parameters at m_t follows the procedure outlined in Sect. 3.2 of [5], with the additional NMSSM parameters $\{\lambda, \kappa, s, \xi_F, \mu'\}$ either initially set to their (user specified) input values, or to zero in the case when κ and s are treated as outputs from the EWSB conditions (section 4.2).

4.1. Running of NMSSM couplings

Following the initial guess at m_t, the two-loop β functions of the $\mathbb{Z}_3$-NMSSM are used to evolve the SUSY preserving parameters to a user specified scale M_X. If gauge unification has been specified as a boundary condition, M_X is revised to leading-log order to provide a more accurate value upon the next iteration:

$$M_X^{\text{new}} = M_X \exp\left(\frac{g_2(M_X) - g_1(M_X)}{g_1'(M_X) - g_2'(M_X)}\right), \tag{36}$$

where primes denote derivatives calculated to two-loop order.

In all stages of the calculation, the evolution of the NMSSM parameters is governed by three family, two-loop renormalization group equations (RGEs), whose form [43, 44] for a general, $N = 1$ semi-simple SUSY gauge theory

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[2]SOFTSUSY does this internally by storing θ_{A^0} in the sPhysical object (see Eq. (Appendix C.4)). Note that the SLHA output gives the 3 by 2 mixing matrix and thus matches SLHA2 conventions.
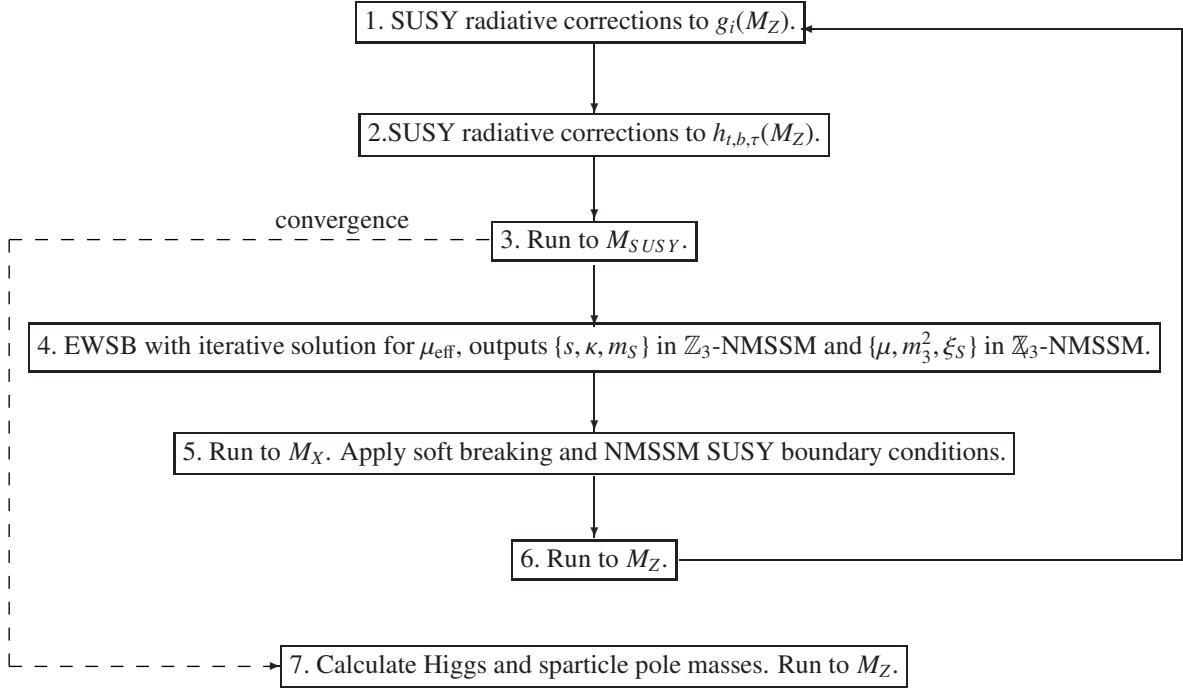
Figure 1: Iterative algorithm used to calculate the NMSSM spectrum. The initial step is the uppermost one. M_{SUSY} is the scale at which the EWSB conditions are imposed, as discussed in the text. M_X is the scale at which the high energy SUSY breaking boundary conditions are imposed.

is known. From these general results, it is possible to derive the explicit expressions of the RGEs in a chosen model (e.g. the work of Martin and Vaughn [43] provides a complete list of the RGEs for the MSSM).

In the case of the NMSSM considered here, it is a simple task to generalize the MSSM expressions [43] to include contributions due to superpotential parameters such as λ and their soft SUSY-breaking counterparts a_λ. (Naturally, the RGEs for such parameters must be derived separately.) The two-loop RGEs for the $\mathbb{Z}_3$-NMSSM are presented in the review by EHT [22], with the the third family approximation

$$Y_U \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \qquad Y_D \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad Y_E \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_e \end{pmatrix}, \tag{37}$$

imposed to simplify the resulting expressions. However, in SOFTSUSY the whole calculation is performed with quark flavor-mixing between all three families, so it is necessary to derive the additional NMSSM contributions from the general RGEs [43, 44]. The resulting expressions are collected in section Appendix D and in each case we have found agreement with the results of EHT [22] once the third family approximation Eq. (37) is enforced. Note that in the SOFTSUSY conventions, all β functions are real. We also incorporate the two-loop running for $\tan\beta$ and the Higgs VEVs $v_{1,2}$ and s. Here, we make use of the results obtained by Sperling et al. [45, 46], where the pure NMSSM contributions are reproduced in section Appendix D. The program can be made to run faster by switching off the two-loop renormalization of the scalar masses and tri-linear scalar couplings. Once the user-supplied boundary conditions are applied at M_X, the whole ensemble of NMSSM soft breaking and SUSY preserving couplings are evolved to M_Z. The inclusion of radiative corrections to the gauge and Yukawa couplings (steps 1 and 2 in Fig. 1), and NMSSM renormalization (step 3) is analogous to MSSM SOFTSUSY — for details we refer the reader to sections 3.3 and 3.4 of the SOFTSUSY manual [5].

4.2. Low energy boundary conditions and electroweak symmetry breaking

The three electroweak symmetry breaking (EWSB) conditions allow the central value of the Z pole mass M_Z to be taken as input, thereby constraining three parameters. Eqs. (13) and (14) can be rewritten in terms of μ_{eff}^2 and $(m_3^2)_{\mathrm{eff}}$,

as in the MSSM. Including tadpole corrections t_i and the transverse self energy Π_{ZZ}^T of the Z boson, the resulting expressions are

$$\mu_{\text{eff}}^2(M_{SUSY}) = \frac{m_{\overline{H_1}}^2(M_{SUSY}) - m_{\overline{H_2}}^2(M_{SUSY})\tan^2\beta(M_{SUSY})}{\tan^2\beta(M_{SUSY}) - 1} - \frac{1}{2}M_Z^2(M_{SUSY}) \tag{38}$$

$$(m_3^2)_{\text{eff}}(M_{SUSY}) = \frac{\sin 2\beta(M_{SUSY})}{2}\left\{\overline{m}_{H_u}^2(M_{SUSY}) + \overline{m}_{H_d}^2(M_{SUSY}) + 2\mu_{\text{eff}}^2(M_{SUSY})\left[1 + \frac{\overline{M}_z^2}{\overline{g}^2 s^2}(M_{SUSY})\right]\right\}. \tag{39}$$

Here, $m_{\overline{H_i}}^2 = m_{H_i}^2 - t_i/v_i$, $M_Z^2(M_{SUSY}) = M_Z^2 + \mathfrak{Re}\Pi_{ZZ}^T(M_{SUSY})$ is the $\overline{DR}$ running (mass)^2 of the Z boson, and $\overline{g} = (g_2^2 + g'^2)^{1/2}$ where g_2 and $g' = \sqrt{3/5}g_1$ are the gauge couplings of $SU(2)_L$ and (unnormalised) $U(1)$ interactions respectively. Through Eqs. (38) and (39) we can fix μ_{eff} and $(m_3^2)_{\text{eff}}$ in a similar manner to the MSSM. Note however, that in this case these are *effective* parameters constructed from several model parameters, so we must select which of the latter are fixed. In the $\mathbb{Z}_3$-NMSSM, we fix s via Eq. 38 and κ via Eq. 39, and use the third EWSB condition to fix m_S^2. In the $\mathbb{Z}_3$-NMSSM, we have more freedom and can choose to fix μ and m_3^2 — as in the MSSM — and use the third EWSB condition to fix ξ_S.

