monty hall workshop

17 oct 2022

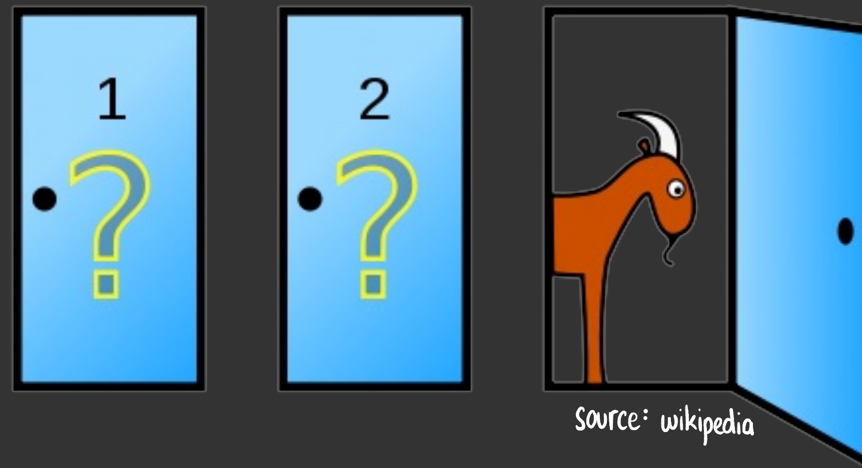
CSCI 373

Ask Marilyn™

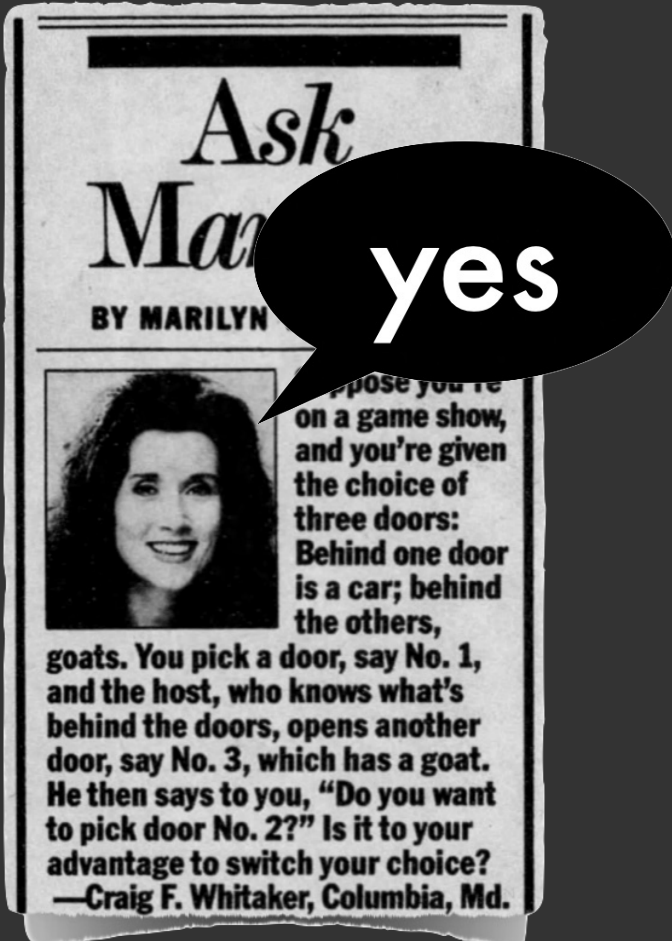
BY MARILYN VOS SAVANT

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

—Craig F. Whitaker, Columbia, Md.



