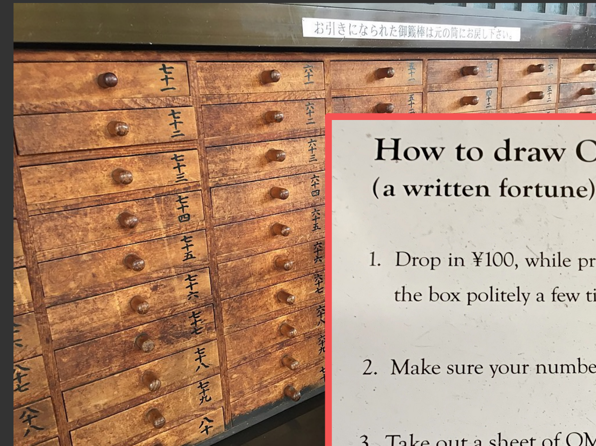
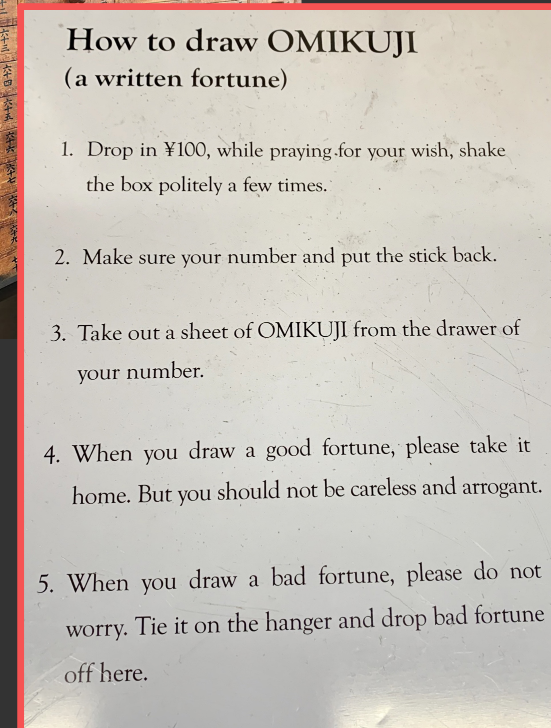
# probability

12 oct 2022

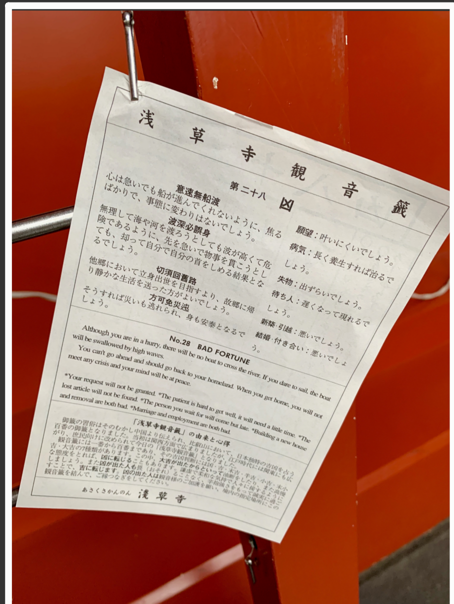
CSCI 373


みくじ 100円




How to draw OMIKUJI
(a written fortune)
1. Drop in ¥100, while praying for your wish, shake the box politely a few times.
2. Make sure your number and put the stick back.
3. Take out a sheet of OMIKUJI from the drawer of your number.
4. When you draw a good fortune, please take it home. But you should not be careless and arrogant.
5. When you draw a bad fortune, please do not worry. Tie it on the hanger and drop bad fortune off here.

omikuji

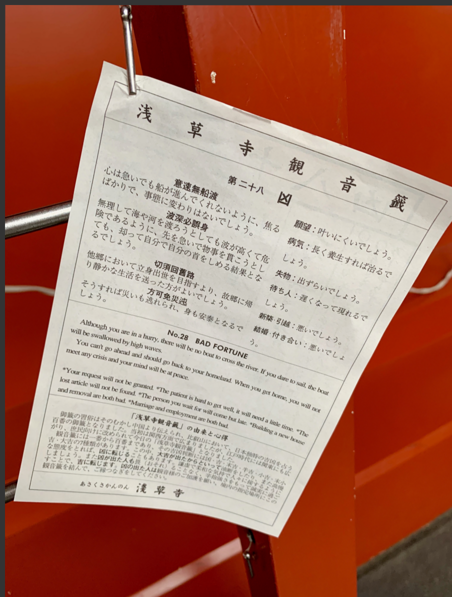



## Fortunes [ edit ]

The standard *Ganzan Daishi Hyakusen* sequence contains the following fortunes (from best to worst):

