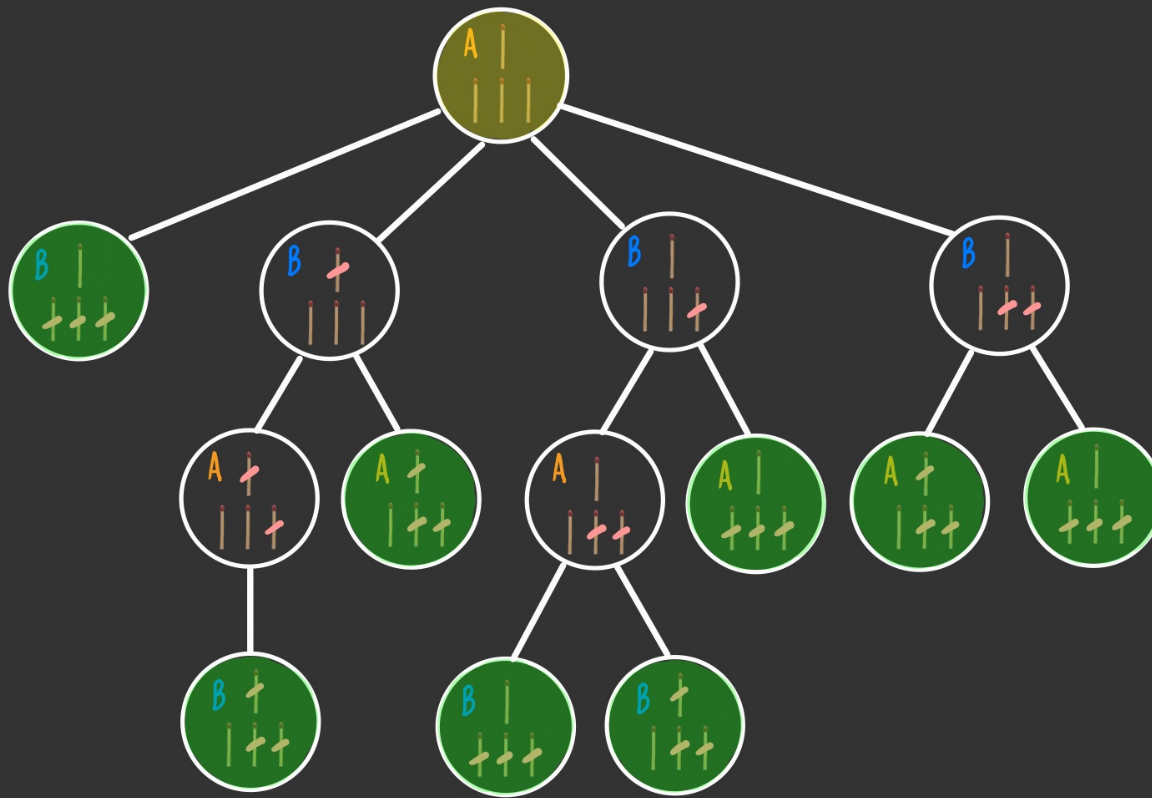
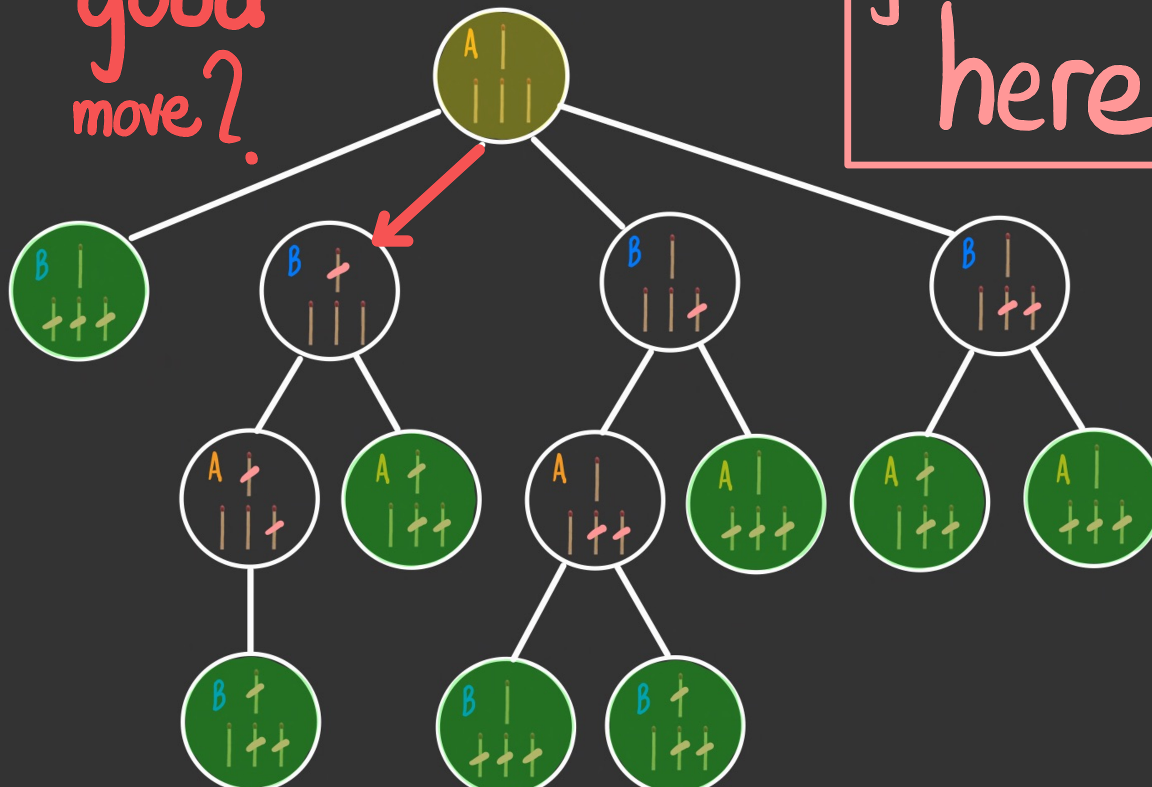
minimax

21 sept
2022

CSCI
373


A
B
B
B
B
A
A
A
A
A
A
B
B
B

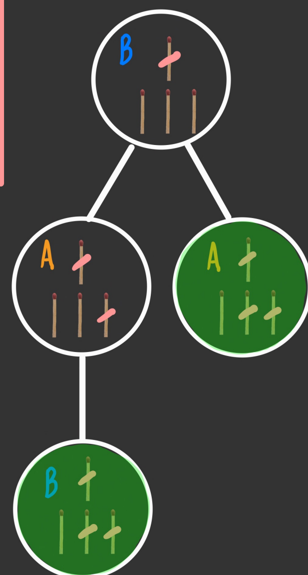
is this a good move?



