Pêches et Océans Canada | Fisheries and Oceans Canada

Limit Reference Points and the Fish Stocks Provisions

Background Material for the *Joint TESA/NOG Workshop*

November 2021

Objectives

- The objectives of this presentation are to:
  - Provide a brief introduction to reference points for age structured models
  - Provide some background information to support the pre-workshop exercises

- The presentation assumes limited knowledge of fisheries and is intended as an optional introduction

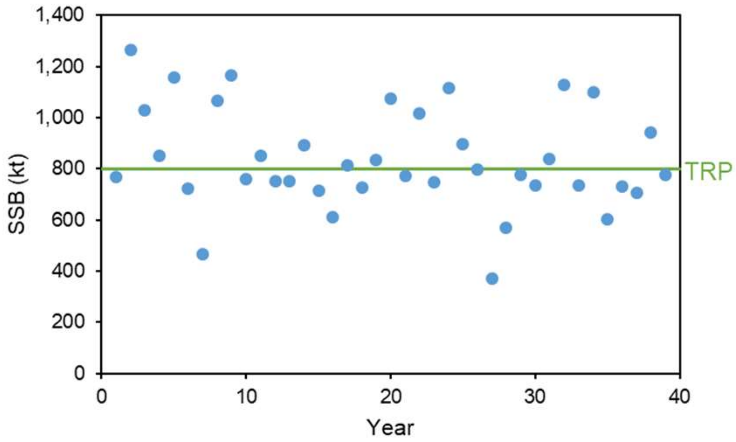
Sections

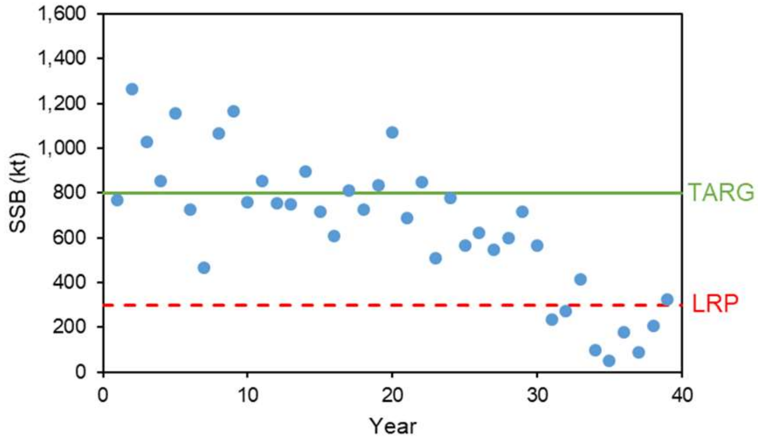
- Introduction:
  - Reference Points (slide 4)
  - LRPs (slide 12)
- Equilibrium Reference Points: Calculations (side 17)
- Per Recruit Reference Points: Calculations (slide 21)
- MSY Reference Points: Calculations (slide 53)
- Pre-Workshop Exercise (slide 71)

Introduction – Reference Points

Reference Points

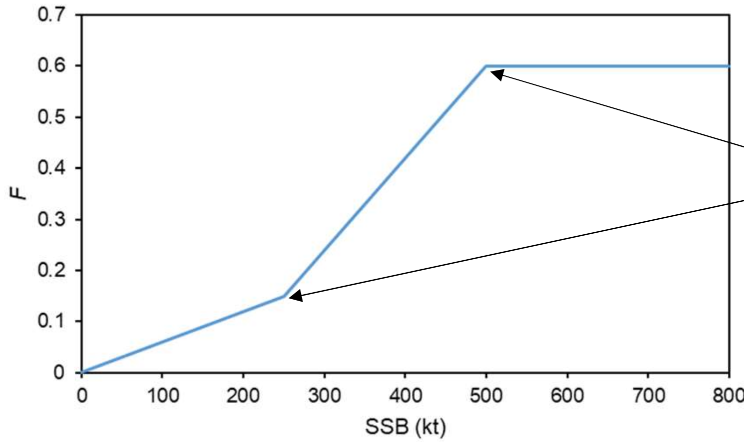
- Target Reference Point (TRP) – desirable state to have the stock approach and fluctuate around
- Limit Reference Point (LRP) – threshold to an undesirable state
- Reference points are generally based on spawning stock biomass (SSB) or the instantaneous fishing mortality rate (*F*)





Reference Points

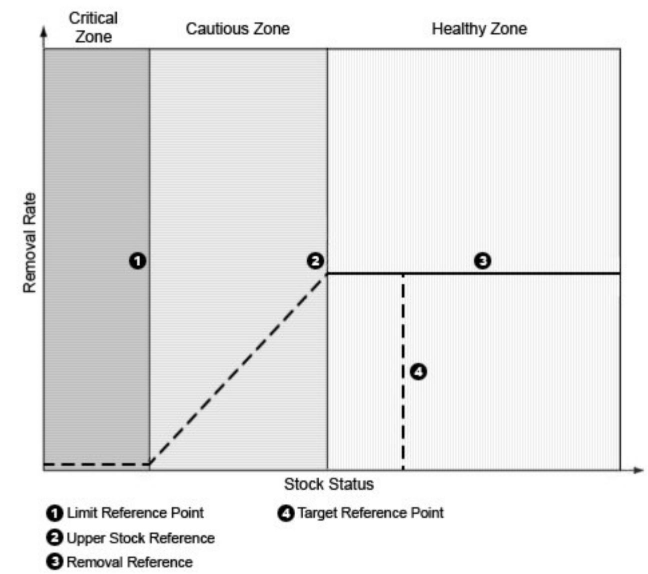
- Used to determine stock status by comparing estimated SSB or *F* to Reference Points
- Used to define performance metrics used to evaluate how well harvest strategies meet (measurable) management objectives
- Can be used as operational control points (OCPs) in harvest control rules (HCRs)



OCPs = points where pre-specified management actions are to be taken (e.g., change in F in a HCR)

DFO – Precautionary Approach (PA) Policy

- A general fishery decision-making framework (DFO 2009) for implementing harvest strategies that incorporate the precautionary approach
- PA Policy applies to key stocks managed by DFO
- Primary components:
    1. reference points and stock status zones;
    2. harvest strategy and harvest decision rules;
    3. the need to take into account uncertainty and risk when developing reference points, and developing and implementing decision rules.



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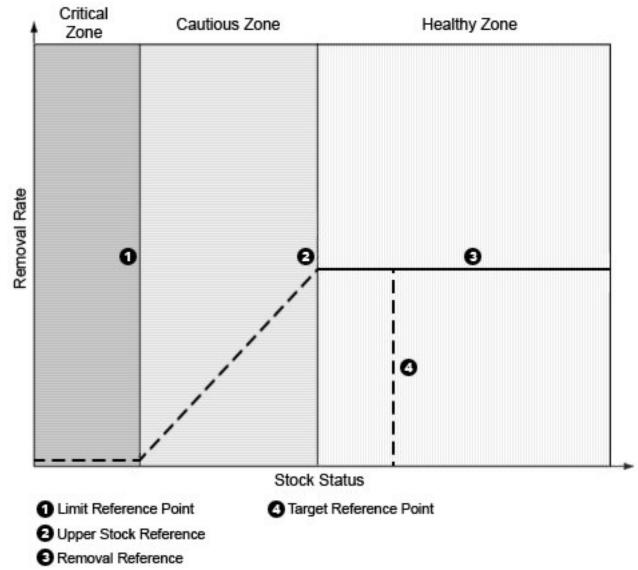


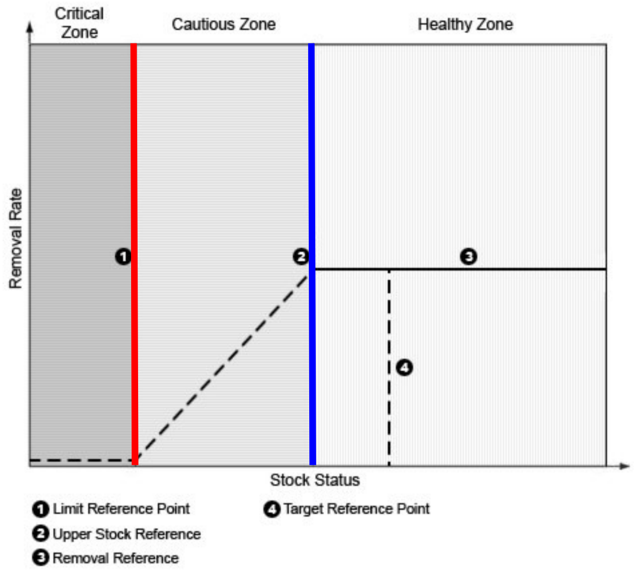
DFO – PA Policy: Reference Points/Status

1. Reference points and stock status zones

- LRP: *"represents the stock level below which there is a high probability that the stock's productivity is so impaired that serious harm will occur"*
- *"At this stock status level, there may also be resultant impacts to the ecosystem, associated species and a long-term loss of fishing opportunities"*
- The LRP is defined by DFO Science through a CSAS peer-review process

DFO – PA Policy: USR

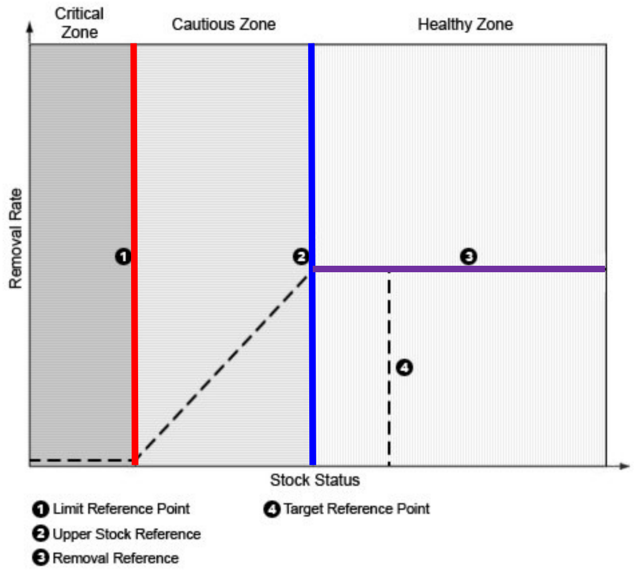
- USR – *"a level below which removals must be progressively reduced so the LRP can be avoided"* OR *"a target reference point"*
- *"USR is defined by resource management, in consultation with stakeholders and with advice and input from DFO Science"*



DFO – PA Policy: Removal Reference

- Removal Reference – limit on the fishing mortality rate (F)

- Focus of the workshop is on LRPs (Role of Science)



Introduction – LRPs

LRP - Definition

"The stock level below which productivity is sufficiently impaired to cause serious harm"

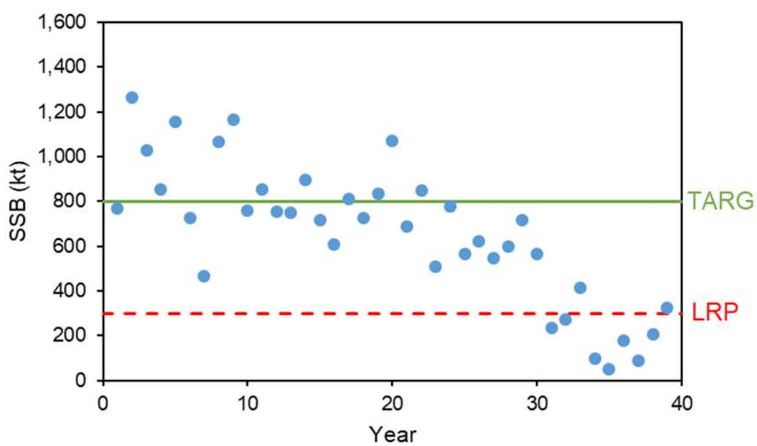
"At this stock status level, there may also be resultant impacts to the ecosystem, associated species and a long-term loss of fishing opportunities"

- Challenge to define a point of "serious harm" before you have reached it
- Often understood as avoiding recruitment overfishing - when adults are removed to such a low level that they cannot reproduce fast enough to replenish the stock

Example: see Shelton and Rice (2002)

LRP - Approaches

- It is egg production (often measured as SSB) that must be maintained to ensure future productivity, so LRPs are frequently based on SSB



Several methods of defining LRPs are based on *F*

LRPs based on F can be interpreted in terms of the equilibrium SSB obtained from fishing at *F* over the long-term

What is Equilibrium?

