CSCI 373

Exam 1: Search-Based Reasoning

Name:

You have until 11pm today (Monday, October 3) to complete the exam. Please submit your completed exam either to me personally, or to my mailbox on the first floor of TCL. The exam must be done without consulting anyone else or the general Internet, however you may consult your own notes, the textbook, my provided lecture notes, and my provided lecture slides.

In the interest of allowing blind grading, please write your name only on this cover sheet, and not on the other exam pages.

		Score	Maximum Score
HW	Blocks		10
	8 Out of 10 Cats Does Countdown		10
	Pacman 1 and 2		10
Exam 1	One		10
	Two		15
	Three		5
	Four		10

One

The object of Sudoku is to fill in the missing entries of a grid such that each row, column, and box contains the digits 1-9 exactly once.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

row

box

column

There's also an easier 4x4 version, where only the digits 1-4 are used, but the same rules apply.

As the CEO of YouDoku (an online Sudoku site), you're always looking for opportunities to diversify your offerings. Your hot new product MoveDoku is based on the 4x4 version, and is advertised as "Sudoku meets the sliding tile puzzle." A player starts with a sliding tile puzzle that contains 3 ones, 4 twos, 4 threes, 4 fours, and a blank space – which is considered the fourth "1". Each move is the same as with a sliding tile puzzle: you can slide an adjacent tile into the blank space.

To the right is an example of a starting state and the first two moves of a game.

	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4

NEWDOKU

2	1	1	1
	2	2	2
3	3	3	3
4	4	4	4

NEWDOKU

2	1	1	1
3	2	2	2
	3	3	3
4	4	4	4

NEWDOKU

A player should slide the tiles until they obtain a valid Sudoku puzzle (i.e. no row, column, or box contains the same digit twice). Again, we treat the blank space as the fourth "1". The goal is to get to a solved Sudoku puzzle from the starting state, using a minimal number of moves.

Consider the following three states:

	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4

A

	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

B

2	4	1	3
1	3	4	2
3	1	2	4
4	2	3	

C

Among these, only state C is a valid solution.

(a) **Formally define an admissible heuristic function H such that $H(A) \geq 14$, $H(B) \geq 18$, and $H(C) \geq 0$ (since C is a final state).** When formalizing your heuristic function, you may use the notation $CELLS(M, d)$ to refer to the set of (i, j)-coordinates such that $M_{ij} = d$ for matrix M (treating a Sudoku puzzle as a 2x2 matrix) and digit d. For example:

$$CELLS(A, 2) = \{(2, 1),\ (2, 2),\ (2, 3),\ (2, 4)\}$$
$$CELLS(B, 2) = \{(1, 3),\ (1, 4),\ (2, 3),\ (2, 4)\}$$
$$CELLS(C, 2) = \{(1, 1),\ (2, 4),\ (3, 3),\ (4, 2)\}$$

(b) **Justify in words why your heuristic function is admissible.** No proof is necessary, just give the basic intuition.

Partial credit: If you can't think of an answer that satisfies the requirements, provide an alternative admissible heuristic function H and compute $H(A),\ H(B),\ H(C)$ for your function. Credit will be awarded based on how close H is to achieving the desired bounds, but H **must be admissible**.

Partial credit: If you can think of an answer but can't quite formalize it, describe your heuristic function as precisely as you can in words.