your answer here

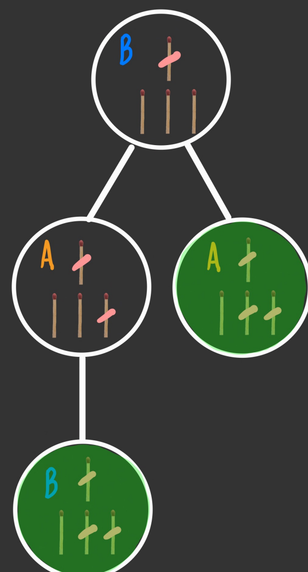
a game with players P is a tuple (M, U) where:

- $M = (Q, \Sigma, \Delta, q_0, F)$ is a state machine where each state $q \in Q$ has the form $(p, q') \in P \times Q'$ for an auxiliary set Q' of states
- utility function $U : F \times P \to \mathbb{R}$ which gives the value of each final state for each player

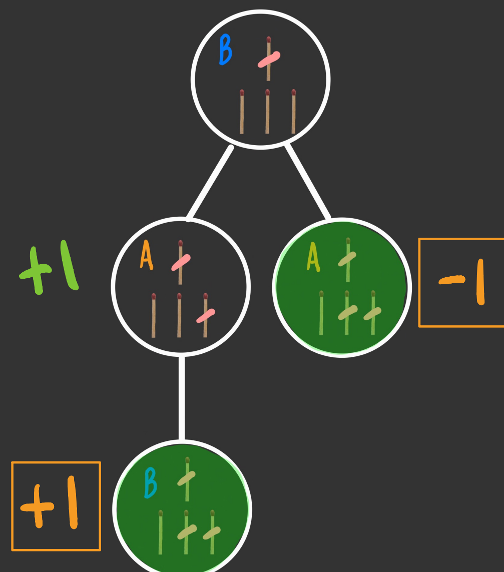


the minimax algorithm is based on the pessimistic principle that our opponent will always choose the move that is worst for us

how might we say this more rigorously?



the minimax algorithm is based on the pessimistic principle that our opponent will always choose the move that gives us the lowest utility



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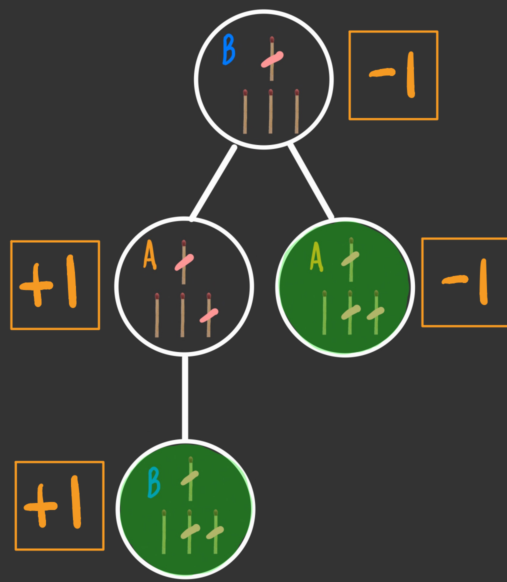

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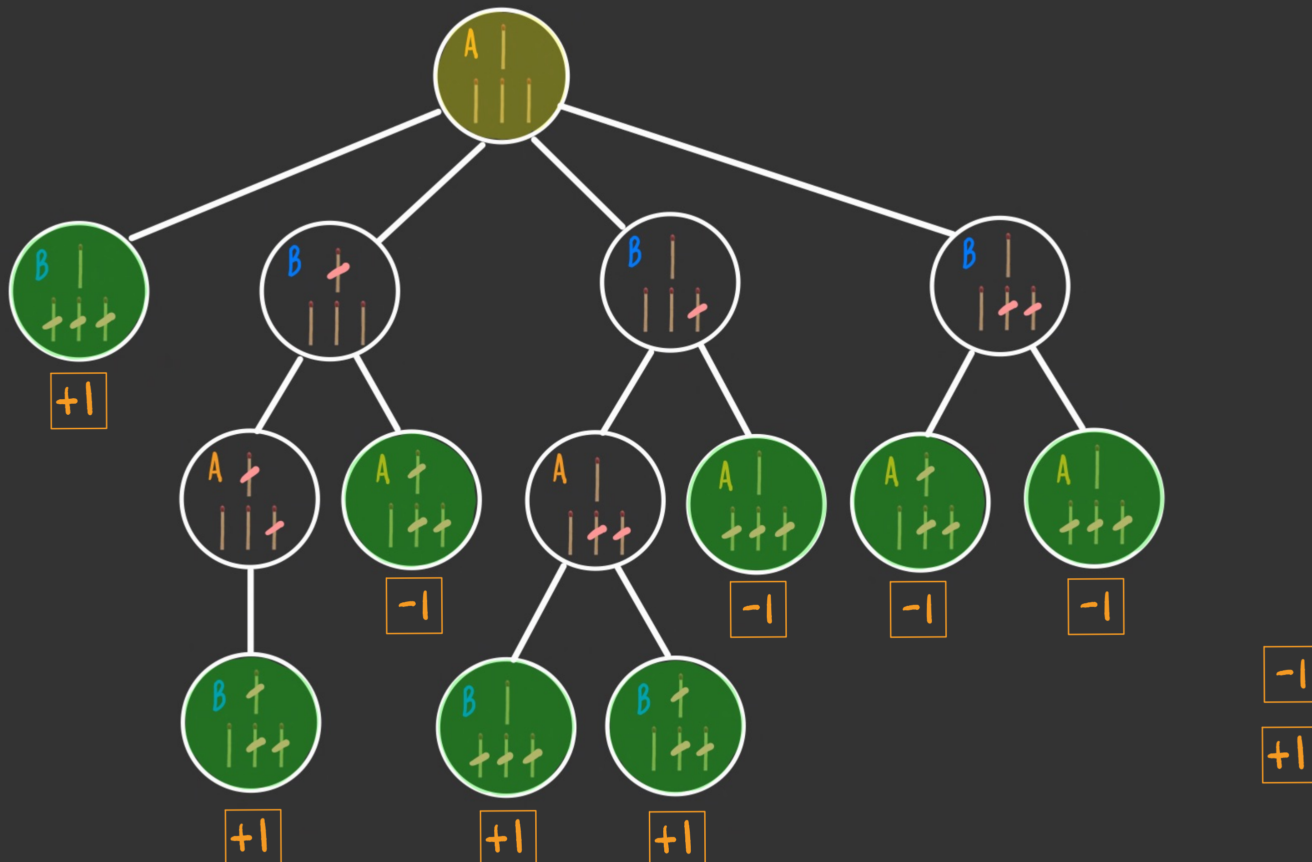
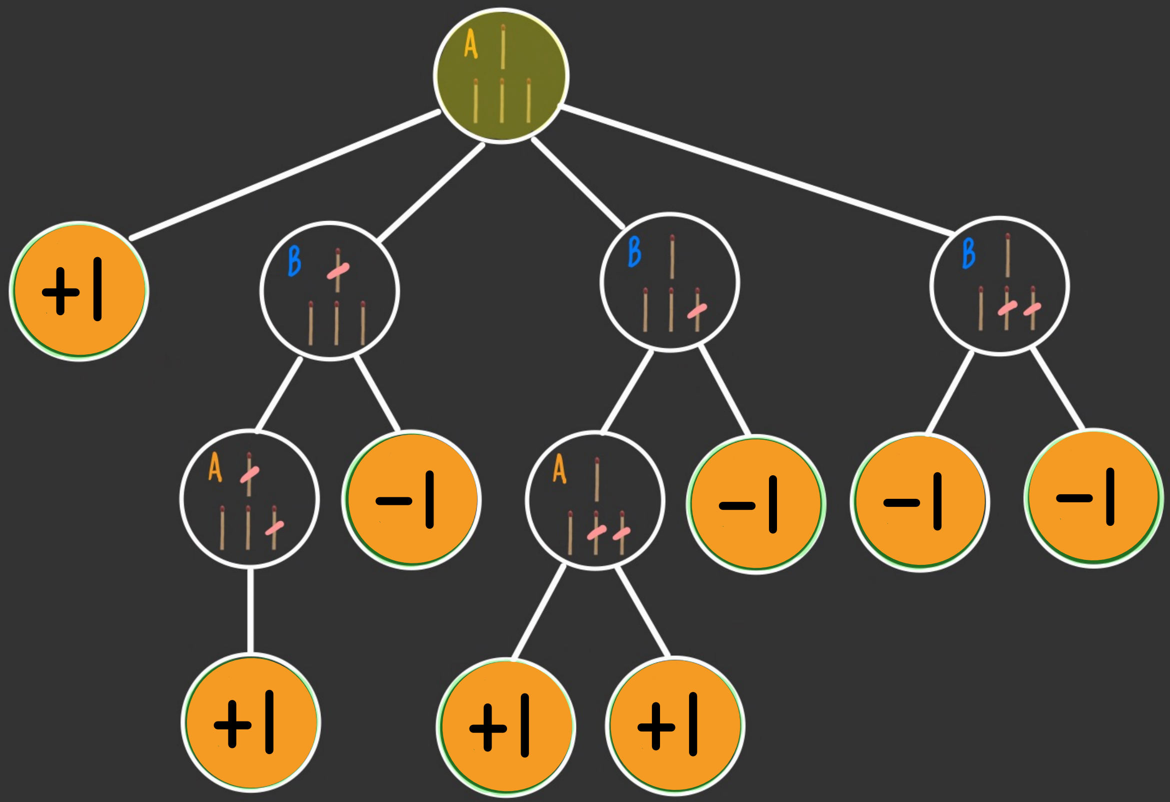