- A theoretical model state that arises when the **fishing mortality** (F), **exploitation pattern** (selectivity) and other fishery or stock characteristics (**growth, natural mortality** (M)**, recruitment**) do not change from year to year
- Simplifying assumptions:
  - Births = Deaths
  - A fish population harvested at a constant *rate* for many years will achieve equilibrium ("steady state")
- A tool to help understand effects of fishing on fish populations

- Reality:
  - Annual variability "noise" (e.g., variable recruitment) – this is accounted for in annual stock assessments
  - Effects of time-varying processes on assessments and reference points are challenging and not well-understood

Provisional Default LRP in the PA Policy

- The provisional default reference points in the PA are based on (spawning) biomass at Maximum Sustainable Yield (MSY)
- LRP provisional default is 40% B_{MSY}
- Many proxies for B_{MSY} exist
- Some background information is provided in the following slides to support the calculations that are conducted in the pre-workshop exercise

Equilibrium Reference Points: Calculations

Examples of Limit Reference Points

SOME EXAMPLES OF LIMIT REFERENCE POINTS[1]

Reference Point	Description	Some Pros	Some Cons
X% B_{MSY} X% SSB_{MSY}	A percentage (X) of the biomass (B) or spawning stock biomass (SSB) that is obtained on average from fishing at rates associated with maximum sustainable yield (MSY)	Takes into account various components of productivity; Many proxies have developed over the years; DFO's PA Policy gives a provisional default LRP of 40% B_{MSY} (other jurisdictions have policy defaults ranging from 30-50% B_{MSY})	May be difficult to estimate; Sensitive to uncertainty in model assumptions concerning recruitment (e.g. steepness), natural mortality rate, selectivity, etc; Relationship between B_{MSY} and B_0 depends on model assumptions
X% B_0 (or K)	A percentage (X) of the biomass (B) or spawning stock biomass (SSB) under conditions of no fishing; sometimes 'carrying capacity' (K) is used	May be more estimable than B_{MSY} for data-poorer stocks; Provisional policy defaults available in other countries (e.g., 20% B_0); 40-50% B_0 are common generic proxies for B_{MSY} (although 30-60% can be used)	May be difficult to estimate; Sensitive to uncertainty in model assumptions (e.g. steepness, natural mortality rate)
Y% Equilibrium B at $F_{X\%SPR}$	A percentage (Y) of the equilibrium biomass (B) at a level of fishing mortality (F) that allows the stock to maintain X% of its maximum spawning potential (i.e. X% spawning potential ratio) that would have been obtained with no fishing	Used as a reference point for recruitment overfishing; Requires fewer assumptions or data (e.g., stock-recruitment relationship is not required); $F_{40\%SPR}$ is a common proxy for F_{MSY} (the equilibrium biomass is therefore a proxy for B_{MSY})	Sensitive to uncertainty in model assumptions (e.g., "assumed" resilience, natural mortality rate); Dynamic pool-type reference points like SPR may not be suitable for stocks with complex spatial structure (e.g., invertebrates)

B_{MSY}

% of B_0 as a proxy for B_{MSY}

% of Equilibrium B at $F_{X\%SPR}$ as a proxy for B_{MSY}

Some Equilibrium Reference Points

- Calculations for an age-structured model:
  1. *SSB* at $F_{X\%SPR}$: equilibrium SSB from fishing at a fishing mortality rate over the long-term that reduces the spawning potential ratio (SPR) to X%

     SPR = ratio of spawning biomass per recruit at a given *F to* spawning biomass per recruit at $F = 0$
  2. SSB_0: equilibrium SSB under conditions of no fishing
  3. F_{MSY} and SSB_{MSY} : fishing mortality rate at which yield is maximized over the long-term and equilibrium SSB from fishing at F_{MSY}
- An Excel spreadsheet and R script are provided in the pre-workshop exercise

Pre-Workshop Example Calculations.xlsx

Read Me | phi0 | phiF SPR YPR | SPR YPR all F | Fx%SPR | SSB at Fx%SPR | SRR and Yield | MSY | Plots | SR Plots with Ref Pts

Read Me tab: Description of each tab in Spreadsheet

Calculations are done for the equilibrium references points on the previous slide using an example data set

The following slides provide some background information to support these calculations

	A	B	C
1	This spreadsheet guides you through some reference point calculations using an example data set with ages 1 to 9+		
2	The example data set is for a haddock stock but the calculations are not species specific and the number of age classes can be modified		
3	Some example plots can be found on the last 2 tabs		
4			
5	Tab	Inputs	Outputs
6	phi0	weight-, maturity-, and natural mortality-at-age	Unfished spawning biomass per recruit (ϕ_0)
7	phiF SPR YPR	weight-, maturity-, vulnerability- and natural mortality-at-age	Fished spawning biomass per recruit (ϕ_F)
8		F	Spawning Potential Ratio (SPR)
9			Yield per Recruit (YPR)
10	SPR YPR all F	weight-, maturity-, vulnerability- and natural mortality-at-age	
11		**calculations over a range of F values**	
12	Fx%SPR	weight-, maturity-, vulnerability- and natural mortality-at-age	$F_{X\%}$ (i.e. F at X% SPR)
13		**calculations over a range of F values**	
14		X corresponding to X% SPR	
15	SSB at Fx%SPR	weight-, maturity-, vulnerability- and natural mortality-at-age	Equilibrium SSB at F_X% (i.e. F at X% SPR)
16		**calculations over a range of F values**	Equilibrium unfished spawning biomass (SSB_0)
17		X corresponding to X% SPR	
18		Equilibrium unfished recruitment	
19	Add (Beverton-Holt) stock recruitment relationship		
20	SRR and Yield	weight-, maturity-, vulnerability- and natural mortality-at-age	Equilibrium unfished spawning biomass (SSB_0)
21		steepness (h) and equilibrium unfished recuitment (R_0) OR Beverton-Holt a and b	Equilibrium recruitment at F=F
22		F	Equilibrium spawning biomass at F=F
23			Yield at F=F
24	MSY	weight-, maturity-, vulnerability- and natural mortality-at-age	Equilibrium MSY, F_{MSY}, SSB_{MSY}
25		steepness and equilibrium unfished recuitment OR Beverton-Holt a and b	
26		**calculations over a range of F values**	
27			
28			
29	Legend for shading:		
30	User defined value (value, biological data, or model output)		
31	Estimated for F=0 (unfished)		
32	Estimated for F=F (fished)		
33	Note difference in formula when dragging cells across (first age class and plus group)		
34	No stock recruitment relationship assumed		

Per Recruit Reference Points: Calculations

Per Recruit Reference Points [Concepts]

1. Mortality Rates
2. Survivorship
3. Spawning biomass per recruit (or eggs per recruit)
4. Spawning potential ratio (*SPR*)
5. Yield per recruit (*YPR*)
6. Equilibrium SSB at a specified *F*

Per Recruit Reference Points 1. Mortality Rates

Notation:

- M = Natural Mortality (instantaneous rate) – removals from predation, disease, old age
- F = Fishing Mortality (instantaneous rate) – removals from fishing
- $Z = M + F$ = Total Mortality (instantaneous rate)

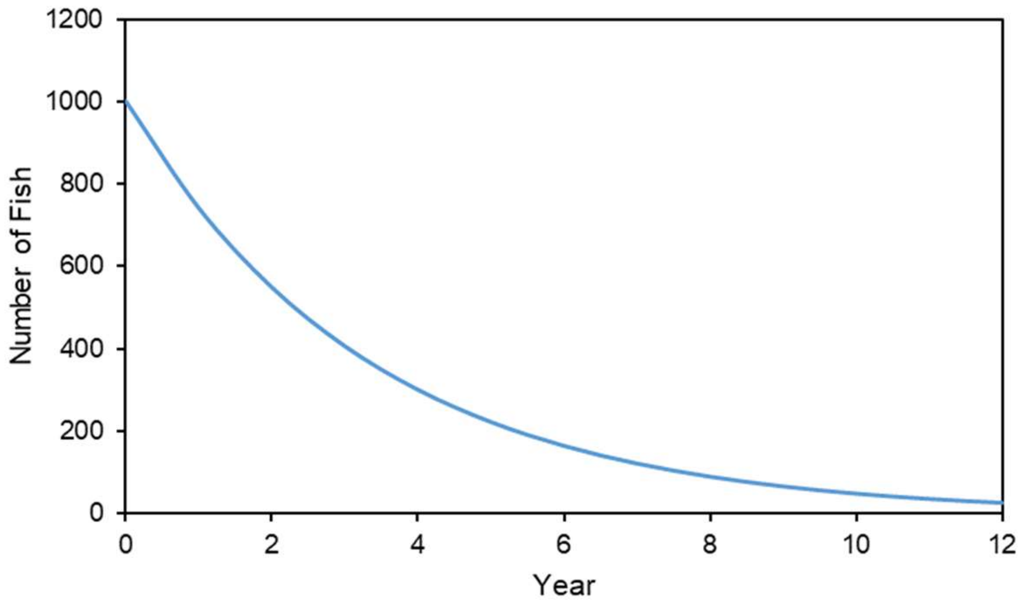
Per Recruit Reference Points 1. Mortality Rates

