

A primer on fitting growth models to age estimates

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von Bertalanffy growth model - background

- original published in 1934, the growth model of von Bertalanffy is the most commonly used in fisheries science
- the basis of the model is that growth is the result of the opposing forces of anabolism and catabolism

Bertalanffy (1934)

von Bertalanffy growth model - notation

Model parameterised with age at length 0:

$$L_a = L_{\infty} \times (1 - \exp(-k(a - t_0)))$$

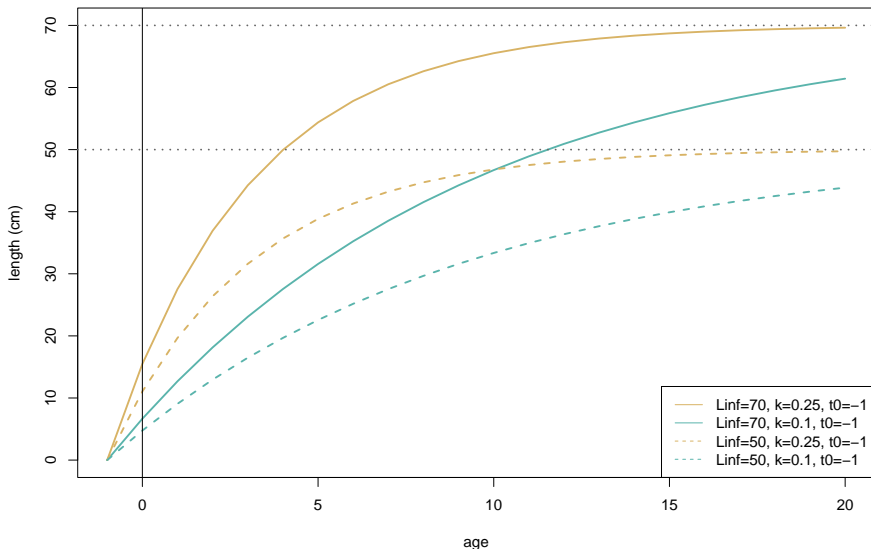
Alternatively, the model can be parameterised with the length at age 0:

$$L_a = ((L_{\infty} - L_0) \times (1 - \exp(-ka))) + L_0$$

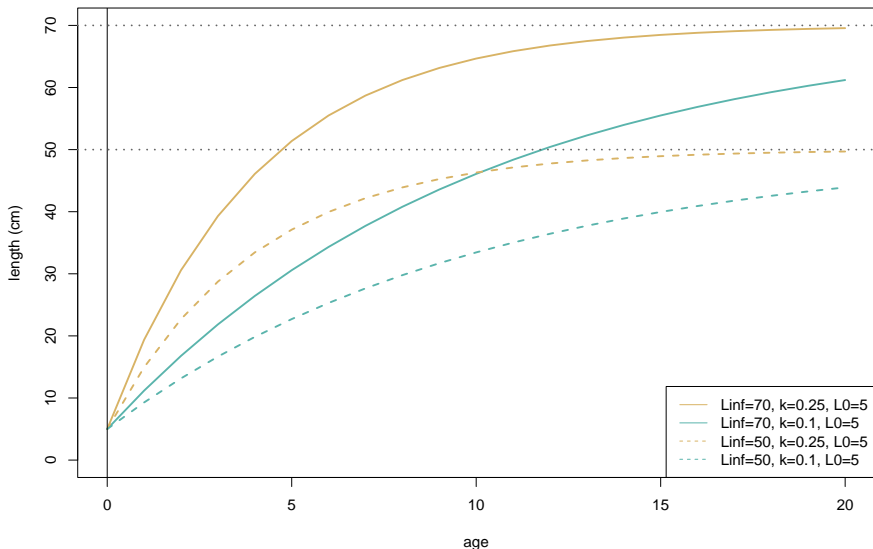
von Bertalanffy growth model - parameters

- L_{∞} is the asymptotic length that an individual will reach as it ages
- k is not a “growth parameter”
- t_0 or L_0 are the model intercept

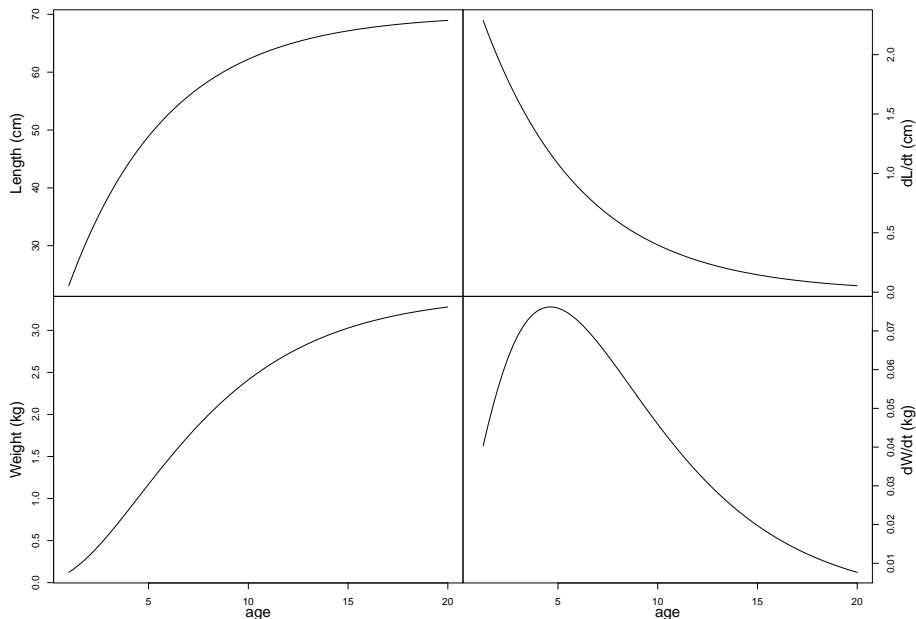
von Bertalanffy growth model - using age at length 0