A
B
B
B
B
+1
A
A
A
A
A
A
-1
-1
-1
-1
B
B
B
+1
+1
+1
-1
+1


A
B
B
B
+1
A
-1
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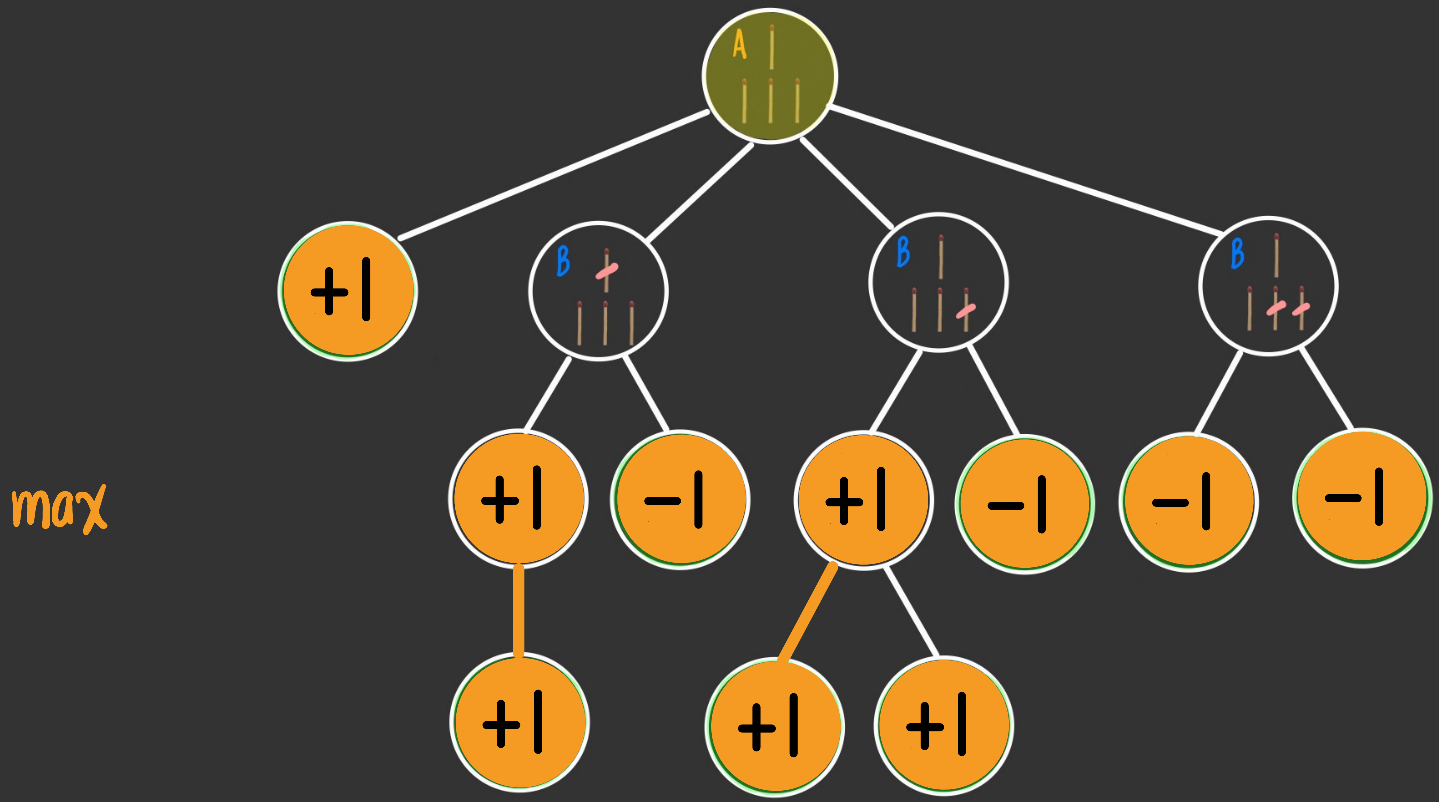
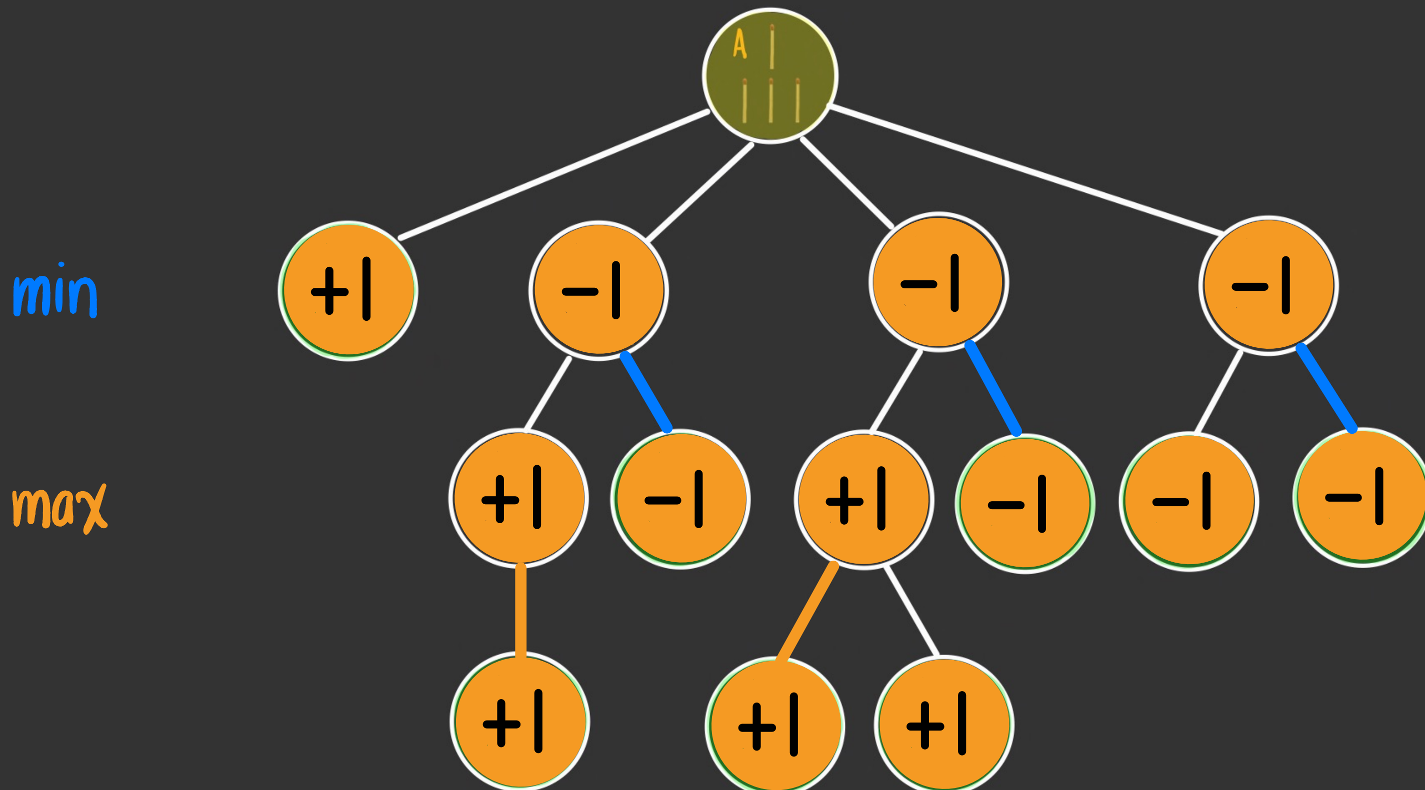
-1
-1
-1
+1
+1
+1