- Mortality rates can be instantaneous or finite (e.g., annual)
- Generally work with instantaneous rates (e.g., Z = 0.3)
- Mortality is applied by decreasing the population by a constant proportion in each time period (t)

$$N_{t+1} = N_t e^{-Z}$$

Per Recruit Reference Points 1. Mortality Rates

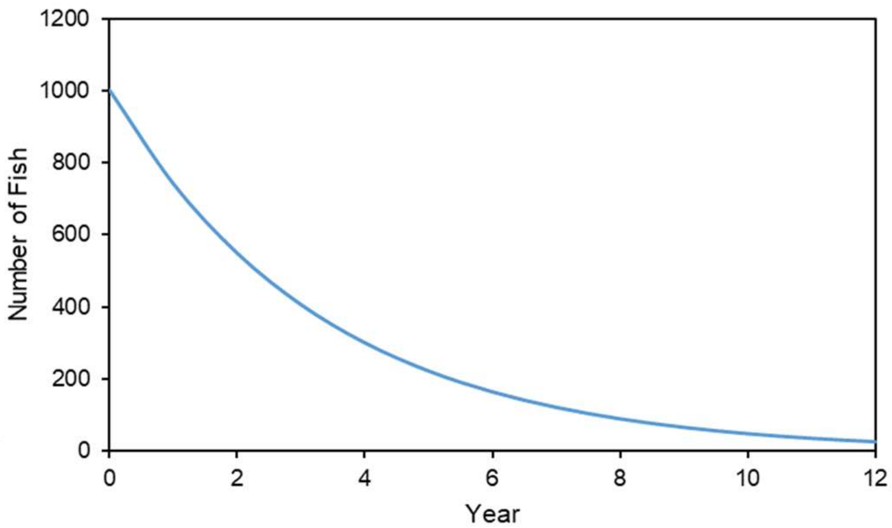
- Example: Z = 0.3, start with 1000 individuals at age 0



Per Recruit Reference Points 1. Mortality Rates

- Example: Z = 0.3, start with 1000 individuals at age 0
- N_{year1} = $N_{year0}e^{-0.3}$ = $1000e^{-0.3}$ = 741
- Annual removal rate = $1 - \frac{741}{1000} = 0.259$
- 25.9% of population removed each year

- In general:
- Annual removal rate = $1 - e^{-Z}$



Per Recruit Reference Points 1. Mortality Rates

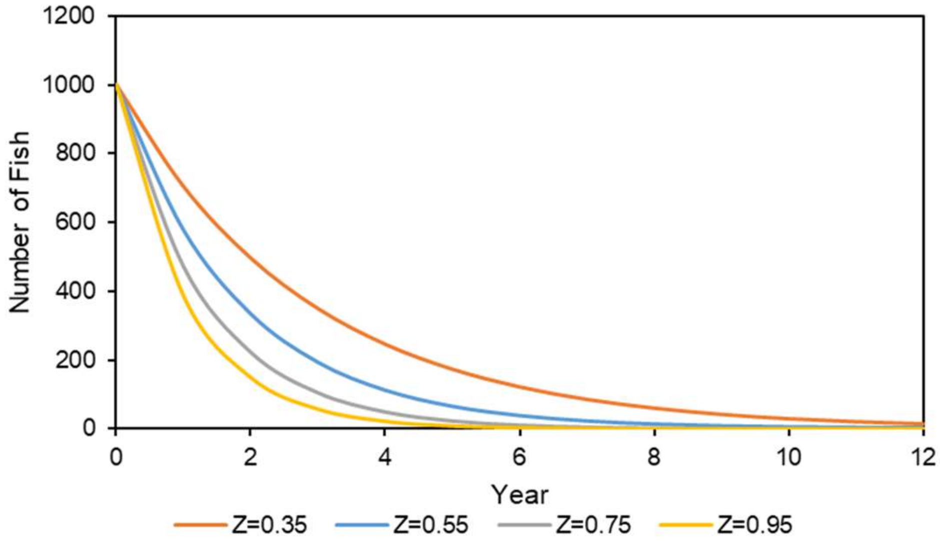
- Harvest rate (or exploitation rate) is the annual proportional change in numbers of fish for a given instantaneous fishing mortality rate (*F*)
- Sometimes expressed as a percentage (%)

F	Harvest Rate = $1-e^{-F}$
0.1	0.095 or 9.5%
0.3	0.259 or 25.9%
0.5	0.393 or 39.9%
0.7	0.503 or 50.3%

Per Recruit Reference Points 1. Mortality Rates

- Add M = 0.25 and calculate total removal rate (from M and F)

F	Harvest Rate = $1-e^{-F}$	M	Z = F+M	Total removal rate = $1-e^{-Z}$
0.1	0.095	0.25	0.35	0.295
0.3	0.259	0.25	0.55	0.423
0.5	0.393	0.25	0.75	0.528
0.7	0.503	0.25	0.95	0.613



Per Recruit Reference Points 2. Survivorship

Notation:

- l_a = survivorship-at-age a = (Probability of surviving to age a)
- v_a = vulnerability (selectivity) -at-age a = proportion of age class that is vulnerable to the fishery
- Age of recruitment = age at first recruitment to the fishery
- Plus group (X^+) = age category that contains all fish age X and older

Per Recruit Reference Points 2. Survivorship

- We track a cohort from the age of recruitment to a plus group
- We begin assuming no fishing mortality (F=0) so removal is only by natural mortality (M)
- We define "survivorship-at-age" (l_a) as the probability of surviving to age a

- Example with age of recruitment = 1, plus group = 9^+, and M = 0.3

P(A and B) = P(A)P(B)
If events A and B are independent

Per Recruit Reference Points 2. Survivorship

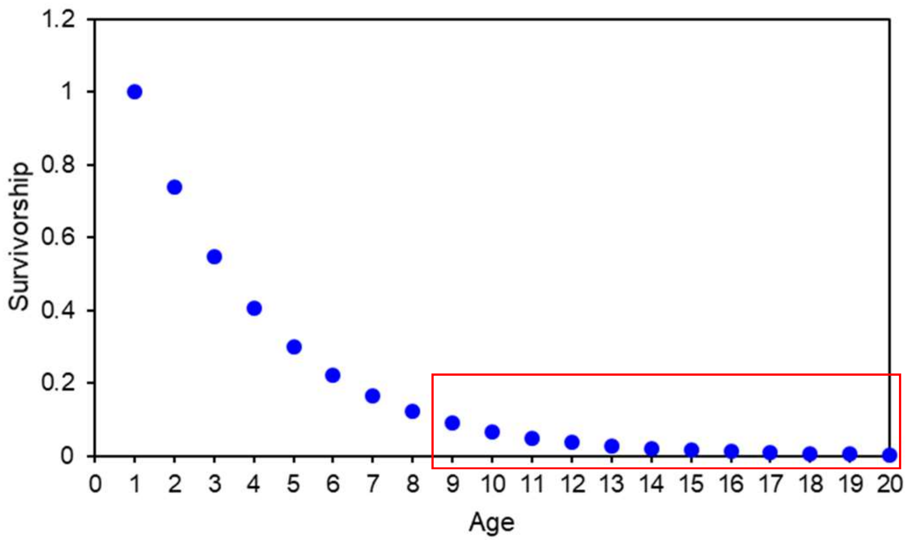
age 1 = age of recruitment

- For F = 0
  - $l_{a=1} = P(surviving\ to\ age\ 1) = 1$
  - $l_{a=2} = P(surviving\ to\ age\ 2)$
    $= P\big(surviving\ to\ age\ 1\ \underline{and}\ from\ age\ 1\ to\ age\ 2\big)$ = 1×e^{-M} = e^{-M}
  - $l_{a=3} = P(surviving\ to\ age\ 3)$
    $= P\big(surviving\ to\ age\ 2\ \underline{and}\ from\ age\ 2\ to\ age\ 3\big)$ = e^{-M}×e^{-M} = e^{-2M}
  - $l_{a=4} =$
    $= P\big(surviving\ to\ age\ 3\ \underline{and}\ from\ age\ 3\ to\ age\ 4\big)$ = e^{-2M}×e^{-M} = e^{-3M}

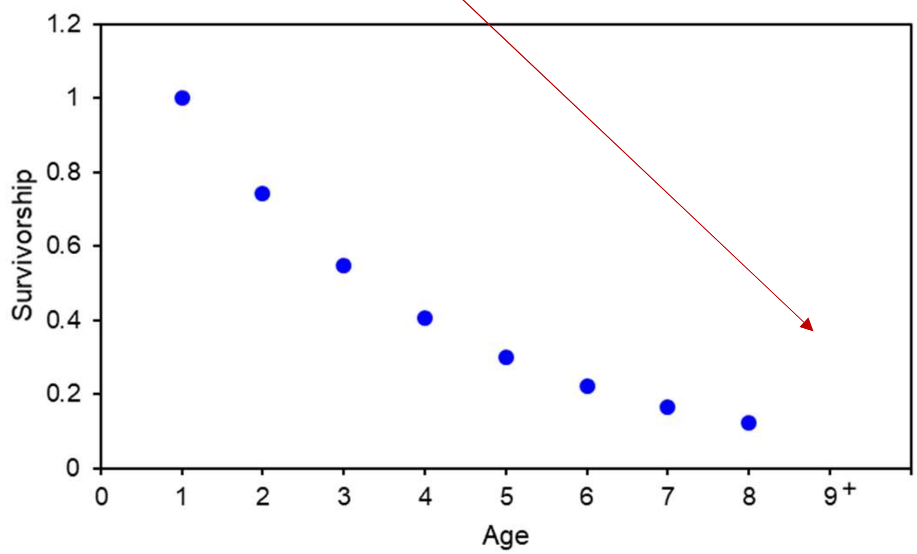
- $l_{a=a}$ = e$^{-(a-1)M}$

Per Recruit Reference Points 2. Survivorship

- For F = 0, M = 0.3

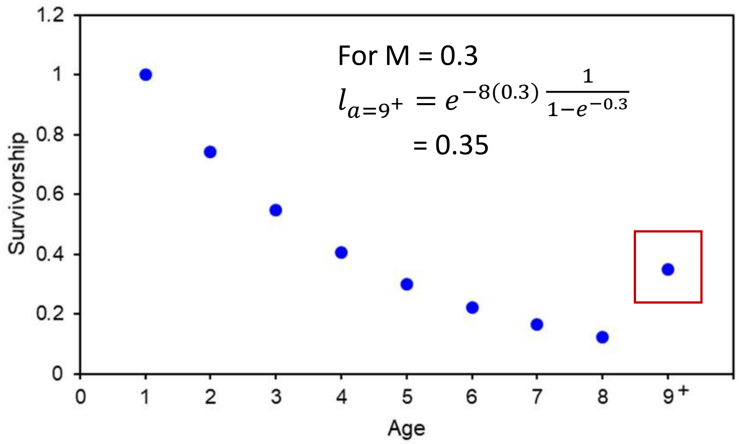


What is survivorship for age 9+?