The full one-loop tadpole corrections from [42] are implemented, along with NMSSM two-loop $O(\alpha_t\alpha_s)$ and $O(\alpha_b\alpha_s)$ contributions [42] to the tadpoles.[3] The two loop corrections from the MSSM are used for $O(\alpha_t^2)$, $O(\alpha_b\alpha_\tau)$, $O(\alpha_b^2)$, $O(\alpha_\tau^2)$ and $O(\alpha_t\alpha_b)$, though it should be noted that these are not complete in the NMSSM. In both one-loop and two-loop cases, the tadpole corrections themselves depend on the output from the EWSB conditions, therefore an iteration is employed to find a self consistent solution. After the EWSB iteration converges, the whole set of NMSSM parameters are run to m_Z. As detailed in Section 3.3 of [5], the gauge couplings g_1, g_2 and g_3 (where g_1 is the GUT normalised gauge coupling of $U(1)_Y$) and third family $\overline{DR}$ Yukawa couplings, y_t, y_b and y_τ are fixed, including the precision corrections at M_Z. Note however, that the expressions for the one-loop self energies of the gauge bosons and fermions are modified to match those given in [42] for the NMSSM.

SOFTSUSY calculates corrections to $\sin\theta_W$ following the procedure outlined in [47]. We use the same procedure in the NMSSM, with self energies generalised to NMSSM and using the NMSSM the lightest Higgs mass and the coupling which will match that of the MSSM when in the MSSM limit. This ensures a simple simple MSSM limit for threshold corrections.

In the $\mathbb{Z}_3$-NMSSM, the parameters κ, s, ξ_F and μ' are reset to their input values at M_Z. The parameters are then evolved back to M_{SUSY} where M_Z^2 and $\tan\beta$ are predicted as part of a consistency check. If the user has specified that λ is to be input at the SUSY scale then λ is set here.

In general, the scalar Higgs potential (in both the $\mathbb{Z}_3$- and $\mathbb{Z}_3$-NMSSM) can possess several local minima [22], so we include a test at M_{SUSY} to determine whether the chosen parameter space point corresponds to a global minimum (as done in the NMSPEC [29] CHECKMIN routine). The test works by comparing the value of the physical potential at the VEVs v_u, v_d, s against scenarios where two or more VEVs are zero. We include one-loop radiative corrections to the effective potential from third generation quarks and squarks; corrections from other sfermions are negligible due to their small Yukawa couplings. The parameters are then evolved back up to M_X and the procedure is repeated until convergence is achieved, as shown in Fig. 1.

4.3. NMSSM spectrum

After the iteration has converged we calculate the pole masses. The Higgs pole masses are calculated using one-loop self energies from Degrassi and Slavich [42], with additional $\mathbb{Z}_3$ contributions to the triple Higgs couplings included (see Appendix A of EHT [22]). Two-loop corrections [42] of $O(\alpha_t\alpha_s)$ and $O(\alpha_b\alpha_s)$ are incorporated via FORTRAN files provided by Pietro Slavich. Contributions of order $O(\alpha_t^2)$, $O(\alpha_b\alpha_\tau)$, $O(\alpha_b^2)$, $O(\alpha_\tau^2)$ and $O(\alpha_t\alpha_b)$ are included from the MSSM FORTRAN files (also supplied by Pietro Slavich), but we note that these expressions receive additional NMSSM contributions which are currently unavailable. Consequently, our calculation is not correct to this order, but rather to $O(\alpha_t\alpha_s)$ and $O(\alpha_b\alpha_s)$. Nevertheless, the higher order MSSM contributions provide (a) a good approximation in the vicinity of the MSSM limit , and (b) easier comparisons against MSSM results.

---

[3]We thank Pietro Slavich for kindly supplying us with the FORTRAN files.

The sfermions, neutralinos and charginos also receive new NMSSM corrections to their self energies. To the best of our knowledge, the required expressions are presented only in [48]. However, we found a number of typographical errors in the published results [48], whose origin[4] was due to the need to manually condense the auto-generated LATEX output from SARAH [30, 31, 32, 33]. In particular, the self energy expressions generated by SARAH do not contain these errors. Therefore, we used a combination of results listed in [48], auto-generated LATEX output from SARAH for the self energies, plus individual checks of our own. Finally, all one-loop self energies, tadpole corrections, and two-loop RGEs were unit tested against code pieces auto-generated from FlexibleSUSY [49], an in development MATHEMATICA package for generating C++ code which makes use of the aforementioned SARAH package.

Acknowledgments

Appendix A. Running SOFTSUSY

SOFTSUSY produces an executable called softpoint.x. For the calculation of the spectrum of single points in parameter space, we recommend the SUSY Les Houches Accord 2 (SLHA2) [34] input/output option. The user must provide a file (e.g. the example file included in the SOFTSUSY distribution rpvHouchesInput), that specifies the model dependent input parameters. The program may then be run with

```
./softpoint.x leshouches < nmssmHouchesInput
```

For the SLHA2 input option, the output will also be given in SLHA2 format. Such output can be used for input into other programs which subscribe to the accord, such as PYTHIA [38] (for simulating sparticle production and decays at colliders), for example. For further details on the necessary format of the input file, see Ref. [34]. It supports the setting of all other SLHA2 input blocks associated with non-complex couplings. The default output is in SLHA2 format, the conventions of which are explained in Ref. [34].

As of SOFTSUSY3.4, the command line interface of softpoint.x has changed, see softpoint.x --help. For the MSSM the new syntax is

```
./softpoint.x <susy-breaking-model> [susy-breaking-model options] [general options]
```

where the available SUSY-breaking models are listed in Table A.1 and the general options are listed in Table A.2. For the NMSSM, the syntax is

```
./softpoint.x nmssm <susy-breaking-model> [NMSSM flags] [NMSSM parameters] [general options]
```

where sugra is the only currently available susy-breaking model. The general options are listed in Table A.2 and the NMSSM flags and parameter options are listed in Table A.3.

---

[4]F. Staub, private communication.

susy-breaking model	susy-breaking model option	description
sugra	--m0=<value>	unified soft scalar mass
	--m12=<value>	unified soft gaugino mass
	--a0=<value>	unified trilinear coupling
amsb	--m0=<value>	unified soft scalar mass
	--m32=<value>	gravitino mass
gmsb	--n5=<value>	number of $SU(5)$ mulitplets
	--mMess=<value>	messenger scale
	--LAMBDA=<value>	SUSY breaking scale $< F_S > / < S >$
	--cgrav=<value>	Gravitino mass scaling factor

Table A.1: MSSM command line options for `softpoint.x`

general option	description
--mbmb=<value>	bottom mass $m_b^{\overline{MS}}(m_b)$
--mt=<value>	top mass m_t^{pole}
--alpha_s=<value>	strong coupling $\alpha_s^{\overline{MS}}(M_Z)$
--QEWSB=<value>	EWSB scale
--alpha_inverse=<value>	inverse electromagnetic coupling $1/\alpha_{\text{em}}^{\overline{MS}}(M_Z)$
--tanBeta=<value>	$\tan \beta$
--sgnMu=<value>	sign μ
--mgut=unified	define unification scale M_X by $g_1(M_X) = g_2(M_X)$
--mgut=msusy	set unification scale to M_{SUSY}
--mgut=<value>	set unification scale to fixed value

Table A.2: General command line options for `softpoint.x`

Appendix B. Calculating decays with `NMSSMTools`

SOFTSUSY has a compatibility mode which interfaces with NMSSMTools to calculate sparticle decays in the NMSSM. To enable it, the user has to first install NMSSMTools and then run the `setup_nmssmtools.sh` script

```
$ cd /path/to/NMSSMTools/
$ wget http://www.th.u-psud.fr/NMHDECAY/NMSSMTools_4.1.2.tgz
$ tar xf NMSSMTools_4.1.2.tgz
$ cd /path/to/softsusy/
$ ./setup_nmssmtools.sh \
      --nmssmtools-dir=/path/to/NMSSMTools/NMSSMTools_4.1.2 \
      --compile
```

The `setup_nmssmtools.sh` script copies `nmProcessSpec.f` and `Makefile.nmssmtools` from the SOFTSUSY directory to the `main/` directory within the NMSSMTools folder. If the `--compile` flag is provided, NMSSMTools is recompiled. Afterwards the user can generate a NMSSM spectrum with SOFTSUSY and use NMSSMTools to calculate the decays. The `softsusy_nmssmtools.x` script combines these two steps:

```
$ ./softsusy_nmssmtools.x leshouches < slhaInput > slhaOutput
```

Here `slhaInput` is an SLHA input file with the SOFTSUSY block entry 15 set to 1. Additional NMSSMTools specific flags can also be used with entries 16 and 17, which are past to NMSSMTools as MODSEL blocks 9 and 10 respectively, following the NMSSMTools convention.

```
Block SOFTSUSY
    15    1       # NMSSMTools compatible output (default: 0)
```

