

bayesian  
networks  
12 oct  
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CSCI  
373

$$P(x_{\text{f}}, x_{\text{o}}, x, y_{\text{f}}, y_{\text{o}}, y, z_{\text{f}}, z_{\text{o}}, z)$$

↑ 46656 rows

can we express this  
more compactly?

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1 | x_2, x_3, x_4) \boxed{y|x} = \frac{P(x_1, x_2, x_3, x_4)}{P(x_2, x_3, x_4)}, \text{ so } P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \overbrace{P(x_2, x_3, x_4)}$$

but what is this?

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1 | x_2, x_3, x_4) \boxed{y|x} = \frac{P(x_1, x_2, x_3, x_4)}{P(x_2, x_3, x_4)}, \text{ so } P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \underline{P(x_2, x_3, x_4)}$$

$$P(x_2 | x_3, x_4) \boxed{y|x} = \frac{P(x_2, x_3, x_4)}{P(x_3, x_4)}, \text{ so } \underline{\underline{P(x_2, x_3, x_4)}} = P(x_2 | x_3, x_4) \underline{\underline{P(x_3, x_4)}}$$

and what  
is this?

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1 | x_2, x_3, x_4) \stackrel{y|x}{=} \frac{P(x_1, x_2, x_3, x_4)}{P(x_2, x_3, x_4)}, \text{ so } P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \overline{P(x_2, x_3, x_4)}$$

$$P(x_2 | x_3, x_4) \stackrel{y|x}{=} \frac{P(x_2, x_3, x_4)}{P(x_3, x_4)}, \text{ so } \overline{P(x_2, x_3, x_4)} = P(x_2 | x_3, x_4) \overline{P(x_3, x_4)}$$

$$P(x_3 | x_4) \stackrel{y|x}{=} \frac{P(x_3, x_4)}{P(x_4)}, \text{ so } \overline{P(x_3, x_4)} = P(x_3 | x_4) \overline{P(x_4)}$$

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \underbrace{P(x_2, x_3, x_4)}$$

$$\overline{P(x_2, x_3, x_4)} = P(x_2 | x_3, x_4) \underbrace{P(x_3, x_4)}$$

$$\overline{\overline{P(x_3, x_4)}} = P(x_3 | x_4) \underbrace{P(x_4)}$$

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \underbrace{P(x_2, x_3, x_4)}$$

$$\underbrace{P(x_2, x_3, x_4)} = P(x_2 | x_3, x_4) \underbrace{P(x_3 | x_4) P(x_4)}$$

$$\underbrace{P(x_3 | x_4)} = P(x_3 | x_4) P(x_4)$$

idea: re-express joint probability as  
a product of conditional probabilities

$$P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) \underbrace{P(x_2 | x_3, x_4)}_{P(x_3 | x_4)} \underbrace{P(x_4)}$$

$$\overline{P(x_2, x_3, x_4)} = P(x_2 | x_3, x_4)$$

$$\overline{P(x_3, x_4)} = \underbrace{P(x_3 | x_4)}_{P(x_4)}$$

$$\overline{P(x_3, x_4)} = P(x_3 | x_4)$$

$$\overline{P(x_3, x_4)} = P(x_3 | x_4)$$

this is called the  
chain rule of probability



$$P(x_1, x_2, x_3, x_4) = P(x_1 | x_2, x_3, x_4) P(x_2 | x_3, x_4) P(x_3 | x_4) P(x_4)$$

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i | x_{i+1}, \dots, x_n)$$



given variables  $X_1, \dots, X_n$  with domains  $D(X_i)$   
and joint probability  $P(x_1, \dots, x_n)$

$$P(x_1, \dots, x_m) = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} P(x_1, \dots, x_n)$$



$$P(x_{m+1}, \dots, x_n | x_1, \dots, x_m) = \frac{P(x_1, \dots, x_n)}{P(x_1, \dots, x_m)}$$

$y | x$

$$P(x_{m+1}, \dots, x_n | x_1, \dots, x_m) = \frac{P(x_1, \dots, x_m | x_{m+1}, \dots, x_n) P(x_{m+1}, \dots, x_n)}{P(x_1, \dots, x_m)}$$



$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i | x_{i+1}, \dots, x_n)$$



$$P(x_{\text{f}}, x_{\text{o}}, x, y_{\text{f}}, y_{\text{o}}, y, z_{\text{f}}, z_{\text{o}}, z)$$

46656 rows

Can we express this  
more compactly?

