1 Batchcomp - Additional Simulation Results

1.1 Results with Accurate Vote Tabulations

The upcoming plots present the number of ballots required to approve each assertion during the audit, both by the ALPHA-batch method and by our Batchcomp method. We assume that the reported batch-level tallies completely match the true ones.

Using the 22nd Knesset Election Results (2019)

The results are presented in Figure 1 and Table 1. Approving the reported winners for this election cycle required auditing virtually all ballots by both methods, due to a single assertion which had a very small margin (47). Without this assertion, the Batchcomp audit would be done after auditing 34% of the ballots, while ALPHA-batch would still require 98%.

Assertion	Margin (% of votes)	Batchcomp (% of votes)	ALPHA (% of votes)
Don't move a seat from Likud & Yamina to UTJ & Shas	47 (0.001%)	4,465,091 (≈ 100%)	4,465,148 (≈ 100%)
Don't move a seat from Blue and White to Yisrael Beiteinu	2,996 (0.07%)	1,512,698 (34%)	4,358,261 (98%)
Don't move a seat from Blue and White & Yisrael Beiteinu to UTJ & Shas	4,919 (0.11%)	1,036,112 (23%)	4,407,034 (99%)

Table 1 The last three assertions to be approved by the Batchcomp method in the 22nd Knesset elections, including their margin and the number of ballots they required to be approved by each method.

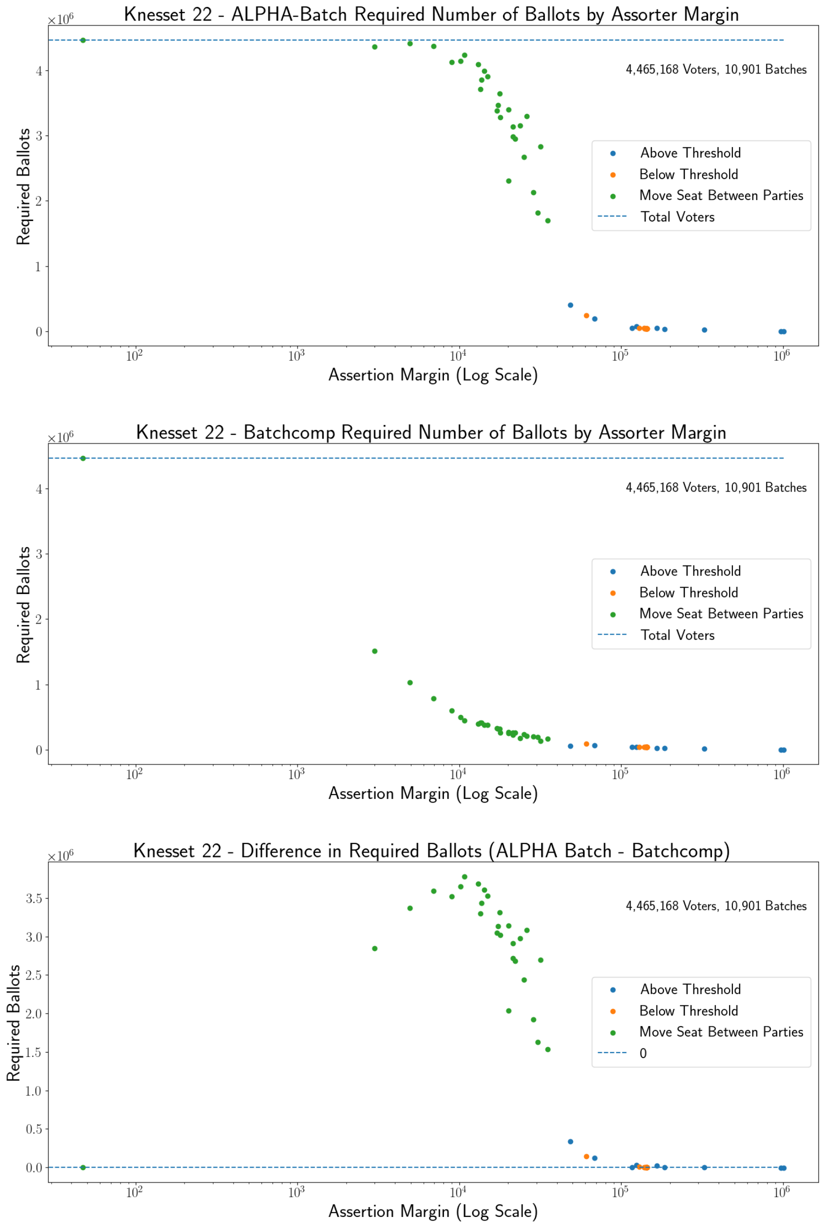

Figure 1 The first two plots present the number of ballots required to approve each assertion during the audit, either by the ALPHA-batch method or by our Batchcomp method. Each point in these plots represents a single assertion, where its value on the x axis is its margin in log-scale (minimal number of ballots that would need to be altered for the assertion to become false), and its value on the y axis is the number of ballots that the audit examined before approving the assertion. Each point in the plot is colored by the type of assertion it represents. The final plot presents the difference in ballots required per assertion between ALPHA-batch and Batchcomp.