von Bertalanffy growth model - using length at age 0



First-order derivative of VB function



Ageing data for American Plaice in the southern Gulf of St. Lawrence



Fitting a single VB growth model for American Plaice

- in R, the *nls* function can be used to fit a VB model to data consisting in age-length pairs
- we will help *nls* by providing starting values for each parameter to be estimated

```
VB.fit <- nls(  
  length ~ Linf * (1-exp(-k * (age-t0))),  
  data=x,  
  start=list(Linf=50, k=0.1, t0=-1)  
)
```

Fitting a single VB growth model for American Plaice

```
print(summary(VB.fit))
```

```
##
```

```
## Formula: length ~ Linf * (1 - exp(-k * (age - t0)))
```

```
##
```

```
## Parameters:
```

```
##           Estimate Std. Error t value Pr(>|t|)
```

```
## Linf 51.8271715  0.1621220   319.7   <2e-16 ***
```

```
## k      0.0965155  0.0006285   153.6   <2e-16 ***
```

```
## t0     -0.8013373  0.0135365   -59.2   <2e-16 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
```

```
## Residual standard error: 3.894 on 137849 degrees of freedom
```

```
##
```

```
## Number of iterations to convergence: 4
```

```
## Achieved convergence tolerance: 5.572e-06
```

Fitting a single VB growth model for American Plaice



Fitting a VB growth model with sex-specific parameter values

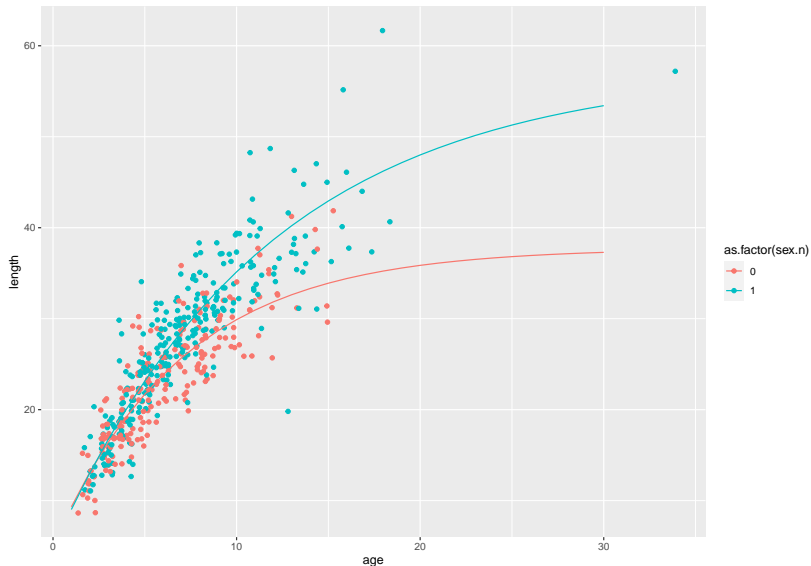
```
VB.fit <- nls(  
  length ~ (Linf+(sex.n*Linf.sex.dev))*  
    (1 - exp((-k+(sex.n*k.sex.dev)) *  
              (age - t0)  
            )  
  ),  
  data=x,  
  start=list(  
    Linf=50,  
    Linf.sex.dev=15,  
    k=0.1,  
    k.sex.dev=0,  
    t0=-1)  
)
```

Fitting a VB growth model with sex-specific parameter values

```
print(summary(VB.fit))
```

```
##
## Formula: length ~ (Linf + (sex.n * Linf.sex.dev)) * (1 - exp(-k * (age - t0)))
##
## Parameters:
##              Estimate Std. Error t value Pr(>|t|)
## Linf          37.7387240  0.1242445  303.75  <2e-16 ***
## Linf.sex.dev  19.6300135  0.1881493  104.33  <2e-16 ***
## k              0.1430340  0.0011530  124.06  <2e-16 ***
## k.sex.dev      0.0566754  0.0008466   66.94  <2e-16 ***
## t0            -0.9821263  0.0179327  -54.77  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Fitting a VB growth model with sex-specific parameter values



References

Bertalanffy, L. von. 1934. Untersuchungen über die gesetzmäßigkeiten des wachstums. 1. Allgemeine grundlagen der theorie. Roux'Arch Entwicklungsmech Org 131: 613–653.