↙ 46656 rows

$$P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z)$$



$$\begin{aligned} &= P(x_{\text{♀}} | x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x_{\text{♂}} | x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x | y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♀}} | y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♀}} | z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♂}} | z) \\ &\cdot P(z) \end{aligned}$$

did this  
save us  
space?

46656 rows

$$P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z)$$



$$\begin{aligned} &= P(x_{\text{♀}} | x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x_{\text{♂}} | x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x | y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♀}} | y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♀}} | z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♂}} | z) \\ &\cdot P(z) \end{aligned}$$

no, not yet,  
but ...



$$P(y, z) = P(z | y) \cdot P(y)$$

y	z	$P(y, z)$
A	A	
A	B	
A	AB	
A	O	
B	A	
B	B	
B	AB	
B	O	
AB	A	
AB	B	
AB	AB	
AB	O	
O	A	
O	B	
O	AB	
O	O	

y	z	$P(z   y)$
A	A	
A	B	
A	AB	
A	O	
B	A	
B	B	
B	AB	
B	O	
AB	A	
AB	B	
AB	AB	
AB	O	
O	A	
O	B	
O	AB	
O	O	

y	$P(y)$
A	
B	
AB	
O	

do we have to specify all these values?



$$P(y, z) = P(z | y) \cdot P(y)$$

y	z	$P(y, z)$
A	A	
A	B	
A	AB	
A	O	
B	A	
B	B	
B	AB	
B	O	
AB	A	
AB	B	
AB	AB	
AB	O	
O	A	
O	B	
O	AB	
O	O	x

y	z	$P(z   y)$
A	A	
A	B	
A	AB	
A	O	x
B	A	
B	B	
B	AB	x
B	O	x
AB	A	
AB	B	
AB	AB	
AB	O	x
O	A	
O	B	
O	AB	
O	O	x

y	$P(y)$
A	
B	
AB	
O	x

15 values

12 values

3 values



$$P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z)$$

46655 rows to specify

$$\begin{aligned} &= P(x_{\text{♀}} | x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x_{\text{♂}} | x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(x | y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♀}} | y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | y, z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(y_{\text{♂}} | z_{\text{♀}}, z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♀}} | z_{\text{♂}}, z) \\ &\cdot P(z_{\text{♂}} | z) \\ &\cdot P(z) \end{aligned}$$



no, not yet,  
but ...

$$P(z, z_{\text{♀}}, z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma})$$



$$\begin{aligned} &= P(z \mid z_{\text{♀}}, z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(z_{\text{♀}} \mid z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(z_{\sigma} \mid y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y \mid y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\text{♀}} \mid y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\sigma} \mid x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x \mid x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x_{\text{♀}} \mid x_{\sigma}) \\ &\cdot P(x_{\sigma}) \end{aligned}$$

first, observe  
that we can put  
the variables in  
any order before  
applying the  
chain rule

$$P(z, z_{\varphi}, z_{\sigma}, y, y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma})$$



$$= P(z \mid z_{\varphi}, z_{\sigma}, y, y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma}) \leftarrow \text{next, observe:}$$

$$\cdot P(z_{\varphi} \mid z_{\sigma}, y, y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma})$$

$$\cdot P(z_{\sigma} \mid y, y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma})$$

$$\cdot P(y \mid y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma})$$

$$\cdot P(y_{\varphi} \mid y_{\sigma}, x, x_{\varphi}, x_{\sigma})$$

$$\cdot P(y_{\sigma} \mid x, x_{\varphi}, x_{\sigma})$$

$$\cdot P(x \mid x_{\varphi}, x_{\sigma})$$

$$\cdot P(x_{\varphi} \mid x_{\sigma})$$

$$\cdot P(x_{\sigma})$$

$$z_{\varphi} = A \quad z_{\sigma} = 0$$



$$z = A$$

$$z \perp\!\!\!\perp \{x, x_{\varphi}, x_{\sigma}, y, y_{\varphi}, y_{\sigma}\} \mid \{z_{\varphi}, z_{\sigma}\}$$