A
B
B
B
+1
+1
-1
+1
-1
-1
-1
+1
+1
+1
max



A
min
max
+1
−1
−1
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+1
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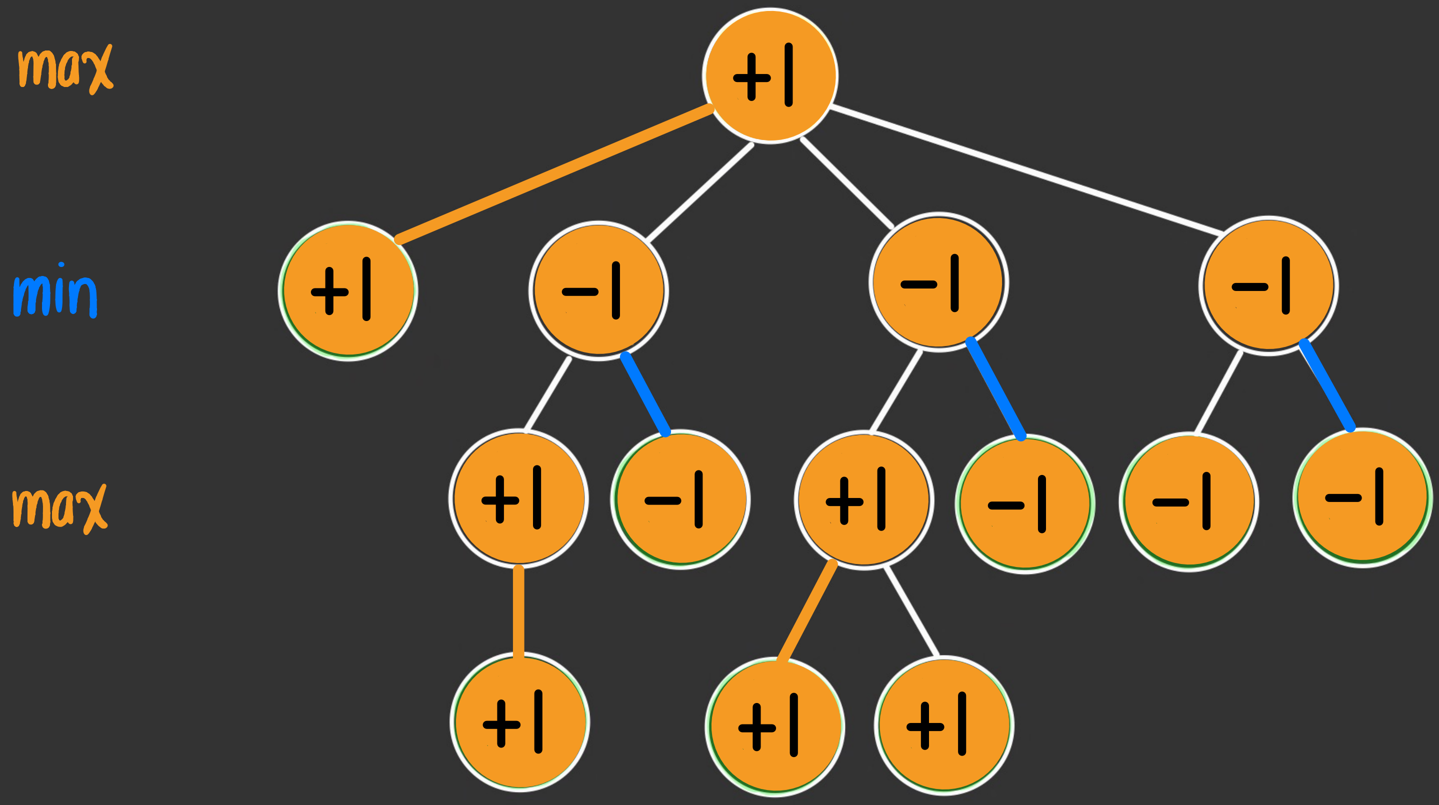
+1
+1
+1