NMSSM flags	description
`--lambdaAtMsusy`	input λ at scale M_{SUSY}

NMSSM parameters	description
`--m0=<value>`	unified soft scalar mass
`--m12=<value>`	unified soft gaugino mass
`--a0=<value>`	unified trilinear coupling
`--tanBeta=<value>`	$\tan\beta$
`--mHd2=<value>`	soft down-type Higgs mass squared $m_{H_1}^2$
`--mHu2=<value>`	soft up-type Higgs mass squared $m_{H_2}^2$
`--mu=<value>`	μ parameter
`--BmuOverCosBetaSinBeta=<value>`	$B\mu/(\cos\beta\sin\beta)$
`--lambda=<value>`	trilinear superpotential coupling λ
`--kappa=<value>`	trilinear superpotential coupling κ
`--Alambda=<value>`	trilinear soft coupling A_λ
`--Akappa=<value>`	trilinear soft coupling A_κ
`--lambdaS=<value>`	$\lambda\langle S \rangle = \lambda s/\sqrt{2}$
`--xiF=<value>`	linear superpotential coupling ξ_F
`--xiS=<value>`	linear soft coupling ξ_S
`--muPrime=<value>`	bilinear superpotential coupling μ'
`--mPrimeS2=<value>`	bilinear soft coupling $m_S'^2$
`--mS2=<value>`	bilinear soft mass m_S^2

Table A.3: NMSSM command line options for `softpoint.x`

```
16    4      # Select Micromegas option for NMSSMTools
             # (default: 0) 0=no, 1=relic density only
             # 2=direct detection + relic density,
             # 3=indirect detection + relic density
             # 4=all
17    1      # 1:sparticle decays via NMSDECAY (default: 0)
```

After `softsusy_nmssmtools.x` is called, the following three output files can be found in the NMSSMTools directory `NMSSMTools_4.1.2/main/`: The file `nmProcessSpec-decay` contains the sparticle decays in form of SLHA DECAY blocks, `nmProcessSpec-omega` will contain the output from `micrOMEGAS` if entry 16 is selected to be non-zero and `nmProcessSpec-spectr` contains the spectrum calculated by NMSSMTools.

Appendix C. Class Structure

We now go on to sketch the NMSSM class hierarchy. Only methods and data which are deemed of possible importance for prospective users are mentioned here, but there are many others within the program itself.

Appendix C.1. General structure

To implement the NMSSM (and other non-minimal supersymmetric models), the SOFTSUSY class hierarchy was generalized with the following requirements in mind:

- The class of supersymmetric parameters (gauge couplings, superpotential parameters and VEVs), whose beta functions are independent of soft-breaking parameters, should be at the top of the class hierarchy. This makes them usable independently of the soft-breaking parameters, for example during the initial guess.

- One should be able to reuse as much MSSM code as possible, for example by inheriting from existing MSSM classes.
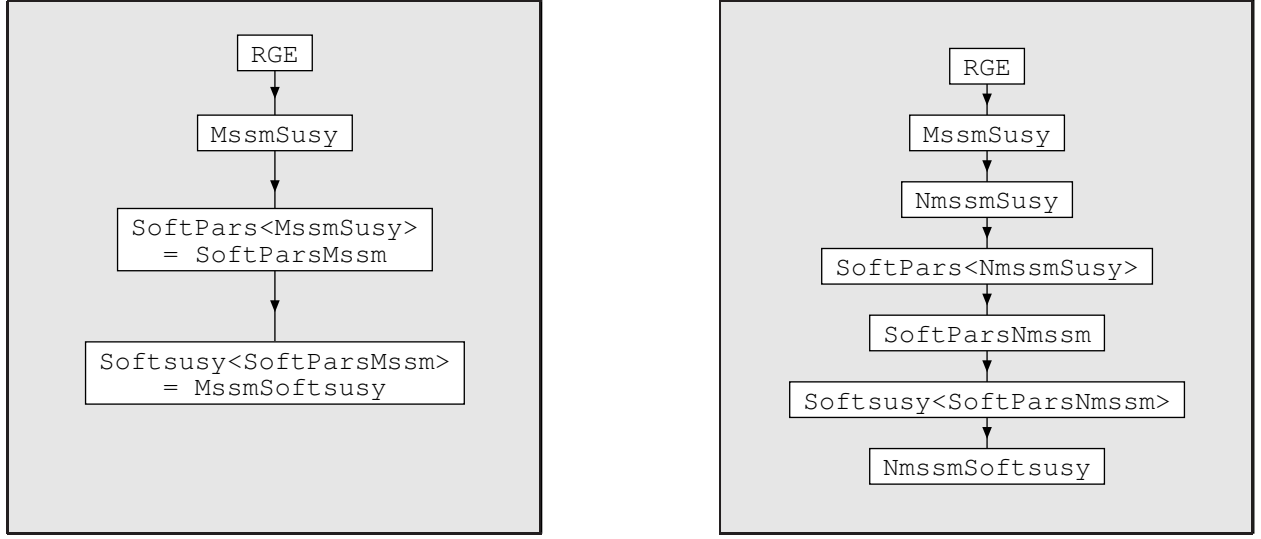
11

Figure C.2: Heuristic high-level class structure of SOFTSUSY. Inheritance is displayed by the arrows and `typedef`s are displayed by the equals signs.

The above requirements were implemented by the following changes:

1. The class of the soft breaking MSSM parameters and their beta functions was converted into the class template `SoftPars<Susy>`. The template parameter represents the class of supersymmetric parameters, from which `SoftPars<Susy>` inherits. The class which contains *all* MSSM parameters and beta functions, `SoftParsMssm`, was made a typdef for `SoftPars<MssmSusy>`, where `MssmSusy` is the class that contains the supersymmetric MSSM parameters and beta functions.

```
template <class Susy>
class SoftPars : public Susy {
   // implementation of soft breaking MSSM parameters
   // and their beta functions
};

typedef SoftPars<MssmSusy> SoftParsMssm;
```

This approach makes it possible to have a class of soft breaking MSSM parameters but with a different set of supersymmetric parameters. This mechanism is used in the NMSSM, see Section Appendix C.3.

2. The class which organises the MSSM mass spectrum calculation was converted into the class template `Softsusy<SoftPars>`. The template parameter represents the class of all model parameters and beta functions, from which `Softsusy<SoftPars>` inherits. `MssmSoftsusy` was made a `typedef` for `Softsusy<SoftParsMssm>`.

```
template <class SoftPars>
class Softsusy : public SoftPars {
   // organisation of MSSM mass spectrum calculation
   // using model parameters in SoftPars
};

typedef Softsusy<SoftParsMssm> MssmSoftsusy;
```

This approach makes it possible to have a MSSM spectrum calculation class but with an arbitrary set of model parameters. This mechanism is used in the NMSSM, see Appendix C.4.

data variable		methods
`double lambda, kappa` λ, κ	trilinear superpotential couplings	`displayLambda` `displayKappa`
`double mupr` μ'	bilinear superpotential coupling	`displayMupr`
`double xiF` ξ_F	linear superpotential coupling	`displayXiF`
`double sVEV` s	VEV of singlet field	`displaySVEV`

Table C.4: `NmssmSusy` class data and accessor methods

data variable		methods
`double alambda, akappa` a_λ, a_κ	trilinear soft parameters	`displayTrialambda` `displayTriakappa`
A_λ	a_λ/λ	`displaySoftAlambda`
A_κ	a_κ/κ	`displaySoftAkappa`
`double mSpsq` $m_S'^2$	bilinear soft parameters	`displayMspSquared`
`double mSsq` m_S^2	soft scalar mass	`displayMsSquared`
`double xiS` ξ_S	linear soft parameters	`displayXiS`

Table C.5: `SoftParsNmssm` class data and accessor methods

Appendix C.2. `NmssmSusy` *class*

The class of supersymmetric NMSSM parameters and beta functions, `NmssmSusy`, inherits from `MssmSusy` to reuse the MSSM parameters and beta functions, see Figure C.2. It adds data members and access methods for the new supersymmetric NMSSM parameters, which can be found in Table C.4.

