Taylor Sullivan
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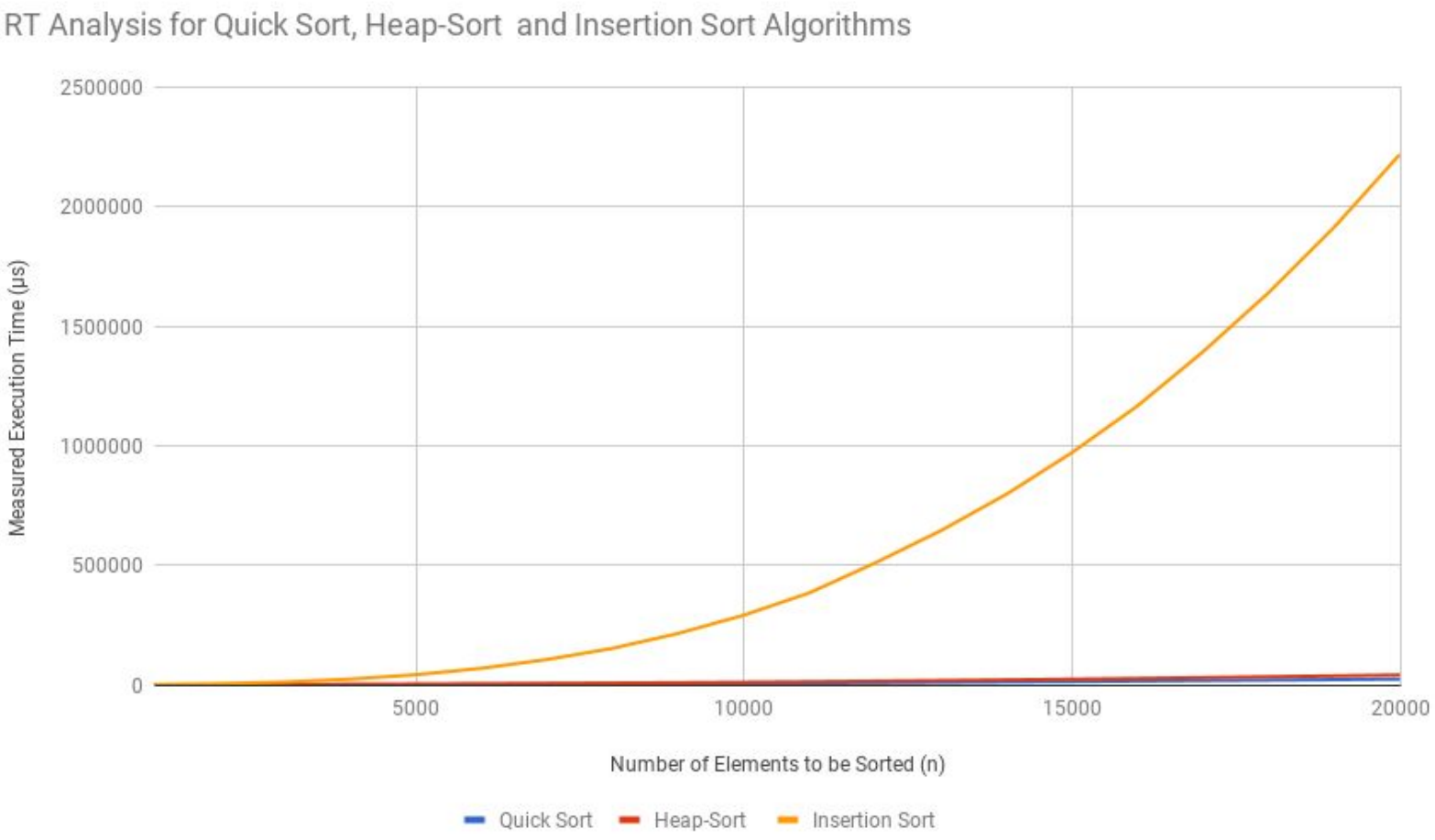