max
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+1
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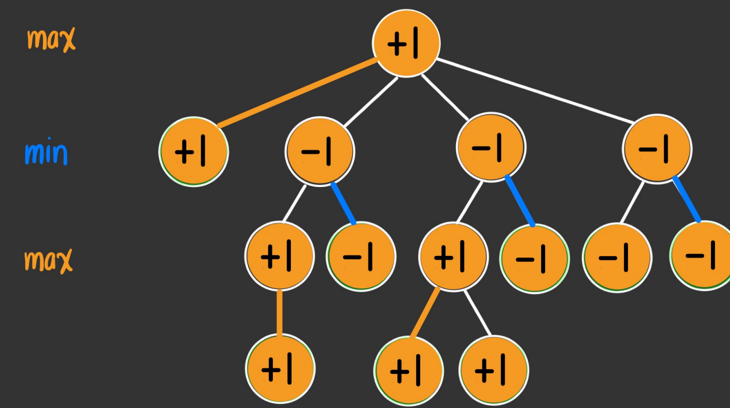
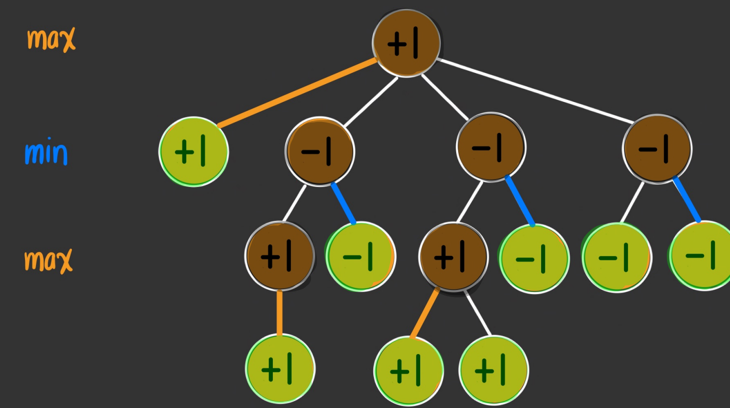
+1
+1
+1

player whose utility we're interested in

$$\text{minimax}(q,p) = \begin{cases} U(q,p) & \text{if } q \in F \\ \max\limits_{q'} \left\{ \text{minimax}(q',p) \;\middle|\; \langle q,\sigma,q' \rangle \in \Delta \right\} & \text{if } q \notin F \text{ and } p(q) = p \\ \min\limits_{q'} \left\{ \text{minimax}(q',p) \;\middle|\; \langle q,\sigma,q' \rangle \in \Delta \right\} & \text{if } q \notin F \text{ and } p(q) \neq p \end{cases}$$





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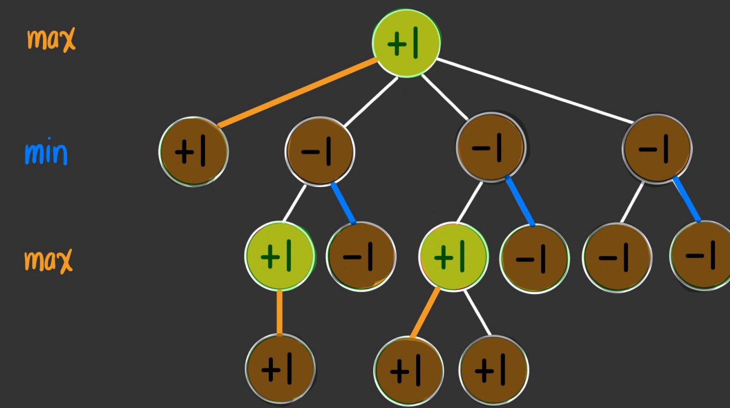

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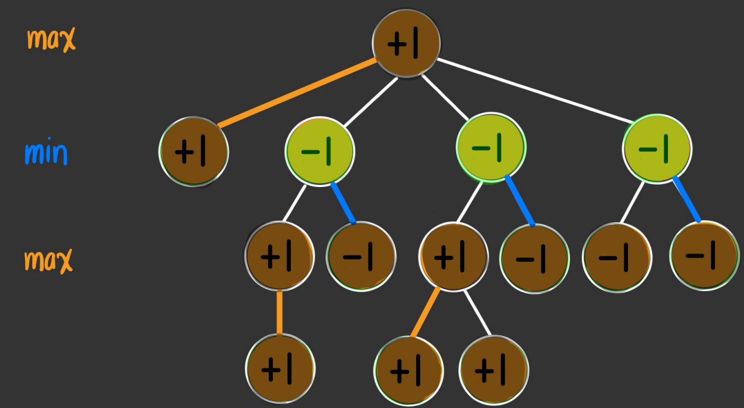
player whose turn it is in state q