Per Recruit Reference Points 2. Survivorship

$$l_{a=9^+} = P(\textit{surviving to at least age } 9)$$
$$= P(\textit{surviving to age } 9) + P(\textit{surviving to age } 10) + P(\textit{surviving to age } 11) + \cdots$$
$$= e^{-8M} + e^{-9M} + e^{-10M} + \ldots$$
$$= e^{-8M}(1 + e^{-M} + (e^{-M})^2 + (e^{-M})^3 + \ldots)$$
$$l_{a=9^+} = e^{-8M}\frac{1}{1-e^{-M}}$$



Maclaurin series expansion

$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$$

Per Recruit Reference Points 2. Survivorship

- In general:

$$l_a = \begin{cases} 1, a \ = a_{rec} \\ l_{a-1} e^{-(M_{a-1} + F v_{a-1})}, a_{rec} < \ a < a_{max} \\ \dfrac{l_{a-1} e^{-(M_{a-1} + F v_{a-1})}}{1 - e^{-(M_a + F v_a)}}, a = \ a_{max} \end{cases}$$

Per Recruit Reference Points 2. Survivorship

- In general:

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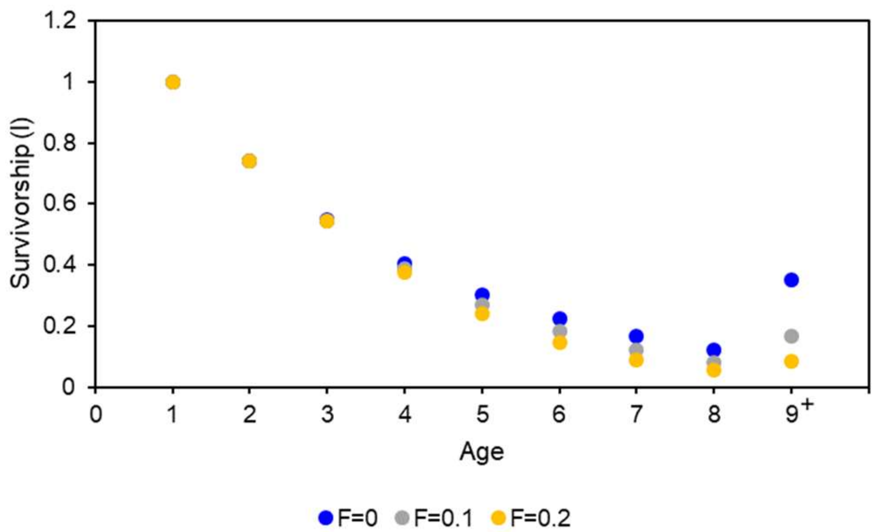
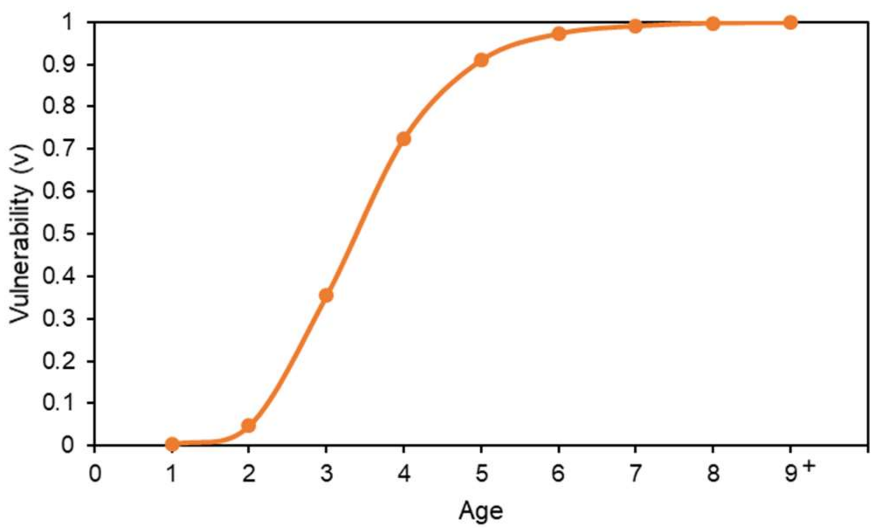




Per Recruit Reference Points 3. Spawning Biomass per Recruit

Notation:

- φ_F= Equilibrium spawning biomass per recruit (spawning biomass produced on average by each recruit over its lifetime, taking into account survival, growth, and maturity)
- w_a = weight-at-age a
- m_a = proportion mature-at-age a
- a_{max} = maximum age (or plus group)
- f_a = relative fecundity-at-age (eggs per unit body weight)

Per Recruit Reference Points 3. Spawning Biomass per Recruit

- Equilibrium spawning biomass per recruit (φ_F) =

- Sum over all ages:
  P(survives to age a) × weight-at-age a × proportion mature-at-age a

$$\varphi_F = \sum_{a=1}^{a_{max}} l_a w_a m_a$$

Per Recruit Reference Points 3. Eggs per Recruit

- If we knew fecundity-at-age, we could calculate "eggs per recruit"
  Equilibrium eggs per recruit (φ_F)
- Sum over all ages:

Eggs/unit body weight

Relative fecundity-at-age a × P(survives to age a) × weight-at-age a × proportion mature-at-age a

$$\varphi_F = \sum_{a=1}^{a_{max}} f_a l_a w_a m_a$$

Per Recruit Reference Points 3. Spawning Biomass per Recruit or Eggs per Recruit

- We generally don't know relative fecundity and assume that SSB is proportional to fecundity as use:

Equilibrium spawning biomass per recruit (φ_F)

Per Recruit Reference Points 4. Spawning Potential Ratio (*SPR*)

Notation:

- SPR = ratio of

the *spawning biomass (or eggs)* per recruit over its lifespan when fishing at given *F*

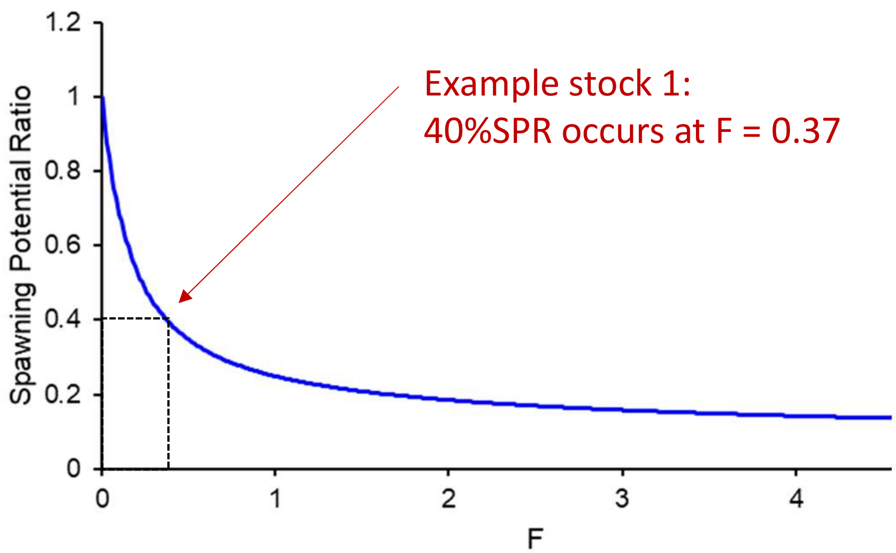
<u>relative to</u>

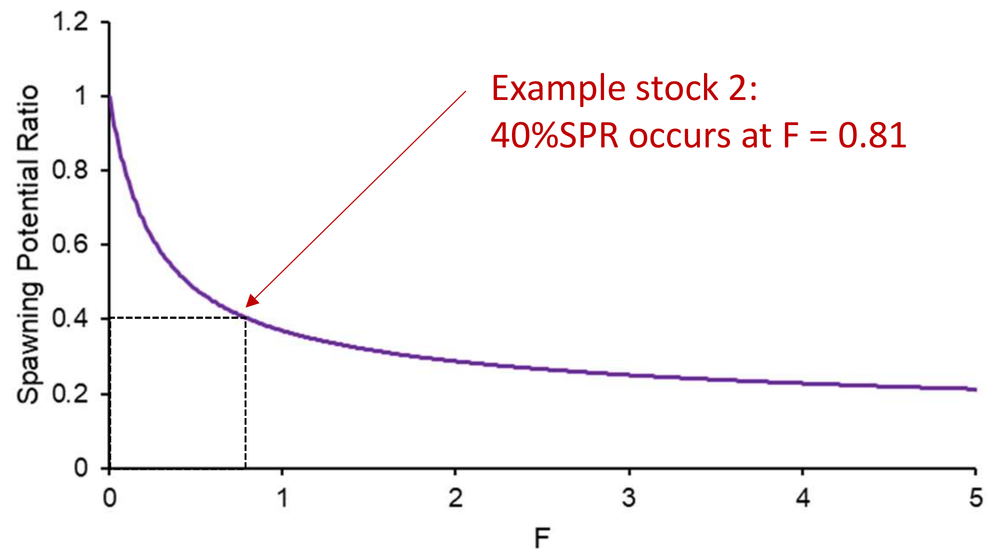
the *spawning biomass (or eggs)* per recruit over its lifespan with no fishing

Per Recruit Reference Points 4. Spawning Potential Ratio (*SPR*)

Reference points based on SPR are common proxies for F_{MSY} and SSB_{MSY}

- $SPR_F = \frac{\varphi_F}{\varphi_0}$
  - φ_F = spawning biomass per recruit when fishing at F=F
  - φ_0 = spawning biomass per recruit at F=0





Per Recruit Reference Points 4. Spawning Potential Ratio (*SPR*)

- What influences $\text{SPR}_F = \frac{\varphi_F}{\varphi_0}$?
  - Survivorship:
    - *M*
    - Vulnerability (selectivity)
    - *F*
  - Weight-at-age
  - Maturity-at-age

$$\varphi_F = \sum_{a=1}^{a_{max}} l_a w_a m_a$$

$$l_a = \begin{cases} 1, a = a_{rec} \\ l_{a-1} e^{-(M_{a-1} + F v_{a-1})}, a_{rec} < a < a_{max} \\ \dfrac{l_{a-1} e^{-(M_{a-1} + F v_{a-1})}}{1 - e^{-(M_a + F v_a)}}, a = a_{max} \end{cases}$$

Per Recruit Reference Points 4. Spawning Potential Ratio (*SPR*)

Read Me | phi0 | phiF SPR YPR | SPR YPR all F | Fx%SPR | SSB at Fx%SPR | SRR and Yield | MSY | Plots | SR Plots with Ref Pts

- What influences $\text{SPR}_F = \frac{\varphi_F}{\varphi_0}$?
  - Explore the Excel Spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	4. Calculate F_X% (i.e. F at X% SPR)													
2			Parameter		1	2	3	4	5	6	7	8	9+	units
3			Weight-at-age	w_age	0.528688	0.917438	1.2655	1.635396	1.943729	2.215542	2.518042	2.795604	3.155688	kg
4			Proportion mature	m_age	0.189883	0.51526	0.830718	0.940038	0.975651	0.989041	0.994701	0.9973	0.99857	-
5			M	M_age	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	-
6	Start with F =	0.1	Unfished survivorship	l_0_age	1	0.818731	0.67032	0.548812	0.449329	0.367879	0.301194	0.246597	1.113794	-
7			Unfished spawning biomass (or eg	ϕ_0	8.645726									kg/recruit
8			Vulnerability (selectivity)	v_age	0.004044	0.048547	0.354896	0.724669	0.908942	0.97144	0.989727	0.996591	0.998065	-
9			Fishing mortality rate	F	0.1									-
10			Fished survivorship	l_F_age	1	0.8184	0.666804	0.526898	0.401232	0.299959	0.222851	0.165261	0.472789	-
11			Fished spawning biomass (or eggs	ϕ_F	5.92523									kg/recruit
12			Spawning potential ratio	SPR	0.685336									-
13			Yield per recruit-at-age	YPR_age	0.000194	0.003296	0.026683	0.05466	0.061508	0.055851	0.048005	0.039785	0.12866	kg
14			Yield per recruit	YPR	0.41864									kg
15														
16														
17				Step 1. Enter X in cell B19										
18				Step 2. Find approximate F when SPR = X (i.e. find F where abs(SPR-x%) is at its minimum										
19				Step 3. Repeat column F beginning in cell F30 to lookup the F corresponding to min[abs(SPR-X)]										
20		4		Step 4. Solution is in cell B20										
21	X =	40	%											
22	F=	0.37												
23														