- Great blessing (大吉, *dai-kichi*)
- Blessing (吉, *kichi*)
- Small blessing (小吉, *shō-kichi*)
- Half-blessing (半吉, *han-kichi*)
- Future blessing (末吉, *sue-kichi*)
- Future small blessing (末小吉, *sue-shō-kichi*)
- Misfortune (凶, *kyō*)

from the wikipedia article "o-mikuji"

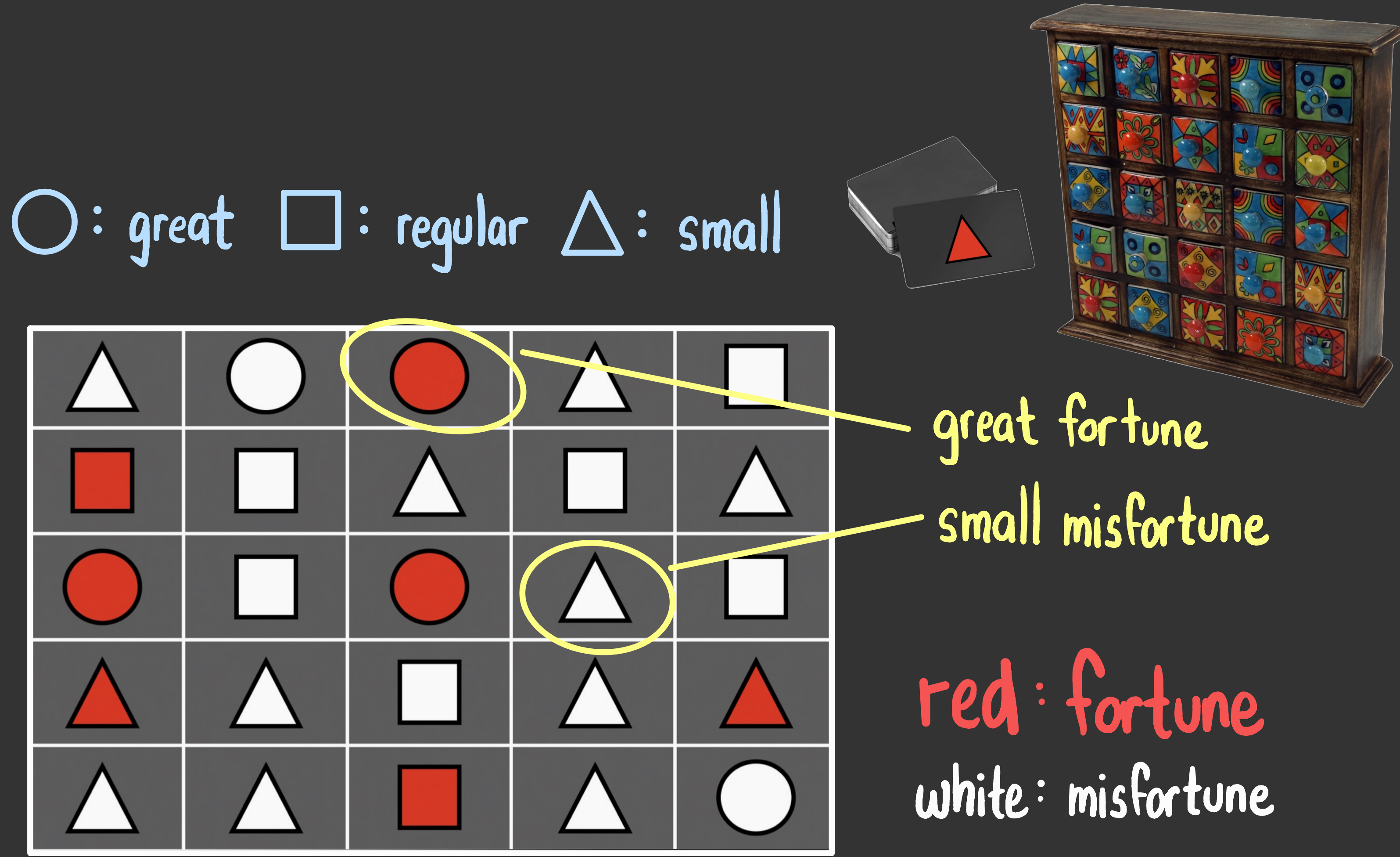



## Fortunes [ edit ]

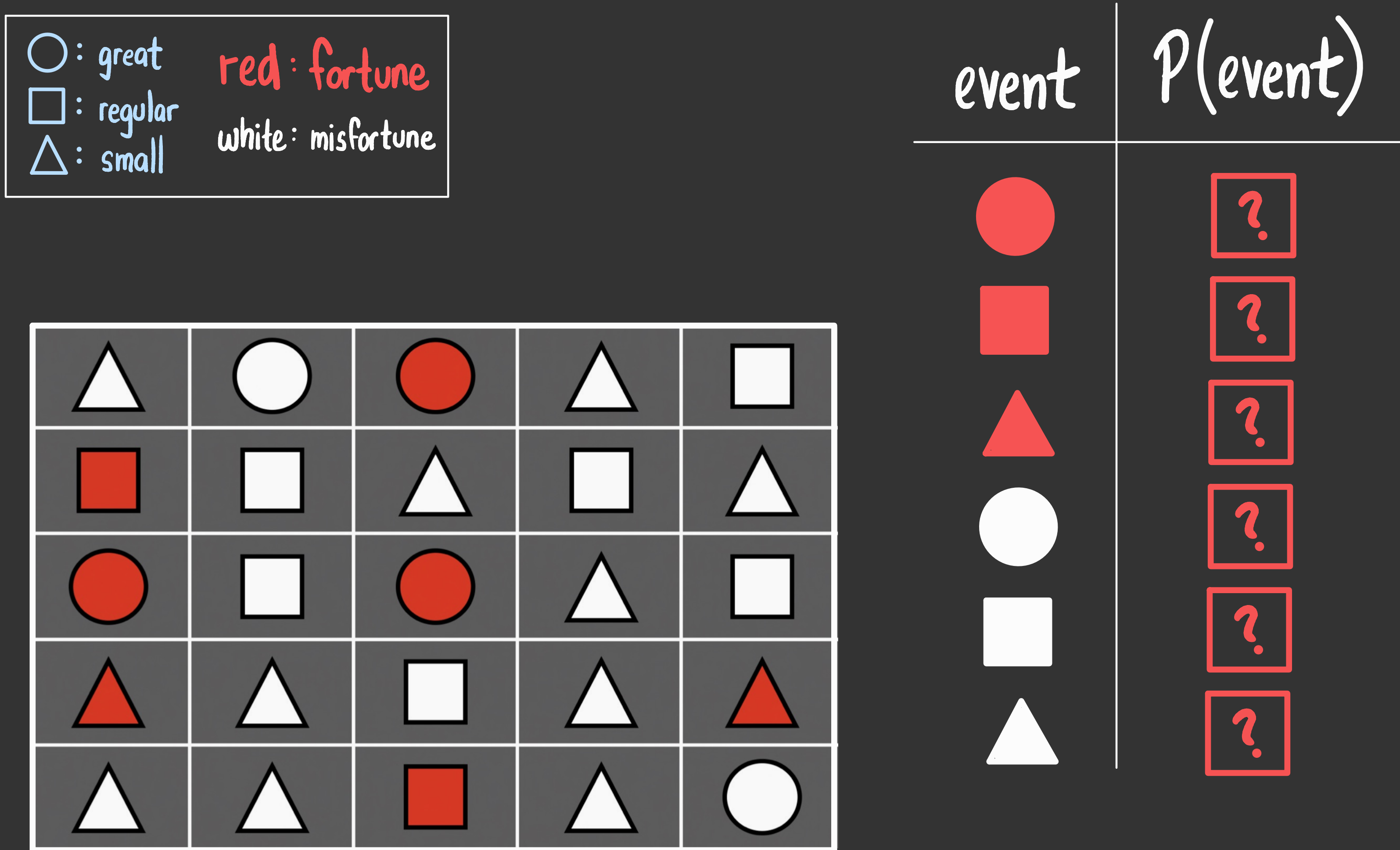
The standard *Ganzan Daishi Hyakusen* sequence contains the following fortunes (from best to worst):

