

# **Land Use Efficiency: How Unit Count and Stories Impact Lot Value\***

Ziheng (Tony) Fang

## **Abstract**

For properties on the real estate market, one may expect that the larger the lot size, the larger the market value of the property. In the real world, is that true? Would the relationship between lot size and market value instead be impacted by the total number of units the property contains and the number of stories the property hosts? Some in the real estate industry would wonder if having more residential units or a higher building would increase or decrease the value of the land the property is built on. Data regarding total assessed value of different residential properties was studied from real estate data provided by the Assessor's Office in San Francisco. Methods utilized included building a regression model, studying the interaction effects, and running a hypothesis test for the interaction effects.

## **Introduction**

In many urban areas across the country, the population is extremely dense, making land among one of the most scarcest and valuable resources for real estate developers. In the eyes of a developer, how efficiently land is utilized, whether it's having multiple units in one property or multiple stories in a unit, significantly impacts the city's housing supply. In cities like San Francisco, where housing is limited and prices are up, land use is a significant problem that city leaders want to address. On top of supply, understanding how lot area would interact with the number of units inside a property or the number of stories a unit has can provide additional insights on how efficient land use can actually have an impact on property value in the eyes of a potential homeowner. From this our hypothesis test in this study, we can also see that contrary to the belief that increases in lot area would increase the assessed value, the two variables are actually not related.

---

\*Project GitHub Repository: <https://github.com/TFang96/MATH261A-Project2>

Prior research often examined how the prices of property are predicted by direct predictors such as the lot area, room count, bedroom count, etc. However, these studies very rarely address how variables regarding efficient land use, such as having multiple units in a building and multiple stories in a unit could impact how our predictors affect property value. In reality, many of these variables are interdependent indicating that while we could utilize land more efficiently by having a residential unit with multiple floors and having multiple residential units in one building, in the eyes of a property owner, the value per unit of lot size may also change significantly.

This paper uses data from the San Francisco Assessor-Recorder office to see how property values are impacted not only by the lot area, but by how the number of units inside a property and the number of stories per unit impacts how lot area affects property values. A multiple linear regression model was utilized to examine both the direct effect of lot area on the assessed value and how the unit count and story count affects the impact lot area has on the assessed value through the interaction effect.

By identifying how vertical expansion and having multiple units inside a building affects how a unit change in lot area affects property value, this paper provides additional insight for developers when considering land use. Insights from this analysis can help developers and policymakers better understand trade-offs when it comes to land use efficiency and real estate value.

The Data section gives further details on data that was utilized for this paper. Details on the statistical methods utilized are listed in the Methods section. The Results section includes our statistical findings. The conclusion we drew can be found in the Discussion section.

## Data

The data set used for this paper, Assessor Historical Secured Property Tax Rolls (*Assessor Historical Secured Property Tax Rolls 2025*), provides details such as the assessed values, lot area, number of units, number of stories, and more for properties within the city of San Francisco since 2016. The data was compiled by the city assessor's office. We have a total of 3.72M observations in the original dataset where each observation represents a real estate property. A sample of 50k observations were taken from the original dataset to address computational limitations while still being large enough for stable coefficient estimates. Variables that were used for the response variable in our study included Assessed.Fixtures.Value, Assessed.Improvement.Value, Assessed.Land.Value, and Assessed.Personal.Property.Value, which were summed up into the AssessedValue. Variables used as our predictor variables included Lot.Area, Number.of.Units, and Number.of.Stores. To allow numerical computations, all commas were removed. Since clients look for very different features when on the market for residential and commercial properties, we only kept data on single family residential and multiple family residential units.

Table 1: Summary statistics for assessed property values, unit count within each building and story count within a building.

	Value
Mean_Assessed_Value	7.739162e+05
Assessed_Value_SD	3.212203e+06
MinAssessedValue	0.000000e+00
MaxAssessedValue	3.090602e+08
Mean_LotArea	4.599074e+03
LotArea_SD	2.633166e+05
Min_LotArea	1.900000e-01
Max_LotArea	5.800145e+07
Mean_NumberOfUnits	2.204700e+00
NumberOfUnits_SD	8.187697e+00
Min_NumberOfUnits	0.000000e+00
Max_NumberOfUnits	4.430000e+02
Mean_NumberOfStories	1.619720e+00
NumberOfStories_SD	4.554172e+00
Min_NumberOfStories	0.000000e+00
Max_NumberOfStores	9.940000e+02

## Methods

We started off by fitting a linear regression model based on the dataset using the `lm()` function in R (R Core Team 2024). The original model is as follows:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} X_{i1} + \beta_3 X_{i1} X_{i2} + \epsilon_i$$