i.e.  $P(z \mid z_{\varphi}, z_{\sigma}, y, y_{\varphi}, y_{\sigma}, x, x_{\varphi}, x_{\sigma}) = P(z \mid z_{\varphi}, z_{\sigma})$

$$P(z, z_{\text{♀}}, z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma})$$



$$\begin{aligned} &= P(z \mid z_{\text{♀}}, z_{\sigma}) \\ &\cdot P(z_{\text{♀}} \mid z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(z_{\sigma} \mid y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y \mid y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\text{♀}} \mid y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\sigma} \mid x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x \mid x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x_{\text{♀}} \mid x_{\sigma}) \\ &\cdot P(x_{\sigma}) \end{aligned}$$

what other  
conditional  
independence  
relationships  
are there?

$$P(z, z_{\text{♀}}, z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma})$$



$$\begin{aligned} &= P(z \mid z_{\text{♀}}, z_{\sigma}) \\ &\cdot P(z_{\text{♀}} \mid z_{\sigma}, y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(z_{\sigma} \mid y, y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y \mid y_{\text{♀}}, y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\text{♀}} \mid y_{\sigma}, x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(y_{\sigma} \mid x, x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x \mid x_{\text{♀}}, x_{\sigma}) \\ &\cdot P(x_{\text{♀}} \mid x_{\sigma}) \\ &\cdot P(x_{\sigma}) \end{aligned}$$

what other  
conditional  
independence  
relationships  
are there?

$$P(z, z_f, z_o, y, y_f, y_o, x, x_f, x_o)$$

$$\begin{aligned} &= P(z \mid z_f, z_o) \\ &\cdot P(z_f \mid x_f, x_o) \\ &\cdot P(z_o \mid y_f, y_o) \\ &\cdot P(y \mid y_f, y_o) \\ &\cdot P(y_f) \\ &\cdot P(y_o) \\ &\cdot P(x \mid x_f, x_o) \\ &\cdot P(x_f) \\ &\cdot P(x_o) \end{aligned}$$

did this  
save us  
space?

↙ 46656 rows

$$P(z, z_f, z_o, y, y_f, y_o, x, x_f, x_o)$$

$$\begin{aligned} &= P(z | z_f, z_o) \leftarrow 4 \cdot 3 \cdot 3 = 36 \text{ rows} \\ &\cdot P(z_f | x_f, x_o) \leftarrow 3 \cdot 3 \cdot 3 = 27 \text{ rows} \\ &\cdot P(z_o | y_f, y_o) \leftarrow 3 \cdot 3 \cdot 3 = 27 \text{ rows} \\ &\cdot P(y | y_f, y_o) \leftarrow 4 \cdot 3 \cdot 3 = 36 \text{ rows} \\ &\cdot P(y_f) \leftarrow 3 \text{ rows} \\ &\cdot P(y_o) \leftarrow 3 \text{ rows} \\ &\cdot P(x | x_f, x_o) \leftarrow 4 \cdot 3 \cdot 3 = 36 \text{ rows} \\ &\cdot P(x_f) \leftarrow 3 \text{ rows} \\ &\cdot P(x_o) \leftarrow 3 \text{ rows} \end{aligned} \quad \left. \right\} 174 \text{ rows}$$

$$P(z, z_f, z_o, y, y_f, y_o, x, x_f, x_o)$$

$$\begin{aligned} &= P(z \mid z_f, z_o) \\ &\cdot P(z_f \mid x_f, x_o) \\ &\cdot P(z_o \mid y_f, y_o) \\ &\cdot P(y \mid y_f, y_o) \\ &\cdot P(y_f) \\ &\cdot P(y_o) \\ &\cdot P(x \mid x_f, x_o) \\ &\cdot P(x_f) \\ &\cdot P(x_o) \end{aligned}$$