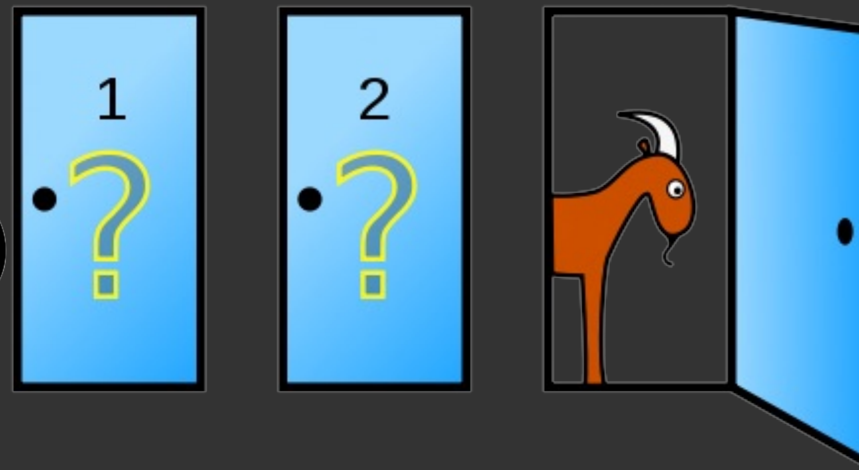
should you switch?



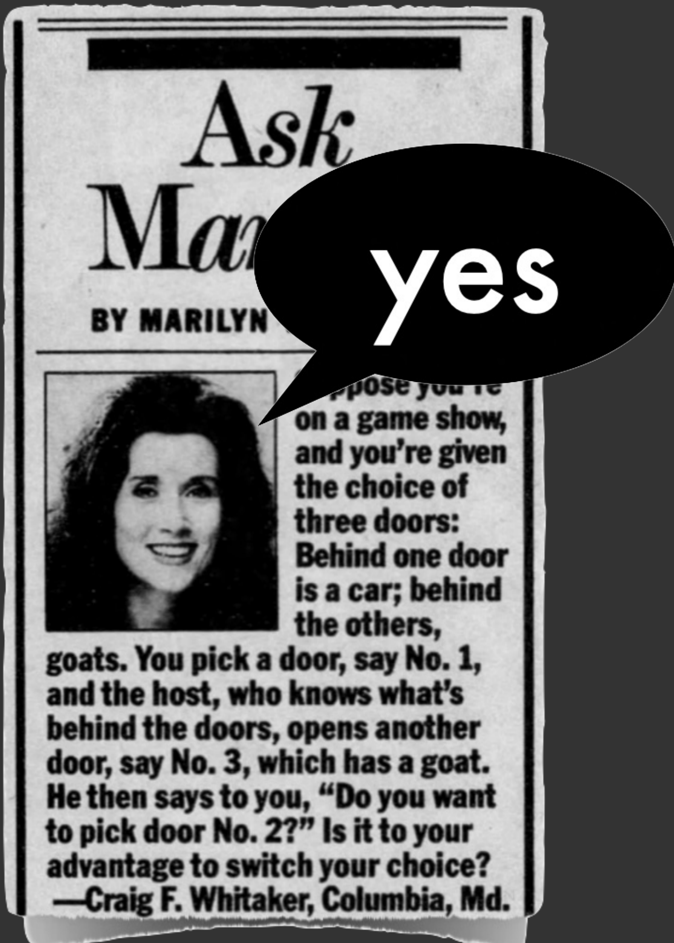
Ask
Ma

BY MARILYN

ppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
—Craig F. Whitaker, Columbia, Md.



should you switch?


Ask

Ma

BY MARILYN

…ppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

—Craig F. Whitaker, Columbia, Md.

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

Scott Smith, University of Florida[3]

Ask Marilyn™

BY MARILYN VOS SAVANT

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
—Craig F. Whitaker, Columbia, Md.

I have been a faithful reader of your column, and I have not, until now, had any reason to doubt you. However, in this matter (for which I do have expertise), your answer is clearly at odds with the truth.
James Rauff, Ph.D.
Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?
Charles Reid, Ph.D.
University of Florida

I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.
W. Robert Smith, Ph.D.
Georgia State University

You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?
E. Ray Bobo, Ph.D.
Georgetown University

I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.
Kent Ford
Dickinson State University

Maybe women look at math problems differently than men.
Don Edwards
Sunriver, Oregon

You are the goat!
Glenn Calkins
Western State College

You made a mistake, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.
Everett Harman, Ph.D.
U.S. Army Research Institute

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

$$P(w, f, g, c) = P(w \mid f, c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

$$
\begin{tikzcd}
 & & C \arrow[dl] \arrow[dd] \\
 & G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$
\begin{aligned}
P(w, f, g, c) &= P(w \mid f, g, c) \\
&\cdot P(f \mid g, c) \\
&\cdot P(g \mid c) \\
&\cdot P(c)
\end{aligned}
$$