In the original regression model, our predictor variables include the lot area of a property ( $X_{i1}$ ), the number of units ( $X_{i2}$ ), and the number of stories ( $X_{i3}$ ). The response variable (Y) is the total assessed value of that property. The intercept ( $\beta_0$ ) represents the hypothetical assessed value of a property that has a lot area of 0 sq. ft, 0 units, and 0 stories. The coefficient for lot area ( $\beta_1$ ) represents the change in assessed value per unit increase in lot area. The coefficient  $\beta_2$  represents the interaction effect between lot area and the number of units, or the change in the effect of lot area for every extra unit in a property. The coefficient  $\beta_3$  represents the interaction effect between lot area and the number of stories, or the change in the effect of lot area for every extra story in a property. We also have an error term,  $\epsilon_i$  which includes variations in assessed value that are not influenced by our predictor variables.

However, upon examining the residual vs. fitted values plot, we can observe heteroskedasticity, reflecting unequal variance among the error terms, which makes our hypothesis tests off. To

address this, we applied a log transformation to the response variable, where the transformed model shows substantially more constant variance.

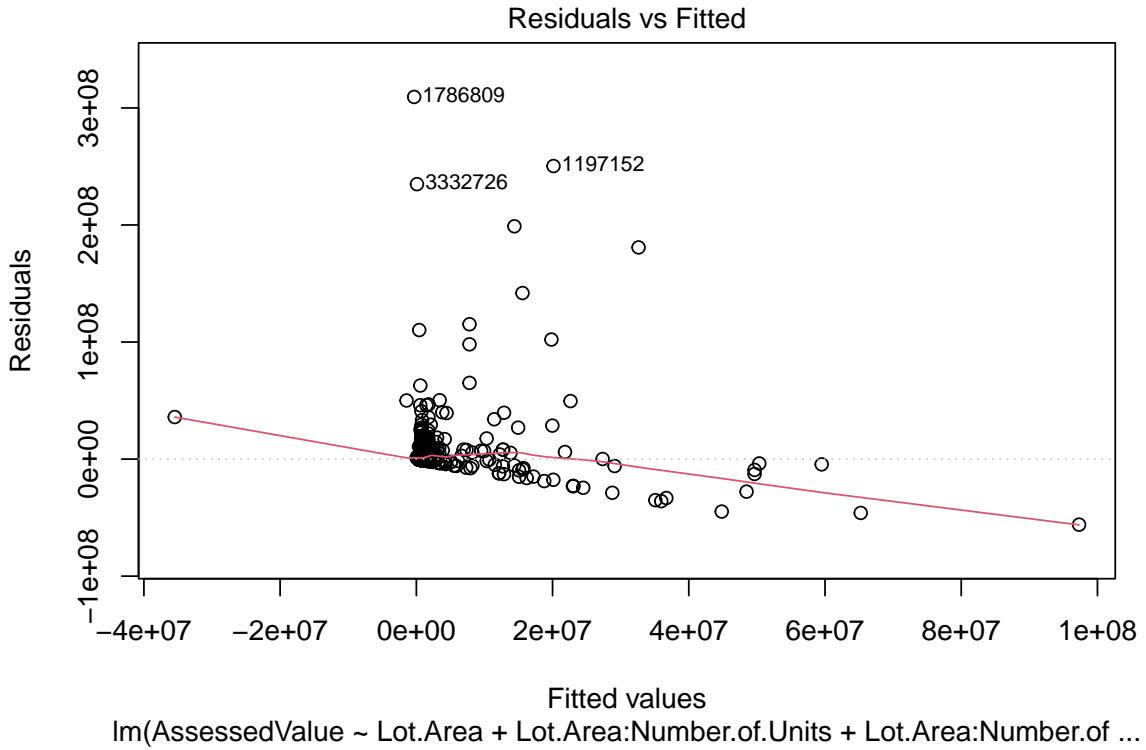


Figure 1: Residual vs. Fitted Values in original model. Observe extreme heteroskedasticity.

The transformed model is as follows:

$$\log(Y_i + 1) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} X_{i1} + \beta_3 X_{i3} X_{i1} + \epsilon_i$$

In our transformed model, our predictor and response variables remain the same. However, when it comes to our coefficients, the intercept ( $\beta_0$ ) represents the predicted log-assessed value when all other predictors are equal to 0, which is not realistic in our context, the coefficient for lot area ( $\beta_1$ ) represents the percentage change in the assessed value for a unit increase in lot area when all other predictors are 0, the coefficient for the interaction effect between lot area and the number of units ( $\beta_2$ ) represents the percentage change in the percentage change in assessed value per unit increase in lot area for every extra unit in the property, and the coefficient for the interaction effect between lot area and the number of stories ( $\beta_3$ ) represents

the change in the percentage change in assessed value per unit increase in lot area for every extra story in the property.

