

expectimax

26 sept
2022

CSCI
373

WOODLE

CEDAR
EBONY
MAPLE
BEECH
BIRCH

the target word
is chosen uniformly
at random but not
revealed to us

W O O D L E

$$\underline{P(t)}$$

CEDAR	$\frac{1}{5}$
EBONY	$\frac{1}{5}$
MAPLE	$\frac{1}{5}$
BEECH	$\frac{1}{5}$
BIRCH	$\frac{1}{5}$

the target word
is chosen uniformly
at random but not
revealed to us

✓ CEDAR	CEDAR	CEDAR	CEDAR	CEDAR
EBONY	✓ EBONY	EBONY	EBONY	EBONY
MAPLE	MAPLE	✓ MAPLE	MAPLE	MAPLE
BEECH	BEECH	BEECH	✓ BEECH	BEECH
BIRCH	BIRCH	BIRCH	BIRCH	✓ BIRCH

is EBONY a good guess? ✓/✗

✓ CEDAR	✗ CEDAR	CEDAR	✗ CEDAR	✗ CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	✗ EBONY
MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	✗ MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	✗ BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH
—	—	—	—	—
2	1	2	1	1
remain	remains	remain	remains	remains

is EBONY a good guess?

✓ CEDAR	✗ CEDAR	CEDAR	✗ CEDAR	✗ CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	✗ EBONY
MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	✗ MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	✗ BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

expected # of remaining words after guessing EBONY = ?

✓ CEDAR	✗ CEDAR	CEDAR	✗ CEDAR	✗ CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	✗ EBONY
MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	✗ MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	✗ BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

expected # of remaining words after guessing EBONY = ?

$$2 \cdot P(\text{CEDAR}) + 1 \cdot P(\text{EBONY}) + 2 \cdot P(\text{MAPLE}) + 1 \cdot P(\text{BEECH}) + 1 \cdot P(\text{BIRCH})$$

✓ CEDAR	✗ CEDAR	CEDAR	✗ CEDAR	✗ CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	✗ EBONY
MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	✗ MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	✗ BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

expected # of remaining words after guessing EBONY = 1.4

$$2 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5} + 2 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5}$$

✓ CEDAR	CEDAR	CEDAR	CEDAR	CEDAR
EBONY	✓ EBONY	EBONY	EBONY	EBONY
MAPLE	MAPLE	✓ MAPLE	MAPLE	MAPLE
BEECH	BEECH	BEECH	✓ BEECH	BEECH
BIRCH	BIRCH	BIRCH	BIRCH	✓ BIRCH

is BIRCH a good guess? ✓ / ✗

✓ CEDAR	✗ CEDAR	✗ CEDAR	✗ CEDAR	CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	EBONY
✗ MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

— — — — —

| | | | |

remains remains remains remains remains

is BIRCH a good guess? ✓ / ✗

✓ CEDAR	✗ CEDAR	✗ CEDAR	✗ CEDAR	CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	EBONY
✗ MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

expected # of remaining words after guessing BIRCH =

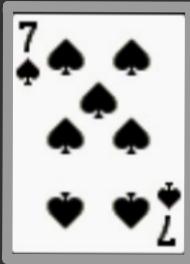
$$1 \cdot P(\text{CEDAR}) + 1 \cdot P(\text{EBONY}) + 1 \cdot P(\text{MAPLE}) + 1 \cdot P(\text{BEECH}) + 1 \cdot P(\text{BIRCH})$$

✓ CEDAR	✗ CEDAR	✗ CEDAR	✗ CEDAR	CEDAR
✗ EBONY	✓ EBONY	✗ EBONY	✗ EBONY	EBONY
✗ MAPLE	✗ MAPLE	✓ MAPLE	✗ MAPLE	MAPLE
✗ BEECH	✗ BEECH	✗ BEECH	✓ BEECH	BEECH
✗ BIRCH	✗ BIRCH	✗ BIRCH	✗ BIRCH	✓ BIRCH

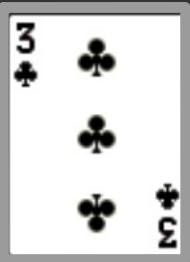
expected # of remaining words after guessing BIRCH = 1.0

$$1 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5} + 1 \cdot \frac{1}{5}$$

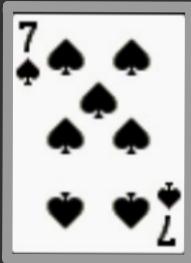
dealer



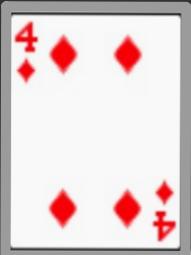
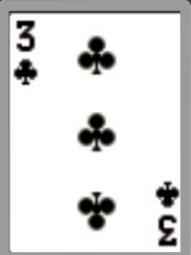
player