player whose utility we're interested in



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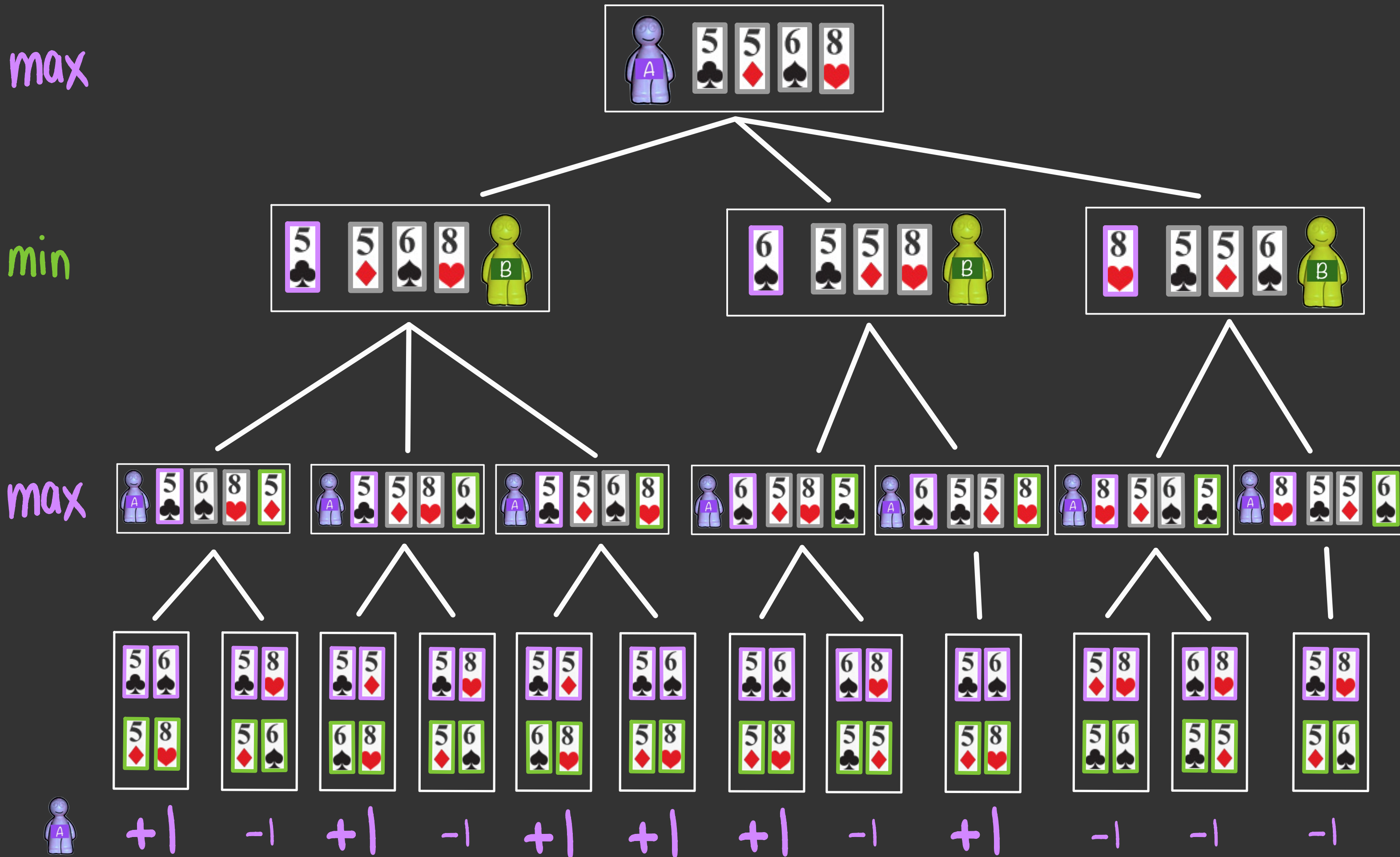
max
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