we can represent our  
independence assumptions  
with a graph

we create a vertex for each variable

$$P(z, z_{\text{♀}}, z_{\text{♂}}, y, y_{\text{♀}}, y_{\text{♂}}, x, x_{\text{♀}}, x_{\text{♂}}) = P(z \mid z_{\text{♀}}, z_{\text{♂}}) \cdot P(z_{\text{♀}} \mid x_{\text{♀}}, x_{\text{♂}}) \cdot P(z_{\text{♂}} \mid y_{\text{♀}}, y_{\text{♂}}) \cdot P(y \mid y_{\text{♀}}, y_{\text{♂}}) \cdot P(y_{\text{♀}}) \cdot P(y_{\text{♂}}) \cdot P(x \mid x_{\text{♀}}, x_{\text{♂}}) \cdot P(x_{\text{♀}}) \cdot P(x_{\text{♂}})$$

$\chi$        $\gamma$   
 $\chi_{\text{♀}}$      $\chi_{\text{♂}}$      $\gamma_{\text{♀}}$      $\gamma_{\text{♂}}$   
 $z_{\text{♀}}$        $z_{\text{♂}}$   
 $z$

the conditioning set of a variable  
are its parents in the graph

$$P(z, z_\varphi, z_\sigma, y, y_\varphi, y_\sigma, x, x_\varphi, x_\sigma)$$

$$= P(z | z_\varphi, z_\sigma)$$

$$\cdot P(z_\varphi | x_\varphi, x_\sigma)$$

$$\cdot P(z_\sigma | y_\varphi, y_\sigma)$$

$$\cdot P(y | y_\varphi, y_\sigma)$$

$$\cdot P(y_\varphi)$$

$$\cdot P(y_\sigma)$$

$$\cdot P(x | x_\varphi, x_\sigma)$$

$$\cdot P(x_\varphi)$$

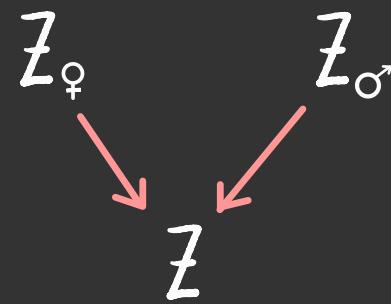
$$\cdot P(x_\sigma)$$

X

