A primer on fitting growth models to ageing data

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von Bertalanffy growth model - background

- original published in 1934, the growth model of vonBertalanffy is the most commonly used in fisheries science
- the basis of the model is that growth is the result of opposing forces on anabolism and catabolism

Bertalanffy (1934)

von Bertalanffy growth model - notation

Model parameterised with age at length 0:

$$L_a = L_\infty \times (1 - exp\,(-k\,(a - t_0)))$$

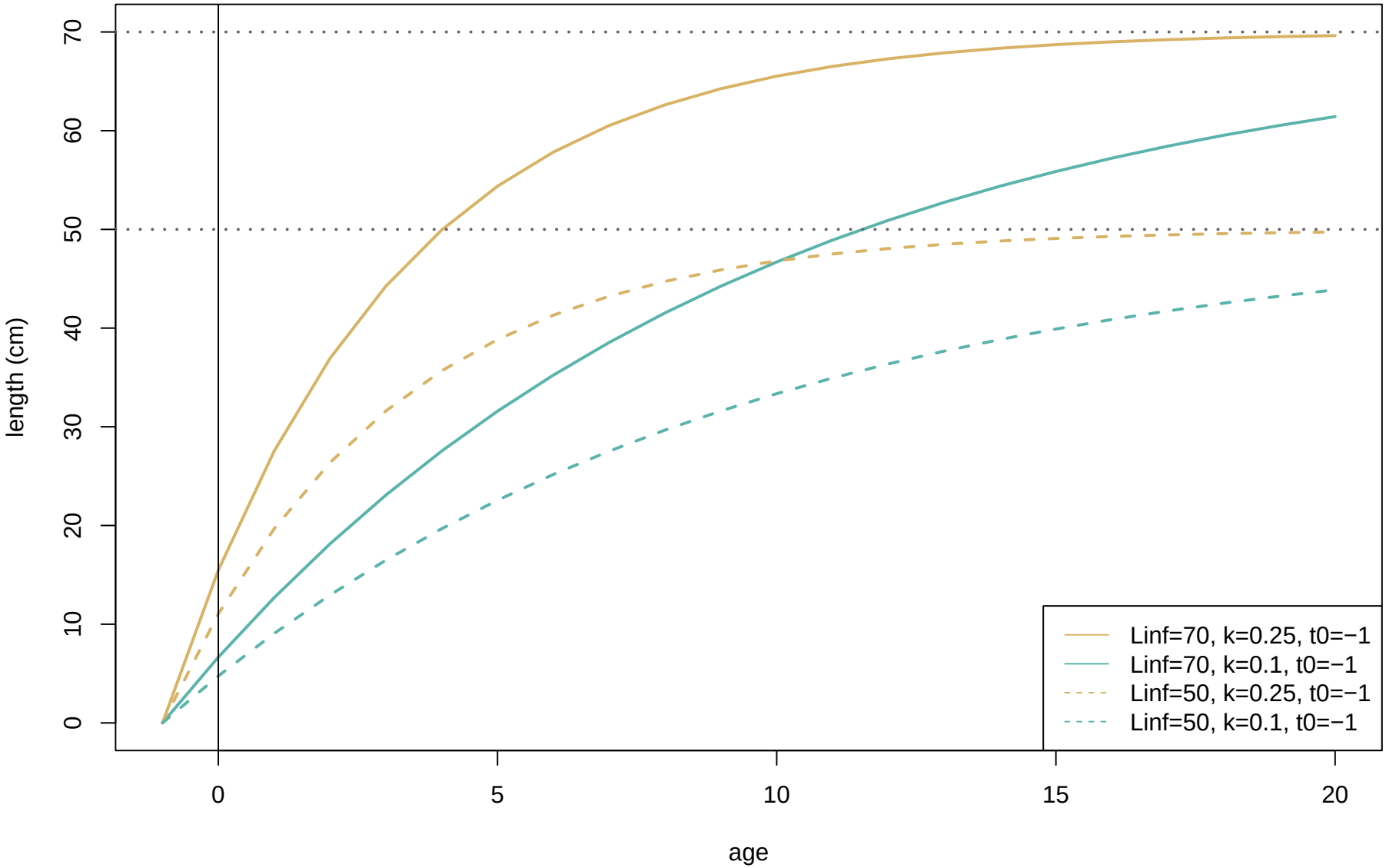
Alternatively, the model can be parameterised with the length at age 0:

$$L_a = ((L_\infty - L_0) \times (1 - exp\,(-ka))) + L_0$$

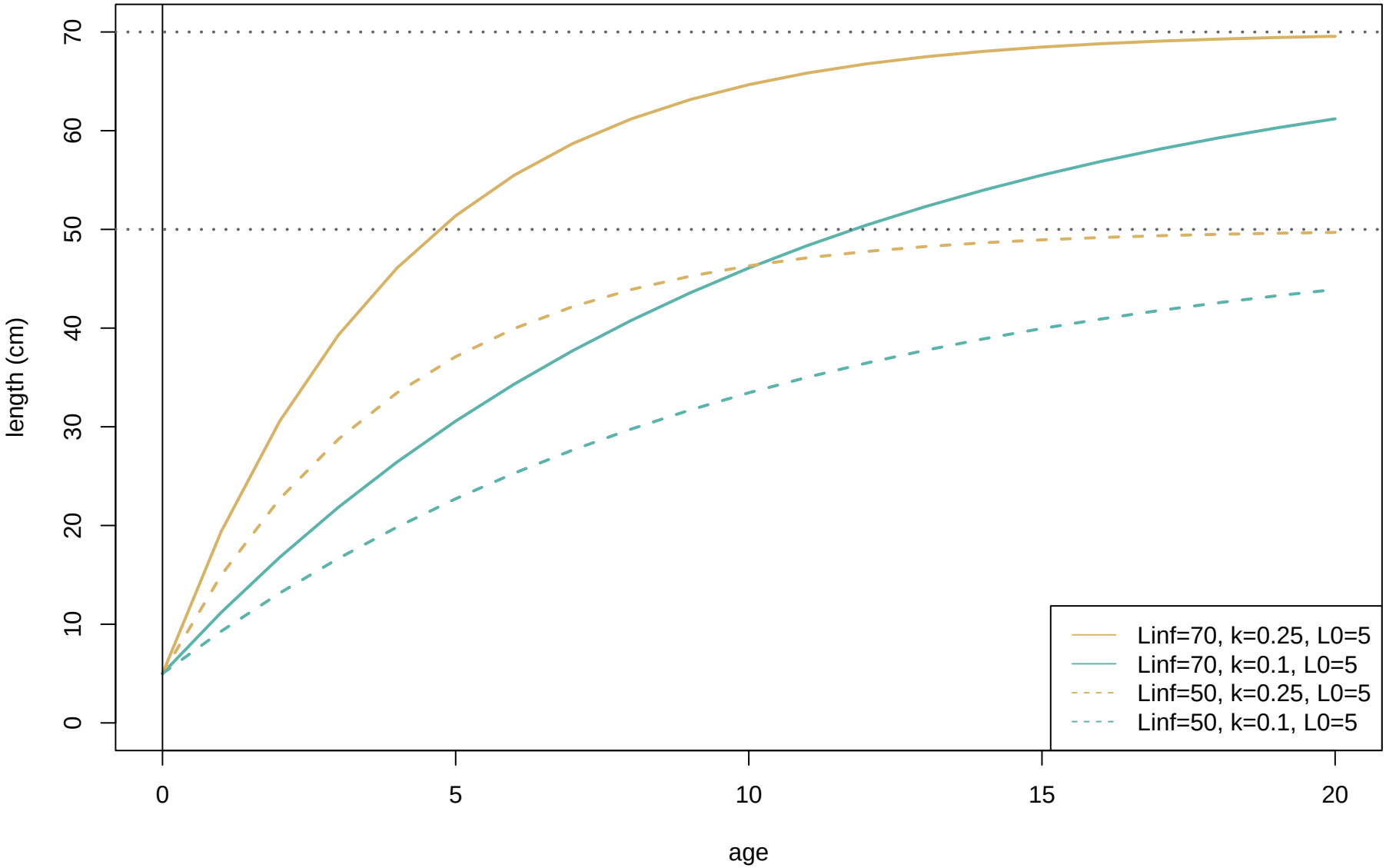
von Bertalanffy growth model - parameters

- L_∞ is the asymptotic length that an individual will reach as it ages
- k is not a "growth parameter"
- t_0 or L_0 are the model intercept