Per Recruit Reference Points 5. Yield per Recruit

Notation:

- Yield per Recruit (*YPR*)-at-age = the expected life time yield per fish recruited into the stock at a specified age

Per Recruit Reference Points 5. Yield per Recruit

- Yield per Recruit (*YPR*) at age for a given *F*

from *M* and *F*

Removals per recruit-at-age = biomass per recruit-at-age × removal rate

$$\text{Removals per recruit at age} = l_a w_a (1 - e^{-(M_a + F v_a)})$$

YPR-at-age = biomass per recruit-at-age × proportion of removal rate that is due to fishing

$$YPR_{F\_a} = l_a w_a (1 - e^{-(M_a + F v_a)}) \frac{F v_a}{M_a + F v_a}$$

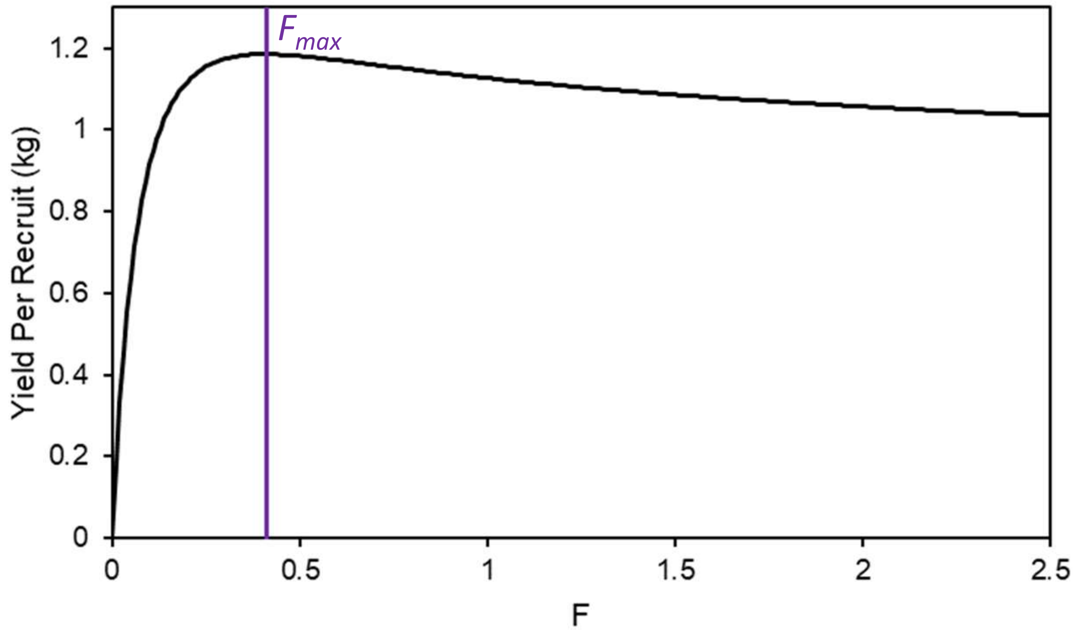
Per Recruit Reference Points 5. Yield per Recruit

- YPR = Sum over all ages: $YPR_{F\_a} = l_a w_a (1 - e^{-(M_a + Fv_a)}) \frac{Fv_a}{M_a + Fv_a}$

$$YPR_F = \sum_{a=1}^{a_{max}} l_a w_a \left(1 - e^{-(M_a + Fv_a)}\right) \frac{Fv_a}{M_a + Fv_a}$$

Per Recruit Reference Points 5. Yield per Recruit

- Can be calculated over a range of F values to estimate F_{max}
- F_{max} = fishing mortality rate corresponding to the maximum YPR



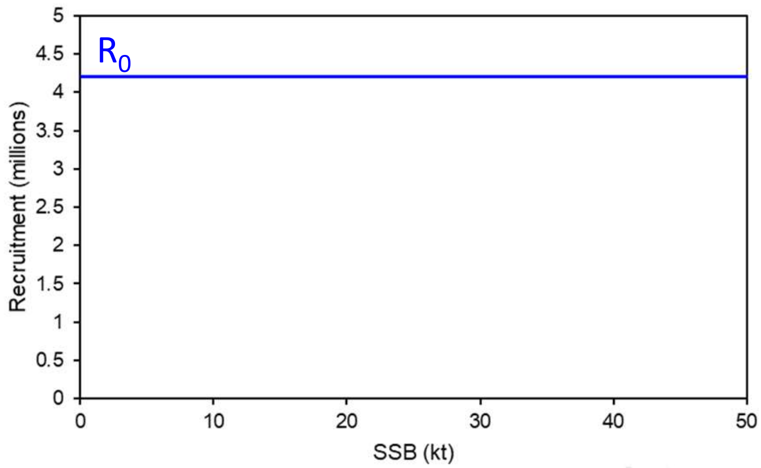
Per Recruit Reference Points 6. Equilibrium SSB at F=F

Notation:

- Equilibrium SSB at $F=F_y$: equilibrium spawning stock biomass obtained from fishing at a fishing mortality rate F_y over the long-term

Per Recruit Reference Points 6. Equilibrium SSB at $F=F_y$

- What SSB is obtained on average from fishing at $F=F_y$?
- Given an unfished equilibrium recruitment (R_0) and no stock recruitment relationship, we can assume average recruitment will be R_0



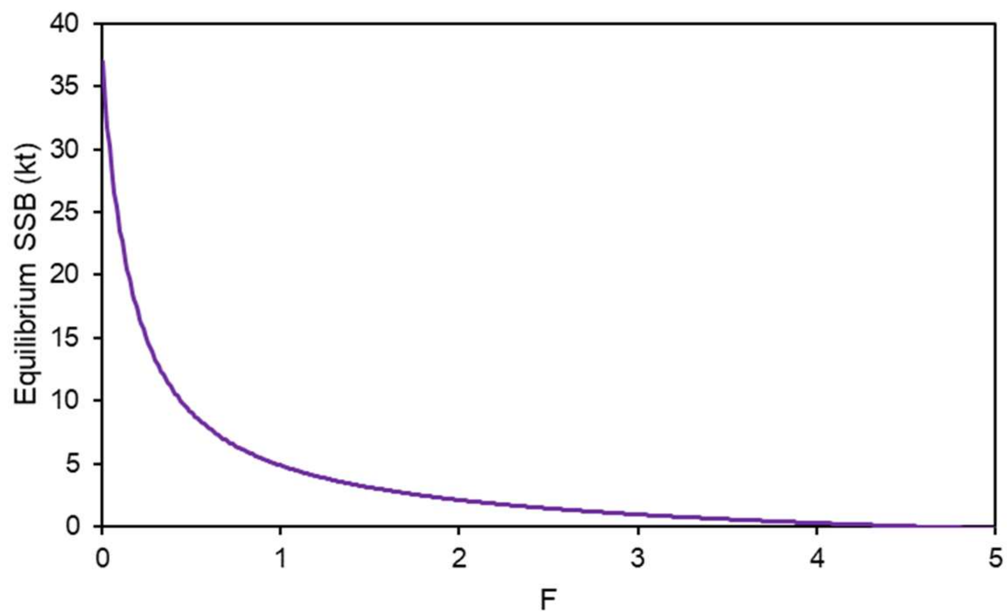
- Equilibrium $SSB_F = \varphi_F R$

Number of recruits

Spawning biomass per recruit

Per Recruit Reference Points 6. Equilibrium SSB at F=F

- Can be calculated over a range of F values

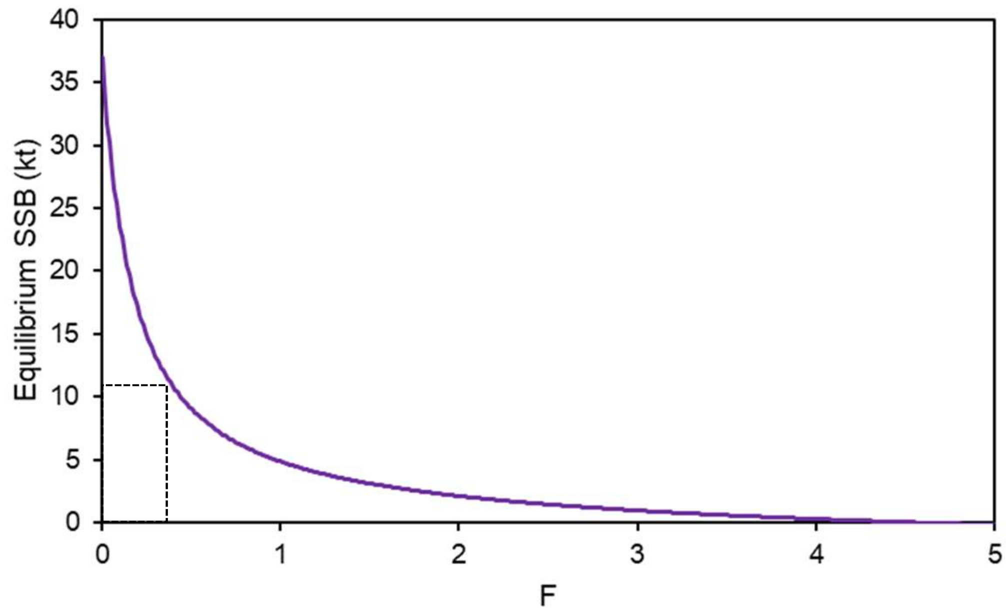


Per Recruit Reference Point: SSB at $F_{X\%SPR}$

- Equilibrium SSB at $F_{40\%SPR}$ (*SPR* = spawning potential ratio)
- $F_{40\%SPR}$ = fishing mortality rate that allows the stock to maintain 40% of its maximum (unfished) spawning potential

- Example: Suppose $F_{40\%SPR}$ = 0.41

- Equilibrium SSB at $F_{40\%SPR}$ = 10.9 kt

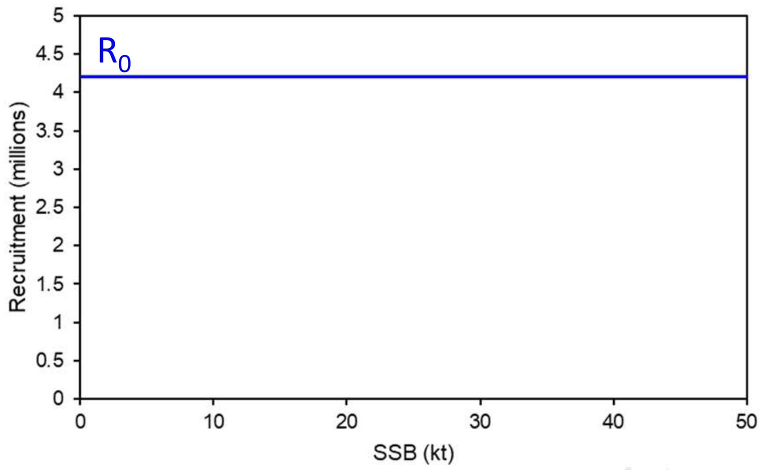


SSB_0 (assuming no stock recruitment relationship)