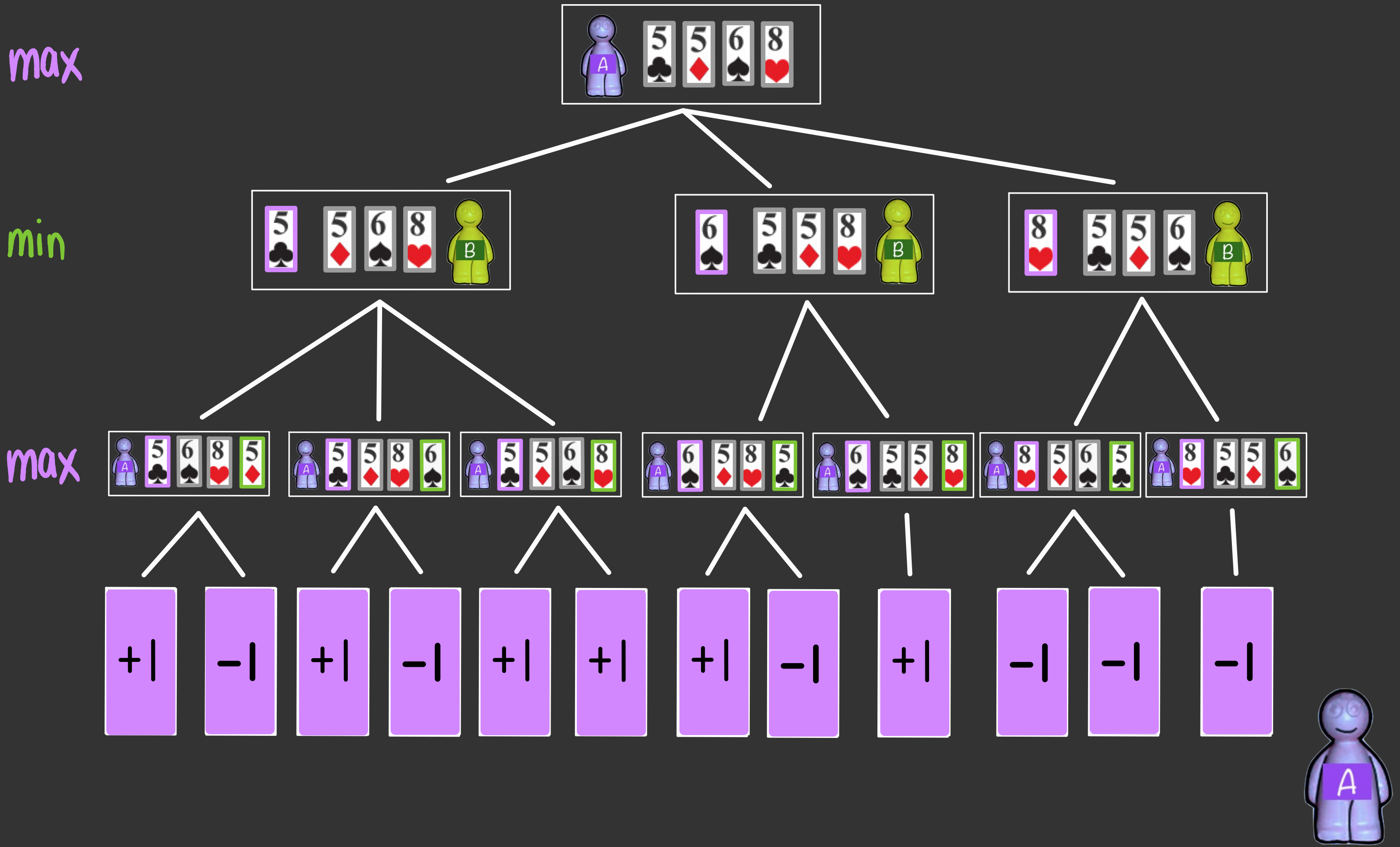
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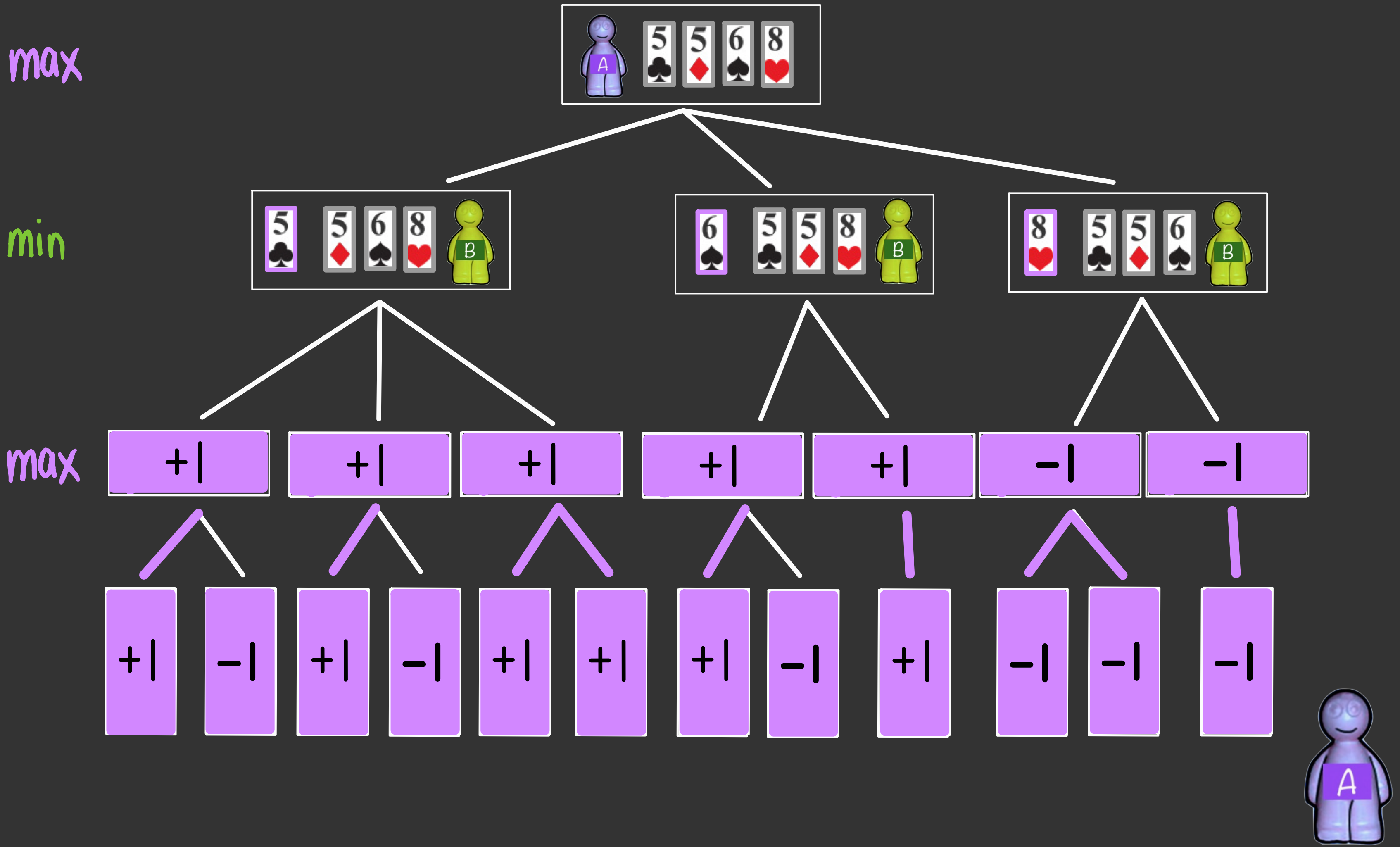
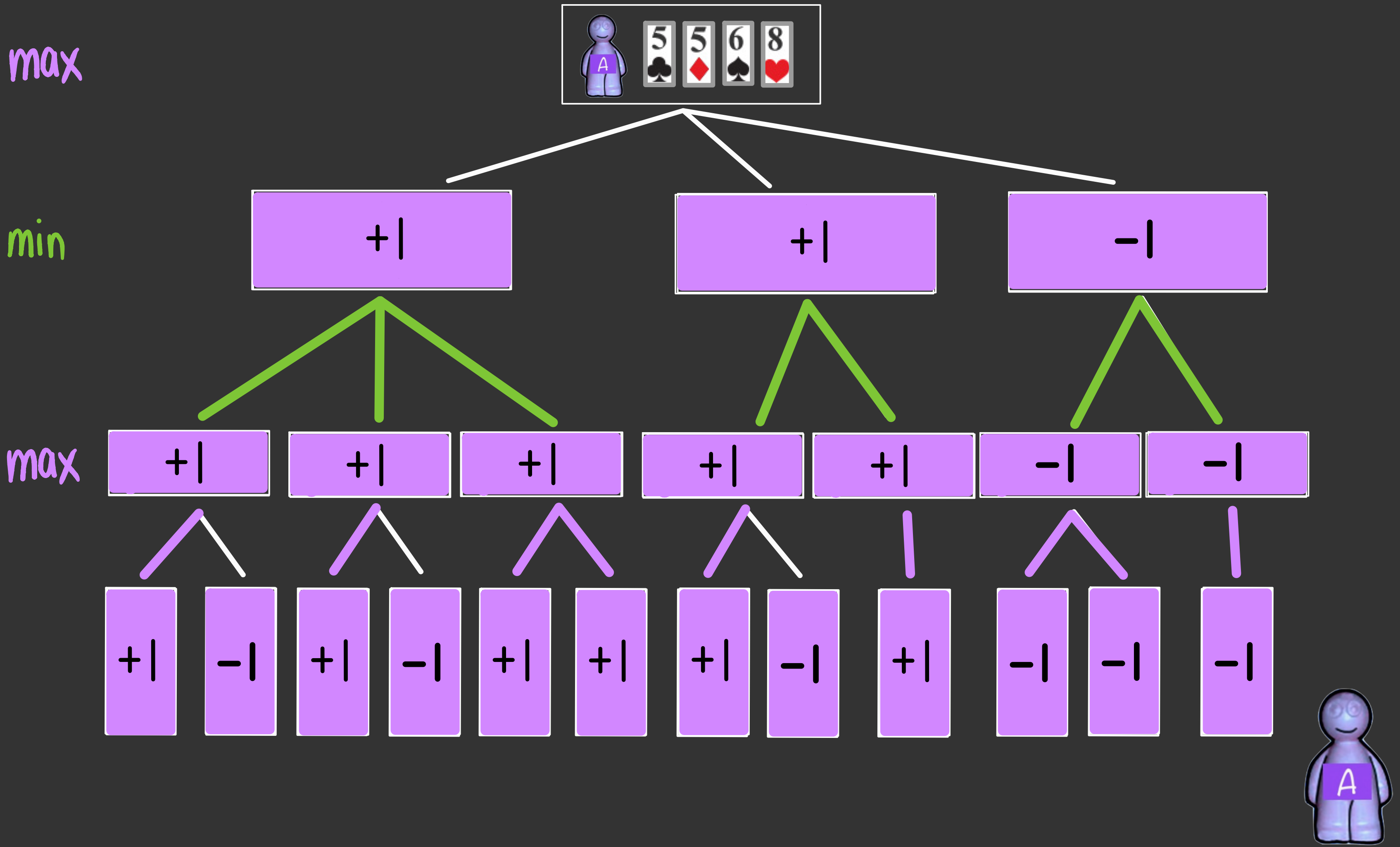