- Great blessing (大吉, *dai-kichi*)
- Blessing (吉, *kichi*)
- Small blessing (小吉, *shō-kichi*)
- ~~Half blessing (半吉, *han-kichi*)~~ Great Misfortune (大凶, *dai-kyō*)
- ~~Future blessing (末吉, *sue-kichi*)~~ Small Misfortune (小凶, *shō-kyō*)
- ~~Future small blessing (末小吉, *sue-shō-kichi*)~~
- Misfortune (凶, *kyō*)

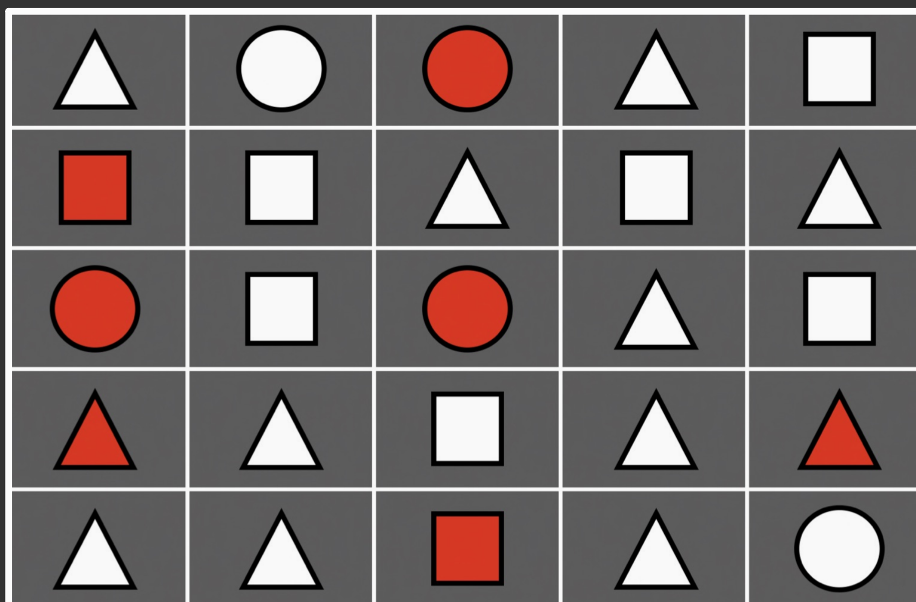
from the wikipedia article "o-mikuji"