dealer

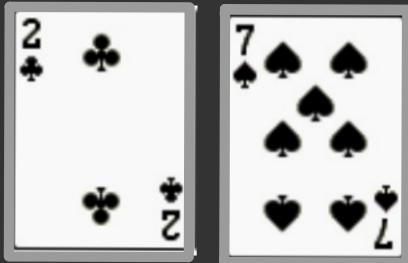


player

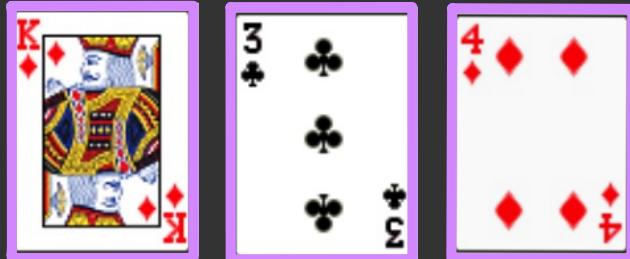


hit

dealer



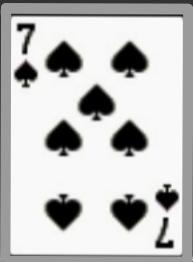
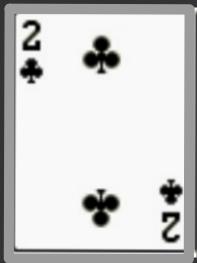
17
player



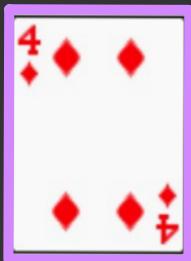
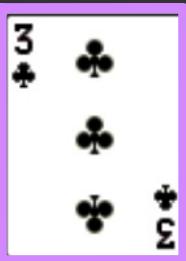
stand

dealer must hit on 16 or less
dealer must stand on 17 or more

dealer

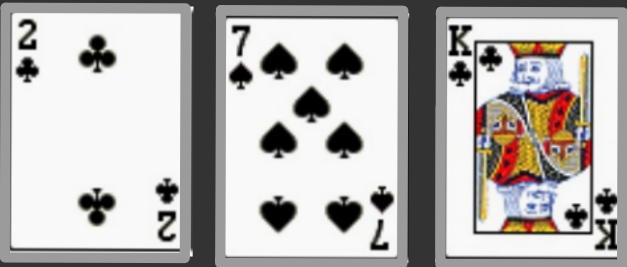


17
player

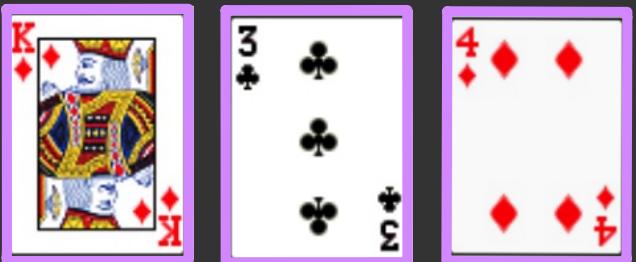


dealer must hit on 16 or less
dealer must stand on 17 or more

dealer



17
player

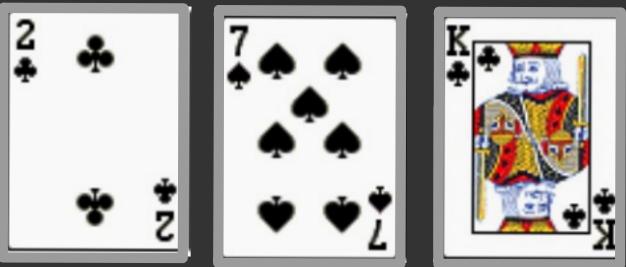


hit

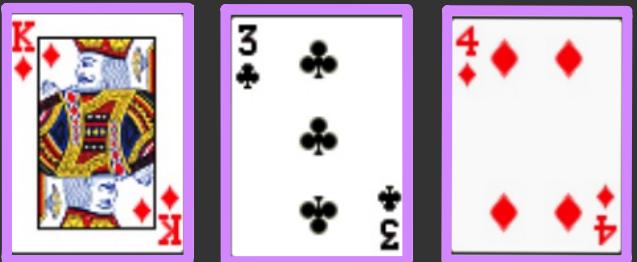
dealer must hit on 16 or less
dealer must stand on 17 or more

dealer

19



17
player

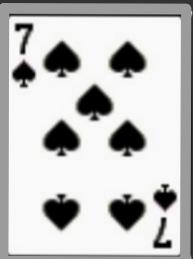
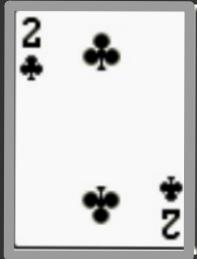


stand

winner

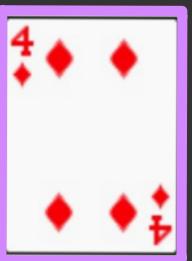
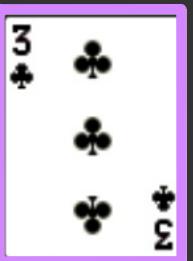
dealer

19



dealer must hit on 16 or less
dealer must stand on 17 or more

17
player

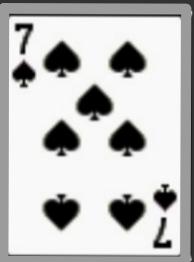
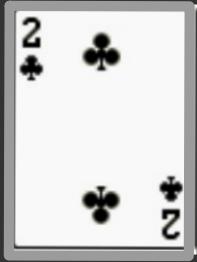


but this is
not really
a two player
game, as the
dealer has no
agency

winner

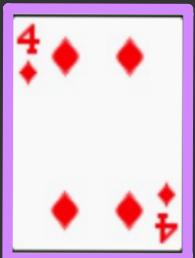
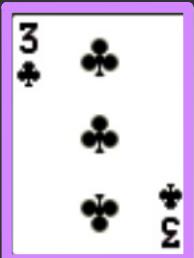
dealer