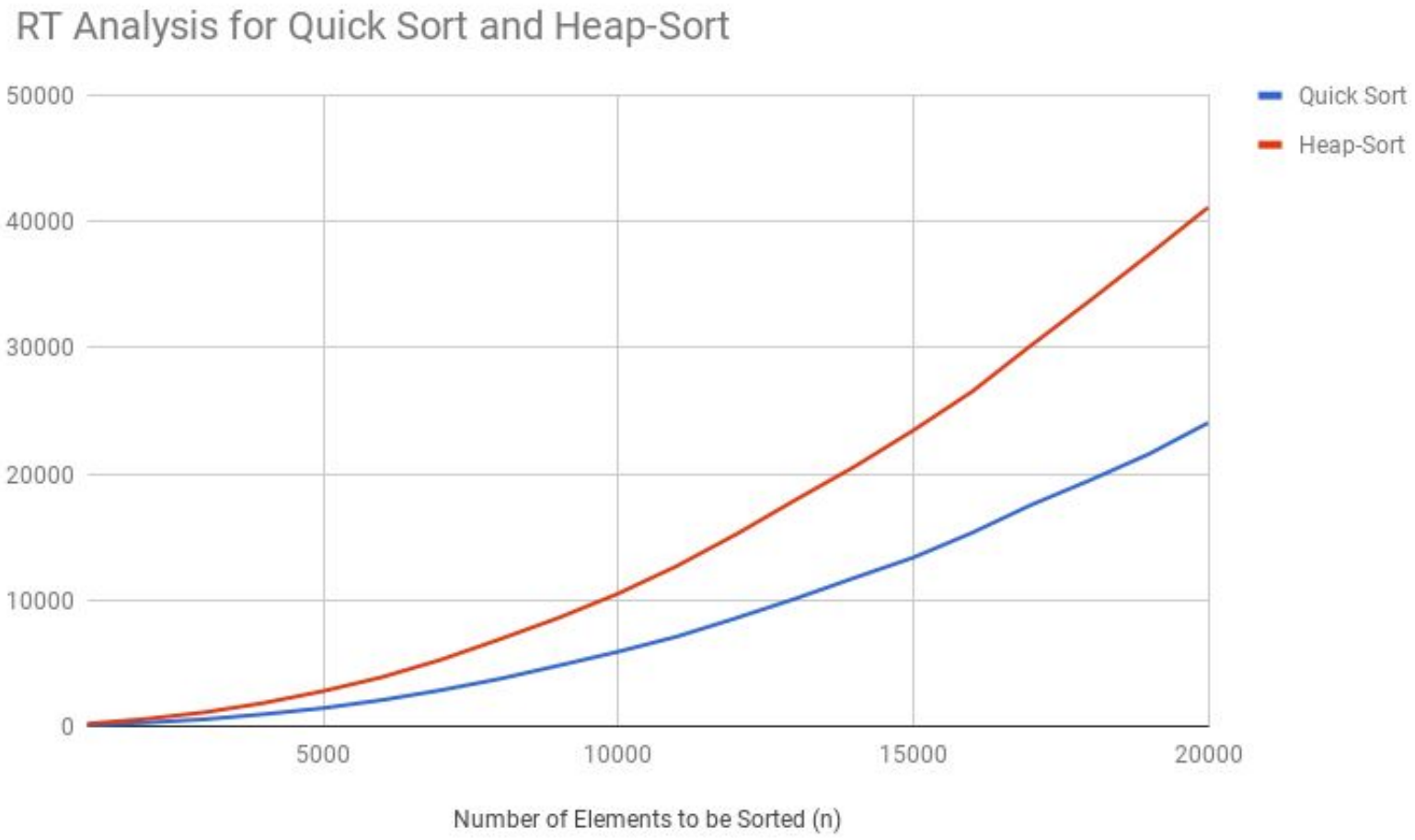
COT 4400-001
Term Project



