EM Polarization Slides

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1 Chapter 1

1.1 Part 1

Starting with the complex wave equation:

$$\tilde{\vec{E}}\left(\vec{r},t\right) = \tilde{\vec{E}}_0 \exp\left(i\vec{k}\cdot\vec{r} - i\omega t\right) \tag{1}$$

where $\tilde{\vec{E}}_0$ is the complex amplitude, $\vec{k}$ is the wave vector, and ω is the angular frequency. Let $\hat{\tilde{E}}_0 = \hat{x}$ and $\hat{k} = \hat{z}$. Note that this satisfies the condition that the electric field is perpendicular to the direction of propagation, i.e. $\hat{E}\cdot\hat{k} = 0$. The wave is linearly polarized in the $\hat{x}$ direction and depends only on the z-component of $\vec{r}$.

$$\tilde{\vec{E}}\left(z,t\right) = \widetilde{E}_0 \exp\left(ikz - i\omega t\right)\hat{x} \tag{2}$$

Here we consider the electric field in the xy-plane, or $z = 0$

$$\tilde{\vec{E}}\left(z,t\right)\bigg|_{z=0} = \widetilde{E}_0 \exp\left(-i\omega t\right)\hat{x} \tag{3}$$

What will the plot of the electric field in the xy-plane look like?

1.2 Part 2

Here we consider the same electric field at $z = 0$, but from a 3-D perspective

1.3 Part 3

Imagine observing a point on the plane wave at light speed. What will the electric field look like from this perspective?

Hint: An observer moving at light speed will have a moving coordinate of $z = ct$, where c is the speed of light. How is the speed of light related to the wave vector and the angular frequency? What does the plane wave equation evaluate to by using the moving coordinate $z = ct$?

$$\left.\vec{\widetilde{E}}(z,t)\right|_{z=ct} = \widetilde{E}_0 \exp(ikct - i\omega t)\,\hat{x} \tag{4}$$
$$= \widetilde{E}_0 \exp\left(ik\frac{\omega}{k}t - i\omega t\right)\hat{x} \tag{5}$$
$$= \widetilde{E}_0 \exp(i\omega t - i\omega t)\,\hat{x} \tag{6}$$
$$= \widetilde{E}_0\hat{x} \tag{7}$$

What will the plot of the electric field look like as time is animated?

1.4 Part 4

Consider the electric field for some fixed z, or $z = z_0$. At some arbitrary time t_0, what is the electric field? After time dt, how far will the electric field move for this fixed z_0? As the electric field at z_0 moves along the z-axis, does the electric field vary or remain constant?

$$dz = c\,dt = \frac{\omega}{k}dt \tag{8}$$

$$\left.\vec{\widetilde{E}}(z,t)\right|_{z=z_0,\; t=t_0} = \widetilde{E}_0 \exp(ikz_0 - i\omega t_0)\,\hat{x} \tag{9}$$

$$\left.\vec{\widetilde{E}}(z,t)\right|_{z=z_0+dz,\; t=t_0+dt} = \widetilde{E}_0 \exp(ik(z_0+dz) - i\omega(t_0+dt))\,\hat{x} \tag{10}$$
$$= \widetilde{E}_0 \exp(ikz_0 + ik\,dz - i\omega t_0 - i\omega\,dt)\,\hat{x} \tag{11}$$
$$= \widetilde{E}_0 \exp\left(ikz_0 + ik\frac{\omega}{k}dt - i\omega t_0 - i\omega\,dt\right)\hat{x} \tag{12}$$
$$= \widetilde{E}_0 \exp(ikz_0 + i\omega\,dt - i\omega t_0 - i\omega\,dt)\,\hat{x} \tag{13}$$
$$= \widetilde{E}_0 \exp(ikz_0 - i\omega t_0)\,\hat{x} \tag{14}$$
$$= \left.\vec{\widetilde{E}}(z,t)\right|_{z=z_0,\; t=t_0} \tag{15}$$

This plot shows the planes $z = z_0$ where the electric field is at a minimum or a maximum. From this plot, what is the wavelength?

1.5 Part 5

Now consider the electric field at time $t = 0$. If the value of the electric field is sampled by varying z, what would the plot look like?

The sampled values of electric field are

$$E_{\text{sample}} \in \widetilde{E}_0 \exp(ikz)\,\hat{x}\,\delta_{z,n\,dz} \tag{16}$$

where dz is the sampling interval, $n \in \mathbb{Z}$, and $\delta_{i,j}$ is the Kronecker delta.

First, the samples of the electric field are plotted

$$\vec{\widetilde{E}}(z,t)\bigg|_{z=n\,dz,\ t=0} = \widetilde{E}_0 \exp\left(ikz\right)\hat{x} \tag{17}$$

Second, the electric field at time $t = 0$ is plotted

$$\vec{\widetilde{E}}(z,t)\bigg|_{t=0} = \widetilde{E}_0 \exp\left(ikz\right)\hat{x} \tag{18}$$

Finally, time is allowed to vary

$$\vec{\widetilde{E}}(z,t) = \widetilde{E}_0 \exp\left(ikz - i\omega t\right)\hat{x} \tag{19}$$

1.6 Part 6

If the plane wave is linearly polarized in the $\hat{x}$-direction, in which direction does the magnetic field point?

$$\vec{\widetilde{B}}(z,t) = \frac{k}{\omega}\left(\hat{k} \times \vec{\widetilde{E}}\right) \tag{20}$$

Here is a plot of the electric and magnetic fields as time varies.