Appendix C.3. `SoftParsNmssm` *class*

To implement the class of soft-breaking NMSSM parameters, `SoftParsNmssm`, the `SoftPars<Susy>` template is instantiated using `NmssmSusy` as template parameter. Thereby one obtains the class of MSSM soft-breaking beta functions, using supersymmetric NMSSM parameters. `SoftParsNmssm` then inherits from `SoftPars<NmssmSusy>` to add extra NMSSM contributions to the soft-breaking beta functions:

```
class NmssmSusy : public MssmSusy {
   // implement supersymmetric NMSSM parameter beta functions
   // by reusing MSSM ones
};

class SoftParsNmssm : public SoftPars<NmssmSusy> {
   // implement soft-breaking NMSSM parameter beta functions
   // by reusing MSSM ones
};
```

Furthermore, `SoftParsNmssm` adds new soft-breaking NMSSM data members and access methods, which are listed in Table C.5.

data variable	description
`DoubleVector mh0,mA0`	vectors of neutral Higgs masses $m_{h^0_{1...n}}$, $m_{A^0_{1...m}}$
	(MSSM: $n = 2, m = 1$, NMSSM: $n = 3, m = 2$)
`double mHpm`	charged Higgs mass $m_{H^\pm}$
`DoubleVector msnu`	vector of $m_{\tilde{\nu}_{i=1...3}}$ masses
`DoubleVector mch,mneut`	vectors of $m_{\chi^\pm_{i=1...2}}$, $m_{\chi^0_{i=1...n}}$ respectively
	(MSSM: $n = 4$, NMSSM: $n = 5$)
`double mGluino`	gluino mass $m_{\tilde{g}}$
`DoubleMatrix mixNeut`	orthogonal neutralino mixing matrix O
	(MSSM: 4 by 4, NMSSM: 5 by 5)
`double thetaL, thetaR`	$\theta_{L,R}$ chargino mixing angles
`double thetat, thetab`	$\theta_{t,b}$ sparticle mixing angles
`double thetatau`	θ_τ sparticle mixing angle
`double thetaH`	CP-even Higgs mixing angle α in the MSSM
`double thetaA0`	CP-odd Higgs mixing angle θ_{A^0} in the NMSSM
`DoubleMatrix mixh0`	orthogonal CP-even Higgs mixing matrix R in the NMSSM
`DoubleMatrix mu, md, me`	(2 by 3) matrices of up squark, down squark and
	charged slepton masses

Table C.6: `sPhysical` structure. Masses are pole masses, and stored in units of GeV. Mixing angles are in radian units.

Appendix C.4. `NmssmSoftsusy` *class*

To create the NMSSM spectrum calculation class, `NmssmSoftsusy`, the `Softsusy<SoftPars>` template class is instantiated using `SoftParsNmssm` as template parameter. Thereby one obtains an NMSSM spectrum calculator, which uses NMSSM parameters and beta functions. `NmssmSoftsusy` then inherits from `Softsusy<SoftParsNmssm>` and overwrites MSSM functions to account for the extra NMSSM particles:

```
class NmssmSoftsusy : public Softsusy<SoftParsNmssm> {
   // organise NMSSM spectrum calculation reusing MSSM functions
};
```

To implement the NMSSM pole masses and mixing matrices, the `sPhysical` struct had to be generalized, as in Table C.6.

Appendix D. Renormalization Group Equations for the NMSSM

In this section, we present the components of the one- and two-loop renormalization group equations (RGEs) which belong exclusively to the NMSSM. Our expressions have been derived in the $\overline{\text{DR}}$ scheme from existing results [43, 44] for general SUSY gauge theories. The complete RGEs are then obtained by combing the expressions below with those for the MSSM [43].

Appendix D.1. Yukawa Couplings

For $t = \ln Q$, the trilinear superpotential parameter Y^{ijk} evolves according to the general expression [43]

$$\frac{d}{dt} Y^{ijk} = Y^{ijp} \Gamma^k_p + Y^{kjp} \Gamma^i_p + Y^{ikp} \Gamma^j_p, \tag{D.1}$$

where

$$\Gamma^j_i = \frac{1}{16\pi^2} \gamma^{(1)j}_i + \frac{1}{(16\pi^2)^2} \gamma^{(2)j}_i, \tag{D.2}$$

and $\gamma_i^{(1,2)j}$ are the one- and two-loop anomalous dimensions respectively. Note that the 3×3 Yukawa matrices $Y_{U,D,E}$ are obtained by identifying indices in Eq. (D.1) with the relevant chiral superfields in the superpotential.[5]

At one-loop order, the only addition to the MSSM expressions [43] for the $Y_{U,D,E}$ RGEs is the inclusion of λ^2 terms which originate from the Higgs anomalous dimensions

$$\gamma_{H_1}^{(1)H_1}\Big|_{\lambda} = \lambda^2 \quad \text{and} \quad \gamma_{H_2}^{(1)H_2}\Big|_{\lambda} = \lambda^2\,. \tag{D.3}$$

At two-loop order, all the gauge-Yukawa contributions from λ cancel for each $\gamma_i^{(2)j}$, so the additional contributions arising in the NMSSM are simply given by

$$\gamma_{L_i}^{(2)L_j}\Big|_{\lambda} = -\lambda^2 (Y_E Y_E^\dagger)_i^j\,, \tag{D.4}$$

$$\gamma_{E_i}^{(2)E_j}\Big|_{\lambda} = -2\lambda^2 (Y_E^\dagger Y_E)_i^j\,, \tag{D.5}$$

$$\gamma_{Q_i}^{(2)Q_j}\Big|_{\lambda} = -\lambda^2 (Y_U Y_U^\dagger)_i^j - \lambda^2 (Y_D Y_D^\dagger)_i^j\,, \tag{D.6}$$

$$\gamma_{D_i}^{(2)D_j}\Big|_{\lambda} = -2\lambda^2 (Y_D^\dagger Y_D)_i^j\,, \tag{D.7}$$

$$\gamma_{U_i}^{(2)U_j}\Big|_{\lambda} = -2\lambda^2 (Y_U^\dagger Y_U)_i^j\,, \tag{D.8}$$

$$\gamma_{H_1}^{(2)H_1}\Big|_{\lambda} = -3\lambda^4 - 2\lambda^2\kappa^2 - 3\lambda^2 \text{Tr}(Y_U Y_U^\dagger)\,, \tag{D.9}$$

$$\gamma_{H_2}^{(2)H_2}\Big|_{\lambda} = -3\lambda^4 - 2\lambda^2\kappa^2 - 3\lambda^2 \text{Tr}(Y_D Y_D^\dagger) - \lambda^2 \text{Tr}(Y_E Y_E^\dagger)\,. \tag{D.10}$$

In a similar manner, the RGEs for λ and κ are obtained from Eq. (D.1), with

$$\frac{d}{dt}\lambda = \lambda(\Gamma_{H_1}^{H_1} + \Gamma_{H_2}^{H_2} + \Gamma_S^S)\,, \tag{D.11}$$

$$\frac{d}{dt}\kappa = 3\kappa\Gamma_S^S\,, \tag{D.12}$$

where the one- and two-loop expressions for the singlet anomalous dimension are given by

$$\gamma_S^{(1)S} = 2\lambda^2 + 2\kappa^2\,, \tag{D.13}$$

$$\gamma_S^{(2)S} = -4\lambda^4 - 8\kappa^4 - 8\kappa^2\lambda^2 - 6\lambda^2 \text{Tr}(Y_U Y_U^\dagger) - 6\lambda^2 \text{Tr}(Y_U Y_U^\dagger) - 2\lambda^2 \text{Tr}(Y_E Y_E^\dagger) + \tfrac{6}{5}g_1^2\lambda^2 + 6g_2^2\lambda^2\,. \tag{D.14}$$

Appendix D.2. Gauge Couplings

In the NMSSM, the one-loop RGEs of the for the gauge couplings g_a are identical to those for the MSSM. At two-loop order however, the λ coupling appears through the term

$$\frac{d}{dt}g_a \ni -\frac{g_a^3}{(16\pi^2)^2}Y_{ijk}Y^{ijk}C_a(k)/d(G_a)\,, \tag{D.15}$$

where $d(G_a)$ is the dimension of the adjoint representation of gauge group G_a. The result is

$$\frac{d}{dt}g_a\Big|_{\lambda} = -\frac{g_a^3}{(16\pi^2)^2}\lambda^2\Lambda_a^{(2)}\,, \qquad \Lambda_a^{(2)} = (\tfrac{6}{5}, 2, 0)\,, \tag{D.16}$$

where we have taken into account the additional factor of 2 which arises from tracing over $SU(2)$ group indices in Eq. (D.15).

---

[5]For example, for $k = H_2$ we have $Y^{ijH_2} \equiv (Y_U)^{ij}$.