- Equilibrium SSB at F=0
- Equilibrium $SSB_0 = \varphi_0 R_0$

Number of recruits

Unfished spawning biomass per recruit

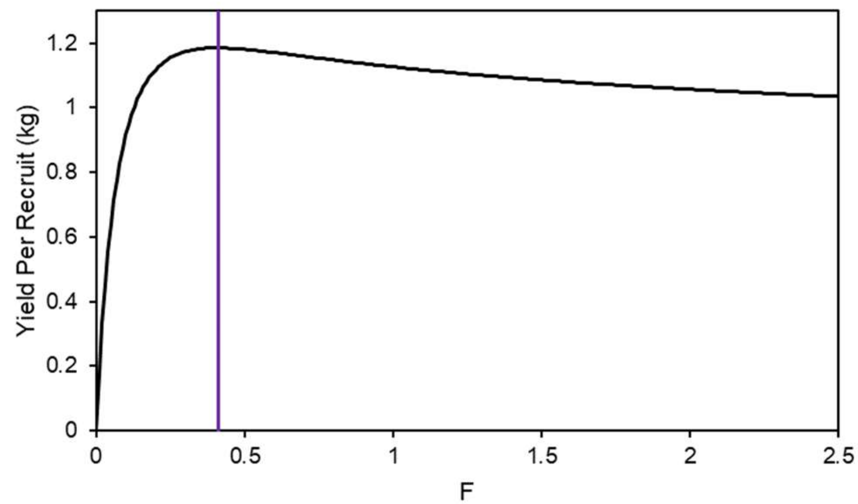


MSY Reference Points: Calculations

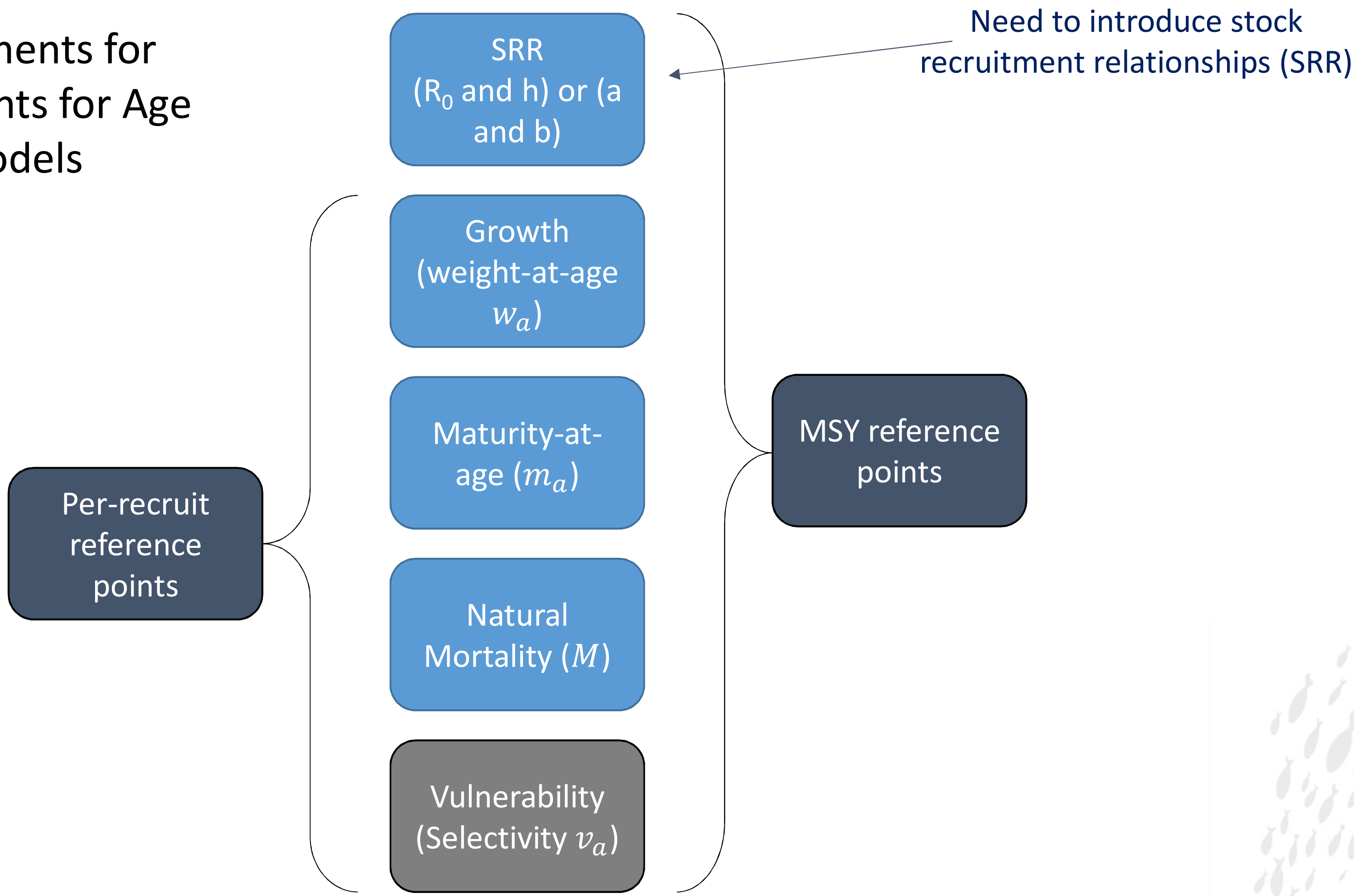
Yield per Recruit → Yield

- Yield = YPR × R
- We already know how to calculate YPR for various F
- Now we need to know how the number of recruits (R) changes with F

Stock Recruitment Relationships



Data Requirements for Reference Points for Age Structured Models



MSY Reference Points

Notation:

- R_0 = average number of recruits in an unfished state (at equilibrium)
- h = steepness of the stock recruitment relationship = proportion of equilibrium unfished recruitment produced by 20% of unfished equilibrium SSB
- a and b = parameters of the Beverton-Holt stock recruitment relationship
- α and β = parameters of the Ricker stock recruitment relationship

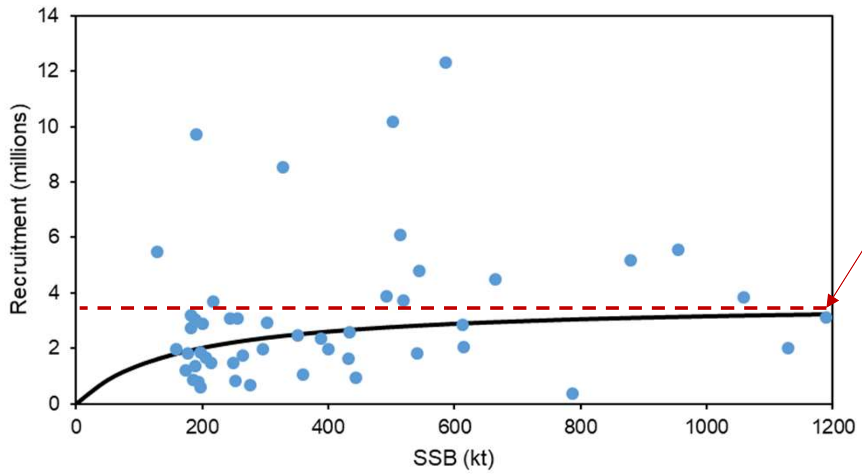
MSY Reference Points [Concepts]

1. Stock Recruitment Relationships (SRR)
2. MSY

MSY Reference Points 1. Stock Recruitment Relationships

- Beverton-Holt

$$R(B) = \frac{aB}{1 + bB}$$

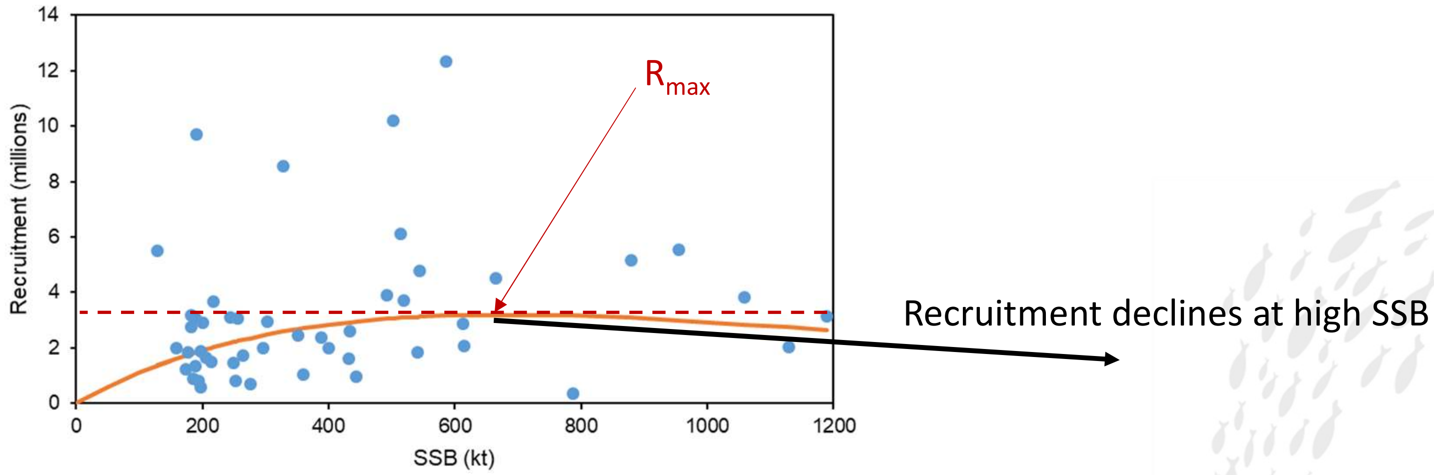


asymptotic recruitment $R_{max} = \frac{a}{b}$

MSY Reference Points 1. Stock Recruitment Relationships

- Ricker

$$R(B) = \alpha B\, e^{-\beta B}$$



MSY Reference Points 1. Stock Recruitment Relationships

- Focus on Beverton-Holt

$$R(B) = \frac{aB}{1 + bB} \xrightarrow{\text{Alternative parameterization}} R(B) = \frac{4R_0 hB}{(1-h)R_0\varphi_0 + (5h-1)B}$$

$$h = \frac{a\varphi_0}{4 + a\varphi_0}$$

h = steepness

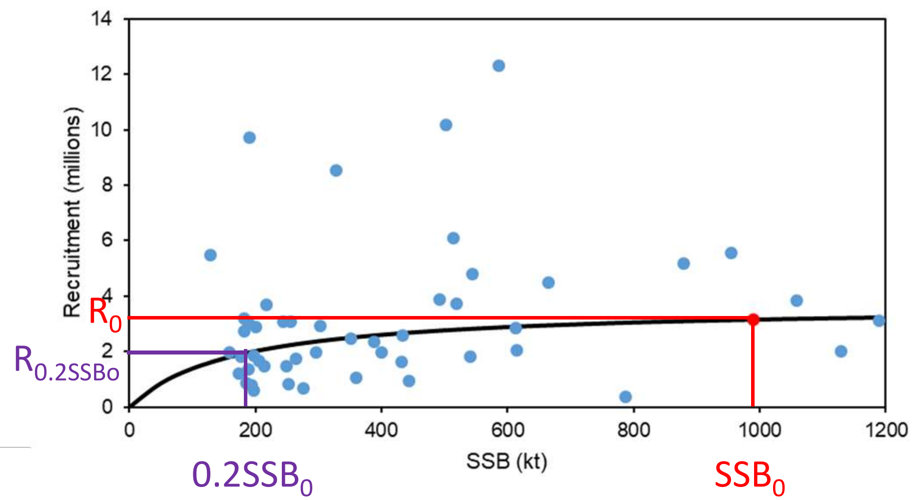
$$R_0 = \frac{1}{b}\left(a - \frac{1}{\varphi_0}\right)$$