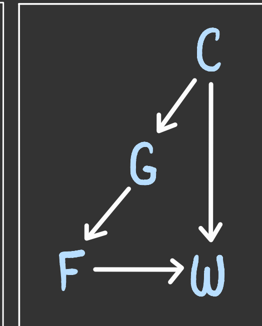
assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins



$$P(w, f, g, c) = P(w \mid f, c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

next, we need to specify these distributions

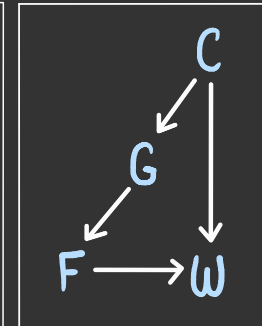
assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins



$$P(w,f,g,c) = P(w|f,c) \cdot P(f|g) \cdot P(g|c) \cdot P(c)$$

f	c	w	$P(w\|f,c)$

how many rows ?

assume the contestant's initial choice is door 1 and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

$$\begin{tikzcd} & & C \arrow[dl] \arrow[dd] \\ & G \arrow[dl] & \\ F \arrow[rr] & & W \end{tikzcd}$$

$$P(w,f,g,c) = P(w \mid f,c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

f	c	w	$P(w \mid f,c)$
1	1	yes	$\boxed{?}$
1	1	no	$\boxed{?}$
1	2	yes	$\boxed{?}$
1	2	no	$\boxed{?}$
1	3	yes	$\boxed{?}$
1	3	no	$\boxed{?}$
2	1	yes	$\boxed{?}$
2	1	no	$\boxed{?}$
2	2	yes	$\boxed{?}$
2	2	no	$\boxed{?}$
2	3	yes	$\boxed{?}$
2	3	no	$\boxed{?}$
3	1	yes	$\boxed{?}$
3	1	no	$\boxed{?}$
3	2	yes	$\boxed{?}$
3	2	no	$\boxed{?}$
3	3	yes	$\boxed{?}$
3	3	no	$\boxed{?}$

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins

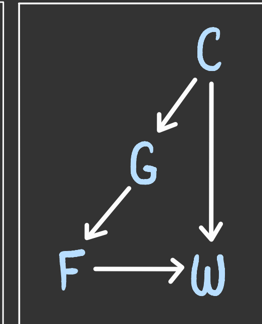
$$
\begin{tikzcd}
 & & C \arrow[dl] \arrow[dd] \\
 & G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$P(w,f,g,c) = P(w|f,c) \cdot P(f|g) \cdot P(g|c) \cdot P(c)$$

f	c	w	$P(w\|f,c)$
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins



$$P(w,f,g,c) = P(w \mid f,c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

g	f	$P(f \mid g)$
2	1	$\boxed{?}$
2	2	$\boxed{?}$
2	3	$\boxed{?}$
3	1	$\boxed{?}$
3	2	$\boxed{?}$
3	3	$\boxed{?}$

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

$$
\begin{tikzcd}
 & & C \arrow[dl] \arrow[dd] \\
 & G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$P(w,f,g,c) = P(w \mid f,c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

g	f	$P(f \mid g)$
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins

$$
\begin{tikzcd}
 & & C \arrow[dl] \arrow[dd] \\
 & G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$
P(w, f, g, c) = P(w \mid f, c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)
$$

c	g	$P(g \mid c)$
1	2	$\boxed{?}$
1	3	$\boxed{?}$
2	2	$\boxed{?}$
2	3	$\boxed{?}$
3	2	$\boxed{?}$
3	3	$\boxed{?}$

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins

$$
\begin{tikzcd}
& & C \arrow[dl] \arrow[dd] \\
& G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$
\begin{aligned}
P(w, f, g, c) = \; & P(w \mid f, c) \\
& \cdot P(f \mid g) \\
& \cdot P(g \mid c) \\
& \cdot P(c)
\end{aligned}
$$

c	g	$P(g \mid c)$
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

$$
\begin{tikzcd}
 & & C \arrow[dl] \arrow[dd] \\
 & G \arrow[dl] & \\
F \arrow[rr] & & W
\end{tikzcd}
$$

$$
P(w, f, g, c) = P(w \mid f, c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)
$$

c	$P(c)$
1	?
2	?
3	?