**Below is a magnification of Quick sort and Heap sort from above graph**

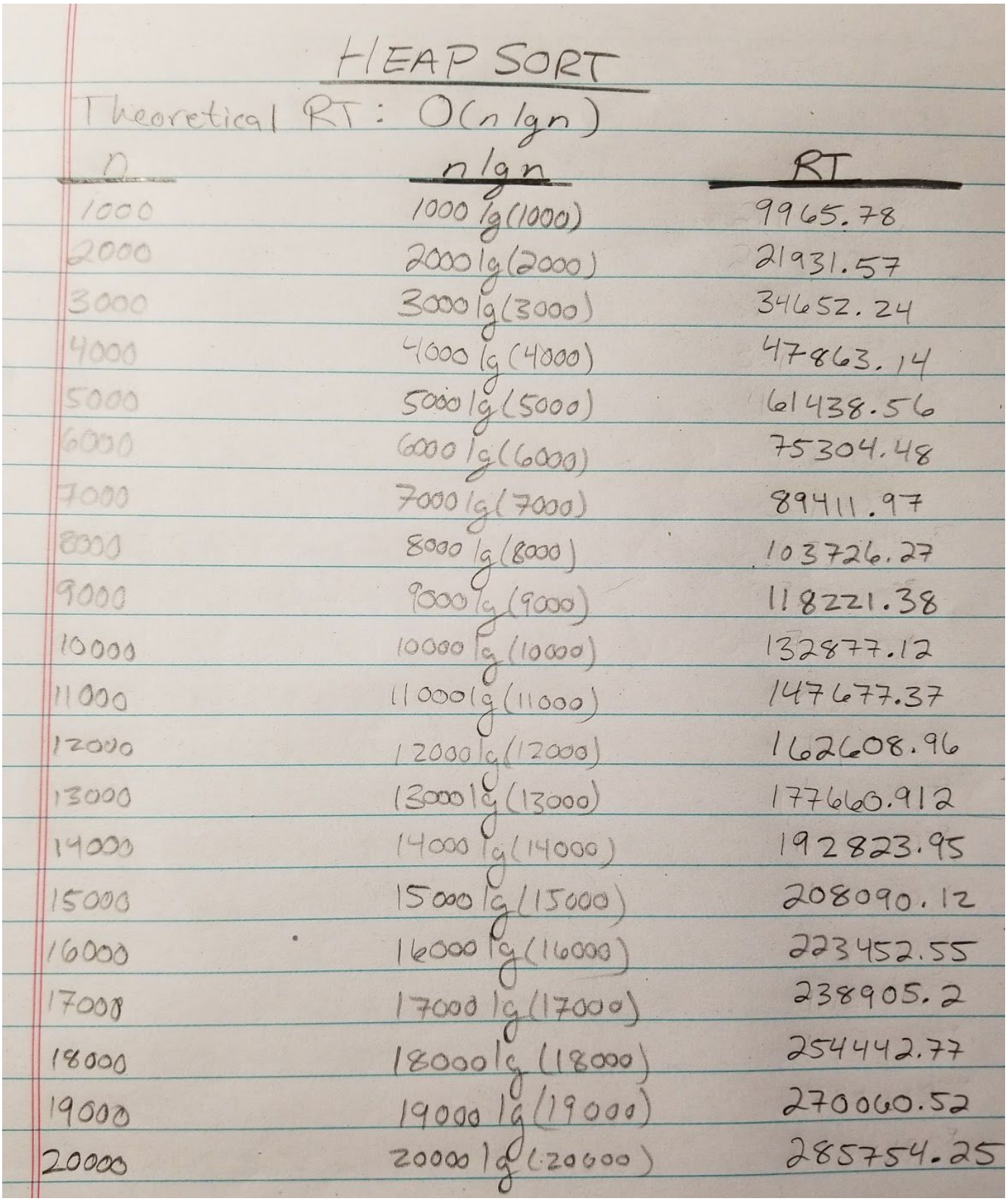


**HEAP SORT RUN TIME TABLE**

| Number of Elements to be Sorted (n) | Heap-Sort RT(μs) | Theoretical RT *(n lgn)* | Constant *C* |
|---|---|---|---|
| 1000 | 213.4 | 9965.78 | 2.1413E-08 |
| 2000 | 602.2 | 21931.57 | 2.7458E-08 |
| 3000 | 1130 | 34652.24 | 3.2609E-08 |
| 4000 | 1870.5 | 47863.14 | 3.9080E-08 |
| 5000 | 2807.7 | 61438.56 | 4.5699E-08 |
| 6000 | 3917.9 | 75304.48 | 5.2027E-08 |
| 7000 | 5298.8 | 89411.97 | 5.9263E-08 |
| 8000 | 6926.7 | 103726.27 | 6.6778E-08 |
| 9000 | 8628 | 118221.38 | 7.2981E-08 |
| 10000 | 10534.4 | 132877.12 | 7.9279E-08 |
| 11000 | 12732 | 147677.37 | 8.6214E-08 |
| 12000 | 15232.4 | 162608.96 | 9.3675E-08 |
| 13000 | 17940.3 | 177660.912 | 1.0098E-07 |
| 14000 | 20577.6 | 192823.95 | 1.0672E-07 |
| 15000 | 23456.8 | 208090.12 | 1.1272E-07 |
| 16000 | 26543.4 | 223452.55 | 1.1879E-07 |
| 17000 | 30201 | 238905.2 | 1.1264E-07 |
| 18000 | 33758 | 254442.77 | 1.3267E-07 |
| 19000 | 37406.4 | 270060.52 | 1.3851E-07 |
| 20000 | 41130.5 | 285754.25 | 1.4394E-07 |