MSY Reference Points 1. Stock Recruitment Relationships

- Steepness (*h*) = proportion of equilibrium unfished recruitment produced by 20% of unfished equilibrium SSB

$$R(B) = \frac{4R_0hB}{(1-h)R_0\varphi_0 + (5h-1)B}$$

$$h = \frac{R_0}{R_{0.2SS\ _0}}$$



MSY Reference Points 1. Stock Recruitment Relationships

- *h* is a measure of the resilience of recruitment to decreases in SSB from SSB_0
- *h* ranges from 0.2 to 1
- *h* is <u>hard to estimate</u>
- *h* = 1 (recruits do not decline as SSB declines from SSB_0)
- *h* = 0.2 (recruits decline linearly as SSB declines from SSB_0)

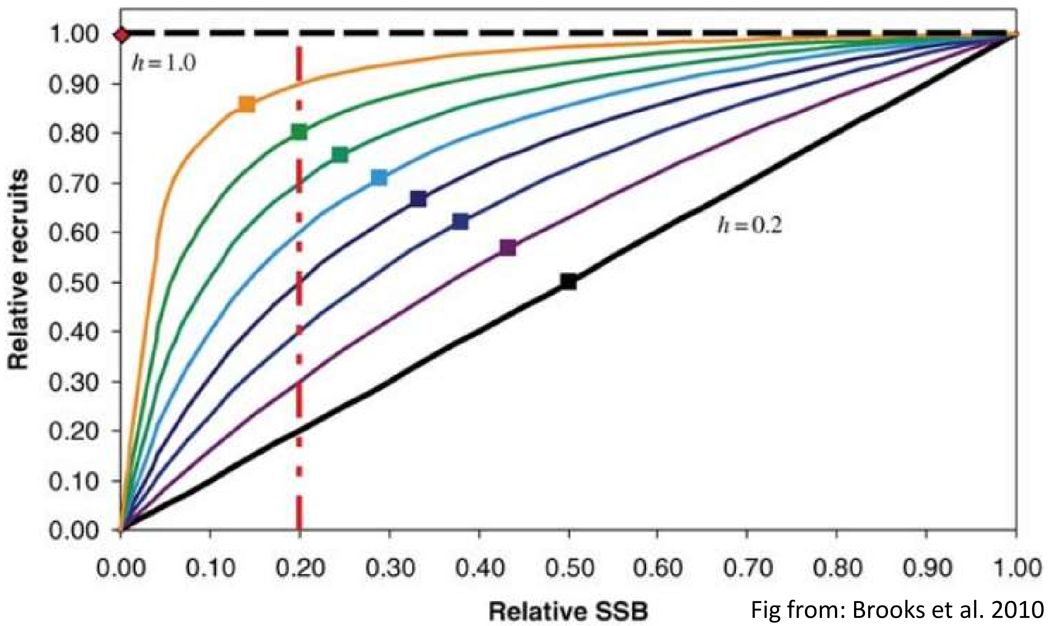
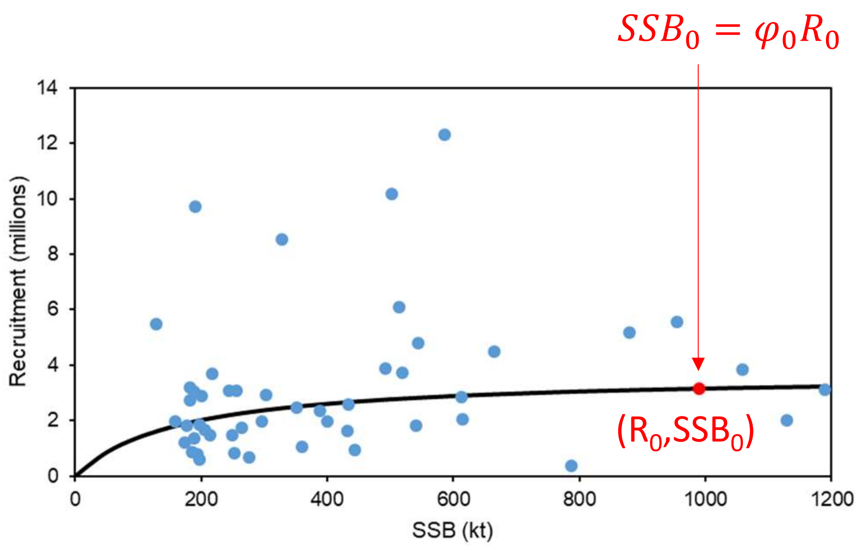


Fig from: Brooks et al. 2010

MSY Reference Points 1. Stock Recruitment Relationships

- Given a SRR we can calculate $SSB_0 = \varphi_0 R_0$

How to get parameter	$R(B) = \dfrac{aB}{1+bB}$	$R(B) = \dfrac{4R_0 hB}{(1-h)R_0\varphi_0 + (5h-1)B}$
φ_0	$\varphi_0 = \sum_{a=1}^{a_{max}} l_a w_a m_a$ Estimated from survivorship-at-age, weight-at-age, maturity-at-age	Estimated from parameterization
R_0	$R_0 = \dfrac{1}{b}\left(a - \dfrac{1}{\varphi_0}\right)$	Estimated from parameterization



MSY Reference Points 1. Stock Recruitment Relationships

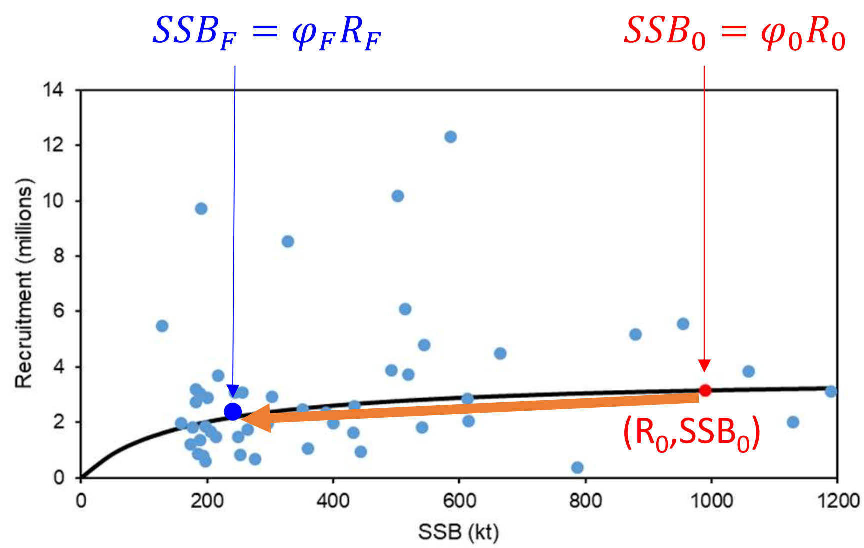
- When *F* is applied, the spawning biomass per recruit (φ_F) decreases from the unfished φ_0

- The equilibrium recruitment for φ_F is: $R_F = \frac{1}{b}\left(a - \frac{1}{\varphi_F}\right)$

- The equilibrium SSB for *F* is: $SSB_F = \varphi_F R_F$

MSY Reference Points 1. Stock Recruitment Relationships

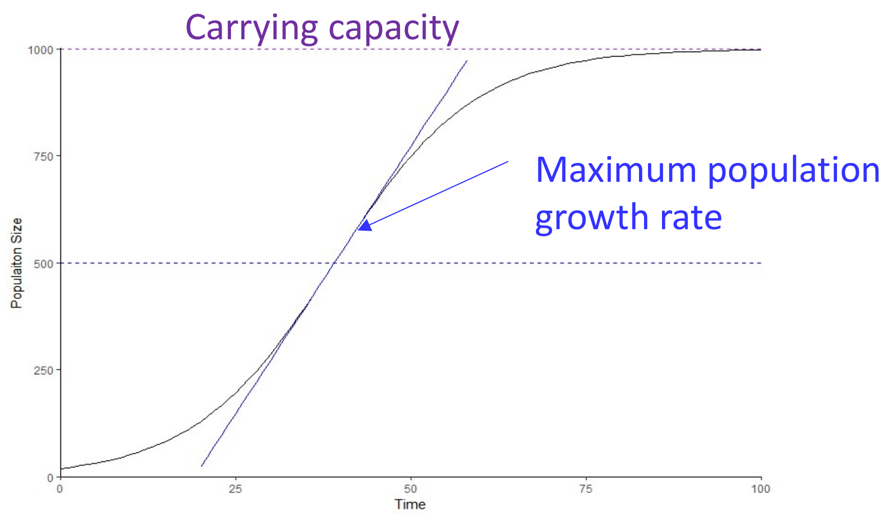
- When F is applied, the equilibrium recruitment and equilibrium SSB "slide down" the SRR from (R_0,SSB_0). Example: F = 0.34
- At F = 0.34 the unfished biomass per recruit decreases to $\varphi_{0.34}$
- Equilibrium recruitment: $R_{F=0.34}$ at $\varphi_{0.34}$
- Equilibrium SSB: $SSB_{F=0.34}$ at $\varphi_{0.34}$



MSY Reference Points 2. MSY

- MSY: the largest catch on average over the long-term that can be continuously removed from the stock assuming constant environmental conditions
- Reference points are based on F_{MSY}
- SSB_{MSY} is the average (equilibrium) SSB from fishing at F_{MSY}

Theory:
- population growth rate is zero at carrying capacity
- as population size is reduced below carry capacity, the population growth rate increases (due to available resources)
- Maximum population growth rate occurs at an intermediate population size
- The "growth" is harvested as surplus production