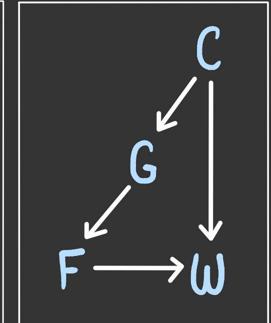
assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

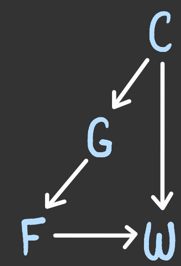


$$P(w,f,g,c) = P(w \mid f,c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

C	$P(c)$
1	$0.\overline{33}$
2	$0.\overline{33}$
3	$0.\overline{33}$

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car
let G be the door that the host opens to reveal a goat
let F be the contestant's final choice
let W be whether the contestant wins



$$P(w,f,g,c) = P(w|f,c) \cdot P(f|g) \cdot P(g|c) \cdot P(c)$$

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

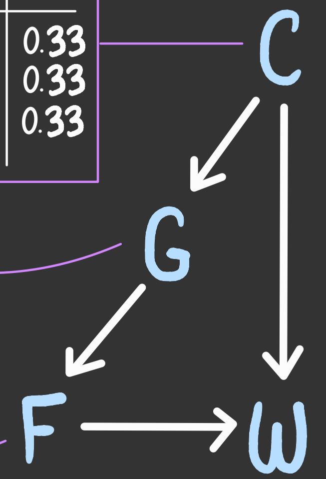
c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

c	
1	0.33
2	0.33
3	0.33

$$P(w,f,g,c) = P(w \mid f,c) \cdot P(f \mid g) \cdot P(g \mid c) \cdot P(c)$$

c	
1	0.33
2	0.33
3	0.33

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0



g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

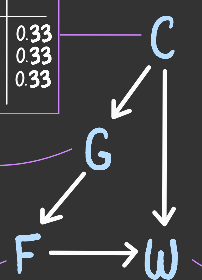
let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins

c	
1	0.33
2	0.33
3	0.33

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0



g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

should the contestant switch?

how do we express this as a probability query?

your answer here

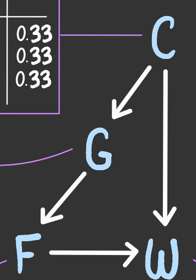
assume the contestant's initial choice is door 1
and that the contestant's strategy is to switch

let C be the door with the car

let G be the door that the host opens to reveal a goat

let F be the contestant's final choice

let W be whether the contestant wins



c	
1	0.33
2	0.33
3	0.33

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

should the contestant switch?

how do we express this as a probability query?

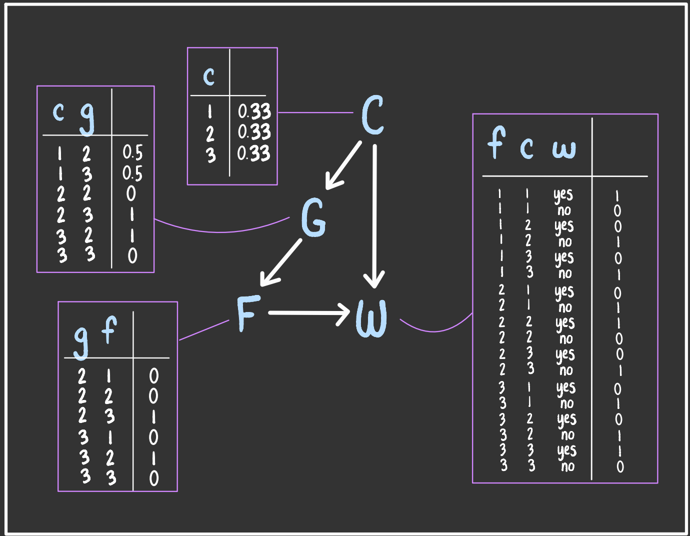
$$P(w)$$

$$P(w) = ?$$

marginal distribution over W

how do we express a marginal probability in terms of a joint probability?

