A primer on fitting growth models to age estimates

Daniel Ricard - DFO Science Gulf Region

2023-02-02 - TESA Workshop

von Bertalanffy growth model - background

- original published in 1934, the growth model of von Bertalanffy is the most commonly used in fisheries science
- the basis of the model is that growth is the result of the opposing forces of anabolism and catabolism

Bertalanffy (1934)

von Bertalanffy growth model - notation

Model parameterised with age at length 0:

$$L_a = L_\infty \times (1 - exp\,(-k\,(a - t_0)))$$

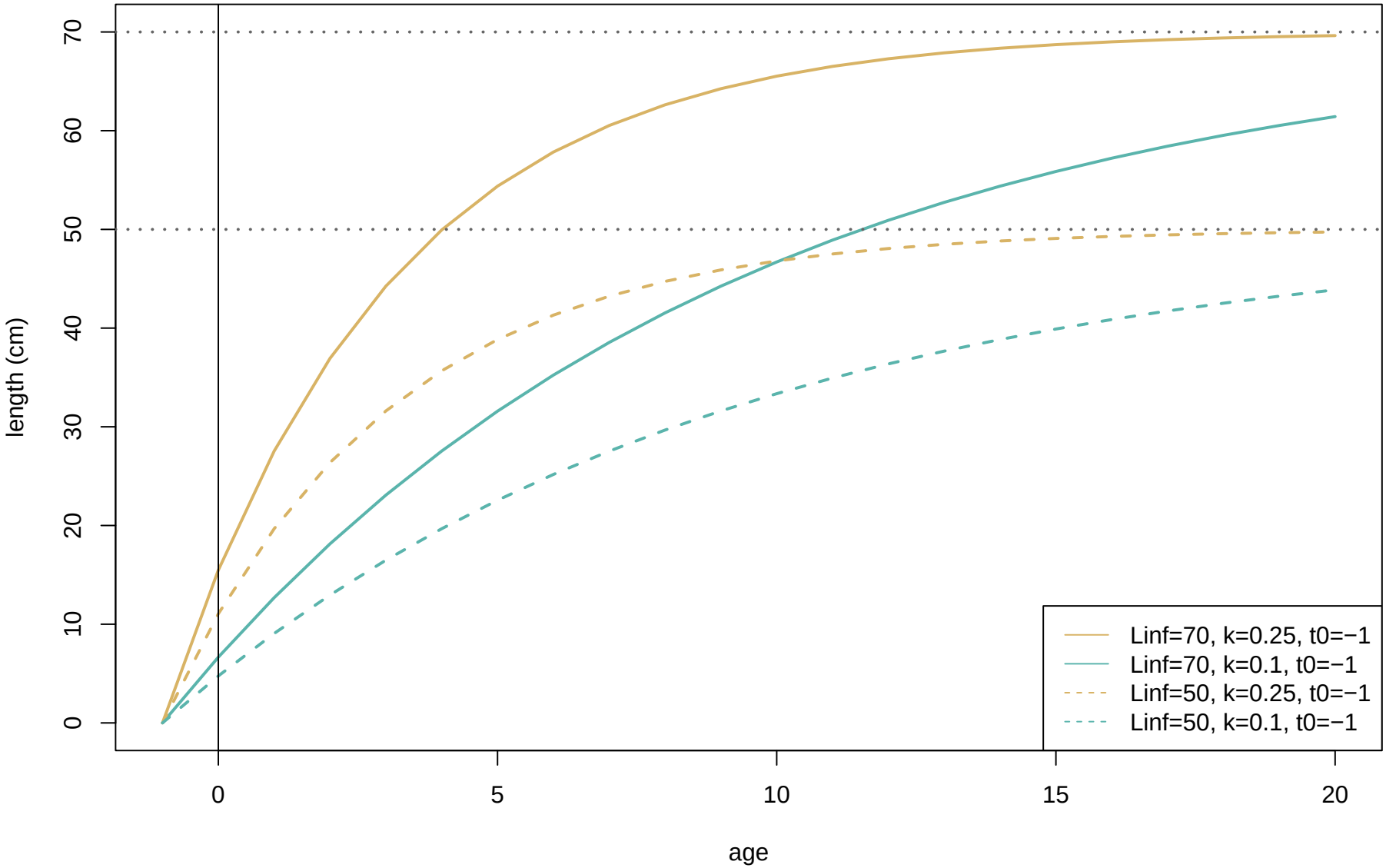
Alternatively, the model can be parameterised with the length at age 0:

$$L_a = ((L_\infty - L_0) \times (1 - exp\,(-ka))) + L_0$$

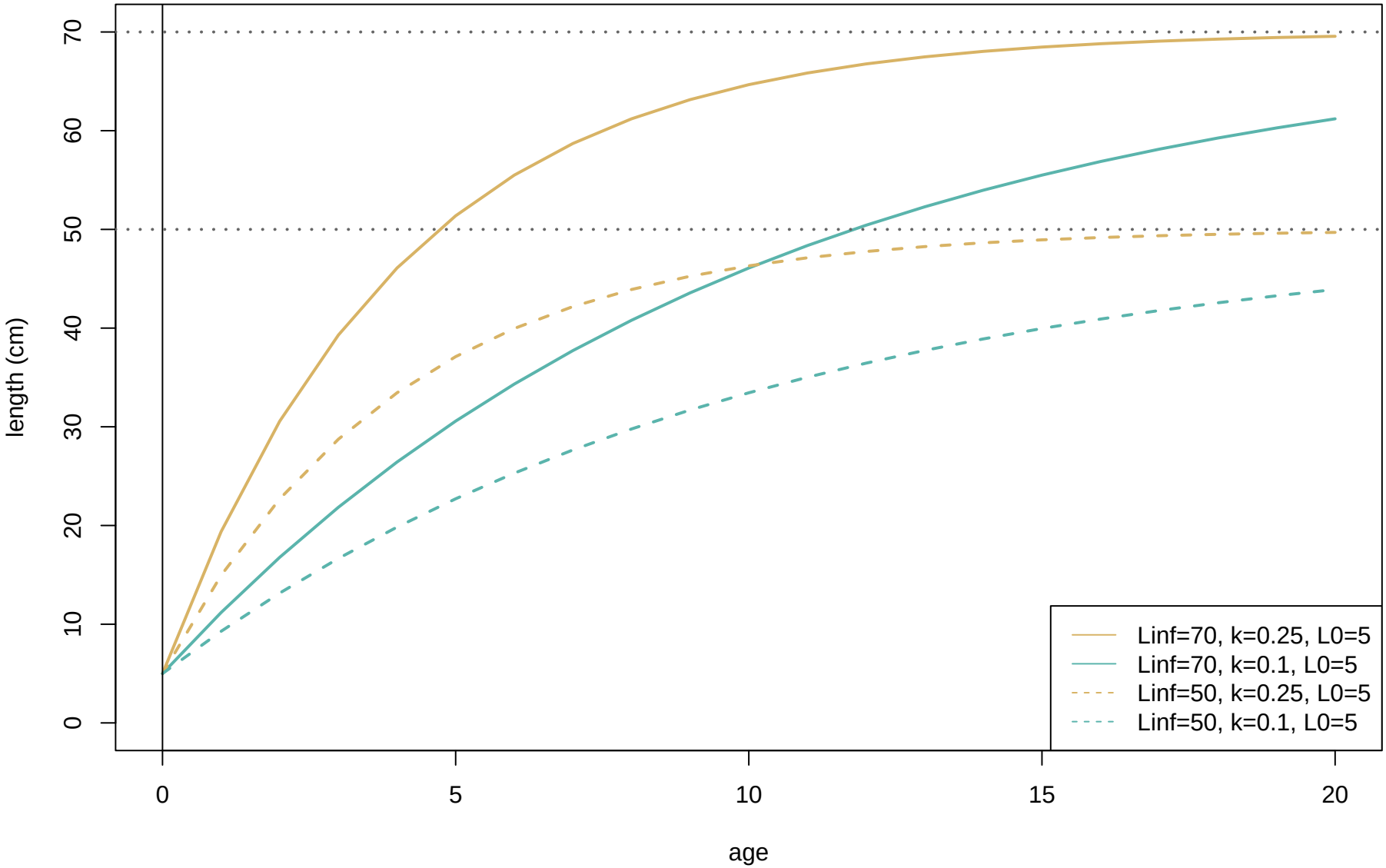
von Bertalanffy growth model - parameters

- L_∞ is the asymptotic length that an individual will reach as it ages
- k is not a "growth parameter"
- t_0 or L_0 are the model intercept