19



dealer must hit on 16 or less
dealer must stand on 17 or more

17
player



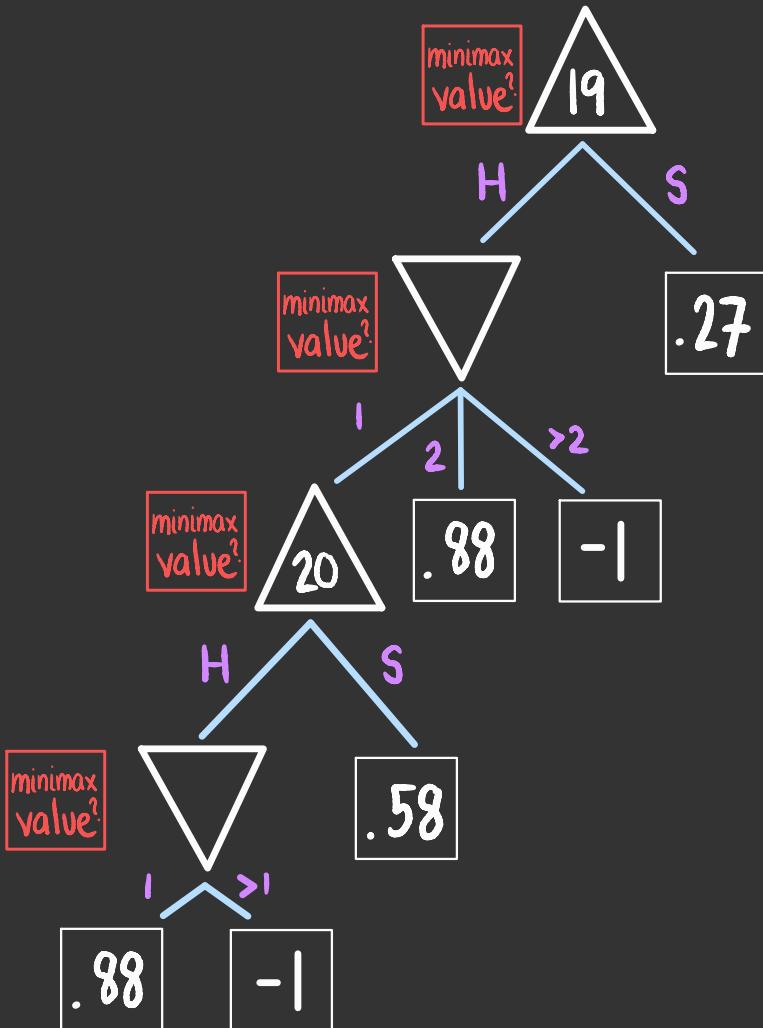
who are we
playing against,
really?



fate

player's final total	probability of			expected utility
	win	draw	loss	$1 \cdot P(\text{win}) + 0 \cdot P(\text{draw}) - 1 \cdot P(\text{loss})$
21	.88	.12	0	.88
20	.70	.18	.12	.58
19	.57	.13	.30	.27
18	.43	.14	.43	0
17	.28	.15	.57	-.29
16	.28	0	.72	-.44
bust	0	0	1	-1

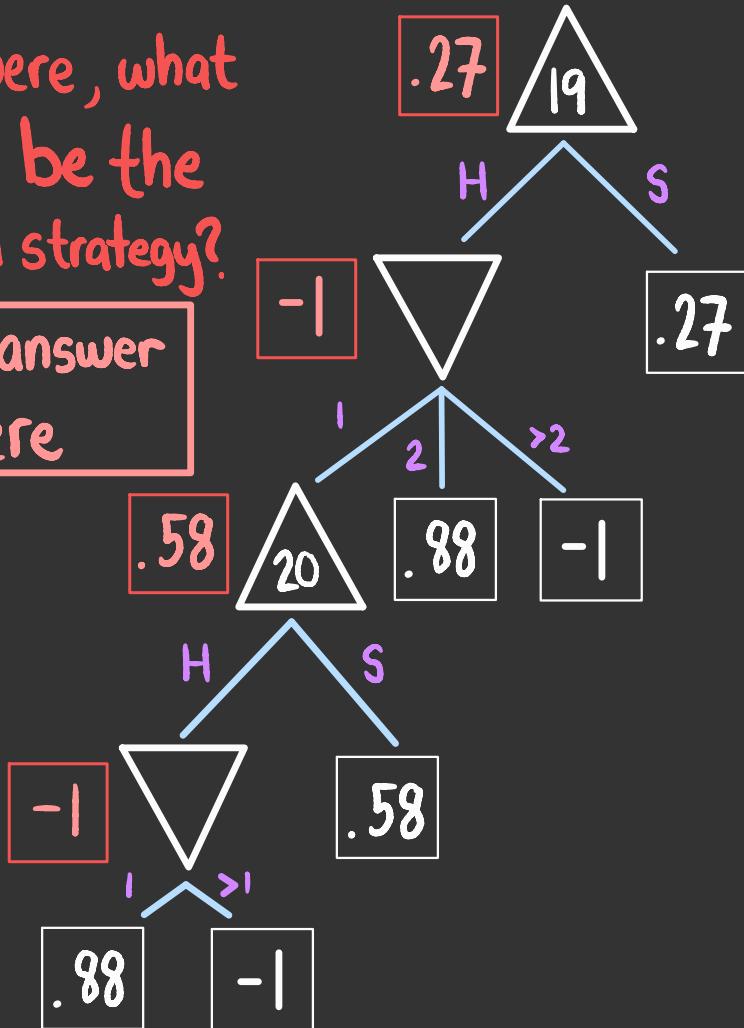
we could model
black jack
as minimax
against fate