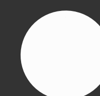


○ : great □ : regular △ : small
great fortune
small misfortune
red : fortune
white : misfortune



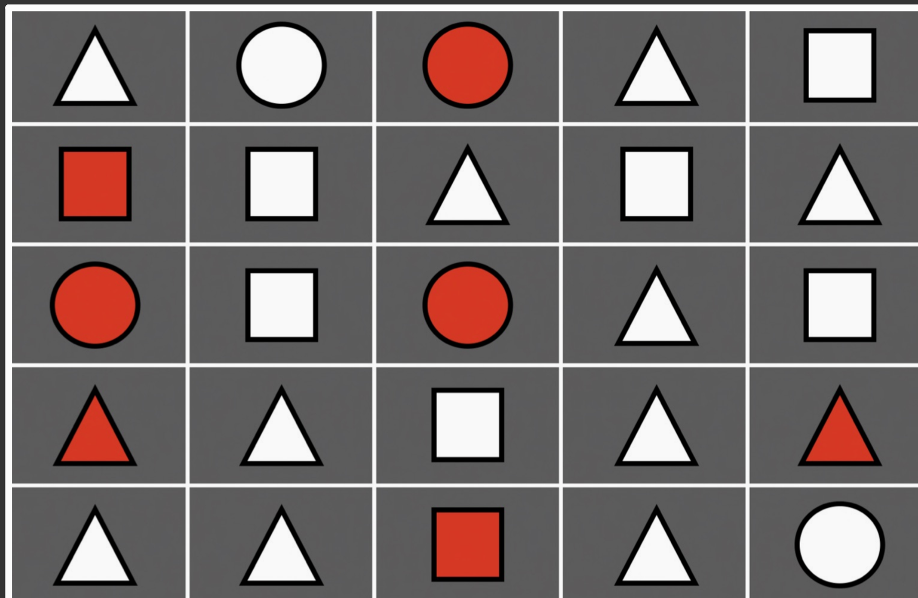
: great
: regular
: small
red : fortune
white : misfortune
event
P(event)
?
?
?
?
?
?

| event | P(event) |
|---|---|
|  | $\frac{3}{25}$ |
|  | $\frac{2}{25}$ |
|  | $\frac{2}{25}$ |
|  | $\frac{2}{25}$ |
|  | $\frac{6}{25}$ |
|  | $\frac{10}{25}$ |