Y

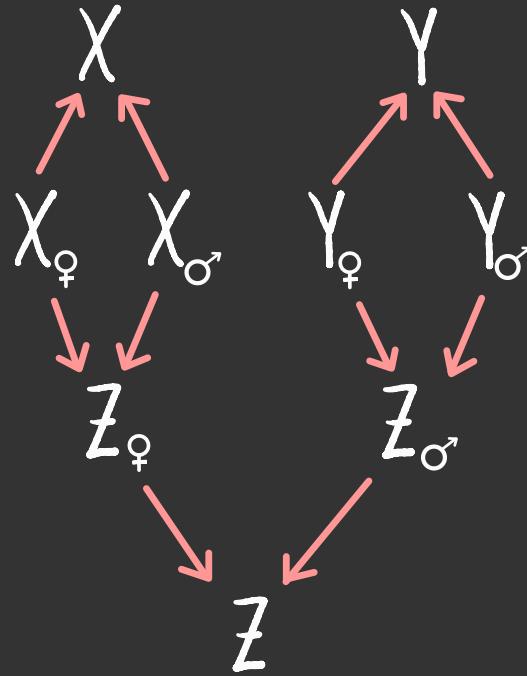
X<sub>φ</sub> X<sub>σ</sub>

Y<sub>φ</sub> Y<sub>σ</sub>



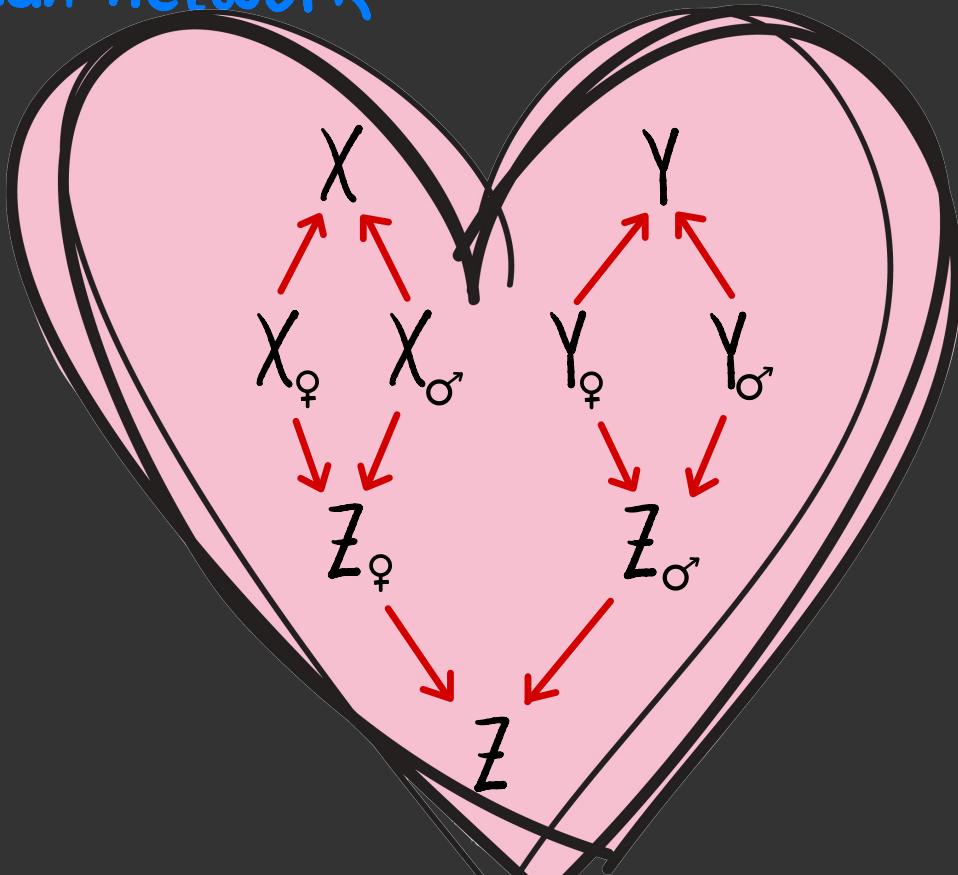
this graph is called a  
bayesian network

$$\begin{aligned} P(z, z_{\text{♀}}, z_{\text{♂}}, y, y_{\text{♀}}, y_{\text{♂}}, x, x_{\text{♀}}, x_{\text{♂}}) &= P(z \mid z_{\text{♀}}, z_{\text{♂}}) \\ &\cdot P(z_{\text{♀}} \mid x_{\text{♀}}, x_{\text{♂}}) \\ &\cdot P(z_{\text{♂}} \mid y_{\text{♀}}, y_{\text{♂}}) \\ &\cdot P(y \mid y_{\text{♀}}, y_{\text{♂}}) \\ &\cdot P(y_{\text{♀}}) \\ &\cdot P(y_{\text{♂}}) \\ &\cdot P(x \mid x_{\text{♀}}, x_{\text{♂}}) \\ &\cdot P(x_{\text{♀}}) \\ &\cdot P(x_{\text{♂}}) \end{aligned}$$



this graph is called a  
bayesian network

$$\begin{aligned} P(z, z_{\text{o}}, z_{\sigma}, y, y_{\text{o}}, y_{\sigma}, x, x_{\text{o}}, x_{\sigma}) &= P(z \mid z_{\text{o}}, z_{\sigma}) \\ &\cdot P(z_{\text{o}} \mid x_{\text{o}}, x_{\sigma}) \\ &\cdot P(z_{\sigma} \mid y_{\text{o}}, y_{\sigma}) \\ &\cdot P(y \mid y_{\text{o}}, y_{\sigma}) \\ &\cdot P(y_{\text{o}}) \\ &\cdot P(y_{\sigma}) \\ &\cdot P(x \mid x_{\text{o}}, x_{\sigma}) \\ &\cdot P(x_{\text{o}}) \\ &\cdot P(x_{\sigma}) \end{aligned}$$