we could model
black jack
as minimax
against fate
but fate isn't
out to get us

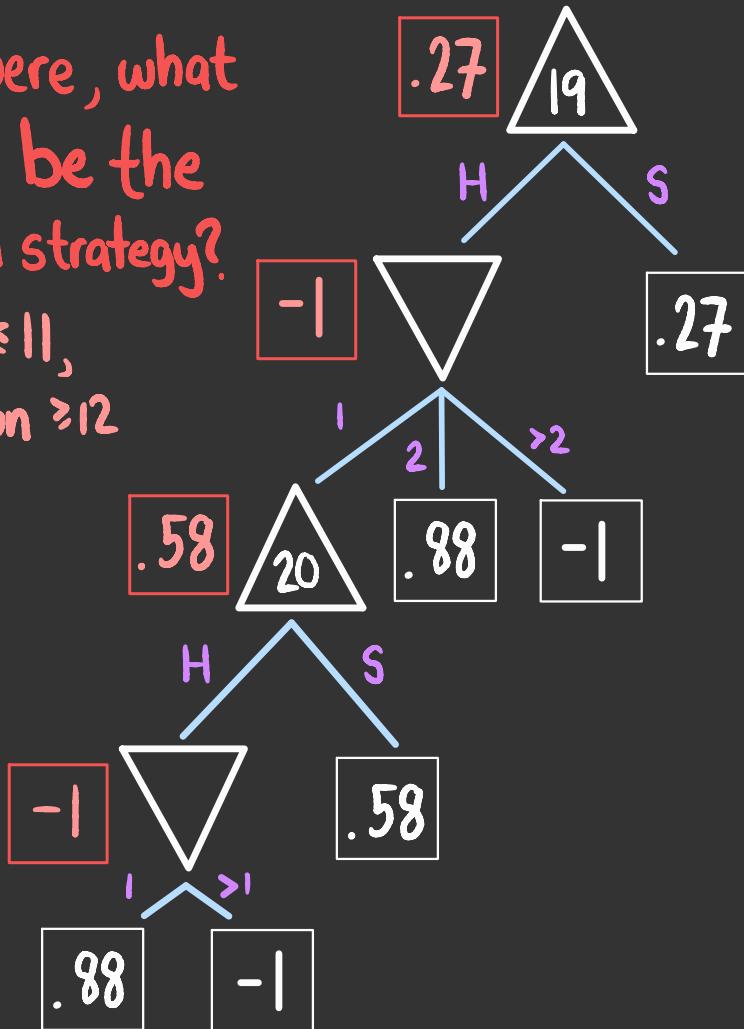
if it were, what
would be the
optimal strategy?

your answer
here



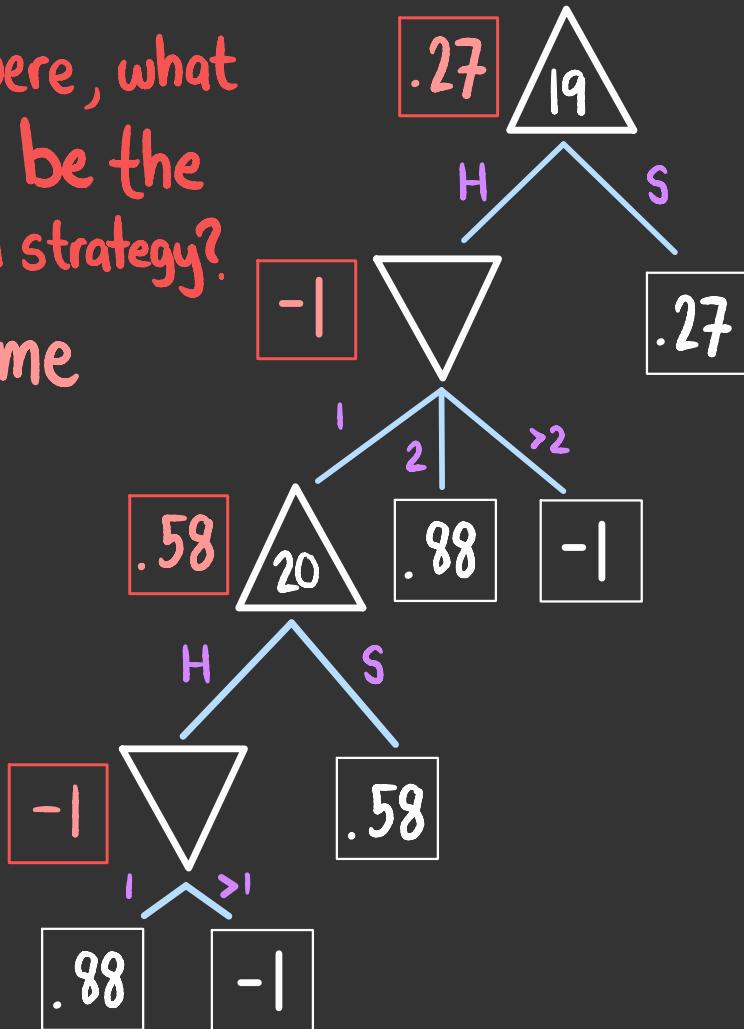
we could model
black jack
as minimax
against fate
but fate isn't
out to get us