Using the 23rd Knesset Election Results (2020)

The results are presented in Figure 2 and Table 2. Approving the reported winners for this election cycle required auditing 35% of ballots by Batchcomp, while requiring 99% by

ALPHA-batch.

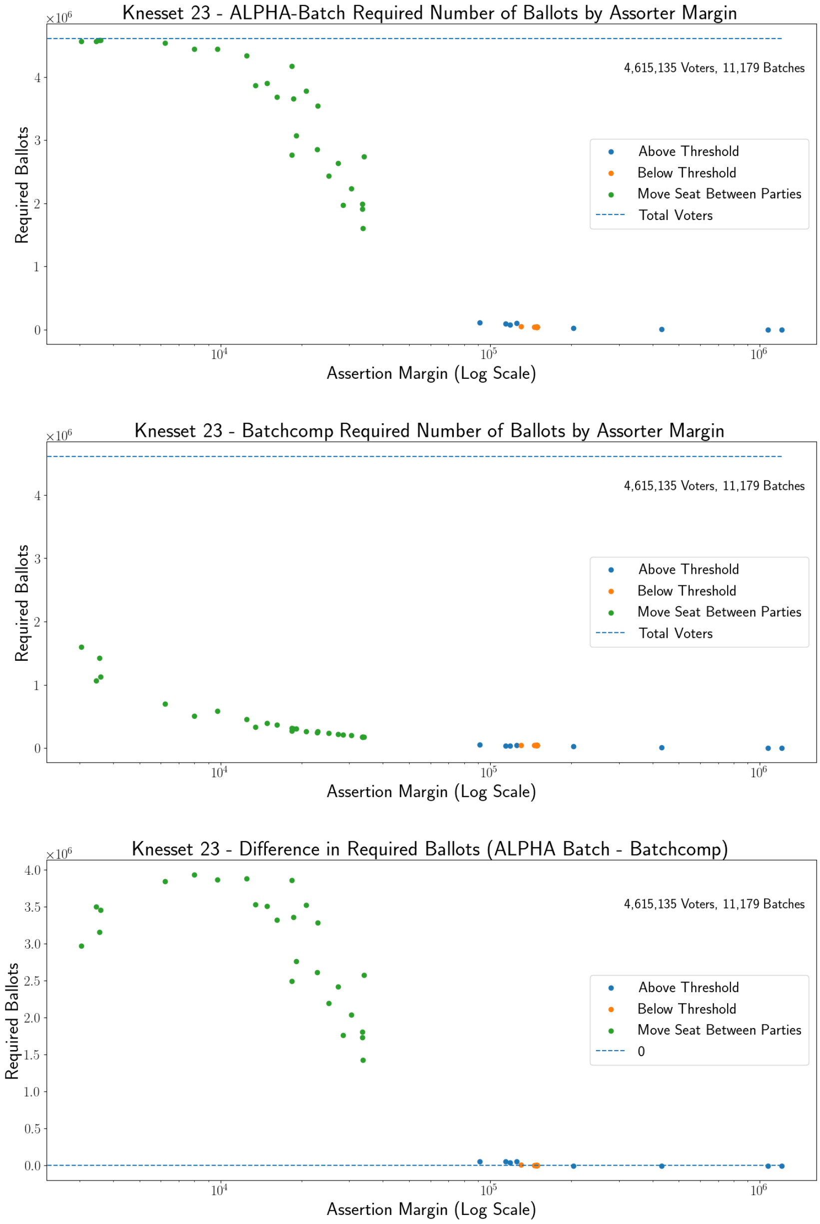

Figure 2 The first two plots present the number of ballots required to approve each assertion during the audit, either by the ALPHA-batch method or by our Batchcomp method. Each point in these plots represents a single assertion, where its value on the x axis is its margin in log-scale (minimal number of ballots that would need to be altered for the assertion to become false), and its value on the y axis is the number of ballots that the audit examined before approving the assertion. Each point in the plot is colored by the type of assertion it represents. The final plot presents the difference in ballots required per assertion between ALPHA-batch and Batchcomp.