Appendix D.3. Gaugino Mass Parameters

As for the gauge couplings above, we need only consider the addition of the λ^2 terms arising from

$$\frac{d}{dt}M_a \ni \frac{2g_a^2}{(16\pi^2)^2}\frac{(T_A^{ijk} - M_a Y^{ijk})Y_{ijk}C_a(k)}{d(G_a)}\,, \tag{D.17}$$

where T_A^{ijk} is a trilinear soft SUSY-breaking parameter. By evaluating the summations in Eq. (D.17), we find

$$\frac{d}{dt}M_a\bigg|_\lambda = \frac{2g_a^2}{(16\pi^2)^2}(\lambda a_\lambda - \lambda^2 M_a)\Lambda_a^{(2)}\,, \tag{D.18}$$

with $\Lambda_a^{(2)}$ as given in (D.16).

Appendix D.4. μ Parameters

The general expression [43, 44] for the SUSY-conserving bilinear terms is given by

$$\frac{d}{dt}\mu^{ij} = \mu^{ip}\Gamma_p^j + \mu^{jp}\Gamma_p^i\,, \tag{D.19}$$

from which we obtain

$$\frac{d}{dt}\mu = \mu(\Gamma_{H_1}^{H_1} + \Gamma_{H_2}^{H_2}),$$
$$\frac{d}{dt}\mu' = 2\mu'\Gamma_S^S \tag{D.20}$$

Appendix D.5. Trilinear Couplings

If we denote T_A^{ijk} as a soft SUSY-breaking trilinear, then the evolution at two-loop is given by

$$\frac{d}{dt}T_A^{ijk} = \frac{1}{16\pi^2}\left[\beta_{T_A}^{(1)}\right]^{ijk} + \frac{1}{(16\pi^2)^2}\left[\beta_{T_A}^{(2)}\right]^{ijk}\,, \tag{D.21}$$

where the explicit expressions for the β functions can be found in [43]. For $T = U, D, E$, the λ contribution to the one-loop β function arises from the following factor

$$\left[\beta_{T_A}^{(1)}\right]^{ij} \ni \frac{1}{2}(T_A)^{ij}Y_{H_\alpha mn}Y^{mnH_\alpha} + (Y_x)^{ij}Y_{H_\alpha mn}T_A^{mnH_\alpha}\,, \tag{D.22}$$

where there is *no summation* over $\alpha = 1, 2$, with the index determined by the choice of T (e.g. if $T = U$ then $\alpha = 2$). Expanding the indices leads to

$$\left[\beta_{T_A}^{(1)}\right]^{ij}\bigg|_\lambda = (T_A)^{ij}\lambda^2 + (Y_x)^{ij}2\lambda a_\lambda\,. \tag{D.23}$$

The two-loop expressions involve a large number of summations so to minimize the proliferation of generation indices we choose to express our results in terms of 3×3 matrices:

$$\beta_{U_A}^{(2)}\bigg|_\lambda = -\lambda^2 U_A[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_D Y_D^\dagger) + \text{Tr}(Y_E Y_E^\dagger)] - \lambda^2[5Y_U Y_U^\dagger U_A + 4U_A Y_U^\dagger Y_U + Y_D Y_D^\dagger U_A + 2D_A Y_D^\dagger Y_U]$$
$$- 2\lambda a_\lambda Y_U[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_D Y_D^\dagger) + \text{Tr}(Y_E Y_E^\dagger)] - 2\lambda^2 Y_U[3\lambda a_\lambda + 2\kappa a_\kappa + 3\text{Tr}(D_A Y_D^\dagger) + \text{Tr}(E_A Y_E^\dagger)]$$
$$- 2\lambda a_\lambda[3Y_U Y_U^\dagger Y_U + Y_D Y_D^\dagger Y_U]\,, \tag{D.24}$$

$$\beta_{D_A}^{(2)}\bigg|_\lambda = -\lambda^2 D_A[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_U Y_U^\dagger)] - \lambda^2[5Y_D Y_D^\dagger D_A + 4D_A Y_D^\dagger Y_D + 2U_A Y_U^\dagger Y_D + Y_U Y_U^\dagger D_A]$$
$$- 2\lambda a_\lambda Y_D[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_U Y_U^\dagger)] - 2\lambda^2 Y_D[3\lambda a_\lambda + 2\kappa a_\kappa + 3\text{Tr}(U_A Y_U^\dagger)] - 2\lambda a_\lambda[3Y_D Y_D^\dagger Y_D + Y_U Y_U^\dagger Y_D]\,, \tag{D.25}$$

$$\beta_{E_A}^{(2)}\bigg|_\lambda = -\lambda^2 E_A[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_U Y_U^\dagger)] - \lambda^2[5Y_E Y_E^\dagger E_A + 4E_A Y_E^\dagger Y_E] - 2\lambda a_\lambda Y_E[3\lambda^2 + 2\kappa^2 + 3\text{Tr}(Y_U Y_U^\dagger)$$
$$- 2\lambda^2 Y_E[3\lambda a_\lambda + 2\kappa a_\kappa + 3\text{Tr}(U_A Y_U^\dagger)] - 6\lambda a_\lambda Y_E Y_E^\dagger Y_E\,. \tag{D.26}$$

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For a_λ, the one-loop β function reads in full

$$\beta_{a_\lambda}^{(1)} = \tfrac{1}{2} a_\lambda (Y_{H_1mn} Y^{mnH_1} + Y_{H_2mn} Y^{mnH_2} + Y_{Smn} Y^{mnS}) + \lambda (Y_{H_1mn} T_A^{mnH_1} + Y_{H_2mn} T_A^{mnH_2} + Y_{Smn} T_A^{mnS})$$
$$- 4 \sum_{a=1,2,3} (a_\lambda - 2M_a \lambda) g_a^2 C_a(H), \tag{D.27}$$

from which the various sums immediately yield

$$\beta_{a_\lambda}^{(1)} = a_\lambda [3\mathrm{Tr}(Y_U Y_U^\dagger) + 3\mathrm{Tr}(Y_D Y_D^\dagger) + \mathrm{Tr}(Y_E Y_E^\dagger) + 12\lambda^2 + 2\kappa^2 - \tfrac{3}{5} g_1^2 - 3 g_2^2]$$
$$+ \lambda [6\mathrm{Tr}(U_A Y_U^\dagger) + 6\mathrm{Tr}(D_A Y_D^\dagger) + 2\mathrm{Tr}(E_A Y_E^\dagger) + 4 a_\kappa \kappa + \tfrac{6}{5} g_1^2 M_1 + 6 g_2^2 M_2]. \tag{D.28}$$

The two-loop expression is given by

$$\beta_{a_\lambda}^{(2)} = - 50\lambda^4 a_\lambda - 36\lambda \mathrm{Tr}(U_A Y_U^\dagger Y_U Y_U^\dagger) - 36\lambda \mathrm{Tr}(D_A Y_D^\dagger Y_D Y_D^\dagger) - 12\lambda \mathrm{Tr}(E_A Y_E^\dagger Y_E Y_E^\dagger) - 9 a_\lambda \mathrm{Tr}(Y_U Y_U^\dagger Y_U Y_U^\dagger)$$
$$- 9 a_\lambda \mathrm{Tr}(Y_D Y_D^\dagger Y_D Y_D^\dagger) - 3 a_\lambda \mathrm{Tr}(Y_E Y_E^\dagger Y_E Y_E^\dagger) - 8\kappa^4 a_\lambda - 32\lambda \kappa^3 a_\kappa - 12\lambda^2 \kappa^2 a_\lambda$$
$$- 18\lambda^3 [(a_\lambda/\lambda)\mathrm{Tr}(Y_U Y_U^\dagger) + \mathrm{Tr}(U_A Y_U^\dagger)] - 18\lambda^3 [(a_\lambda/\lambda)\mathrm{Tr}(Y_D Y_D^\dagger) + \mathrm{Tr}(D_A Y_D^\dagger)]$$
$$- 6\lambda^3 [(a_\lambda/\lambda)\mathrm{Tr}(Y_E Y_E^\dagger) + \mathrm{Tr}(E_A Y_E^\dagger)] - 24\lambda^3 \kappa^2 [(a_\lambda/\lambda) + (a_\kappa/\kappa)] - 12\lambda [\mathrm{Tr}(U_A Y_U^\dagger Y_D Y_D^\dagger) + \mathrm{Tr}(D_A Y_D^\dagger Y_U Y_U^\dagger)]$$
$$- 3\lambda^2 a_\lambda [3\mathrm{Tr}(Y_U Y_U^\dagger) + 3\mathrm{Tr}(Y_D Y_D^\dagger) + \mathrm{Tr}(Y_E Y_E^\dagger)] - 6 a_\lambda \mathrm{Tr}(Y_U Y_U^\dagger Y_D Y_D^\dagger) + \tfrac{12}{5} g_1^2 \lambda^2 [\tfrac{3}{2} a_\lambda - \lambda M_1]$$
$$+ \tfrac{8}{5} g_1^2 \lambda [\mathrm{Tr}(U_A Y_U^\dagger) - M_1 \mathrm{Tr}(Y_U Y_U^\dagger)] - \tfrac{4}{5} g_1^2 \lambda [\mathrm{Tr}(D_A Y_D^\dagger) - M_1 \mathrm{Tr}(Y_D Y_D^\dagger)] + \tfrac{12}{5} g_1^2 \lambda [\mathrm{Tr}(E_A Y_E^\dagger) - M_1 \mathrm{Tr}(Y_E Y_E^\dagger)]$$
$$+ \tfrac{2}{5} g_1^2 a_\lambda [2\mathrm{Tr}(Y_U Y_U^\dagger) - \mathrm{Tr}(Y_D Y_D^\dagger) + 3\mathrm{Tr}(Y_E Y_e^\dagger)] + 12 g_2^2 \lambda^2 [\tfrac{3}{2} a_\lambda - \lambda M_2]$$
$$+ 32 g_3^2 \lambda [\mathrm{Tr}(U_A Y_U^\dagger) - M_3 \mathrm{Tr}(Y_U Y_U^\dagger)] + 32 g_3^2 \lambda [\mathrm{Tr}(D_A Y_D^\dagger) - M_3 \mathrm{Tr}(Y_D Y_D^\dagger)] + 16 g_3^2 a_\lambda [\mathrm{Tr}(Y_U Y_U^\dagger) + \mathrm{Tr}(Y_D Y_D^\dagger)]$$
$$+ \tfrac{1}{50} g_1^4 \lambda [207(a_\lambda/\lambda) - 828 M_1] + \tfrac{1}{2} g_2^4 \lambda [15(a_\lambda/\lambda) - 60 M_2] + \tfrac{9}{5} g_1^2 g_2^2 \lambda [(a_\lambda/\lambda) - 2(M_1 + M_2)]. \tag{D.29}$$