if it were, what
would be the
optimal strategy?
hit on ≤ 11 ,
stand on ≥ 12

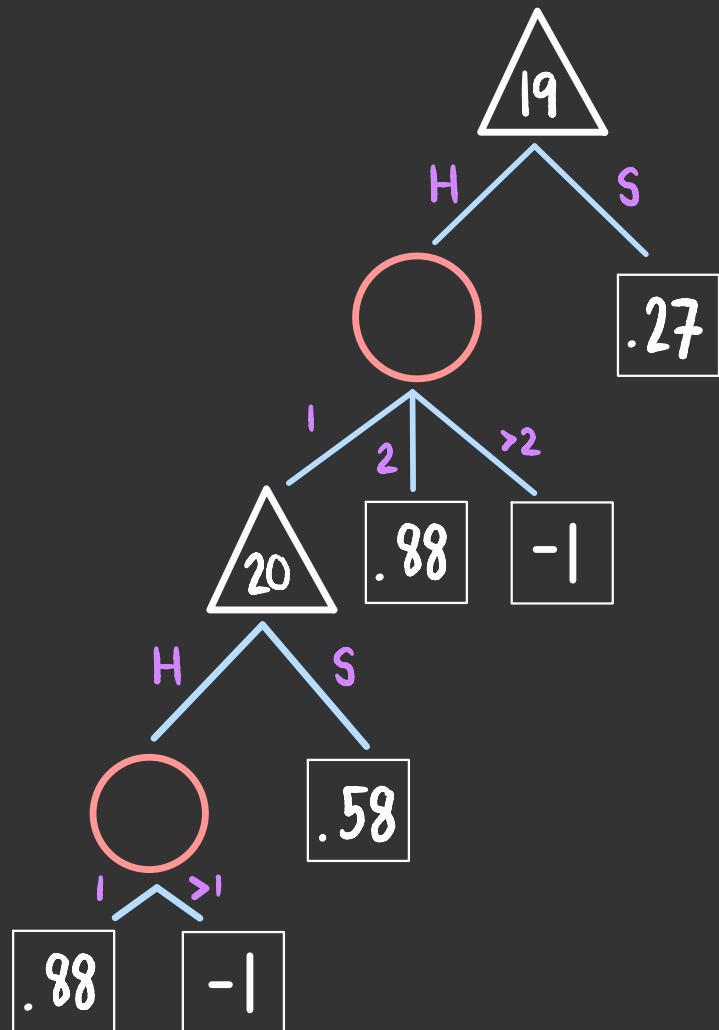


we could model
black jack
as minimax
against fate
but fate isn't
out to get us

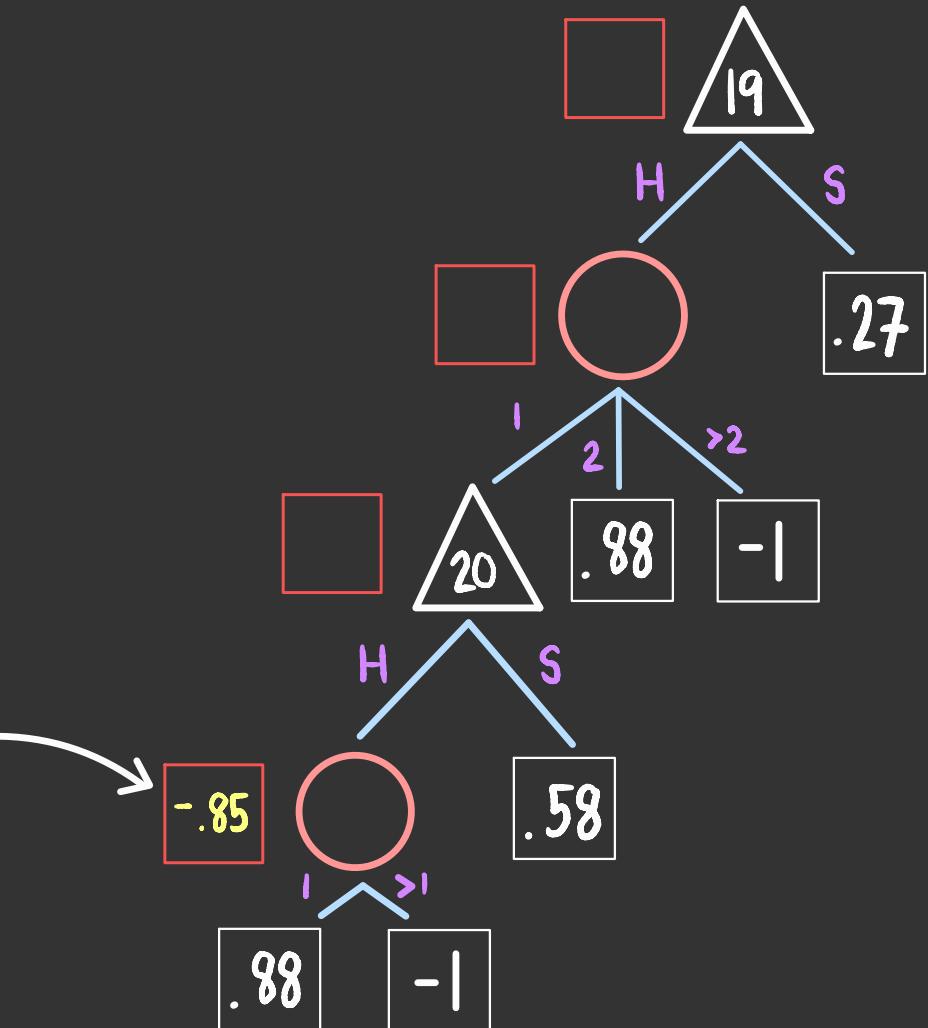
if it were, what
would be the
optimal strategy?
go home