Assertion	Margin (% of votes)	Batchcomp (% of votes)	ALPHA (% of votes)
Don't move a seat from Emet & Blue and White to Likud & Yamina	3,042 (0.07%)	1,594,986 (35%)	4,564,821 (99%)
Don't move a seat from Emet & Blue and White to UTJ & Shas	3,545 (0.08%)	1,424,603 (31%)	4,581,169 (99%)
Don't move a seat from Yisrael Beiteinu to UTJ & Shas	3,591 (0.08%)	1,126,277 (24%)	4,564,821 (99%)

Table 2 The last three assertions to be approved by the Batchcomp method in the 23rd Knesset elections, including their margin and the number of ballots they required to be approved by each method.

1.2 Results with Inaccurate vote tabulations

In addition to checking the Batchcomp method's efficiency under "perfect" conditions, we examine its tolerance to small counting errors. For this purpose, we compare the number of ballots it requires to approve each assertion under two conditions:

1. When each ballot has a probability of 0.01 to be misread in the reported tally. If a ballot is misread, it either becomes invalid (w.p. 0.1) or is counted towards a party drawn uniformly at random.
2. When the reported vote tallies of all batches are completely accurate, as examined previously in Section 1.1.

In Figure 3 and Figure 4, we present the same plots from Section 1.1 for each condition, as well as one additional plot which shows the difference in ballots required between the two conditions. Note that the assertion margins presented in these plots (the x axis) are calculated according to the reported results and not the true ones, since the true margin changes in each repetition of the simulation.

The choice of 0.01 probability for miscounting each ballot is inspired by historical data. Unless critical errors occur, both manual and electronic vote tabulations typically miscount less than 1% of ballots [2, 1].

The 22nd Knesset elections require very small counting errors to change their results. For this reason, it's extremely unlikely for the seat-allocation to remain identical if approximately 1% of the votes are miscounted. Since we are only interested in Batchcomp's performance when the reported and true winners match, this section only examines the 23rd and 24th Knesset elections.

Examining both election cycles shows that most assertions are not significantly effected by the existence of small counting Errors. Meaning, the number of ballots that are required to approve them remains similar.