Solution to Question One

(a)

$$H(m) = \sum_{d=2}^{4} \sum_{r=1}^{4} \min\{|x-r| \mid (x,y) \in CELLS(m,d)\}$$
$$+ \sum_{d=2}^{4} \sum_{c=1}^{4} \min\{|y-c| \mid (x,y) \in CELLS(m,d)\}$$

(b)

☆ any given move can do at most one of the following six things:

- change the row of a 2-tile by one
- change the row of a 3-tile by one
- change the row of a 4-tile by one
- change the column of a 2-tile by one
- change the column of a 3-tile by one
- change the column of a 4-tile by one

note that this is not true of 1-tiles: since the empty space represents a 1-tile, 1-tiles can change position **simultaneously** with another tile

	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

- this column needs a 2, a 3, and a 4
- it will take at least 2 moves to get a 2 into the column
- it will take at least 0 moves to get a 3 into the column
- it will take at least 2 moves to get a 4 into the column
- from ☆, none of these moves can occur simultaneously
- so it will take at least 4 moves to get a 2, 3, and 4 into the column

each of the rows and columns need to have this condition independently satisfied

so the sum of these eight bounds is a lower bound on an optimal solution depth

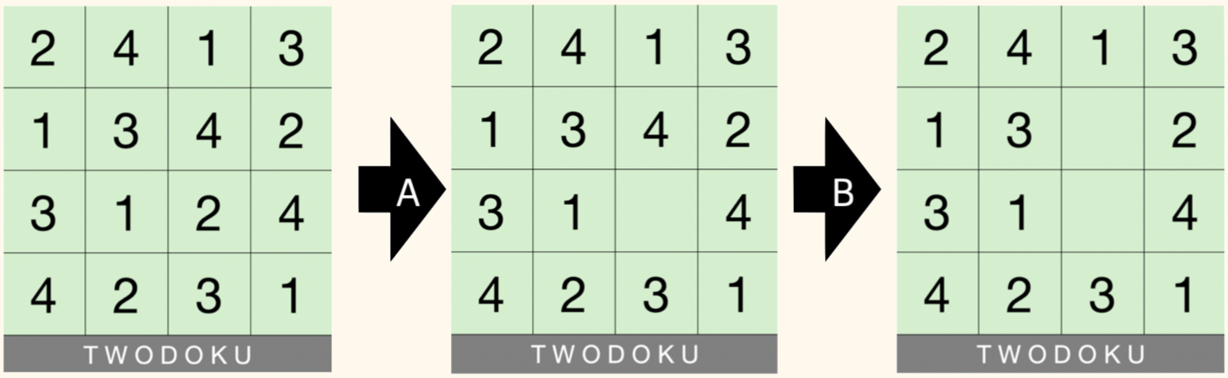
	1	2	2	≥ 4 +
1	1	2	2	≥ 2 +
3	3	4	4	≥ 1 +
3	3	4	4	≥ 2

$\geq 4 + \geq 2 + \geq 1 + \geq 2$

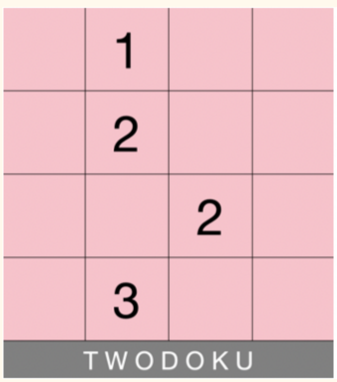
≥ 18

Two

Given the lucrative nature of the multiplayer gaming market, YouDoku's R&D division has also developed a new two-player game called TwoDoku. TwoDoku is advertised as "Sudoku meets Jenga." Starting with a complete board, the players take turns erasing a single digit from the board.



