

# CSE 252D: Advanced Computer Vision

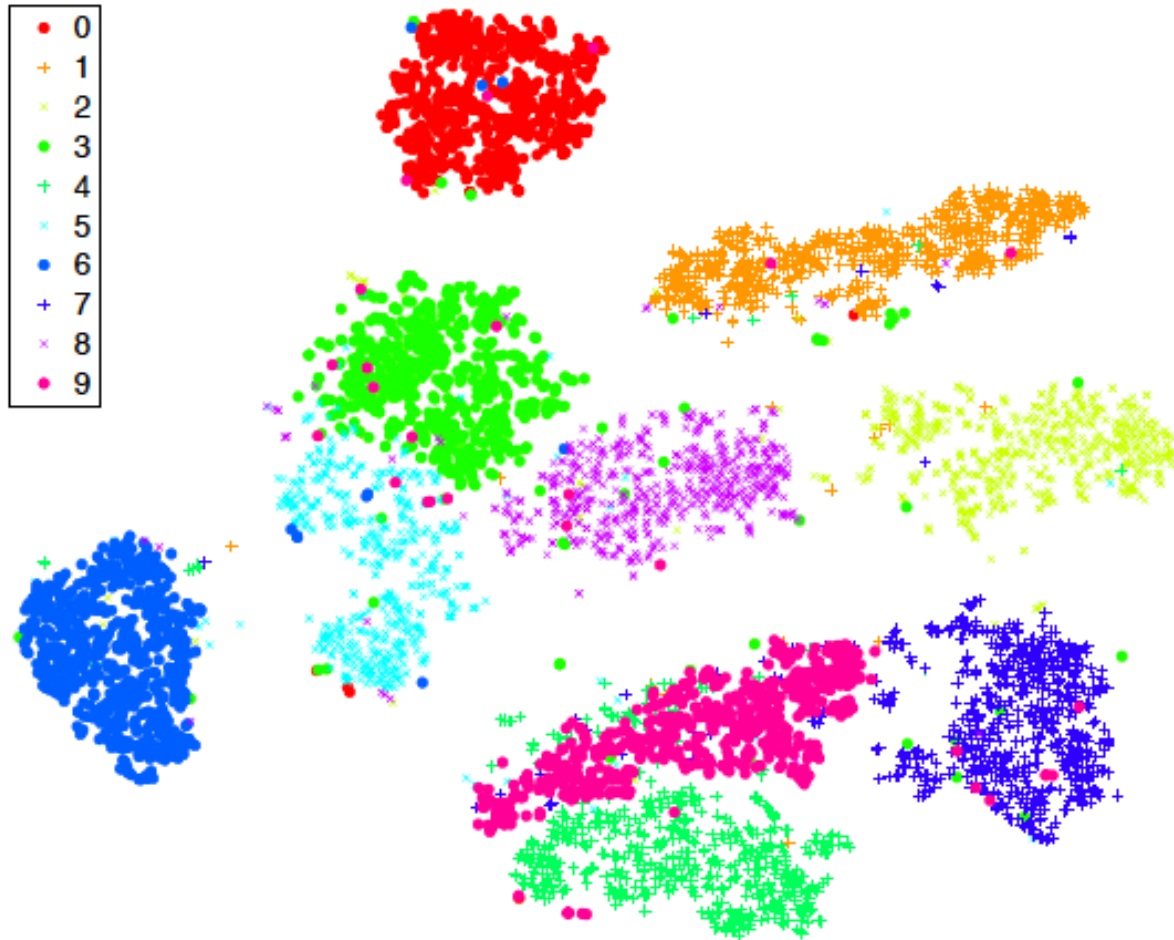
Manmohan Chandraker

## Supplement: Visualization with t-SNE



# Visualization with t-SNE

- Map high-dimensional data with t-distributed stochastic neighbor embedding



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- Map high-dimensional data with t-distributed stochastic neighbor embedding
- Represent similarity for data points as a probability
- Assume neighbor  $x_j$  for point  $x_i$  picked based on Gaussian density

$$p_{ij} = \frac{\exp(-\|x_i - x_j\|^2 / 2\sigma^2)}{\sum_{k \neq i} \exp(-\|x_i - x_k\|^2 / 2\sigma^2)}$$

“Perplexity” parameter, usual in range [5, 50]

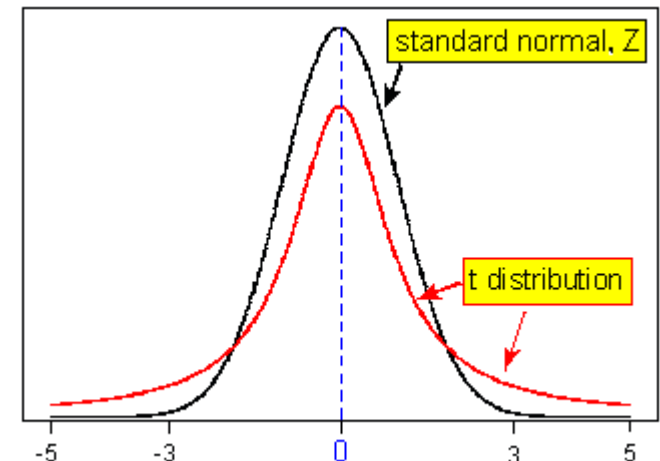
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$$q_{ij} = \frac{(1 + \|y_i - y_j\|^2)^{-1}}{\sum_{k \neq l} (1 + \|y_k - y_l\|^2)^{-1}}$$



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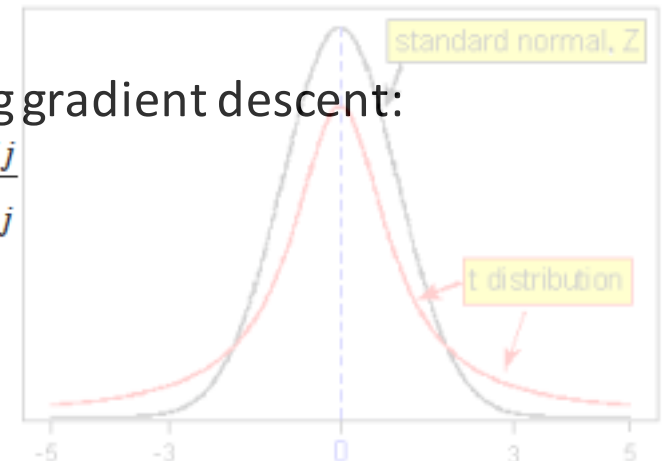
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- Heavier tail means points moderately far in  $x$  are mapped further in  $y$ 
  - More volume at greater distance in high dimensions
  - Mapping nearby points in low dimension requires pushing out far points