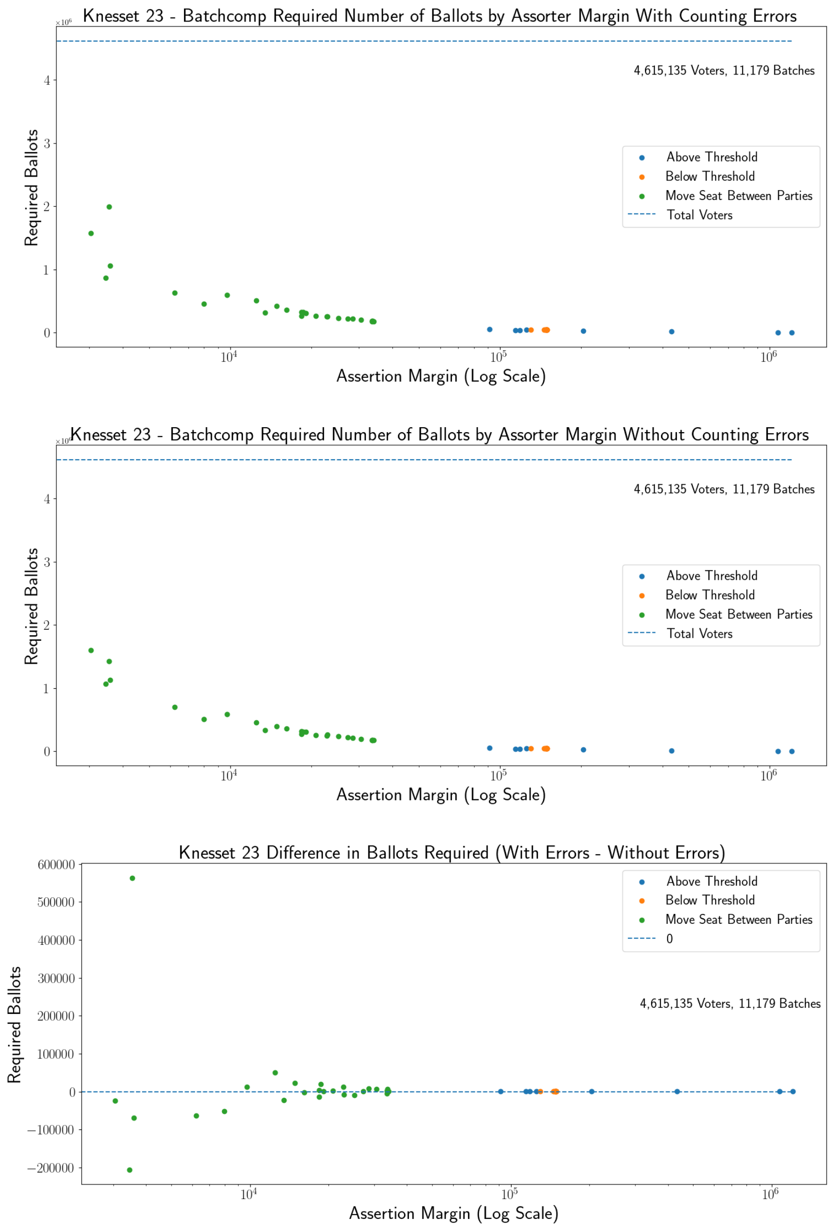

Figure 3 Each of the first two plots presents the number of ballots required to approve each assertion during the audit by Batchcomp, either with or without counting errors. Each point in these plots represents a single assertion, where its value on the x axis is its margin in log-scale, and its value on the y axis is the number of ballots that the audit examined before approving the assertion. Each point in the plot is colored by the type of assertion it represents. The final plot presents the difference in ballots required with and without counting errors.

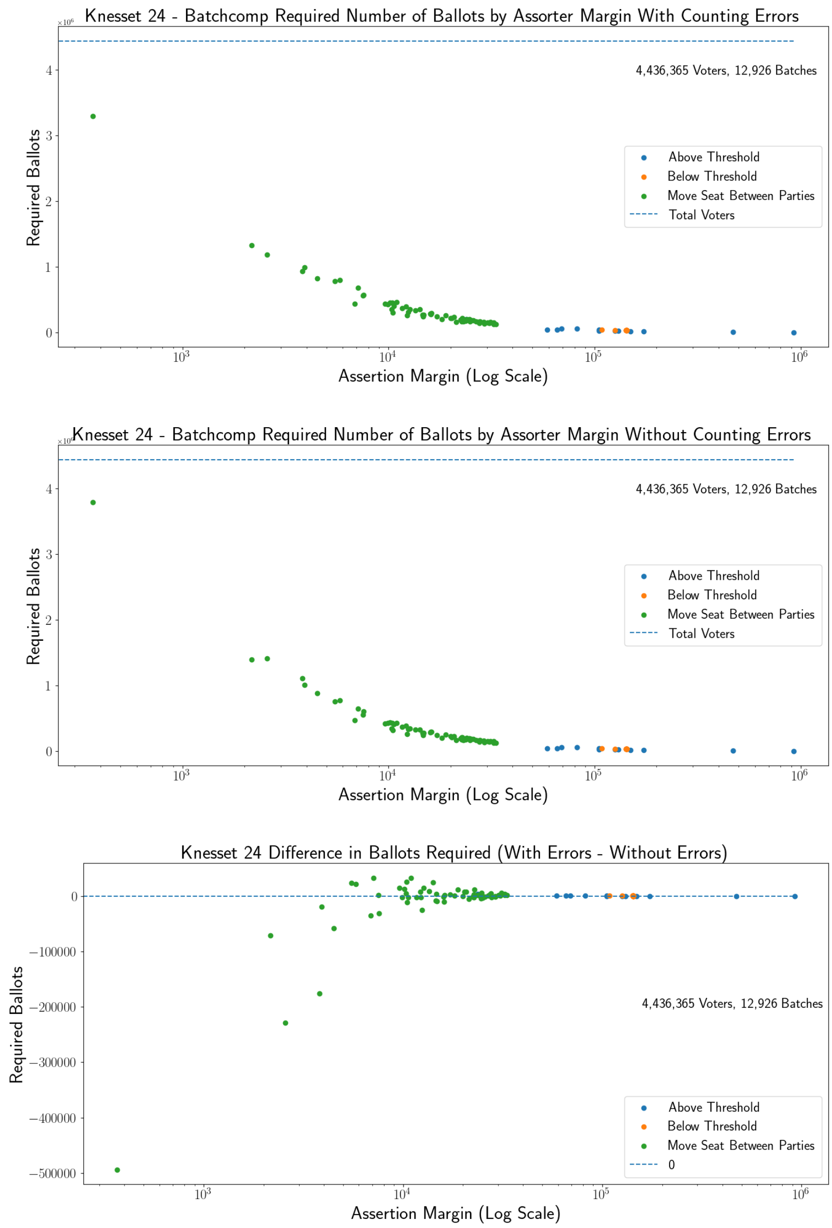

Figure 4 Each of the first two plots presents the number of ballots required to approve each assertion during the audit by Batchcomp, either with or without counting errors. Each point in these plots represents a single assertion, where its value on the x axis is its margin in log-scale, and its value on the y axis is the number of ballots that the audit examined before approving the assertion. Each point in the plot is colored by the type of assertion it represents. The final plot presents the difference in ballots required with and without counting errors.