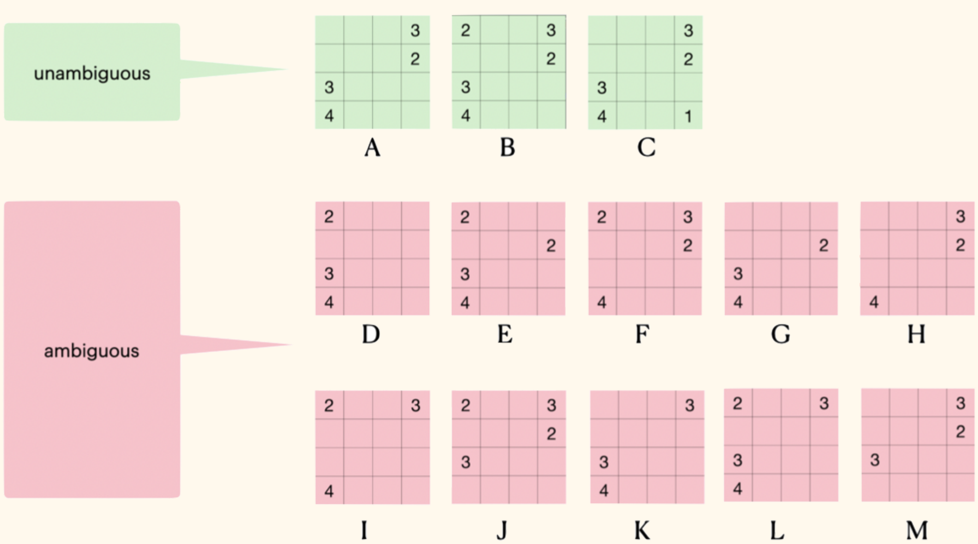
In the above example, Player (i) begins by removing a "2" from cell (3,3). Then Player (ii) responds by removing a "4" from cell (2,3).



You lose the game **if you are the first player to introduce ambiguity into the puzzle**, i.e. you remove a number and the resulting puzzle has more than one solution. If you lose, then your opponent gets points equal to the sum of the values remaining in the puzzle.

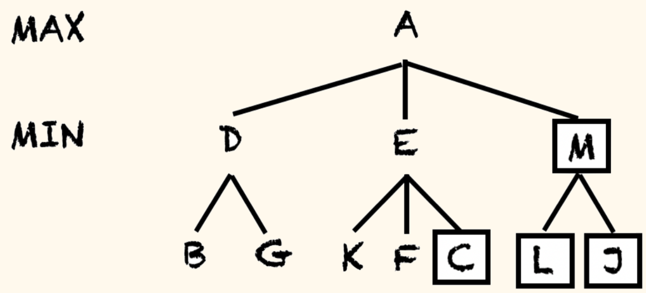
For instance, on the left is a final state in which the winning player would receive a utility of 8 (1+2+2+3), while the losing player would receive a corresponding utility of -8.

For your convenience in solving the upcoming questions, here is a set of states, grouped by whether they are unambiguous (green) or ambiguous (pink).



Emulate the style on the right for your answers to (b) and (d) [This is an example, not a correct answer]:

- Use the alphabetic labels A-M from the previous page to refer to the game states.
- Denote which layers are MIN layers and which layers are MAX layers.
- Put boxes around nodes that are pruned by alpha-beta pruning.



The correct solutions do not necessarily use all the states from the previous page.

Please answer the following:

(a) If the current player is Player (ii), what is the minimax value of state B for Player (i)?
(b) Draw a minimax tree that solves question (a) and maximizes the number of nodes pruned by alpha-beta pruning (assuming left-to-right evaluation of children). Use the alphabetic labels from the previous page to refer to the game states. Put boxes around nodes that are pruned by alpha-beta pruning. If no alpha-beta pruning is possible, write "NO PRUNING IS POSSIBLE" underneath the minimax tree.
(c) Suppose that Player (ii) is actually an AI that plays randomly (i.e. among available moves, it chooses one uniformly at random). If the current player is Player (ii), what is the expectimax value of state B for Player (i)?
(d) Now consider an alternative version of the game where you **win the game** if you are the first player to introduce ambiguity into the puzzle (same point values apply). If the current player is Player (ii), what is the minimax value of state B for Player (i)?
(e) Draw a minimax tree that solves question (d) and maximizes the number of nodes pruned by alpha-beta pruning (assuming left-to-right evaluation of children). Use the alphabetic labels from the previous page to refer to the game states. If no alpha-beta pruning is possible, write "NO PRUNING IS POSSIBLE" underneath the minimax tree.