○ : great
□ : regular
△ : small

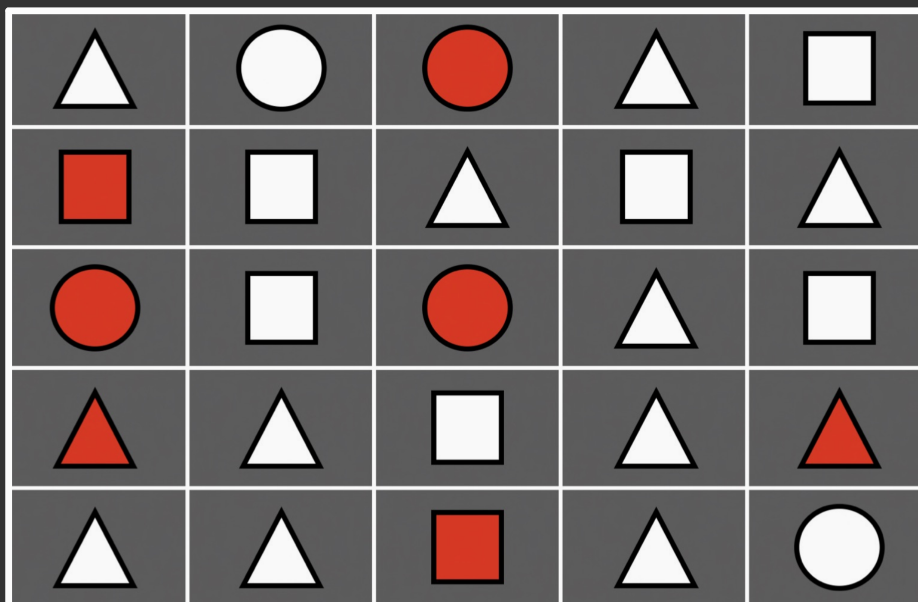
red : fortune
white : misfortune



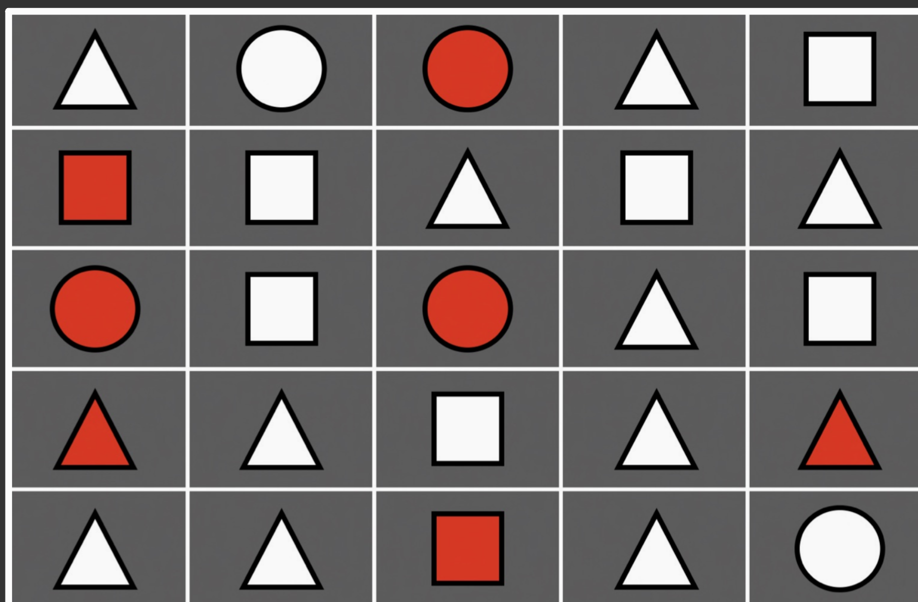
| C | S | $P(c,s)$ |
|---|---|---|
| red | ● | $\frac{3}{25}$ |
| red | ■ | $\frac{2}{25}$ |
| red | ▲ | $\frac{2}{25}$ |
| white | ● | $\frac{2}{25}$ |
| white | ■ | $\frac{6}{25}$ |
| white | ▲ | $\frac{10}{25}$ |

a joint distribution over variables $\{C, S\}$ → 

| C | S | $P(c,s)$ |
|---|---|---|
| red | ● | $\frac{3}{25}$ |
| red | ■ | $\frac{2}{25}$ |
| red | ▲ | $\frac{2}{25}$ |
| white | ● | $\frac{2}{25}$ |
| white | ■ | $\frac{6}{25}$ |
| white | ▲ | $\frac{10}{25}$ |



# what is the probability of misfortune?



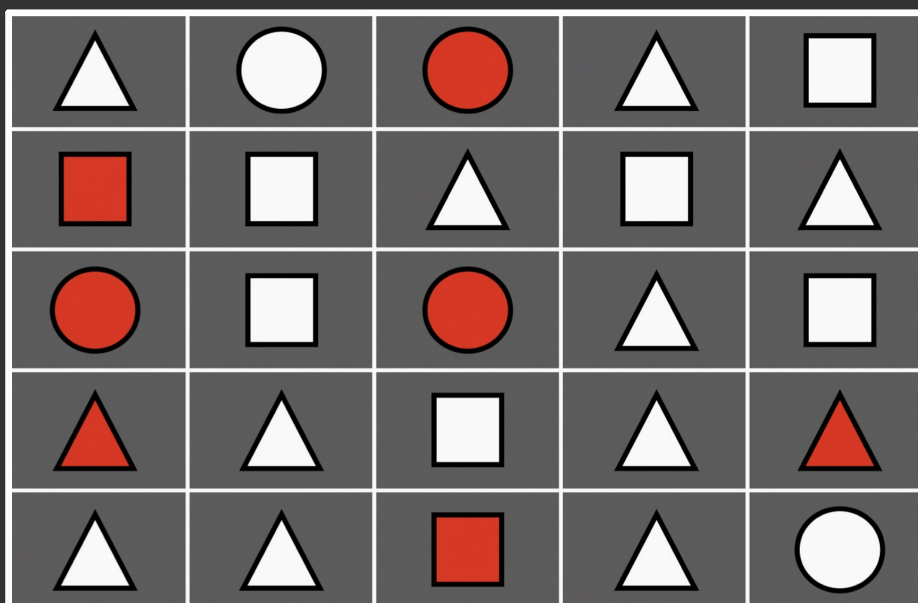
| C | S | $P(c,s)$ |
|---|---|---|
| red | ○ | $\frac{3}{25}$ |
| red | □ | $\frac{2}{25}$ |
| red | △ | $\frac{2}{25}$ |
| white | ○ | $\frac{2}{25}$ |
| white | □ | $\frac{6}{25}$ |
| white | △ | $\frac{10}{25}$ |

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

what is the probability of misfortune?