For a_κ, the one-loop calculation is similar to that of a_λ, with the result

$$\beta_{a_\kappa}^{(1)} = 18 a_\kappa \kappa^2 + 12 a_\lambda \kappa \lambda + 6 a_\kappa \lambda^2. \tag{D.30}$$

At two-loop we have

$$\beta_{a_\kappa}^{(2)} = - 120\kappa^4 a_\kappa - 12\lambda^4 a_\kappa - 48\lambda^3 \kappa a_\lambda - 48\lambda^2 \kappa^3 [(a_\lambda/\lambda) + (a_\kappa/\kappa)] - 24\lambda^2 \kappa^2 a_\kappa$$
$$- 36\lambda^2 \kappa [\mathrm{Tr}(U_A Y_u^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_u Y_u^\dagger)] - 36\lambda^2 \kappa [\mathrm{Tr}(D_A Y_d^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_d Y_d^\dagger)]$$
$$- 12\lambda^2 \kappa [\mathrm{Tr}(E_A Y_e^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_e Y_e^\dagger)] - 6\lambda^2 a_\kappa [3\mathrm{Tr}(Y_u Y_u^\dagger) + 3\mathrm{Tr}(Y_d Y_d^\dagger) + \mathrm{Tr}(Y_e Y_e^\dagger)]$$
$$+ \tfrac{36}{5} g_1^2 \lambda^2 \kappa [(a_\lambda/\lambda) + \tfrac{1}{2}(a_\kappa/\kappa) - M_1] + 36 g_2^2 \lambda^2 \kappa [(a_\lambda/\lambda) + \tfrac{1}{2}(a_\kappa/\kappa) - M_2]. \tag{D.31}$$

Appendix D.6. Higgs Masses

To determine the λ and κ contributions to the Higgs masses, it is useful to define [22] the following quantities

$$M_\lambda^2 = m_{H_1}^2 + m_{H_2}^2 + m_S^2 + a_\lambda^2/\lambda^2,$$
$$M_\kappa^2 = 3 m_S^2 + a_\kappa^2/\kappa^2,$$
$$M_u^2 = \mathrm{Tr}(m_{\tilde{Q}}^2 Y_u Y_u^\dagger) + \mathrm{Tr}(Y_u m_{\tilde{u}}^2 Y_u^\dagger) + m_{H_2}^2 \mathrm{Tr}(Y_u Y_u^\dagger) + \mathrm{Tr}(U_A U_A^\dagger),$$
$$M_d^2 = \mathrm{Tr}(m_{\tilde{Q}}^2 Y_d Y_d^\dagger) + \mathrm{Tr}(Y_d m_{\tilde{d}}^2 Y_d^\dagger) + m_{H_1}^2 \mathrm{Tr}(Y_d Y_d^\dagger) + \mathrm{Tr}(D_A D_A^\dagger),$$
$$M_e^2 = \mathrm{Tr}(m_{\tilde{L}}^2 Y_e Y_e^\dagger) + \mathrm{Tr}(Y_e m_{\tilde{e}}^2 Y_e^\dagger) + m_{H_1}^2 \mathrm{Tr}(Y_e Y_e^\dagger) + \mathrm{Tr}(E_A E_A^\dagger). \tag{D.32}$$

Both the up- and down-type Higgs masses m_{H_2} and m_{H_1} receive the same λ contribution at one-loop order,

$$\beta_{m_{H_\alpha}^2}^{(1)} \Big|_\lambda = 2\lambda^2 M_\lambda^2, \qquad \alpha = 1, 2. \tag{D.33}$$

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The two-loop expressions for $m_{H_2}^2$ are

$$
\begin{aligned}
\beta_{m_{H_2}^2}^{(2)}\Big|_\lambda = & -12\lambda^4\{M_\lambda^2 + (a_\lambda/\lambda)^2\} - 6\lambda^2\{M_d^2 + M_\lambda^2\mathrm{Tr}(Y_dY_d^\dagger) + 2(a_\lambda/\lambda)\mathrm{Tr}(D_AY_d^\dagger)\} \\
& - 2\lambda^2\{M_e^2 + M_\lambda^2\mathrm{Tr}(Y_eY_e^\dagger) + 2(a_\lambda/\lambda)\mathrm{Tr}(E_AY_e^\dagger)\} - 4\lambda^2\kappa^2\{M_\lambda^2 + M_\kappa^2 + 2(a_\lambda/\lambda)(a_\kappa/\kappa)\} \\
& + \tfrac{6}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\,,
\end{aligned}
\tag{D.34}
$$

with a similar result for $m_{H_1}^2$,

$$
\begin{aligned}
\beta_{m_{H_1}^2}^{(2)}\Big|_\lambda = & -12\lambda^4\{M_\lambda^2 + (a_\lambda/\lambda)^2\} - 6\lambda^2\{M_u^2 + M_\lambda^2\mathrm{Tr}(Y_uY_u^\dagger) + 2(a_\lambda/\lambda)\mathrm{Tr}(U_AY_u^\dagger)\} \\
& - 4\lambda^2\kappa^2\{M_\lambda^2 + M_\kappa^2 + 2(a_\lambda/\lambda)(a_\kappa/\kappa)\} - \tfrac{6}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\,.
\end{aligned}
\tag{D.35}
$$

For the singlet mass m_S, the one-loop result is

$$
\beta_{m_S^2}^{(1)} = Y_{Spq}Y^{pqS}m_S^2 + 2Y_{Spq}Y^{Spr}(m^2)_r^q + h_{Spq}h^{Spq}\,,
\tag{D.36}
$$

where

$$
\begin{aligned}
Y_{Spq}Y^{Spr}(m^2)_r^q &= 2\lambda^2(m_{H_1}^2 + m_{H_2}^2) + 4\kappa^2m_S^2\,, \\
h_{Spq}h^{Spq} &= 4a_\lambda^2 + 4a_\kappa^2\,,
\end{aligned}
\tag{D.37}
$$

and thus Eq. (D.36) becomes

$$
\beta_{m_S^2}^{(1)} = 4\lambda^2M_\lambda^2 + 4\kappa^2M_\kappa^2\,.
\tag{D.38}
$$