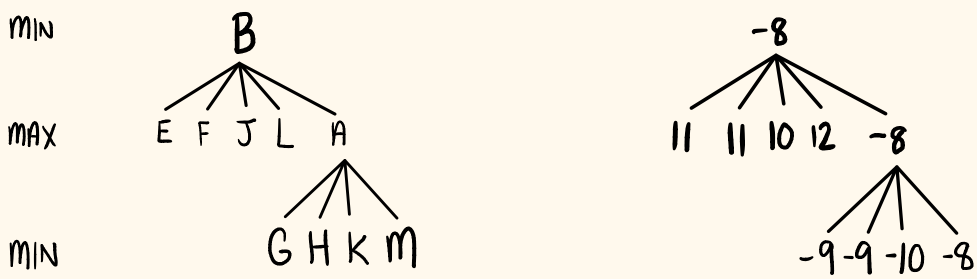
Tips:

- **The answers to (b) and (e) may not be unique. If not, just provide one correct answer.**
- **Be sure to read the game rules and the questions carefully to understand exactly what is being asked.**

Solution to Question Two

(a) -8

(b)

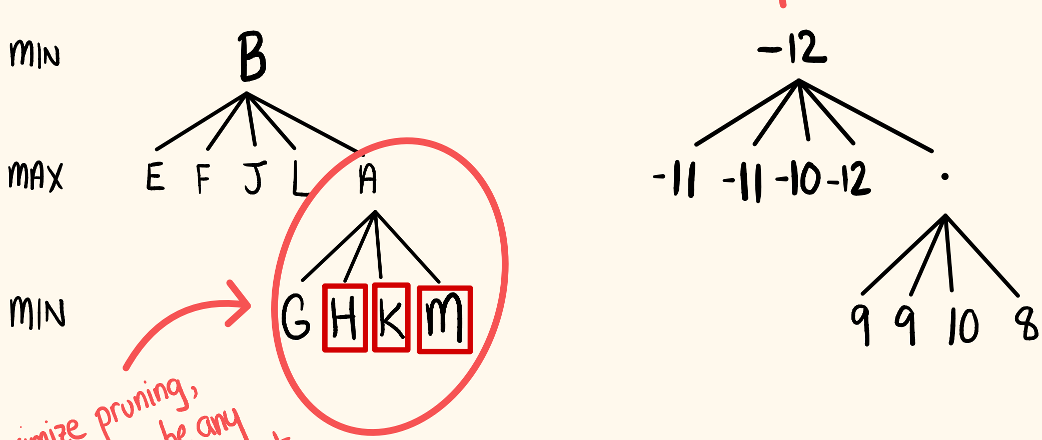


NO PRUNING IS POSSIBLE

(c) $$\frac{11+11+10+12-8}{5} = 7.2$$

(d) -12

(e)



to maximize pruning, this subtree can be any child of the root except the leftmost

Three

Complete weighted state machine $M = (Q, \Sigma, \Delta, q_0, F, w)$ such that depth-first search has runtime $O(n)$ on the state machine, whereas iterative deepening search has runtime $O(n^2)$.

$$Q = \{q_0, q_1, \ldots, q_n\}$$

$$\Sigma = \boxed{\{next\}}$$

$$\Delta = \boxed{\{\langle q_i, next, q_{i+1}\rangle \mid i \in \{0, \ldots, n-1\}\}}$$