# what is $P(C = \text{white})$?



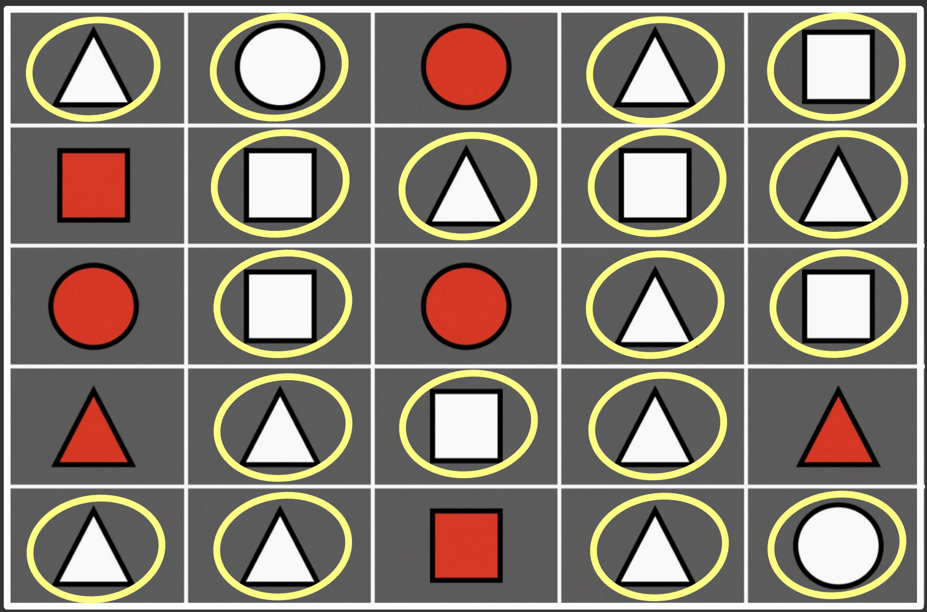
| C | S | $P(c, s)$ |
|---|---|---|
| red | ○ | $\frac{3}{25}$ |
| red | □ | $\frac{2}{25}$ |
| red | △ | $\frac{2}{25}$ |
| white | ○ | $\frac{2}{25}$ |
| white | □ | $\frac{6}{25}$ |
| white | △ | $\frac{10}{25}$ |

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

what is the probability of misfortune?

# what is $P(C = \text{white})$?



| C | S | $P(c,s)$ |
|---|---|---|
| red | ○ | $\frac{3}{25}$ |
| red | □ | $\frac{2}{25}$ |
| red | △ | $\frac{2}{25}$ |
| white | ○ | $\frac{2}{25}$ |
| white | □ | $+\ \frac{6}{25}$ |
| white | △ | $+\ \frac{10}{25}$ |

$$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

| C | S | P(c,s) | |
|---|---|---|---|
| red | ● | $\frac{3}{25}$ | |
| red | ■ | $\frac{2}{25}$ | $= \frac{7}{25}$ |
| red | ▲ | $\frac{2}{25}$ | |
| white | ● | $\frac{2}{25}$ + | |
| white | ■ | $\frac{6}{25}$ + | $= \frac{18}{25}$ |
| white | ▲ | $\frac{10}{25}$ | |

joint probability

| C | P(c) |
|---|---|
| red | $\frac{7}{25}$ |
| white | $\frac{18}{25}$ |

marginal probability

# the law of total probability

| C | S | $P(c,s)$ |
|---|---|---|
| red | ● | $\frac{3}{25}$ |
| red | ■ | $\frac{2}{25}$ |
| red | ▲ | $\frac{2}{25}$ |
| white | ● | $\frac{2}{25}$ |
| white | ■ | $\frac{6}{25}$ |
| white | ▲ | $\frac{10}{25}$ |