At two-loop we get

$$
\begin{aligned}
\beta_{m_S^2}^{(2)} = & -16\lambda^4\{M_\lambda^2 + (a_\lambda/\lambda)^2\} - 32\kappa^4\{M_\kappa^2 + (a_\kappa/\kappa)^2\} - 12\lambda^2\{M_\lambda^2\mathrm{Tr}(Y_uY_u^\dagger) + M_u^2 + 2(a_\lambda/\lambda)\mathrm{Tr}(U_AY_u^\dagger)\} \\
& - 12\lambda^2\{M_\lambda^2\mathrm{Tr}(Y_dY_d^\dagger) + M_d^2 + 2(a_\lambda/\lambda)\mathrm{Tr}(D_AY_d^\dagger)\} - 4\lambda^2\{M_\lambda^2\mathrm{Tr}(Y_eY_e^\dagger) + M_e^2 + 2(a_\lambda/\lambda)\mathrm{Tr}(E_AY_e^\dagger)\} \\
& - 16\lambda^2\kappa^2\{M_\lambda^2 + M_\kappa^2 + 2(a_\lambda/\lambda)(a_\kappa/\kappa)\} + \tfrac{12}{5}g_1^2\lambda^2\{M_\lambda^2 - 2M_1[(a_\lambda/\lambda) - M_1]\} \\
& + 12g_2^2\lambda^2\{M_\lambda^2 - 2M_2[(a_\lambda/\lambda) - M_2]\}
\end{aligned}
\tag{D.39}
$$

Appendix D.7. Squark and Slepton Masses

The squark and slepton masses only receive contributions from λ, κ at two-loop order. The results are listed below, where $\mathbf{1}$ is a 3×3 unit matrix.

$$
\begin{aligned}
\beta_{m_{\tilde{Q}}^2}^{(2)}\Big|_\lambda = & -\lambda^2\{2Y_u^\dagger m_{\tilde{u}}^2 Y_u + m_{\tilde{Q}}^2 Y_uY_u^\dagger + Y_uY_u^\dagger m_{\tilde{Q}}^2 + 2m_{H_2}^2 Y_uY_u^\dagger + 2U_AU_A^\dagger + 2M_\lambda^2 Y_uY_u^\dagger + 2a_\lambda/\lambda(Y_uU_A^\dagger + U_AY_u^\dagger)\} \\
& - \lambda^2\{2Y_d^\dagger m_{\tilde{d}}^2 Y_d + m_{\tilde{Q}}^2 Y_dY_d^\dagger + Y_dY_d^\dagger m_{\tilde{Q}}^2 + 2m_{H_1}^2 Y_dY_d^\dagger + 2D_AD_A^\dagger + 2M_\lambda^2 Y_dY_d^\dagger + 2a_\lambda/\lambda(Y_dD_A^\dagger + D_AY_d^\dagger)\} \\
& + \tfrac{2}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\mathbf{1}\,,
\end{aligned}
\tag{D.40}
$$

$$
\begin{aligned}
\beta_{m_{\tilde{u}}^2}^{(2)}\Big|_\lambda = & -2\lambda^2\{2Y_u^\dagger m_{\tilde{Q}}^2 Y_u + m_{\tilde{u}}^2 Y_u^\dagger Y_u + Y_u^\dagger Y_u m_{\tilde{u}}^2 + 2m_{H_2}^2 Y_u^\dagger Y_u + 2U_A^\dagger U_A + 2M_\lambda^2 Y_u^\dagger Y_u + 2a_\lambda/\lambda(Y_u^\dagger U_A + U_A^\dagger Y_u)\} \\
& - \tfrac{8}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\mathbf{1}\,,
\end{aligned}
\tag{D.41}
$$

$$
\begin{aligned}
\beta_{m_{\tilde{d}}^2}^{(2)}\Big|_\lambda = & -2\lambda^2\{2Y_d^\dagger m_{\tilde{Q}}^2 Y_d + m_{\tilde{d}}^2 Y_d^\dagger Y_d + Y_d^\dagger Y_d m_{\tilde{d}}^2 + 2m_{H_1}^2 Y_d^\dagger Y_d + 2D_A^\dagger D_A + 2M_\lambda^2 Y_d^\dagger Y_d + 2a_\lambda/\lambda(Y_d^\dagger D_A + D_A^\dagger Y_d)\} \\
& + \tfrac{4}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\mathbf{1}\,,
\end{aligned}
\tag{D.42}
$$

$$\beta_{m_{\tilde{L}}^2}^{(2)}\Big|_\lambda = -\lambda^2\{2Y_e^\dagger m_{\tilde{e}}^2 Y_e + m_{\tilde{L}}^2 Y_e Y_e^\dagger + Y_e Y_e^\dagger m_{\tilde{L}}^2 + 2m_{H_1}^2 Y_e Y_e^\dagger + 2E_A E_A^\dagger + 2M_\lambda^2 Y_e Y_e^\dagger + 2a_\lambda/\lambda(Y_e E_A^\dagger + E_A Y_e^\dagger)\}$$
$$- \tfrac{6}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\mathbf{1}\,, \tag{D.43}$$

$$\beta_{m_{\tilde{e}}^2}^{(2)}\Big|_\lambda = -2\lambda^2\{2Y_e^\dagger m_{\tilde{L}}^2 Y_e + m_{\tilde{e}}^2 Y_e^\dagger Y_e + Y_e^\dagger Y_e m_{\tilde{e}}^2 + 2m_{H_1}^2 Y_e^\dagger Y_e + 2E_A^\dagger E_A + 2M_\lambda^2 Y_e^\dagger Y_e + 2a_\lambda/\lambda(Y_e^\dagger E_A + E_A^\dagger Y_e)\}$$
$$+ \tfrac{12}{5}g_1^2\lambda^2(m_{H_1}^2 - m_{H_2}^2)\mathbf{1}\,. \tag{D.44}$$

Appendix D.8. Tadpole Terms

The general RGE for a SUSY-conserving tadpole term reads

$$\frac{d}{dt}L^i = L^p\Gamma_p^i\,, \tag{D.45}$$

and thus for $i = S$ one has

$$\frac{d}{dt}\xi_F = \xi_F\Gamma_S^S\,. \tag{D.46}$$

For the soft SUSY-breaking term ξ_S, we use the general RGE from [44] because Martin and Vaughn [43] do not include the tadpole as part of $\mathcal{L}_{\text{soft}}$. The relevant RGE reads

$$\frac{d}{dt}\xi_S = \frac{1}{16\pi^2}\beta_{\xi_S}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{\xi_S}^{(2)}\,, \tag{D.47}$$

where the one-loop β function is given by

$$\beta_{\xi_S}^{(1)} = 2(\lambda^2 + \kappa^2)\xi_S + 4(\lambda a_\lambda + \kappa a_\kappa)\xi_F + 2\mu'(2\lambda m_3^2 + \kappa m_S'^2)$$
$$+ 4[\lambda\mu(m_{H_2}^2 + m_{H_1}^2) + \kappa\mu' m_S^2]\mu_{jl} + 4a_\lambda m_3^2 + 2a_\kappa m_S'^2\,. \tag{D.48}$$

At two-loop we obtain

$$\beta_{\xi_S}^{(2)} = -4\lambda^4\{\xi_S + 4(a_\lambda/\lambda)\xi_F\} - 8\kappa^4\{\xi_S + 4(a_\kappa/\kappa)\xi_F\} - 6\lambda^2\{\xi_S\text{Tr}(Y_u Y_u^\dagger) + 2[(a_\lambda/\lambda)\text{Tr}(Y_u Y_u^\dagger) + \text{Tr}(U_A Y_u^\dagger)]\}\xi_F$$
$$- 6\lambda^2\{\xi_S\text{Tr}(Y_d Y_d^\dagger) + 2[(a_\lambda/\lambda)\text{Tr}(Y_d Y_d^\dagger) + \text{Tr}(D_A Y_d^\dagger)]\}\xi_F$$
$$- 2\lambda^2\{\xi_S\text{Tr}(Y_e Y_e^\dagger) + 2[(a_\lambda/\lambda)\text{Tr}(Y_e Y_e^\dagger) + \text{Tr}(E_A Y_e^\dagger)]\}\xi_F$$
$$- 8\lambda^2\kappa^2\{\xi_S + 2[(a_\lambda/\lambda) + (a_\kappa/\kappa)]\xi_F\}$$
$$- 12\lambda\{m_3^2[(a_\lambda/\lambda) + \mu']\text{Tr}(Y_u Y_u^\dagger) + m_3^2\text{Tr}(U_A Y_u^\dagger) + \mu\{M_u^2 + [(a_\lambda/\lambda) + \mu']\text{Tr}(U_A Y_u^\dagger) + [m_{H_1}^2 + m_{H_2}^2]\text{Tr}(Y_u Y_u^\dagger)\}\}$$
$$- 12\lambda\{m_3^2[(a_\lambda/\lambda) + \mu']\text{Tr}(Y_d Y_d^\dagger) + m_3^2\text{Tr}(D_A Y_d^\dagger) + \mu\{M_d^2 + [(a_\lambda/\lambda) + \mu']\text{Tr}(D_A Y_d^\dagger) + [m_{H_1}^2 + m_{H_2}^2]\text{Tr}(Y_d Y_d^\dagger)\}\}$$
$$- 4\lambda\{m_3^2[(a_\lambda/\lambda) + \mu']\text{Tr}(Y_e Y_e^\dagger) + m_3^2\text{Tr}(E_A Y_e^\dagger) + \mu\{M_e^2 + [(a_\lambda/\lambda) + \mu']\text{Tr}(E_A Y_e^\dagger) + [m_{H_1}^2 + m_{H_2}^2]\text{Tr}(Y_e Y_e^\dagger)\}\}$$
$$- 8\lambda^3\{m_3^2[2(a_\lambda/\lambda) + \mu'] + \mu[M_\lambda^2 + (a_\lambda/\lambda)[(a_\lambda/\lambda) + \mu'] + m_{H_1}^2 + m_{H_2}^2]\}$$
$$- 8\lambda^2\kappa\{m_S'^2[(a_\lambda/\lambda) + (a_\kappa/\kappa) + \mu'] + \mu'[M_\lambda^2 + (a_\lambda/\lambda)[(a_\kappa/\kappa) + \mu'] + 2m_S^2]\}$$
$$- 8\kappa^3\{m_S'^2[2(a_\kappa/\kappa) + \mu'] + \mu'[M_\kappa^2 + (a_\kappa/\kappa)[(a_\kappa/\kappa) + \mu'] + 2m_S^2]\}$$
$$+ \tfrac{6}{5}\lambda g_1^2\{3m_3^2[(a_\lambda/\lambda) + \mu' - M_1] + 2\mu[m_{H_1}^2 + m_{H_2}^2 - (a_\lambda/\lambda)M_1 - \mu' M_1 + 2M_1^2] + \lambda[2\xi_F[(a_\lambda/\lambda) - M_1] + \xi_S]\}$$
$$+ 3\lambda g_2^2\{3m_3^2[(a_\lambda/\lambda) + \mu' - M_2] + 2\mu[m_{H_1}^2 + m_{H_2}^2 - (a_\lambda/\lambda)M_2 - \mu' M_2 + 2M_2^2] + \lambda[2\xi_F[(a_\lambda/\lambda) - M_2] + \xi_S]\}\,. \tag{D.49}$$