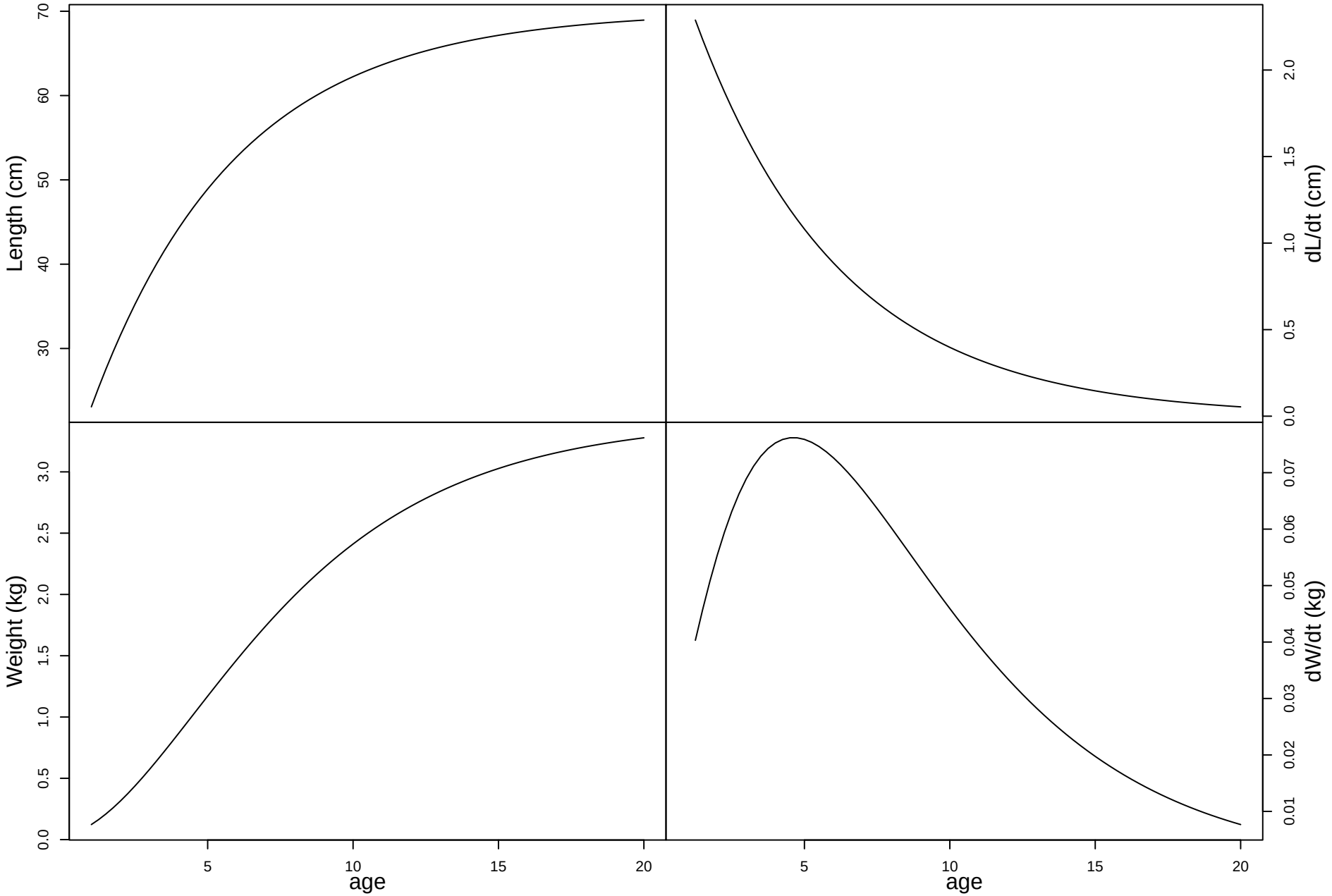
von Bertalanffy growth model - using age at length 0



von Bertalanffy growth model - using length at age 0



First-order derivative of VB function



Replace length with weight

$$W = \alpha L^{\beta} \tag{2}$$

Ageing data for American Plaice in the southern Gulf of St. Lawrence

Fitting a single VB growth model for American Plaice

- in R, the *nls* function can be used to fit a VB model to data consisting in age-length pairs
- we will help *nls* by providing starting values for each parameter to be estimated

Fitting a single VB growth model for American Plaice

```
##
## Formula: length ~ Linf * (1 - exp(-k * (age - t0)))
##
## Parameters:
##       Estimate Std. Error t value Pr(>|t|)
## Linf 51.8271715  0.1621220   319.7   <2e-16 ***
## k     0.0965155  0.0006285   153.6   <2e-16 ***
## t0   -0.8013373  0.0135365   -59.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## Residual standard error: 3.894 on 137849 degrees of freedom
##
## Number of iterations to convergence: 4
## Achieved convergence tolerance: 5.572e-06
```

Fitting a single VB growth model for American Plaice

Fitting a VB growth model with sex-specific parameter values

```
##
## Formula: length ~ (Linf + (sex.n * Linf.sex.dev)) * (1 - ex
##     k.sex.dev)) * (age - t0)))
##
## Parameters:
##                Estimate Std. Error t value Pr(>|t|)
## Linf         37.7387240  0.1242445  303.75   <2e-16 ***
## Linf.sex.dev 19.6300135  0.1881493  104.33   <2e-16 ***
## k             0.1430340  0.0011530  124.06   <2e-16 ***
## k.sex.dev     0.0566754  0.0008466   66.94   <2e-16 ***
## t0           -0.9821263  0.0179327  -54.77   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## Residual standard error: 3.539 on 128641 degrees of freedom
##
```

Fitting a VB growth model with sex-specific parameter values

Fitting a VB growth model with sex-specific and decadal parameter values

Alternatively, implement as a Bayesian model in JAGS

Alternatively, implement in TMB

Individual growth trajectories

Fitting a mixed effects VB model to growth trajetories

Using length frequencies to estimate growth

What if no ageing is available? What if we could use length frequencies to track cohorts and estimate their growth?

Other growth models

- Gompertz model
-

Questions, comments, suggestions

References

Bertalanffy, L. von. 1934. Untersuchungen über die gesetzlichkeiten des wachstums. 1. Allgemaine grundlagen der theorie. Roux'Arch Entwicklungsmech Org 131: 613–653.