MSY Reference Points 2. MSY

- MSY: the largest catch on average over the long-term that can be continuously removed from the stock assuming constant environmental conditions
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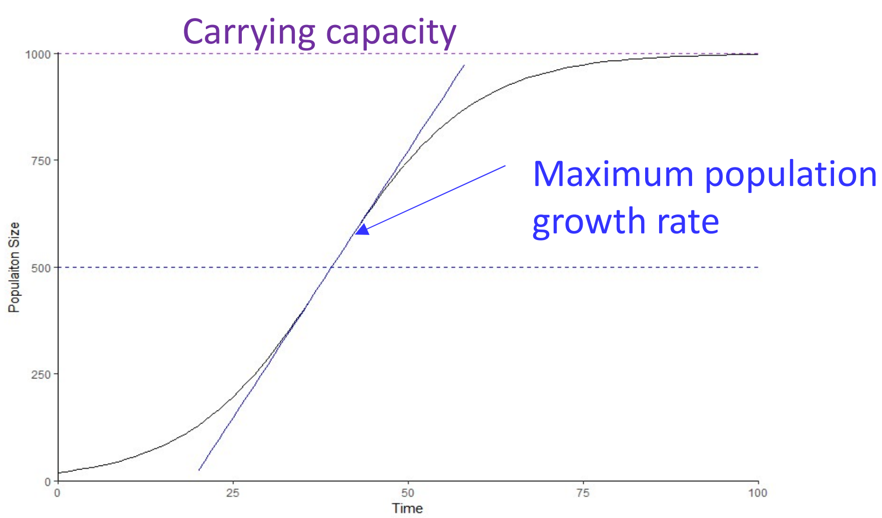
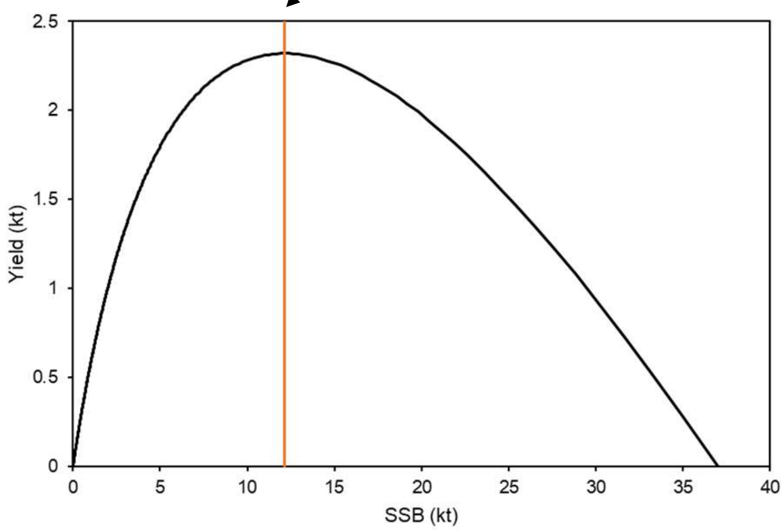


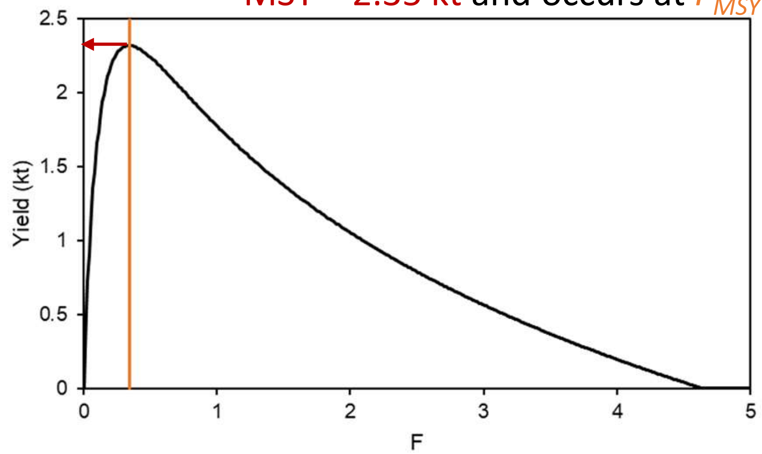
Here the maximum yield occurs at 12kt $= SSB_{MSY}$



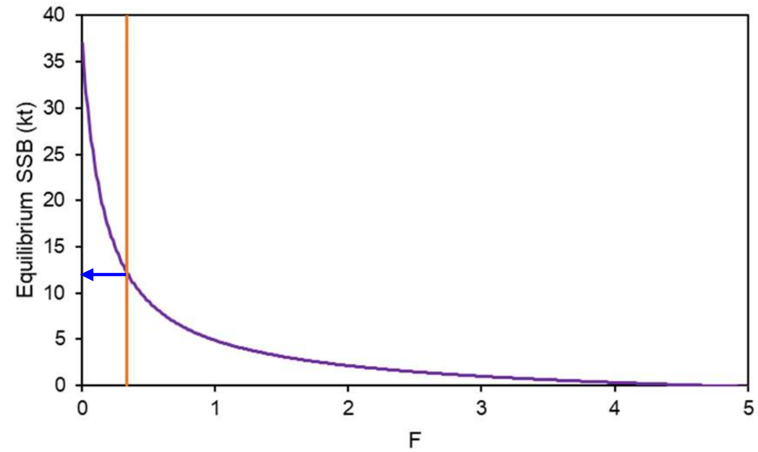
MSY Reference Points 2. MSY

- F_{MSY} = Fishing mortality rate that results in MSY over the long term
- MSY = the largest catch that can be continuously removed from the stock assuming constant environmental conditions (equilibrium)
- SSB_{MSY} = average SSB from fishing at F_{MSY} over the long term
- Example:

MSY = 2.35 kt and occurs at F_{MSY} = 0.3



The equilibrium SSB at F = 0.3 is SSB_{MSY} = 12kt



MSY Reference Point: SSB at F_{MSY}

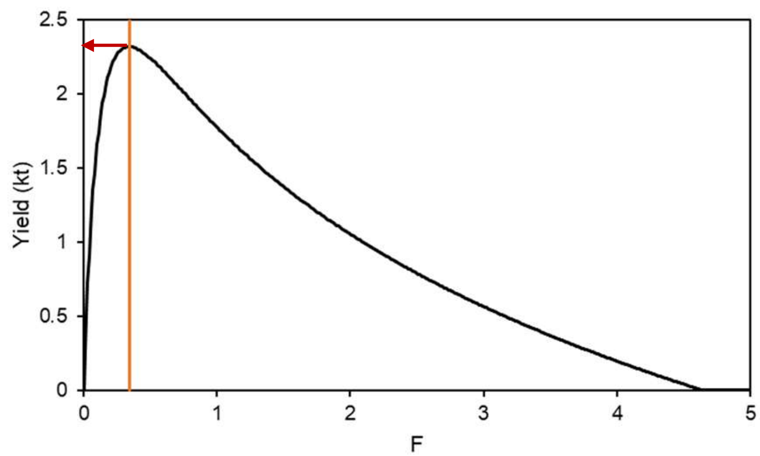
- $\color{red}{Yield_F} = \color{green}{YPR_F} \times \color{purple}{R_F}$

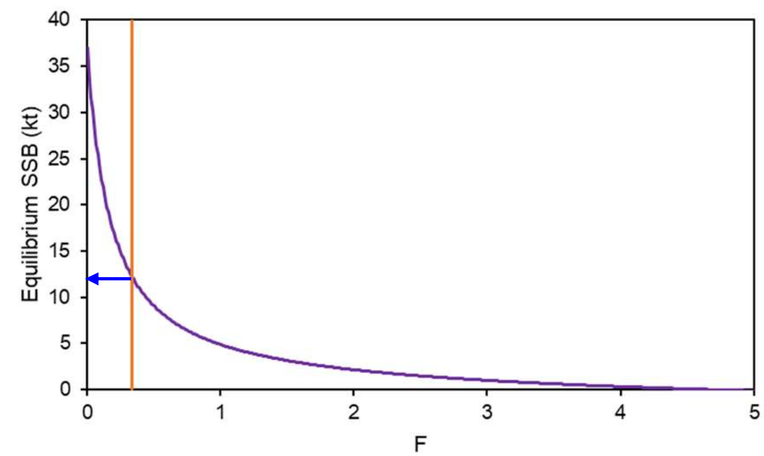
Equilibrium Yield at a specified *F*

Yield-per-recruit at a specified *F*

Equilibrium Recruitment at a specified *F*

- To find SSB at F_{MSY}: calculate *Yield* for various *F* and identify *F* where *Yield* is at its maximum





MSY Reference Points 2. MSY

Read Me | phi0 | phiF SPR YPR | SPR YPR all F | Fx%SPR | SSB at Fx%SPR | SRR and Yield | MSY | Plots | SR Plots with Ref Pts

- Excel Sheet:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	10. Add What-If Analysis Data Table													
2	to calculate Yield for various F and		Parameter		1	2	3	4	5	6	7	8	9+	units
3	estimate equilibrium MSY		Weight-at-age	w_age	0.528688	0.917438	1.2655	1.635396	1.943729	2.215542	2.518042	2.795604	3.155688	kg
4	reference points		Proportion mature	m_age	0.189883	0.51526	0.830718	0.940038	0.975651	0.989041	0.994701	0.9973	0.99857	-
5			M	M_age	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	-
6	Start with F =	0.1	Unfished survivorship	l_0_age	1	0.818731	0.67032	0.548812	0.449329	0.367879	0.301194	0.246597	1.113794	-
7			Unfished spawning biomass (or eg	ϕ_0	8.645726									kg/recruit
8			Vulnerability (selectivity)	v_age	0.004044	0.048547	0.354896	0.724669	0.908942	0.97144	0.989727	0.996591	0.998065	-
9			Fishing mortality rate	F	0.1									-
10			Fished survivorship	l_F_age	1	0.8184	0.666804	0.526898	0.401232	0.299959	0.222851	0.165261	0.472789	-
11			Fished spawning biomass (or eggs	ϕ_F	5.92523									kg/recruit
12			Spawning potential ratio	SPR	0.685336									-
13			Yield per recruit-at-age	YPR_age	0.000194	0.003296	0.026683	0.05466	0.061508	0.055851	0.048005	0.039785	0.12866	kg
14			Yield per recruit	YPR	0.41864									kg
15			Equilibrium unfished recruitment	R_0	4.275858									10^6 recruits
16			Steepness	h	0.65									-
17			Beverton-Holt a	a	0.859219									recruit/kg
18			Beverton-Holt b	b	0.173896									/kg
19			Equilibrium unfished SSB	SSB_0	36.9679									kt
20			Equilibrium fished recruitment	Eq_rec$_F$	3.97047									10^6 recruits
21			Equilibrium fished SSB	Eq_SSB$_F$	23.52595									kt
22			Yield	Yield	1.662199									kt
23			Maximum recruitment from SRR	Rmax	4.940991									10^6 recruits
24			Equilibrium SSB at 50%Rmax	SSB_{Rmax}	5.750562									kt
25														
26		10	MSY	MSY	2.320329									-
27		10	Fishing mortality at MSY	F_{MSY}	0.34									kt
28		10	Equilibrium SSB_{MSY}	SSB_{MSY}	12.14738									kt
29														
30			Ratio SSB_{MSY}/SSB_0	SSB_{MSY}/SSB_0	0.328593									

Pre-Workshop Exercise

Pre-Workshop Exercise

- Calculations for an age-structured model:
  1. *SSB* at $F_{X\%SPR}$
  2. SSB_0
  3. F_{MSY} and SSB_{MSY}

See "Pre-Workshop Example Calculations.xlxs"

See "Pre-Workshop Exercise.R"

- An Excel spreadsheet and R script are provided at assist with the calculations in the pre-workshop exercise
- Explore the influence of various parameters (e.g., *M*, *h*) on MSY reference points and the ratio of SSB_{MSY}/SSB_0