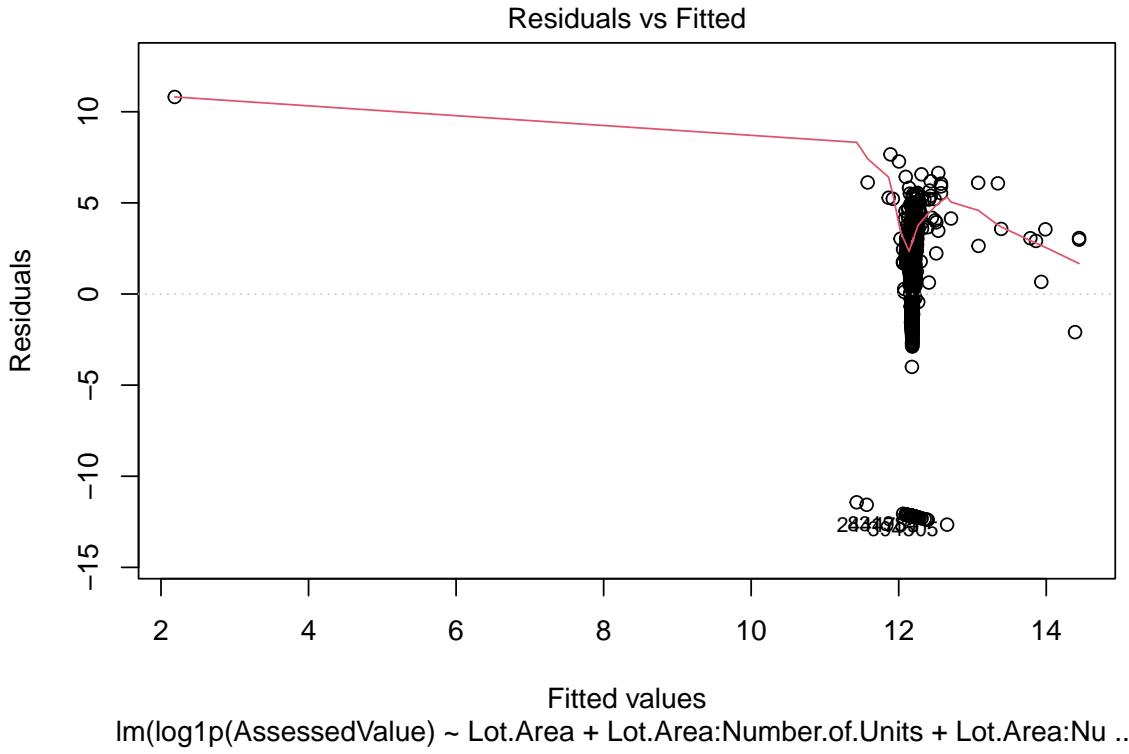


Figure 2: Residuals vs. fitted values in log transformed model. Observe reduced heteroskedasticity.

A limitation we still have is that while the log transformation reduced the heteroskedasticity in a substantial amount, some heteroskedasticity remains, particularly at the more extreme values. This would mean that the hypothesis tests using the p-values might still be slightly off.

## Results

For each coefficient ( $\beta_1, \beta_2, \beta_3$ ), we ran a t-test as our hypothesis test.

For  $\beta_1$ , which represents the percentage change in assessed value per unit increase in lot area, we had a null hypothesis  $H_0$ , which assumed  $\beta_1$  to be equal to 0. This would indicate that there is no relationship between lot area and the assessed value. Our alternative hypothesis,

$H_a$ , would assume that  $\beta_1$  was not equal to 0, indicating a relationship between lot area and assessed value. The test statistic we used was:

$$t = \frac{b_1 - 0}{SE(b_1)}$$

We observed a p-value of less than  $2 * 10^{-16}$ , which leads us to reject the null hypothesis. Observing an estimated coefficient of  $-3.022 * 10^{-6}$ , we can estimate that an extra square foot in lot size leads to a decrease in assessed value by  $-3.022 * 10^{-4}\%$ , which is marginal.

For  $\beta_2$ , which represents the change in the percentage change in assessed value per extra square foot of lot area for each extra unit in the building, we also had a null hypothesis  $H_0$ , which assumed  $\beta_2$  to be equal to 0. This would indicate that the rate of change in the assessed value per extra square foot in lot area is not affected by extra units in the property. In contrast, the alternative hypothesis  $H_a$ , would assume that  $\beta_2$  is not equal to 0, indicating that the rate of change in the assessed value per extra square foot in lot area is affected by extra units in the property. The test statistic we used was:

$$t = \frac{b_2 - 0}{SE(b_2)}$$

We observed a p-value of less than  $4.05 * 10^{-12}$ , which leads us to reject the null hypothesis. Observing an estimated coefficient of  $-6.352 * 10^{-9}$ , we can estimate that each extra unit inside a property leads to the percentage change in the assessed value per extra square foot in lot area to decrease by  $-6.352 * 10^{-7}\%$ .