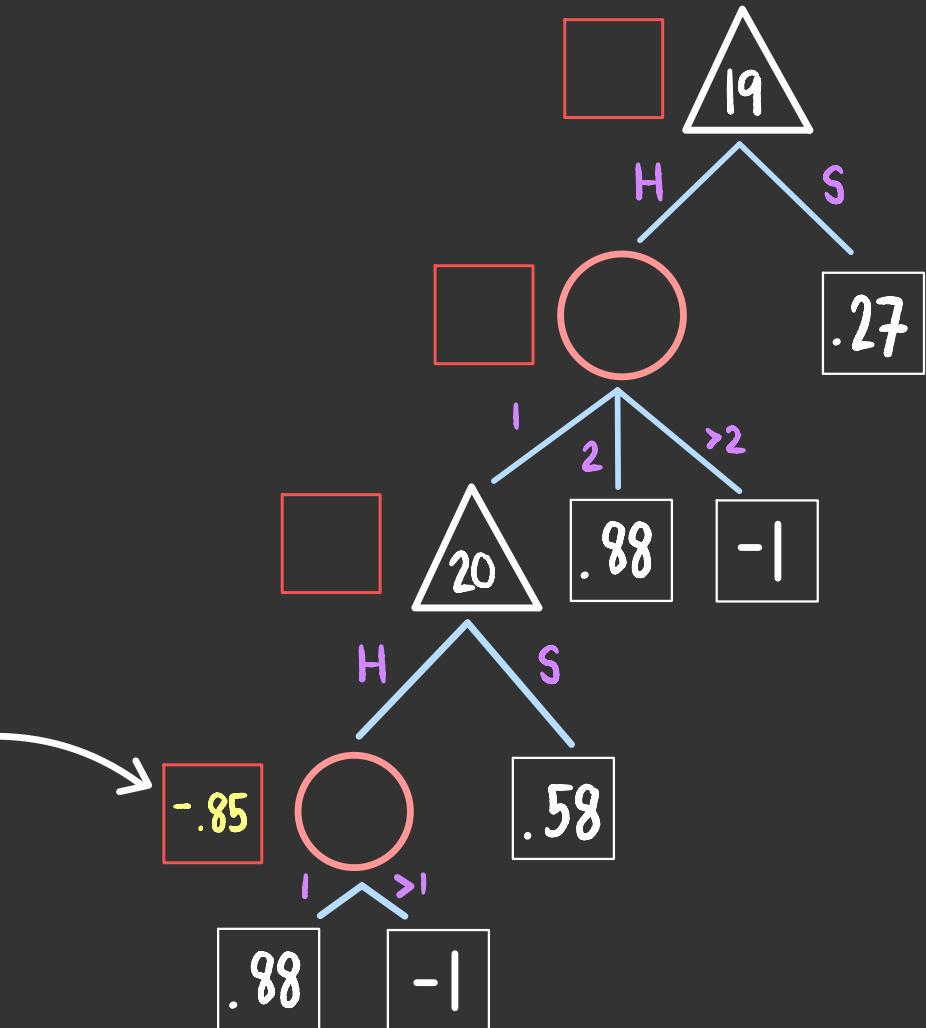
rather than having
fate **oppose** us,
let's have fate play
randomly