a representation of the joint distribution



c	
1	0.33
2	0.33
3	0.33

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

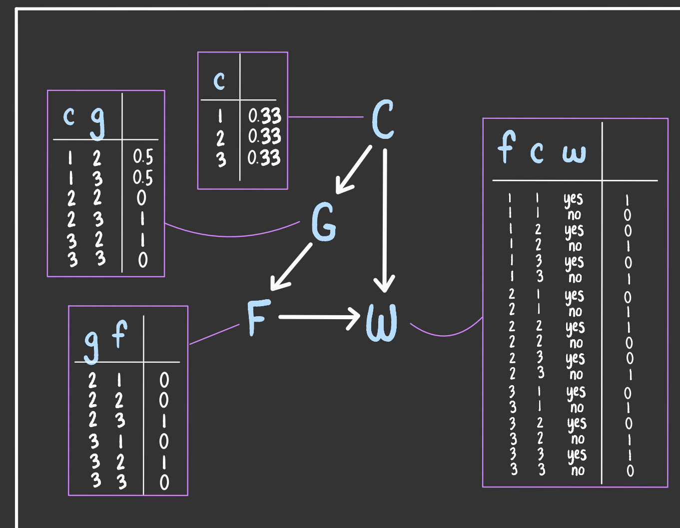
g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

$$P(w) = \sum_{c}\sum_{f}\sum_{g} P(w,f,g,c)$$

marginal distribution over W

a representation of the joint distribution



c	
1	0.33
2	0.33
3	0.33

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

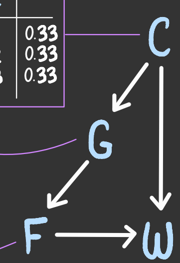
f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

$$P(w) = \sum_c \sum_f \sum_g P(w,f,g,c)$$

$$= \sum_c \sum_f \sum_g P(w|f,c)P(f|g)P(g|c)P(c)$$

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

c	
1	0.33
2	0.33
3	0.33



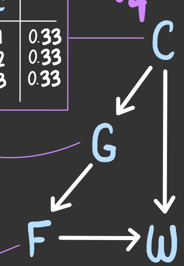
g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

$$P(w) = \sum_c \sum_f \sum_g P(w, f, g, c)$$

$$= \sum_c \sum_f \sum_g \underbrace{P(w \mid f, c)}_{h_1(f,c,w)} \underbrace{P(f \mid g)}_{h_2(g,f)} \underbrace{P(g \mid c)}_{h_3(c,g)} \underbrace{P(c)}_{h_4(c)}$$

these are all just **multivariable functions**



h_4

c	
1	0.33
2	0.33
3	0.33

h_3

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

h_2

g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

h_1

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

these are all just multivariable functions

$P(g|c)$

$h_3(c, g)$

h_3

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

c	g	$P(g \mid c)$
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

these are all just multivariable functions

c	g	$h_3(c, g)$ $\cancel{P(g \mid c)}$
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

these are all just multivariable functions so computing a probability boils down to computing a sum of products

$$P(w) = \sum_c \sum_f \sum_g P(w \mid f, c)\, P(f \mid g)\, P(g \mid c)\, P(c)$$

so computing a probability boils down to computing a sum of products

$$P(w) = \sum_c \sum_f \sum_g \overset{h_1(f,c,w)}{\cancel{P(w|f,c)}}\ \overset{h_2(g,f)}{\cancel{P(f|g)}}\ \overset{h_3(c,g)}{\cancel{P(g|c)}}\ \overset{h_4(c)}{\cancel{P(c)}}$$

$$P(w) = \sum_{c}\sum_{f}\sum_{g} h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$P(w) = \sum_c \sum_f \sum_g \underline{h_1(f,c,w)}\, h_2(g,f)\, h_3(c,g)\, \underline{h_4(c)}$$

not all of the functions

involve g

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \sum_g h_2(g,f)\, h_3(c,g)$$

$$\longrightarrow$$

$$\boxed{\sum_x k f(x) = k \sum_x f(x)}$$

e.g. $\sum_{x=1}^{3} 10x^2 = 10 \cdot 1^2 + 10 \cdot 2^2 + 10 \cdot 3^2 = 10(1^2 + 2^2 + 3^2) = 10 \sum_{x=1}^{3} x^2$