$\frac{3}{25} + \frac{2}{25} + \frac{2}{25} = \frac{7}{25}$

$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$

joint probability

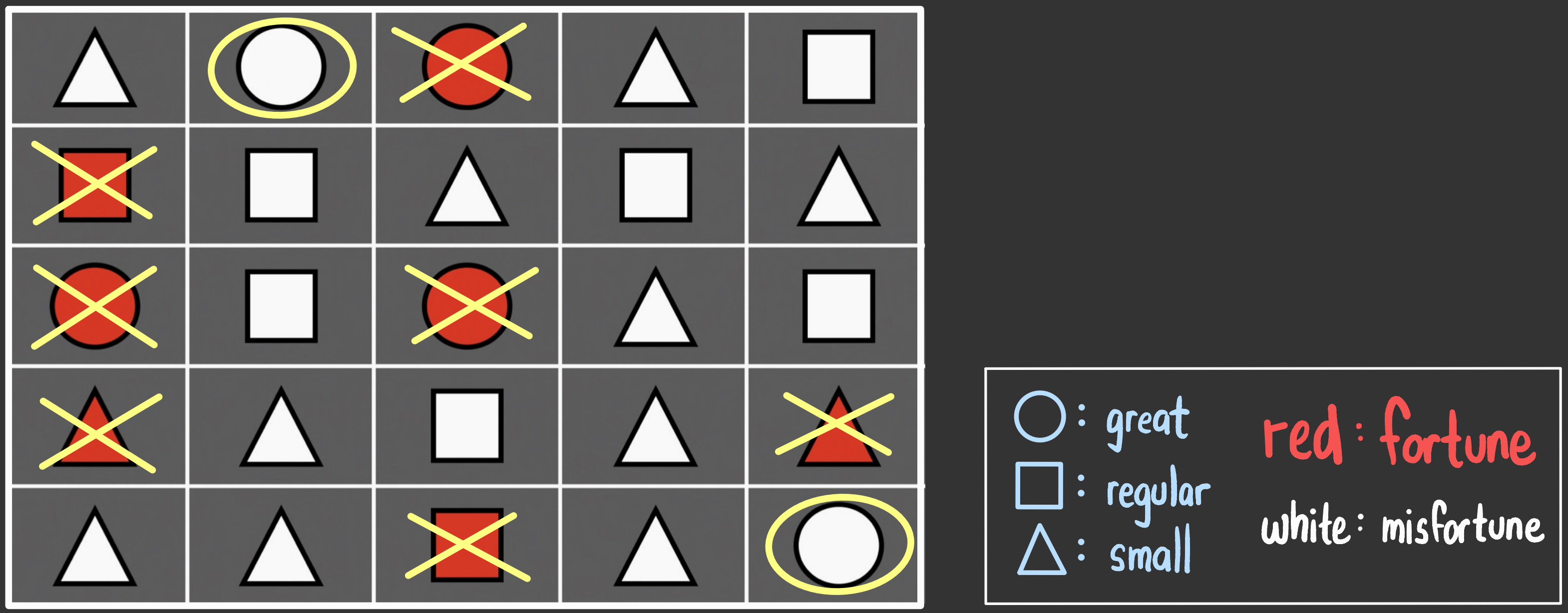
| C | $P(c)$ |
|---|---|
| red | $\frac{7}{25}$ |
| white | $\frac{18}{25}$ |

marginal probability

$$\underbrace{P(x_1, \ldots, x_m)}_{\text{marginal probability}} = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} \underbrace{P(x_1, \ldots, x_n)}_{\text{joint probability}}$$

domain of $x_n$

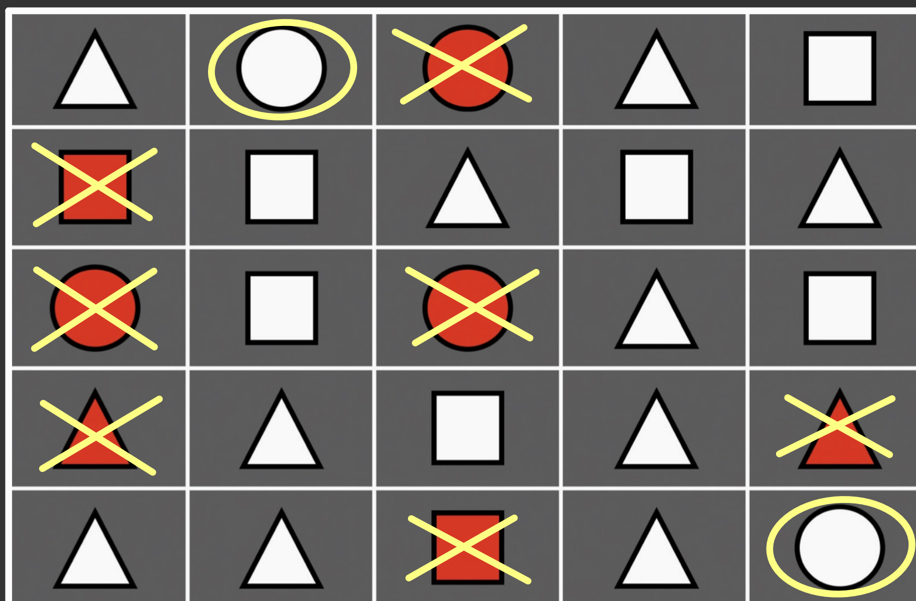
what is the probability of great misfortune, given we draw some kind of misfortune?



what is the probability of great misfortune, given we draw some kind of misfortune?

what is $P(S = \bigcirc \mid C = \text{white})$?

$$\frac{2 \; \bigcirc}{18 \text{ white}} = \frac{1}{9}$$



what is the probability of great misfortune, given we draw some kind of misfortune?

what is $P(S = \text{●} \mid C = \text{white})$?