**MAXIMUM C VALUE : C = 1.4394** $\times 10^{-7}$

**CALCULATIONS FOR HEAP SORT ANALYSIS**

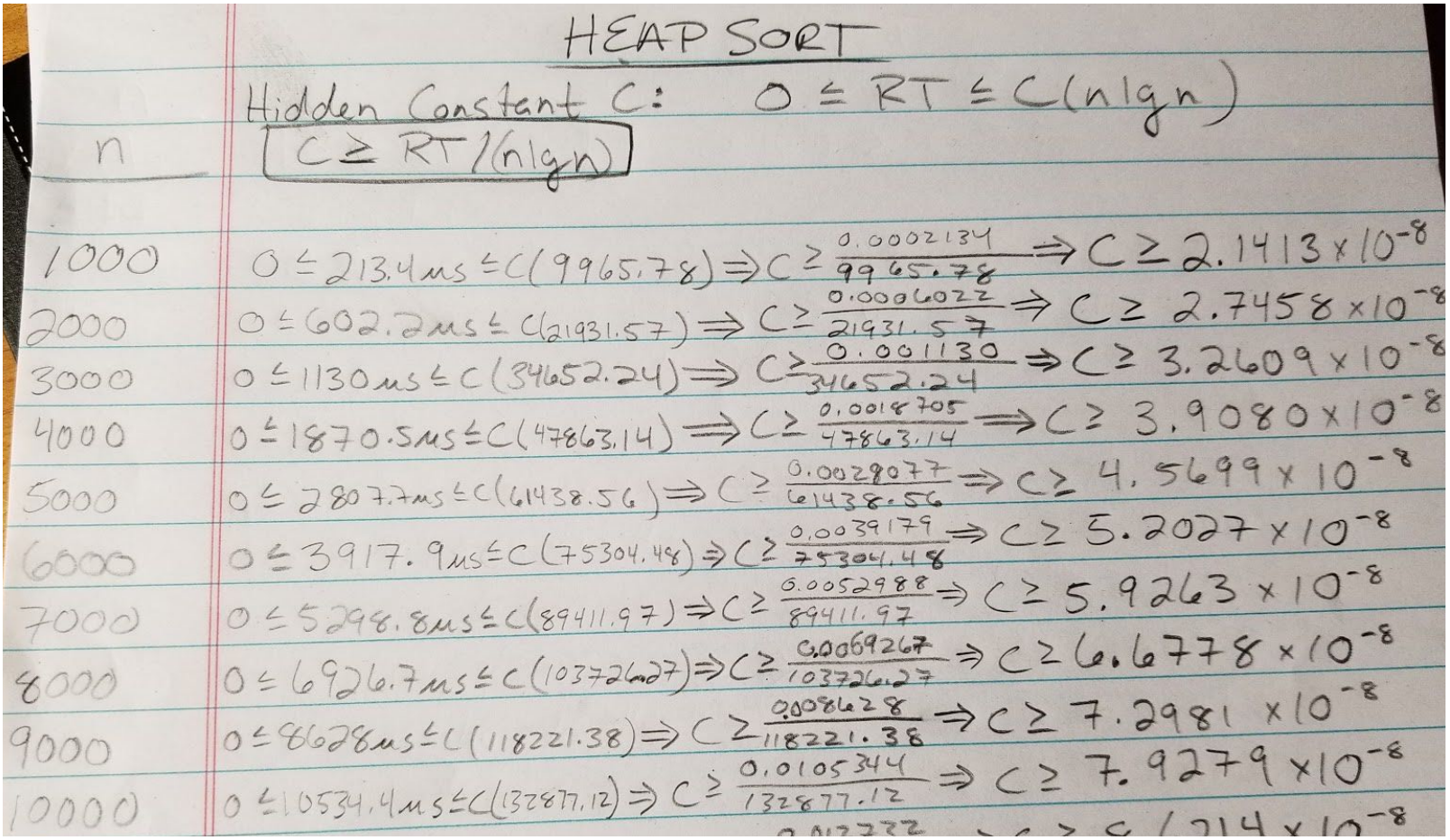


HEAP SORT

Theoretical RT: O(n lgn)

| n | n lgn | RT |
|---|---|---|
| 1000 | 1000 lg(1000) | 9965.78 |
| 2000 | 2000 lg(2000) | 21931.57 |
| 3000 | 3000 lg(3000) | 34652.24 |
| 4000 | 4000 lg(4000) | 47863.14 |
| 5000 | 5000 lg(5000) | 61438.56 |
| 6000 | 6000 lg(6000) | 75304.48 |
| 7000 | 7000 lg(7000) | 89411.97 |
| 8000 | 8000 lg(8000) | 103726.27 |
| 9000 | 9000 lg(9000) | 118221.38 |
| 10000 | 10000 lg(10000) | 132877.12 |
| 11000 | 11000 lg(11000) | 147677.37 |
| 12000 | 12000 lg(12000) | 162608.96 |
| 13000 | 13000 lg(13000) | 177660.912 |
| 14000 | 14000 lg(14000) | 192823.95 |
| 15000 | 15000 lg(15000) | 208090.12 |
| 16000 | 16000 lg(16000) | 223452.55 |
| 17000 | 17000 lg(17000) | 238905.2 |
| 18000 | 18000 lg(18000) | 254442.77 |
| 19000 | 19000 lg(19000) | 270060.52 |
| 20000 | 20000 lg(20000) | 285754.25 |