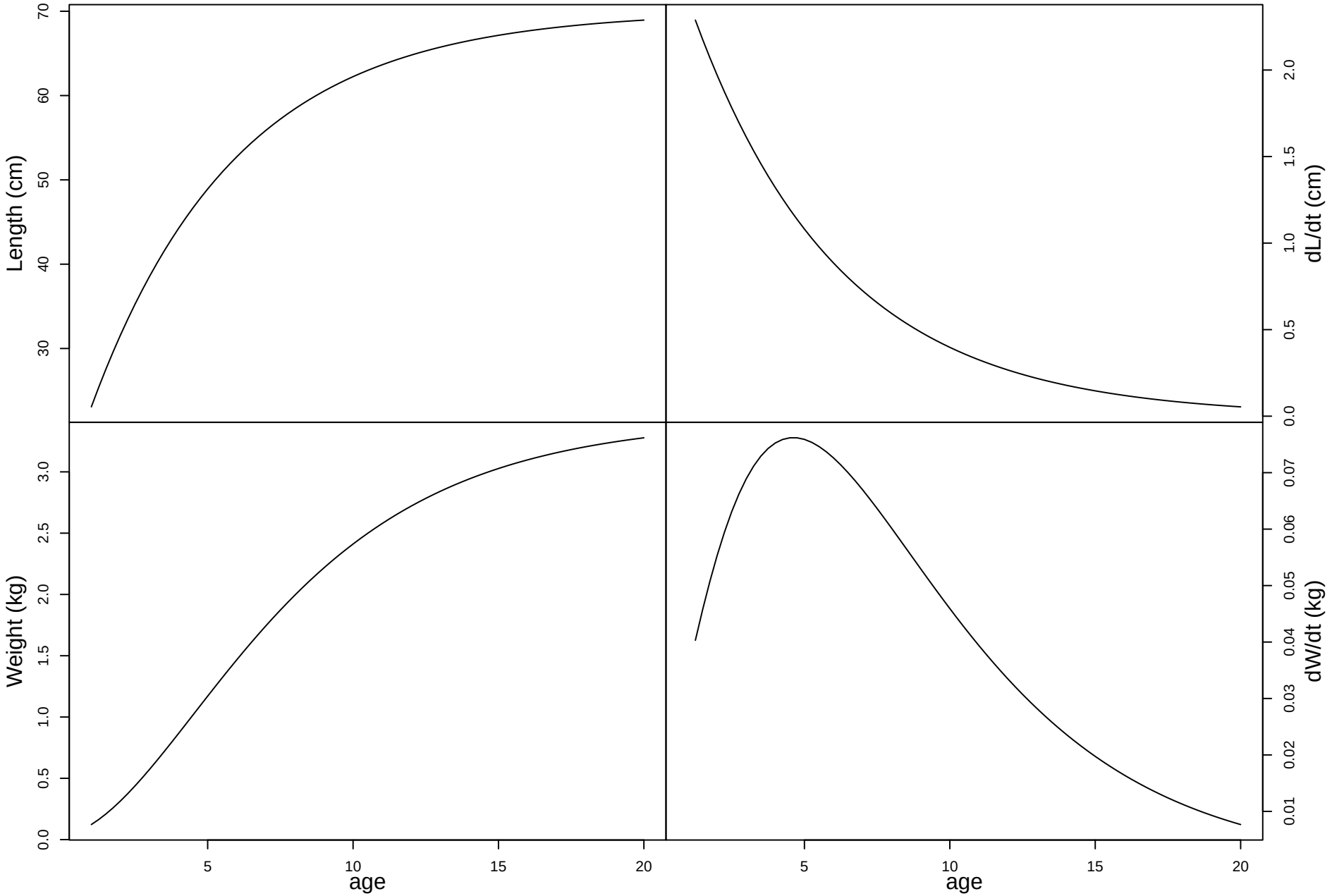
von Bertalanffy growth model - using age at length 0



von Bertalanffy growth model - using length at age 0



First-order derivative of VB function



Ageing data for American Plaice in the southern Gulf of St. Lawrence

Fitting a single VB growth model for American Plaice

- in R, the *nls* function can be used to fit a VB model to data consisting in age-length pairs
- we will help *nls* by providing starting values for each parameter to be estimated

```
VB.fit <- nls(
  length ~ Linf * (1-exp(-k * (age-t0))),
  data=x,
  start=list(Linf=50, k=0.1, t0=-1)
  )
```

Fitting a single VB growth model for American Plaice

```
print(summary(VB.fit))
```

```
##
## Formula: length ~ Linf * (1 - exp(-k * (age - t0)))
##
## Parameters:
##       Estimate Std. Error t value Pr(>|t|)
## Linf 51.8271715  0.1621220   319.7   <2e-16 ***
## k     0.0965155  0.0006285   153.6   <2e-16 ***
## t0   -0.8013373  0.0135365   -59.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## Residual standard error: 3.894 on 137849 degrees of freedom
##
## Number of iterations to convergence: 4
## Achieved convergence tolerance: 5.572e-06
```