$$\frac{2\ \text{●}}{18 \text{ white}} = \frac{1}{9}$$

| C | S | P(c, s) |
|---|---|---|
| red | ● | $\frac{3}{25}$ |
| red | ■ | $\frac{2}{25}$ |
| red | ▲ | $\frac{2}{25}$ |
| white | ● | $\frac{2}{25}$ |
| white | ■ | $\frac{6}{25}$ |
| white | ▲ | $\frac{10}{25}$ |

$$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$$

or:

$$\frac{P(C = \text{white}, S = \text{●})}{P(C = \text{white})}$$

$$= \frac{\frac{2}{25}}{\frac{18}{25}} = \frac{1}{9}$$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

| C | S | $P(c,s)$ |
|---|---|---|
| red | ● | $\frac{3}{25}$ |
| red | ■ | $\frac{2}{25}$ |
| red | ▲ | $\frac{2}{25}$ |
| white | ● | $\frac{2}{25}$ |
| white | ■ | $+\ \frac{6}{25}$ |
| white | ▲ | $+\ \frac{10}{25}$ |

$$\frac{3}{25}\ \frac{2}{25}\ \frac{2}{25} = \frac{7}{25}$$

$$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$$

joint probability

| C | $P(c)$ |
|---|---|
| red | $\frac{7}{25}$ |
| white | $\frac{18}{25}$ |

marginal probability

| C | S | $P(s \mid c) = \frac{P(c,s)}{P(c)}$ |
|---|---|---|
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

$$P(x_1,\ldots,x_m \mid x_{m+1},\ldots,x_n) = \frac{P(x_1,\ldots,x_n)}{P(x_{m+1},\ldots,x_n)}$$

| C | S | $P(s \mid c) = \frac{P(c,s)}{P(c)}$ |
|---|---|---|
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $P(s|c) = \dfrac{P(c|s)P(s)}{P(c)}$

| C | S | $P(s\|c) = \frac{P(c,s)}{P(c)}$ |
|---|---|---|
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

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which means: $P(s|c) = \dfrac{P(c|s)P(s)}{P(c)}$

| C | S | $P(s\|c) = \frac{P(c,s)}{P(c)}$ |
| --- | --- | --- |
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

## observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

**so:** $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

**therefore:** $P(s|c)P(c) = P(c|s)P(s)$

**which means:** $P(s|c) = \dfrac{P(c|s)P(s)}{P(c)}$

| C | S | $P(s\|c) = \frac{P(c,s)}{P(c)}$ |
|---|---|---|
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $$P(s|c) = \frac{P(c|s)P(s)}{P(c)}$$

| C | S | $P(s\|c) = \frac{P(c,s)}{P(c)}$ |
| --- | --- | --- |
| red | ● | $\frac{3}{7}$ |
| red | ■ | $\frac{2}{7}$ |
| red | ▲ | $\frac{2}{7}$ |
| white | ● | $\frac{2}{18}$ |
| white | ■ | $\frac{6}{18}$ |
| white | ▲ | $\frac{10}{18}$ |

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $\boxed{P(s|c) = \frac{P(c|s)P(s)}{P(c)}}$

this is called

bayes rule

given variables $X, Y$ with domains $D(X)$ and $D(Y)$
and joint probability $P(x, y)$

$$P(x) = \sum_{y \in D(Y)} P(x, y)$$

marginal probability of X

$$P(y|x) = \frac{P(x, y)}{P(x)}$$

conditional probability of Y given X

$$P(y|x) = \frac{P(x|y)\,P(y)}{P(x)}$$

bayes rule

given variables $X_1, \ldots, X_n$ with domains $D(X_i)$

and joint probability $P(x_1, \ldots, x_n)$

$$P(x_1, \ldots, x_m) = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} P(x_1, \ldots, x_n)$$ marginal probability

$$P(x_{m+1}, \ldots, x_n \mid x_1, \ldots, x_m) = \frac{P(x_1, \ldots, x_n)}{P(x_1, \ldots, x_m)}$$ conditional probability

$$P(x_{m+1}, \ldots, x_n \mid x_1, \ldots, x_m) = \frac{P(x_1, \ldots, x_m \mid x_{m+1}, \ldots, x_n)\, P(x_1, \ldots, x_m)}{P(x_{m+1}, \ldots, x_n)}$$ bayes rule

given variables $X_1, \dots, X_n$ with domains $D(X_i)$

and joint probability $P(x_1, \dots, x_n)$

$$P(x_1, \dots, x_m) = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} P(x_1, \dots, x_n)$$

$$P(x_{m+1}, \dots, x_n \mid x_1, \dots, x_m) = \frac{P(x_1, \dots, x_n)}{P(x_1, \dots, x_m)}$$

$y \mid x$

$$P(x_{m+1}, \dots, x_n \mid x_1, \dots, x_m) = \frac{P(x_1, \dots, x_m \mid x_{m+1}, \dots, x_n)\, P(x_1, \dots, x_m)}{P(x_{m+1}, \dots, x_n)}$$