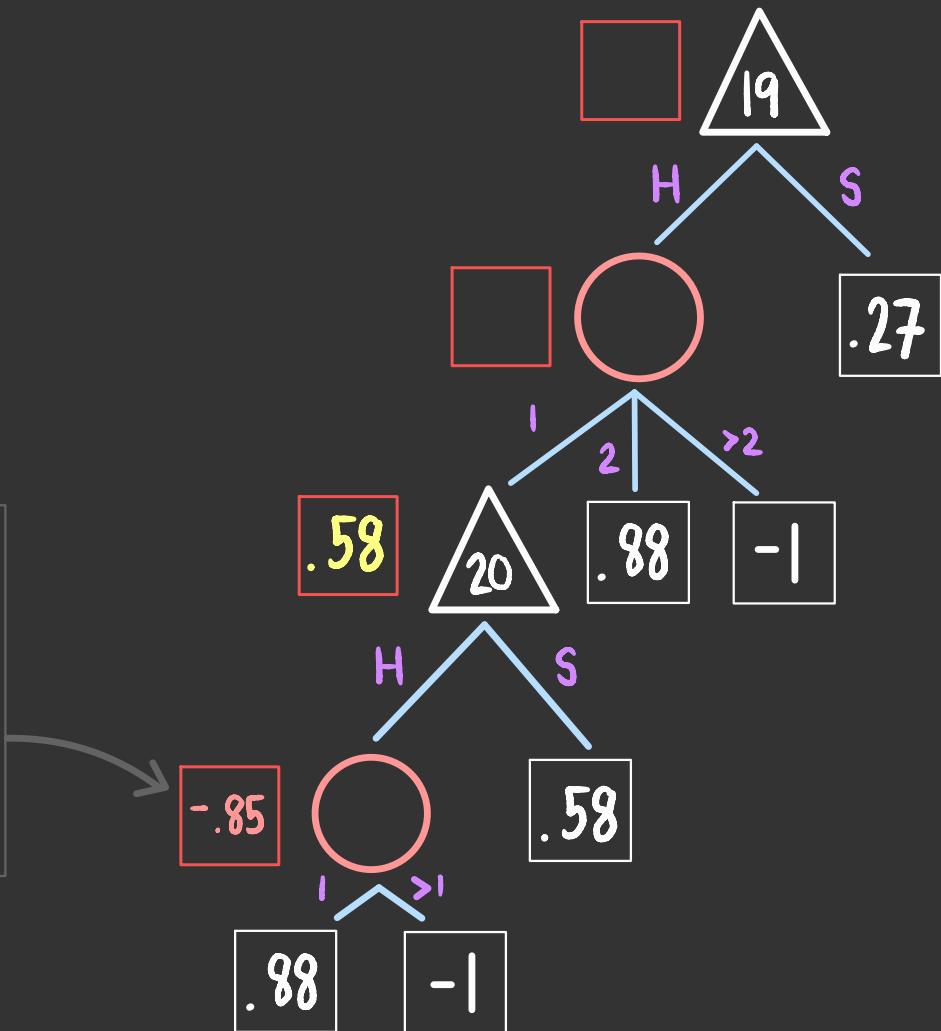
$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & [?] \cdot .88 + [?] \cdot -1 \\
 = & [?]
 \end{aligned}$$



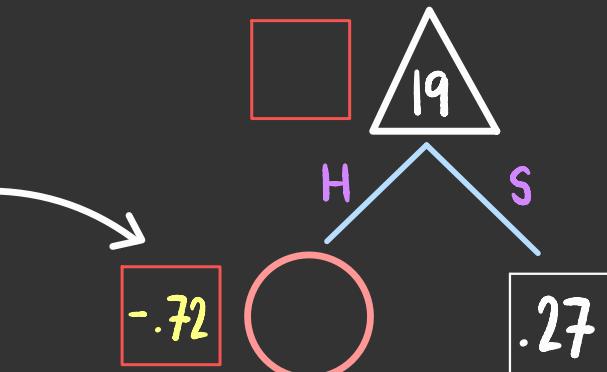
$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & \frac{1}{13} \cdot .88 + \frac{12}{13} \cdot -1 \\
 = & -.85
 \end{aligned}$$



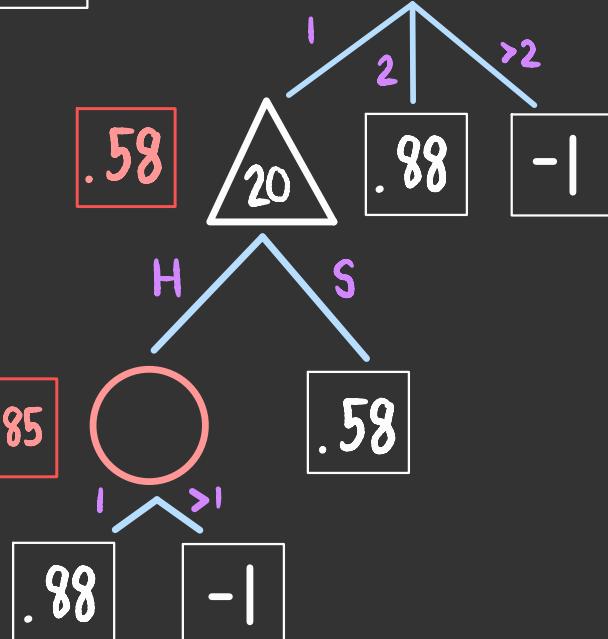
$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & \frac{1}{13} \cdot .88 + \frac{12}{13} \cdot -1 \\
 = & -.85
 \end{aligned}$$



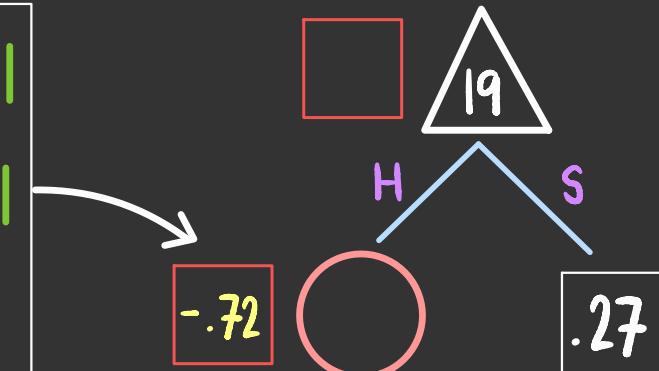
$$\begin{aligned}
 & P(1) \cdot .58 + P(2) \cdot .88 + P(>2) \cdot -1 \\
 = & \boxed{?} \cdot .58 + \boxed{?} \cdot .88 + \boxed{?} \cdot -1 \\
 = & \boxed{?}
 \end{aligned}$$