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Appendix D.9. Additional Parameters

Here we list the λ and κ contributions to the RGEs for the scalar masses $m_3^2 \equiv B\mu$ and $m_S'^2 \equiv B'\mu'$, and the evolution of the Higgs VEVs $v_{1,2,s}$. For the former, we get at one-loop

$$\beta_{m_3^2}^{(1)}\Big|_\lambda = 2\lambda(3\lambda m_3^2 + 2\mu a_\lambda) + 2\lambda\kappa m_S'^2 \,. \tag{D.50}$$

At two-loop we have

$$
\begin{aligned}
\beta_{m_3^2}^{(2)}\Big|_\lambda = &-2\lambda^4(7m_3^2 + 16\mu a_\lambda/\lambda) - 3\lambda^2\{5m_3^2\mathrm{Tr}(Y_u Y_u^\dagger) + 2\mu[3\mathrm{Tr}(U_A Y_u^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_u Y_u^\dagger)]\} \\
&- 3\lambda^2\{5m_3^2\mathrm{Tr}(Y_d Y_d^\dagger) + 2\mu[3\mathrm{Tr}(D_A Y_d^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_d Y_d^\dagger)\} \\
&- \lambda^2\{5m_3^2\mathrm{Tr}(Y_e Y_e^\dagger) + 2\mu[3\mathrm{Tr}(E_A Y_e^\dagger) + (a_\lambda/\lambda)\mathrm{Tr}(Y_e Y_e^\dagger)]\} \\
&- 4\lambda^2\kappa^2\{m_3^2 + 2\mu[(a_\lambda/\lambda) + (a_\kappa/\kappa)]\} - 8\lambda^3\kappa\{m_S'^2 + \mu'(a_\lambda/\lambda)\} - 8\lambda\kappa^3\{m_S'^2 + \mu'(a_\kappa/\kappa)\} \\
&+ \tfrac{12}{5}g_1^2\lambda^2(m_3^2 - \mu M_1) + 12g_2^2\lambda^2(m_3^2 - \mu M_2) \,.
\end{aligned}
\tag{D.51}
$$

For $m_S'^2$, the one-loop β function reads

$$\beta_{m_S'^2}^{(1)} = 4\lambda(\lambda m_S'^2 + 2\mu' a_\lambda) + 8\kappa(\kappa m_S'^2 + \mu' a_\kappa) + 8\lambda\kappa m_3^2 \,. \tag{D.52}$$

At two-loop we have

$$
\begin{aligned}
\beta_{m_S'^2}^{(2)} = &-8\lambda^4\{m_S'^2 + 4\mu'(a_\lambda/\lambda)\} - 16\kappa^4\{2m_S'^2 + 5\mu'(a_\kappa/\kappa)\} - 16\lambda^2\kappa^2\{2m_S'^2 + \mu'[3(a_\lambda/\lambda) + 2(a_\kappa/\kappa)]\} \\
&- 12\lambda^2\{m_S'^2\mathrm{Tr}(Y_u Y_u^\dagger) + 2\mu'[(a_\lambda/\lambda)\mathrm{Tr}(Y_u Y_u^\dagger) + \mathrm{Tr}(U_A Y_u^\dagger)]\} \\
&- 12\lambda^2\{m_S'^2\mathrm{Tr}(Y_d Y_d^\dagger) + 2\mu'[(a_\lambda/\lambda)\mathrm{Tr}(Y_d Y_d^\dagger) + \mathrm{Tr}(D_A Y_d^\dagger)]\} \\
&- 4\lambda^2\{m_S'^2\mathrm{Tr}(Y_e Y_e^\dagger) + 2\mu'[(a_\lambda/\lambda)\mathrm{Tr}(Y_e Y_e^\dagger) + \mathrm{Tr}(E_A Y_e^\dagger)]\} - 16\lambda^3\kappa\{m_3^2 + \mu(a_\lambda/\lambda)\} \\
&- 24\lambda\kappa\{m_3^2\mathrm{Tr}(Y_u Y_u^\dagger) + \mu\mathrm{Tr}(U_A Y_u^\dagger)\} - 24\lambda\kappa\{m_3^2\mathrm{Tr}(Y_d Y_d^\dagger) + \mu\mathrm{Tr}(D_A Y_d^\dagger)\} \\
&- 8\lambda\kappa\{m_3^2\mathrm{Tr}(Y_e Y_e^\dagger) + \mu\mathrm{Tr}(E_A Y_e^\dagger)\} + \tfrac{24}{5}\lambda\kappa g_1^2\{m_3^2 - \mu M_1\} + 24\lambda\kappa g_2^2\{m_3^2 - \mu M_2\} \\
&+ \tfrac{12}{5}\lambda^2 g_1^2\{m_S'^2 + 2\mu'[(a_\lambda/\lambda) - M_1]\} + 12\lambda^2 g_2^2\{m_S'^2 + 2\mu'[(a_\lambda/\lambda) - M_2]\} \,.
\end{aligned}
\tag{D.53}
$$

At one-loop, the up- and down-type Higgs VEVs $v_{u,d}$ receive additional contributions solely from λ [45],

$$\beta_{v_\alpha}^{(1)}\Big|_\lambda = -v_\alpha\lambda^2 \,, \qquad \alpha = 1, 2 \,, \tag{D.54}$$

while the β function for the singlet VEV s is given by

$$\beta_s^{(1)} = -2s(\lambda^2 + \kappa^2) \,. \tag{D.55}$$

At two-loop, the β functions are given by [45, 46]

$$\beta_{v_1}^{(2)} = v_1\left\{\gamma_{H_1}^{(2)H_1} - \left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right)\left[3\mathrm{Tr}(Y_D Y_D^\dagger) + \mathrm{Tr}(Y_E Y_E^\dagger) + \lambda^2\right] + \tfrac{9}{2}g_2^4\right\}, \tag{D.56}$$

$$\beta_{v_2}^{(2)} = v_2\left\{\gamma_{H_2}^{(2)H_2} - \left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right)\left[3\mathrm{Tr}(Y_U Y_U^\dagger) + \lambda^2\right] + \tfrac{9}{2}g_2^4\right\}, \tag{D.57}$$

$$\beta_s^{(2)} = s\gamma_S^{(2)S} \,. \tag{D.58}$$

The one-loop β function for $\tan\beta$ is the same in the NMSSM as the MSSM. At two-loop, one has

$$\beta_{t_\beta}^{(2)} = \tan\beta\left\{\gamma_{H_2}^{(2)H_2} - \gamma_{H_1}^{(2)H_1} + \left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right)\frac{\beta_{t_\beta}^{(1)}}{\tan\beta}\right\}. \tag{D.59}$$

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