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \boxed{\sum_g h_2(g,f)\, h_3(c,g)}$$

this subexpression is a

function

of which variables?

your answer here

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \boxed{\sum_g h_2(g,f)\, h_3(c,g)}$$

this subexpression is a

function

of which variables?

f, c

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c) \sum_g h_2(g,f)\,h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c)\;h_5(f,c) \longleftarrow \text{let } h_5(f,c) = \sum_g h_2(g,f)\,h_3(c,g)$$

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c) \sum_g h_2(g,f)\,h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c)\ h_5(f,c) \longleftarrow \text{let } h_5(f,c) = \sum_g h_2(g,f)\,h_3(c,g)$$

compute and store all 9 values of this function ⟶

F	C	$h_5(f,c)$
1	1	0
1	2	0
1	3	0
2	1	.5
2	2	1
2	3	0
3	1	.5
3	2	0
3	3	1

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c)\ h_5(f,c) \longleftarrow \text{let } \underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\, h_3(c,g)$$

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c) \sum_g h_2(g,f)\,h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c)\;h_5(f,c)$$

let $\underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\,h_3(c,g)$

$$= \sum_c h_4(c) \boxed{\sum_f h_1(f,c,w)\,h_5(f,c)}$$

this subexpression is a

function

of which variables?

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c)\; h_5(f,c) \longleftarrow \text{let } \underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c h_4(c) \sum_f h_1(f,c,w)\; h_5(f,c)$$

$$= \sum_c h_4(c)\, h_6(w,c) \longleftarrow \text{let } h_6(w,c) = \sum_f h_1(f,c,w)\, h_5(f,c)$$

compute and store all 6 values of this function $\longrightarrow$

W	C	$h_6(w,s,c)$
yes	1	0
yes	2	1
yes	3	1
no	1	1
no	2	0
no	3	0

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c)\, h_5(f,c) \longleftarrow \text{let } \underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c h_4(c) \sum_f h_1(f,c,w)\, h_5(f,c)$$

$$= \sum_c h_4(c)\, h_6(w,c) \longleftarrow \text{let } \underbrace{h_6(w,c)}_{6 \text{ computed values}} = \sum_f h_1(f,c,w)\, h_5(f,c)$$

$$P(w) = \sum_{c}\sum_{f}\sum_{g} h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$= \sum_{c}\sum_{f} h_1(f,c,w)\,h_4(c)\sum_{g} h_2(g,f)\,h_3(c,g)$$

$$= \sum_{c}\sum_{f} h_1(f,c,w)\,h_4(c)\,h_5(f,c)$$

let $\underbrace{h_5(f,c)}_{9\text{ computed values}} = \sum_{g} h_2(g,f)\,h_3(c,g)$

$$= \sum_{c} h_4(c)\sum_{f} h_1(f,c,w)\,h_5(f,c)$$

$$\boxed{= \sum_{c} h_4(c)\,h_6(w,c)}$$

let $\underbrace{h_6(w,c)}_{6\text{ computed values}} = \sum_{f} h_1(f,c,w)\,h_5(f,c)$

this subexpression is a **function** of which variables?

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\,h_2(g,f)\,h_3(c,g)\,h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c) \sum_g h_2(g,f)\,h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\,h_4(c)\;h_5(f,c)$$

let $\underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\,h_3(c,g)$

$$= \sum_c h_4(c) \sum_f h_1(f,c,w)\;h_5(f,c)$$

$$= \sum_c h_4(c)\,h_6(w,c)$$

let $\underbrace{h_6(w,c)}_{6 \text{ computed values}} = \sum_f h_1(f,c,w)\,h_5(f,c)$

$$= h_7(w)$$

let $\underbrace{h_7(w)}_{2 \text{ computed values}} = \sum_c h_4(c)\,h_6(w,c)$

$$P(w) = \sum_c \sum_f \sum_g h_1(f,c,w)\, h_2(g,f)\, h_3(c,g)\, h_4(c)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c) \sum_g h_2(g,f)\, h_3(c,g)$$