2 Census RLA - Additional Simulation Results and Technical Details

2.1 Data Generation

The data used to perform this simulation is based on the population census conducted in 2021 [4]. The Statistical Service of Cyprus publicly reports the total number of residents in every district, but not the individual household data, which the census RLA requires. To generate this data, we assumed that the number of residents per household distributes as it does in the United States, as reported by its census [3]. We additionally assumed that 1% of households do not respond to the census and are counted as if they have no residents. The per-household data used in these simulations was generated as follows:

1. The number of households per district was calculated by dividing the district's population by the expected number of residents per household.
2. The number of residents in each household was drawn from the distribution specified in the US census [3].
3. Due to the randomness involved in the previous step, the real census and our generated one might disagree on the population of the districts. To balance this, the constant of each district (c_s) was set as the difference between the population of the district according to the real census and according to our generated one. With this definition, the allocations of representatives to districts by the real census and by our generated one are necessarily identical.

2.2 Audit Parameters and Other Details

We allocated representatives to districts using the D'Hondt method. The audit was run assuming that each household holds 15 residents at most, and with $\delta = 10^{-10}$. The simulation's code is available at `https://github.com/TGKar/Batch-and-Census-RLA`.

2.3 Results with Small Enumeration Errors

Figure 5 shows the census RLA results when the census and PES potentially disagree on 5% of households. For these 5% of households, which are selected uniformly at random, the number of residents according to the PES is re-drawn from the distribution of residents per household described in Section 2.1. The simulation was run over census and full PES results that lead to the same allocation of representatives to states. During the simulated census RLA, the audit only receives the PES results over a subset of randomly selected households.

The results in Figure 5 appear very similar to the results where all reported tallies are accurate, where the census and PES did agree on the number of residents at all households. With the described rate of disagreement between the PES and the census, a PES which surveys 0.77% of households is required for the audit to approve the allocation with a risk limit of 10%, compared to 0.66% if there were no enumeration disagreements. To get a risk-limit of 5%, the PES would need to survey 1.06% of households, compared to 0.85% with no enumeration errors.

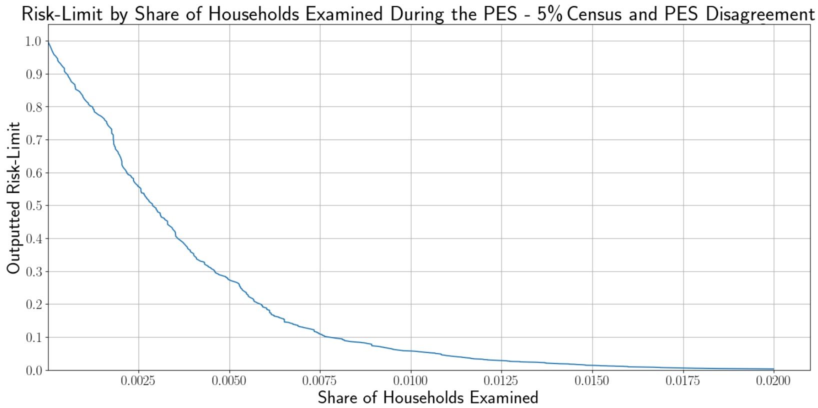

Figure 5 The census RLA output when the census and PES potentially disagree on the number of residents at 5% of households, as a factor of the share of households examined by the PES.

References

1 Stephen Ansolabehere, Barry C. Burden, Kenneth R. Mayer, and Charles Stewart III. Learning from recounts. *Election Law Journal: Rules, Politics, and Policy*, 17(2):100–116, 2018.

2 Michelle Blom, Andrew Conway, Peter J. Stuckey, Vanessa J. Teague, and Damjan Vukcevic. Random errors are not necessarily politically neutral. In *International Joint Conference on Electronic Voting*, pages 19–35. Springer, 2020.

3 United States Census Bureau. Historical households tables. `https://www.census.gov/data/tables/time-series/demo/families/households.html`, 2022. Table HH-4.

4 Statistical Service of the Republic of Cyprus. Census of population and housing 2021: Preliminary results. `https://www.pio.gov.cy/en/press-releases-article.html?id=27965`, 2022.