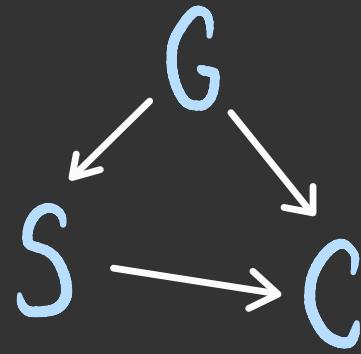


let  $S$  represent whether a person smokes

let  $C$  represent whether a person gets lung cancer

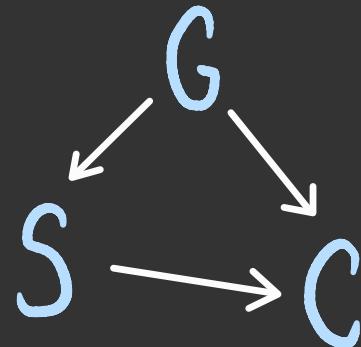
let  $G$  represent a particular gene

what claims does  
this bayesian network  
make?



$$P(g, s, c) = P(g)P(s|g)P(c|s, g)$$

what claims does  
this bayesian network  
make?



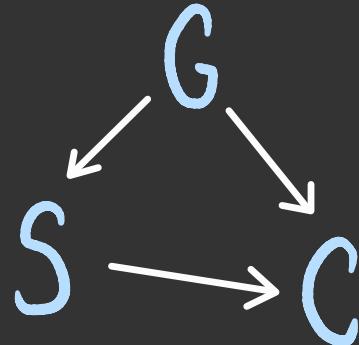


this is just the chain rule

$$P(g, s, c) = P(g)P(s|g)P(c|s, g)$$

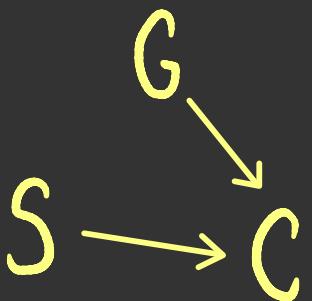
which holds for  
**every**  
joint distribution