$$= \sum_c \sum_f h_1(f,c,w)\, h_4(c)\, h_5(f,c)$$

let $\underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\, h_3(c,g)$

$$= \sum_c h_4(c) \sum_f h_1(f,c,w)\, h_5(f,c)$$

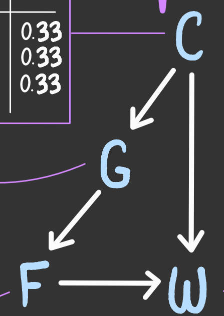
$$= \sum_c h_4(c)\, h_6(w,c)$$

let $\underbrace{h_6(w,c)}_{6 \text{ computed values}} = \sum_f h_1(f,c,w)\, h_5(f,c)$

$$= h_7(w)$$

let $\underbrace{h_7(w)}_{2 \text{ computed values}} = \sum_c h_4(c)\, h_6(w,c)$

we've computed the marginal!



h_4

c	
1	0.33
2	0.33
3	0.33

h_3

c	g	
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

h_2

g	f	
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

h_1

f	c	w	
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

compute these

let $\underbrace{h_5(f,c)}_{\text{9 computed values}} = \sum_g h_2(g,f)\,h_3(c,g)$

let $\underbrace{h_6(w,c)}_{\text{6 computed values}} = \sum_f h_1(f,c,w)\,h_5(f,c)$

let $\underbrace{h_7(w)}_{\text{2 computed values}} = \sum_c h_4(c)\,h_6(w,c)$

f	c	w	h_1
1	1	yes	1
1	1	no	0
1	2	yes	0
1	2	no	1
1	3	yes	0
1	3	no	1
2	1	yes	0
2	1	no	1
2	2	yes	1
2	2	no	0
2	3	yes	0
2	3	no	1
3	1	yes	0
3	1	no	1
3	2	yes	0
3	2	no	1
3	3	yes	1
3	3	no	0

g	f	h_2
2	1	0
2	2	0
2	3	1
3	1	0
3	2	1
3	3	0

c	g	h_3
1	2	0.5
1	3	0.5
2	2	0
2	3	1
3	2	1
3	3	0

c	h_4
1	0.33
2	0.33
3	0.33

compute these

$$\text{let } \underbrace{h_5(f,c)}_{9 \text{ computed values}} = \sum_g h_2(g,f)\,h_3(c,g)$$

$$\text{let } \underbrace{h_6(w,c)}_{6 \text{ computed values}} = \sum_f h_1(f,c,w)\,h_5(f,c)$$

$$\text{let } \underbrace{h_7(w)}_{2 \text{ computed values}} = \sum_c h_4(c)\,h_6(w,c)$$

to compute $P(w) = h_7(w)$

compute $h_5(f, c)$

F	C	$h_5(f,c)$
1	1	?
1	2	?
1	3	?
2	1	?
2	2	?
2	3	?
3	1	?
3	2	?
3	3	?

compute $h_6(w, c)$

W	C	$h_6(w,c)$
yes	1	?
yes	2	?
yes	3	?
no	1	?
no	2	?
no	3	?

compute $h_7(w)$

W	$h_7(w)$
yes	?
no	?

to compute $P(w) = h_7(w)$

compute $h_5(f, c)$

F	C	$h_5(f,c)$
1	1	0
1	2	0
1	3	0
2	1	.5
2	2	1
2	3	0
3	1	.5
3	2	0
3	3	1

compute $h_6(w, c)$

W	C	$h_6(w,c)$
yes	1	0
yes	2	1
yes	3	1
no	1	1
no	2	0
no	3	0

compute $h_7(w)$

W	$h_7(w)$
yes	$\frac{2}{3}$
no	$\frac{1}{3}$

to compute $P(w) = h_7(w)$

compute $h_5(f, c)$

F	C	$h_5(f, c)$
1	1	0
1	2	0
1	3	0
2	1	.5
2	2	1
2	3	0
3	1	.5
3	2	0
3	3	1

compute $h_6(w, c)$

W	C	$h_6(w, c)$
yes	1	0
yes	2	1
yes	3	1
no	1	1
no	2	0
no	3	0

compute $h_7(w)$

W	$h_7(w)$
yes	$\frac{2}{3}$
no	$\frac{1}{3}$

you should switch!