Fitting a single VB growth model for American Plaice

Fitting a VB growth model with sex-specific parameter values

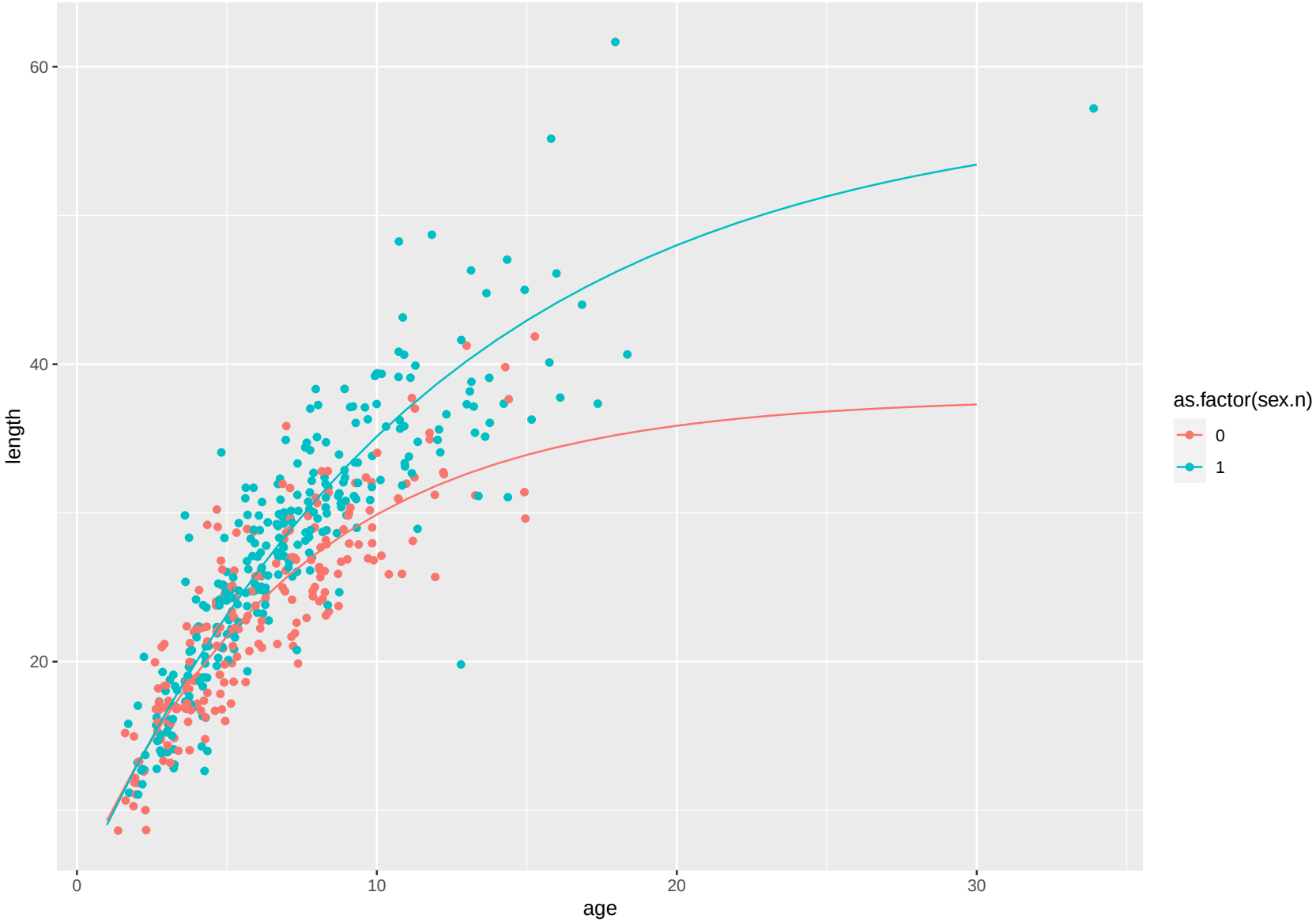
```
VB.fit <- nls(
  length ~ (Linf+(sex.n*Linf.sex.dev))*
    (1 - exp((-k+(sex.n*k.sex.dev)) *
                (age - t0)
              )
     ),
  data=x,
  start=list(
    Linf=50,
    Linf.sex.dev=15,
    k=0.1,
    k.sex.dev=0,
    t0=-1)
  )
```

Fitting a VB growth model with sex-specific parameter values

```
print(summary(VB.fit))
```

```
##
## Formula: length ~ (Linf + (sex.n * Linf.sex.dev)) * (1 - ex
##     k.sex.dev)) * (age - t0)))
##
## Parameters:
##                Estimate Std. Error t value Pr(>|t|)
## Linf         37.7387240  0.1242445  303.75   <2e-16 ***
## Linf.sex.dev 19.6300135  0.1881493  104.33   <2e-16 ***
## k             0.1430340  0.0011530  124.06   <2e-16 ***
## k.sex.dev     0.0566754  0.0008466   66.94   <2e-16 ***
## t0           -0.9821263  0.0179327  -54.77   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
```

Fitting a VB growth model with sex-specific parameter values



References

Bertalanffy, L. von. 1934. Untersuchungen über die gesetzlichkeiten des wachstums. 1. Allgemaine grundlagen der theorie. Roux'Arch Entwicklungsmech Org 131: 613–653.