so this bayesian  
network makes  
no claims

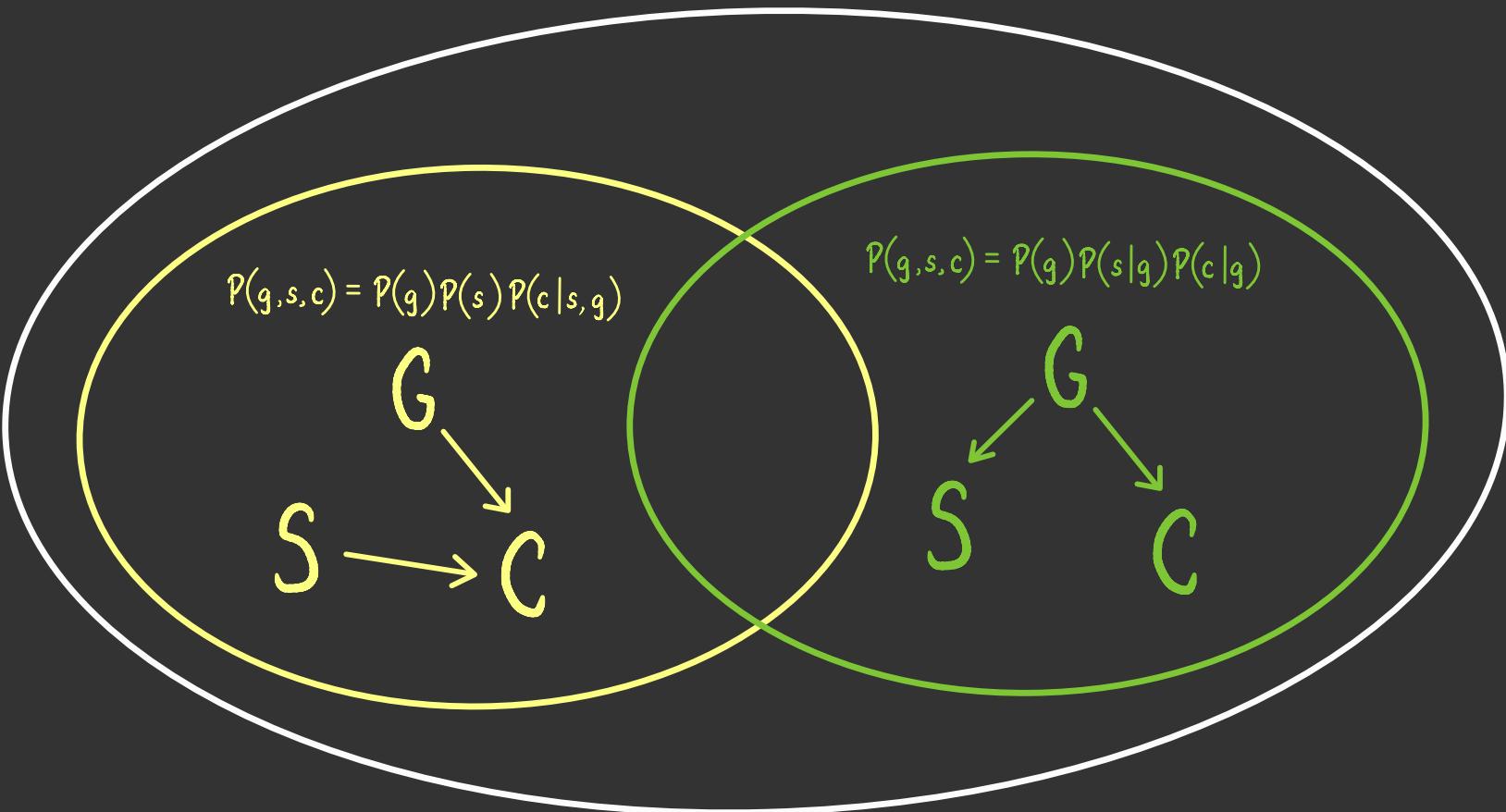


the space of all joint distributions  $P(g, s, c)$

$$P(g, s, c) = P(g)P(s)P(c|s, g)$$

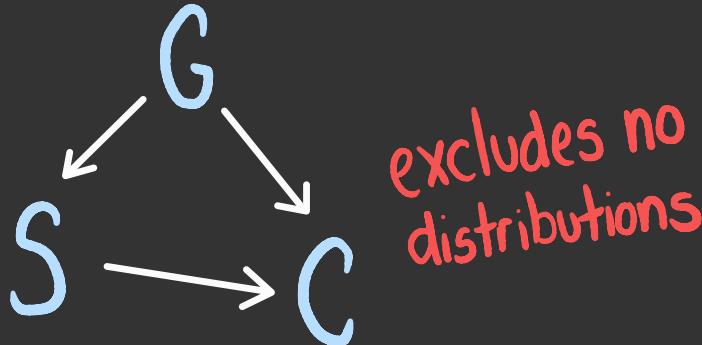


the space of all joint distributions  $P(g, s, c)$



the space of all joint distributions  $P(g, s, c)$

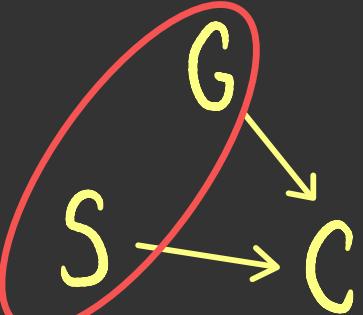

$$P(g, s, c) = P(g)P(s|g)P(c|s, g)$$



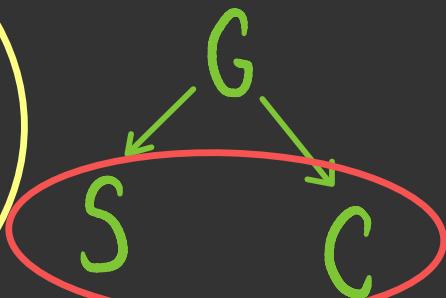
in a bayesian network, the *absent edges*