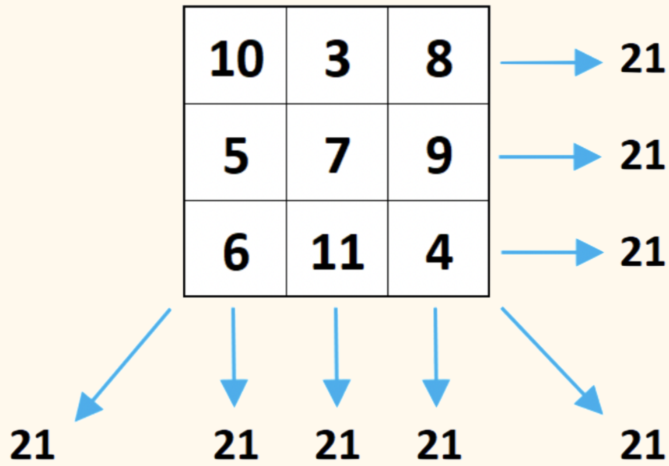
$$q_0 = q_0$$

$$F = \{q_n\}$$

$$\forall d \in \Delta : w(\delta) = 1$$

Four

A magic square is a 3x3 matrix of distinct positive integers where the sum of each diagonal, row, and column is equivalent. On the right is an example where each diagonal, row, and column sums to 21.



(a) Define a state machine $M = (Q, \Sigma, \Delta, q_0, F)$ for finding a magic square whose rows, columns, and diagonals all sum to integer k. **Design the state machine so that it has branching factor $b = 2$ and solution depth $d = 9 + 3k$.**

$$Q = \left\{ (n, M) \;\middle|\; n \in \{1, \ldots, 10\},\ M \in \mathbb{Z}^{3\times 3} \right\}$$

$$\Sigma = \{ \textsc{Increment}, \textsc{Freeze} \}$$

$$\Delta = \left\{ \langle (n,M), \textsc{Increment}, (n, M + E_{ij}) \rangle \;\middle|\; (n,M) \in Q,\ i = 1 + \left\lfloor \frac{n-1}{3} \right\rfloor,\ j = 1 + n - 1 \bmod 3 \right\} \cup \left\{ \langle (n,M), \textsc{Freeze}, (n+1, M) \rangle \;\middle|\; 1 \leq n \leq 9,\ M \in \mathbb{Z}^{3\times 3} \right\}$$

$$q_0 = \left(1, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)$$

$$F = \left\{ (10, M) \;\middle|\; M \in \mathbb{Z}^{3\times 3},\ \begin{array}{l} \sum_{i=1}^{3} M_{ij} = k \ \ \forall j \in \{1,2,3\} \\ \sum_{j=1}^{3} M_{ij} = k \ \ \forall i \in \{1,2,3\} \end{array},\ \begin{array}{l} M_{11} + M_{22} + M_{33} = k \\ M_{31} + M_{22} + M_{13} = k \\ M_{ij} \neq M_{kl} \text{ if } (i,j) \neq (k,l) \end{array} \right\}$$

Tips for part (a):

$$E_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lfloor 3.15 \rfloor = 3$$

$$16 \mod 3 = 1$$

- Refer to set of all $m \times n$ integer matrices as $Z^{m \times n}$. For a matrix M, use the notation M_{ij} to refer to element (i,j) of M.
- You may use the notation E_{ij} to represent a 3x3 matrix whose only nonzero element is element (i,j), which is equal to 1. This is sometimes called a **matrix unit**[1]. An example is provided on the right.
- You may find the floor and modulo operations useful. Examples are provided on the right.
- **You are allowed to re-express the initial state if desired. The provided initial state is simply the one that I used.**

(b) What search algorithm would be best suited to find a solution using your state machine? DFS, BFS, iterative deepening, or depth-limited DFS? Justify your answer.

the space complexity of bfs would be $O(2^{9+3k})$, which would exceed the memory capacity of a computer for even moderate values of k, so <u>not</u> bfs.

iterative deepening would be a decent choice, but since we know the solution depth upfront, the first $8+3k$ iterations would be pure theatre

the best choice, then, is depth-limited search, i.e. run dfs to the solution depth

[1] https://en.wikipedia.org/wiki/Matrix_unit. You may break the rule about consulting the Internet for this one webpage (though it doesn't provide much insight beyond what's already stated above).