For  $\beta_3$ , which represents the change in the percentage change in assessed value per extra square foot of lot area for each extra story in the building, we also had a null hypothesis  $H_0$ , which assumed  $\beta_3$  to be equal to 0. This would indicate that the rate of change in the assessed value per extra square foot in lot area is not affected by extra stories in the building. Whereas, our alternative hypothesis  $H_a$ , would assume that  $\beta_3$  is not equal to 0, indicating that the rate of change in assessed value per unit increase in lot area is impacted by an additional story. The test statistic we used was:

$$t = \frac{b_3 - 0}{SE(b_3)}$$

We observed a p-value of less than  $2 * 10^{-16}$ , which leads us to reject the null hypothesis. Observing an estimated coefficient of  $1.53 * 10^{-6}$ , we can estimate that each extra story on a property leads to the percentage change in the assessed value per extra square foot in lot area to increase by  $1.53 * 10^{-4}\%$ .

## Discussion

To look into how common methods used to efficiently use land for housing can impact how an additional square foot of lot area affects the assessed value of housing properties in San Francisco, we analyzed 2378663 residential units. The total assessed value was considered to be the response variable of lot area, as the predictor variable, with units inside the property and the story count of the property as interaction affects.

To determine how additional units in square feet of lot area impacted the assessed value, and how additional units inside the property and additional stories affected the impact lot area had on the assessed value, we ran a t-test as our hypothesis test with a test size of 0.05. For each respective coefficient, we considered a null hypothesis which would indicate that there is no relation between that particular predictor variable and the assessed value or that a particular variable had no impact on how an additional unit of another predictor variable affected the assessed value. The alternative hypothesis would conclude that a particular predictor variable did have an impact on the assessed value or that a particular variable did impact how another variable affected the assessed value through an interaction affect. The estimated coefficients would then tell us about how much of an affect that predictor variable had on the assessed value or how much a variable affected another variable's impact on the assessed value through an interaction affect.

Looking into how an additional square foot of lot area affects the assessed value, we observe a p-value that is significantly smaller than our test size. Thus, we reject the null hypothesis and conclude that the lot area is related to the assessed value of a property. The estimated coefficient also tells us that considering no units inside a property and no stories, we can expect the assessed value to decrease by 0.0003022% for every square foot of lot area.

When we consider how additional units inside a building would impact how each additional square foot in lot area affects the assessed value, we observe a p-value that is also significantly smaller than our test size. In this case, we also reject the null hypothesis and conclude that the number of units a property has will impact how each additional square feet in lot area affects the assessed value. The estimated coefficient tells us that considering no stories, every extra unit inside a building will decrease the change in assessed value per additional square foot in lot area by 0.0000006352%.

If we were to consider how additional stories to a building would impact how each additional square foot in lot area affects the assessed value, by looking at the p-value, we also observe a value that is significantly smaller than our test size. From that we would reject the null hypothesis and conclude that the number of stories a property has will impact how each additional square feet in lot area affects the assessed value. The estimated coefficient tells us that considering no units, every extra story on a building will increase the change in assessed value per additional square foot in lot area by 0.000153%.

For a developer looking into how they can efficiently utilize land while also maximizing profit in real estate, this study would suggest that by building taller buildings will not only be an

efficient use of land, but will have a positive effect on how each additional square foot of lot area used affects the assessed value. On the other hand, while having multiple units inside a building can save land area, it would have a negative effect on how each additional square foot of lot area used affects the assessed value.

However, this research has a limitation that the dataset used only contains properties in San Francisco, an urban city with significant population density. In an urban area, it can be reasonable to see that a taller building might positively affect how an additional square foot of lot area impacts the assessed value, simply because we are trying to save space. However, in more rural areas, many homeowners might prefer their property to be on one story. To make our findings more conclusive for properties regardless of location, we could take samples of real estate properties in urban, suburban, and rural areas.

## References

- Assessor Historical Secured Property Tax Rolls.* 2025. Assessor's Office, City of San Francisco. <https://data.sfgov.org/Housing-and-Buildings/Assessor-Historical-Secured-Property-Tax-Rolls/wv5m-vpq2>.
- R Core Team. 2024. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. <https://www.R-project.org/>.