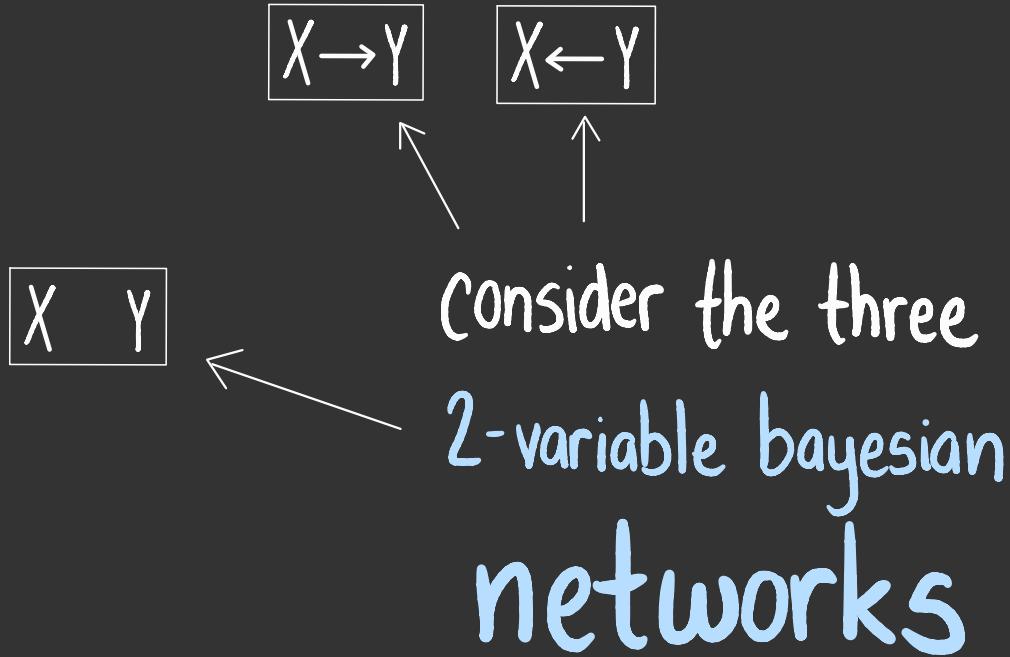
make the claims

$$P(g, s, c) = P(g)P(s)P(c|s, g)$$



$$P(g, s, c) = P(g)P(s|g)P(c|g)$$





claims that:

$$P(x,y) = P(x)P(y|x)$$



claims that:

$$P(x,y) = P(y)P(x|y)$$



claims that:

$$P(x,y) = P(x)P(y)$$



Consider the three  
2-variable bayesian  
networks

$X \rightarrow Y$  or  $X \leftarrow Y$

all joint  
distributions  
 $P(x,y)$

$X$     $Y$

joint distributions  
such that  
 $P(x,y) = P(x)P(y)$

$X$	$Y$	$P(x,y)$
0	0	?
0	1	?
1	0	?
1	1	?

$X$	$Y$	$P(x,y)$
0	0	?
0	1	?
1	0	?
1	1	?

$$P(X=1, Y=1) = .4$$

$$P(X=1)P(Y=1) = .7 \cdot .6 = .42$$

$X \rightarrow Y$  or  $X \leftarrow Y$

all joint  
distributions  
 $P(x,y)$

$$P(x,y) \neq P(x)P(y)$$

$X$      $Y$

joint distributions  
such that  
 $P(x,y) = P(x)P(y)$

$X$	$Y$	$P(x,y)$
0	0	.72
0	1	.18
1	0	.08
1	1	.02

$X$	$Y$	$P(x,y)$
0	0	.1
0	1	.2
1	0	.3
1	1	.4