max
min
max
A
5 5 6 8
5 5 6 8 B
6 5 5 8 B
8 5 5 6 B
+1 +1 +1 +1 +1 −1 −1
+1 −1 +1 −1 +1 +1 +1 −1 +1 −1 −1 −1
A


max
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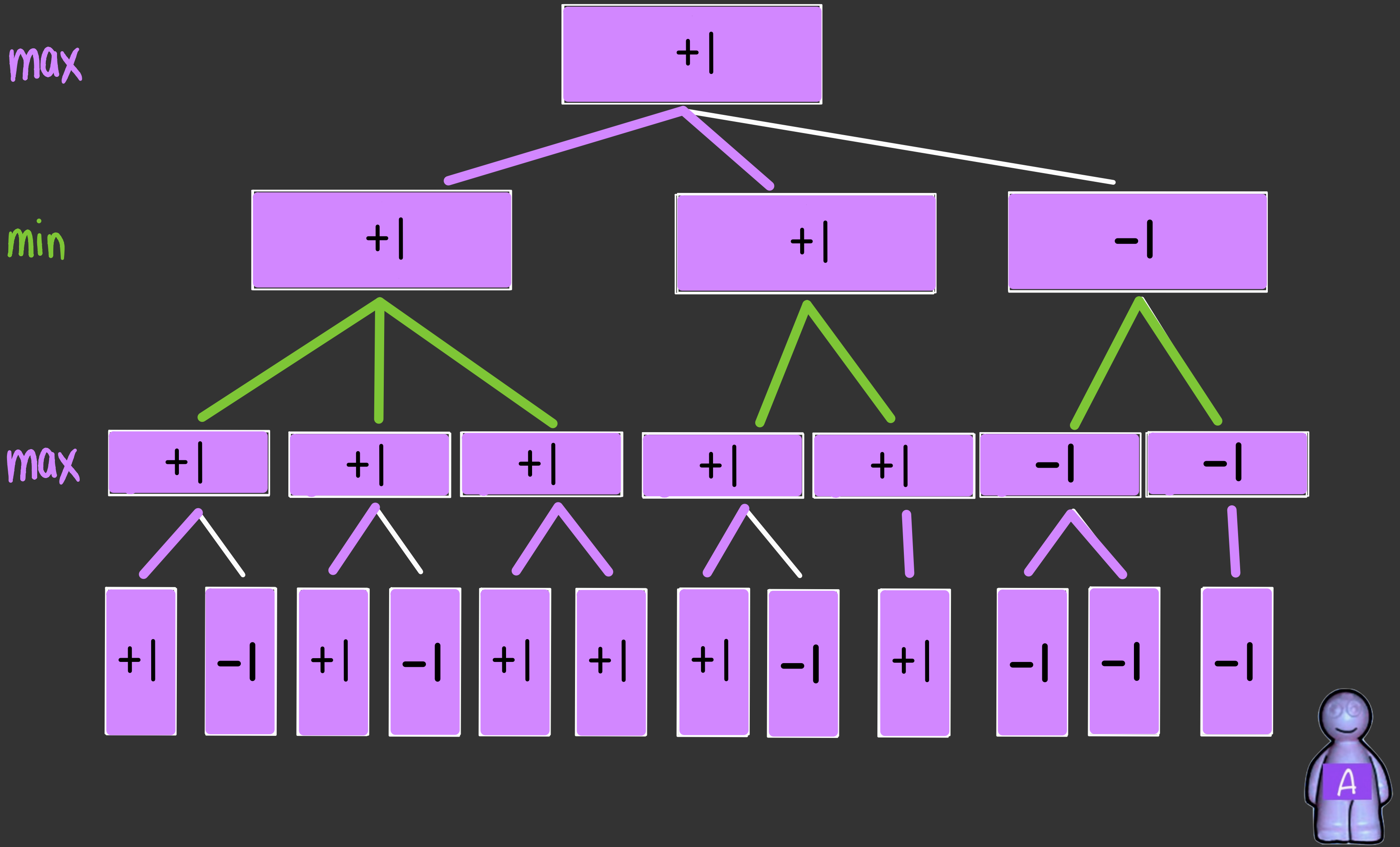
-1
A



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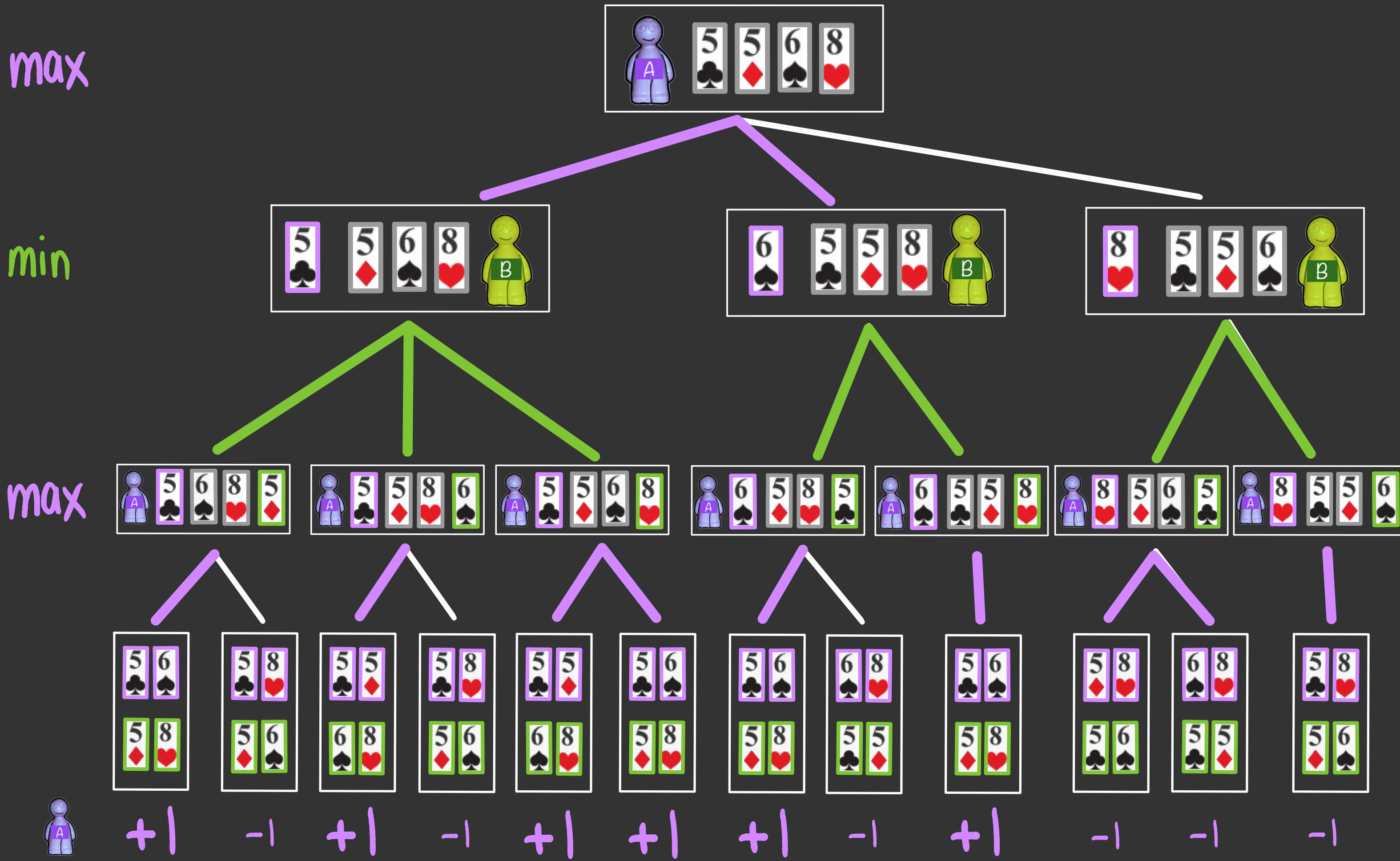
-1
A



max
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-1

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or, as pseudocode:

MINIMAX $(q, p, (m, U))$:

- if $q \in F$: return $U(q,p)$
- children = $\{\text{minimax}(q', p, (m,U)) \mid \langle q,\sigma,q'\rangle \in \Delta(m)\}$
- return max(children) if $p(q) = p$ else min(children)

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what is this?

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recall

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