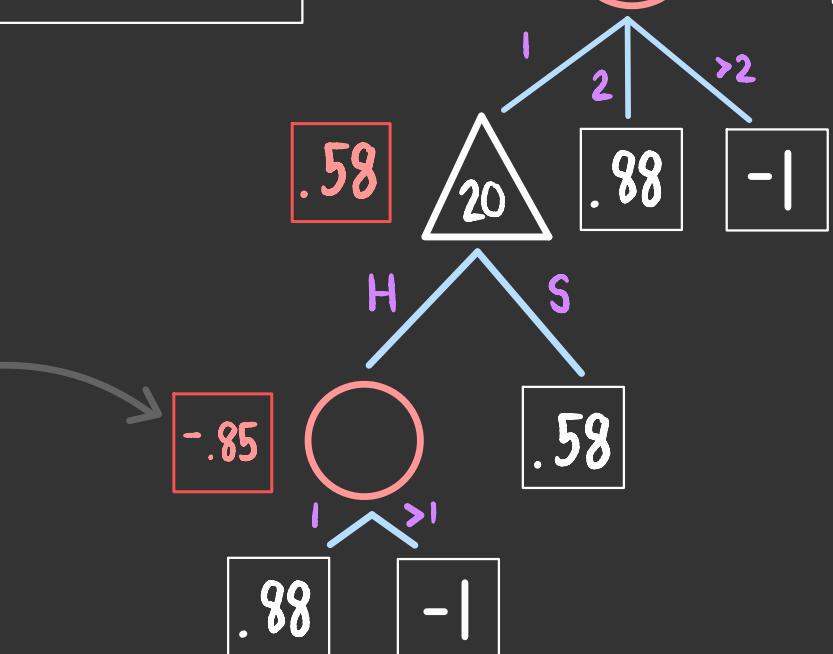
$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & \frac{1}{13} \cdot .88 + \frac{12}{13} \cdot -1 \\
 = & -.85
 \end{aligned}$$



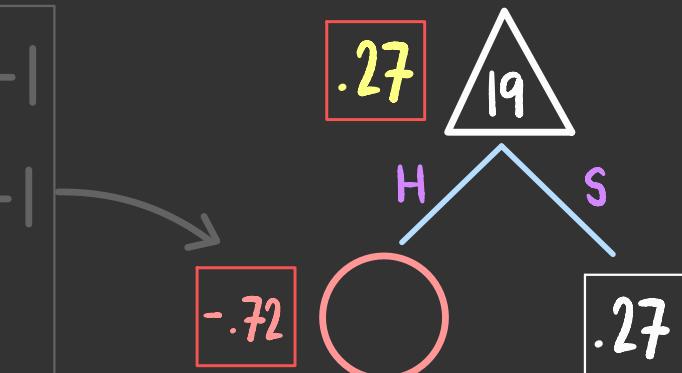
$$\begin{aligned}
 & P(1) \cdot .58 + P(2) \cdot .88 + P(>2) \cdot -1 \\
 = & \frac{1}{13} \cdot .58 + \frac{1}{13} \cdot .88 + \frac{11}{13} \cdot -1 \\
 = & -.72
 \end{aligned}$$



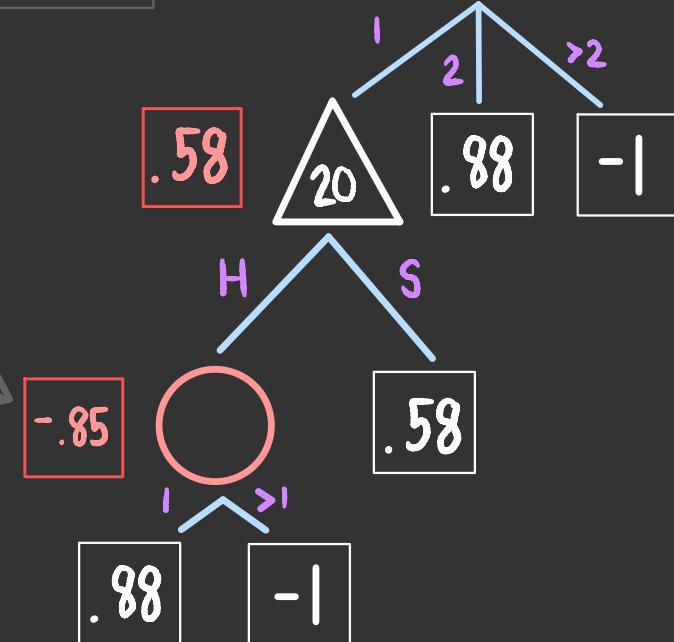
$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & \frac{1}{13} \cdot .88 + \frac{12}{13} \cdot -1 \\
 = & -.85
 \end{aligned}$$



$$\begin{aligned}
 & P(1) \cdot .58 + P(2) \cdot .88 + P(>2) \cdot -1 \\
 = & \frac{1}{13} \cdot .58 + \frac{1}{13} \cdot .88 + \frac{11}{13} \cdot -1 \\
 = & -.72
 \end{aligned}$$



$$\begin{aligned}
 & P(1) \cdot .88 + P(>1) \cdot -1 \\
 = & \frac{1}{13} \cdot .88 + \frac{12}{13} \cdot -1 \\
 = & -.85
 \end{aligned}$$



this variation of
minimax is called
expectimax