**HIDDEN CONSTANT CALCULATIONS**



HEAP SORT

Hidden Constant C: $0 \leq RT \leq C(n \lg n)$

$C \geq RT/(n \lg n)$

| n | |
|---|---|
| 1000 | $0 \leq 213.4\,ms \leq C(9965.78) \Rightarrow C \geq \frac{0.0002134}{9965.78} \Rightarrow C \geq 2.1413 \times 10^{-8}$ |
| 2000 | $0 \leq 602.2\,ms \leq C(21931.57) \Rightarrow C \geq \frac{0.0006022}{21931.57} \Rightarrow C \geq 2.7458 \times 10^{-8}$ |
| 3000 | $0 \leq 1130\,ms \leq C(34652.24) \Rightarrow C \geq \frac{0.001130}{34652.24} \Rightarrow C \geq 3.2609 \times 10^{-8}$ |
| 4000 | $0 \leq 1870.5\,ms \leq C(47863.14) \Rightarrow C \geq \frac{0.0018705}{47863.14} \Rightarrow C \geq 3.9080 \times 10^{-8}$ |
| 5000 | $0 \leq 2807.7\,ms \leq C(61438.56) \Rightarrow C \geq \frac{0.002807}{61438.56} \Rightarrow C \geq 4.5699 \times 10^{-8}$ |
| 6000 | $0 \leq 3917.9\,ms \leq C(75304.48) \Rightarrow C \geq \frac{0.0039179}{75304.48} \Rightarrow C \geq 5.2027 \times 10^{-8}$ |
| 7000 | $0 \leq 5298.8\,ms \leq C(89411.97) \Rightarrow C \geq \frac{0.0052988}{89411.97} \Rightarrow C \geq 5.9263 \times 10^{-8}$ |
| 8000 | $0 \leq 6926.7\,ms \leq C(103726.27) \Rightarrow C \geq \frac{0.0069267}{103726.27} \Rightarrow C \geq 6.6778 \times 10^{-8}$ |
| 9000 | $0 \leq 8628\,ms \leq C(118221.38) \Rightarrow C \geq \frac{0.008628}{118221.38} \Rightarrow C \geq 7.2981 \times 10^{-8}$ |
| 10000 | $0 \leq 10534.4\,ms \leq C(132877.12) \Rightarrow C \geq \frac{0.0105344}{132877.12} \Rightarrow C \geq 7.9279 \times 10^{-8}$ |

| | |
|---|---|
| 10000 | $0 \leq 10534.4\,ms \leq C(132877.12) \Rightarrow C \geq \frac{0.0105344}{132877.12} \Rightarrow C \geq 7.9279 \times 10^{-8}$ |
| 11000 | $0 \leq 12732\,ms \leq C(147677.37) \Rightarrow C \geq \frac{0.012732}{147677.37} \Rightarrow C \geq 8.6214 \times 10^{-8}$ |
| 12000 | $0 \leq 15232.4\,ms \leq C(162608.96) \Rightarrow C \geq \frac{0.0152324}{162608.96} \Rightarrow C \geq 9.3675 \times 10^{-8}$ |
| 13000 | $0 \leq 17940.3\,ms \leq C(177660.912) \Rightarrow C \geq \frac{0.0179403}{177660.912} \Rightarrow C \geq 1.0098 \times 10^{-7}$ |
| 14000 | $0 \leq 20577.6\,ms \leq C(192823.95) \Rightarrow C \geq \frac{0.0205776}{192823.95} \Rightarrow C \geq 1.0672 \times 10^{-7}$ |
| 15000 | $0 \leq 23456.8\,ms \leq C(208090.12) \Rightarrow C \geq \frac{0.0234568}{208090.12} \Rightarrow C \geq 1.1272 \times 10^{-7}$ |
| 16000 | $0 \leq 26543.4\,ms \leq C(223452.55) \Rightarrow C \geq \frac{0.0265434}{223452.55} \Rightarrow C \geq 1.1879 \times 10^{-7}$ |
| 17000 | $0 \leq 30201\,ms \leq C(238905.2) \Rightarrow C \geq \frac{0.030201}{238905.2} \Rightarrow C \geq 1.12641 \times 10^{-7}$ |
| 18000 | $0 \leq 33758\,ms \leq C(254442.77) \Rightarrow C \geq \frac{0.033758}{254442.77} \Rightarrow C \geq 1.3267 \times 10^{-7}$ |
| 19000 | $0 \leq 37406.4\,ms \leq C(270060.52) \Rightarrow C \geq \frac{0.0374064}{270060.52} \Rightarrow C \geq 1.3851 \times 10^{-7}$ |
| 20000 | $0 \leq 41130.5\,ms \leq C(285754.25) \Rightarrow C \geq \frac{0.0411305}{285754.25} \Rightarrow C \geq 1.4394 \times 10^{-7}$ |

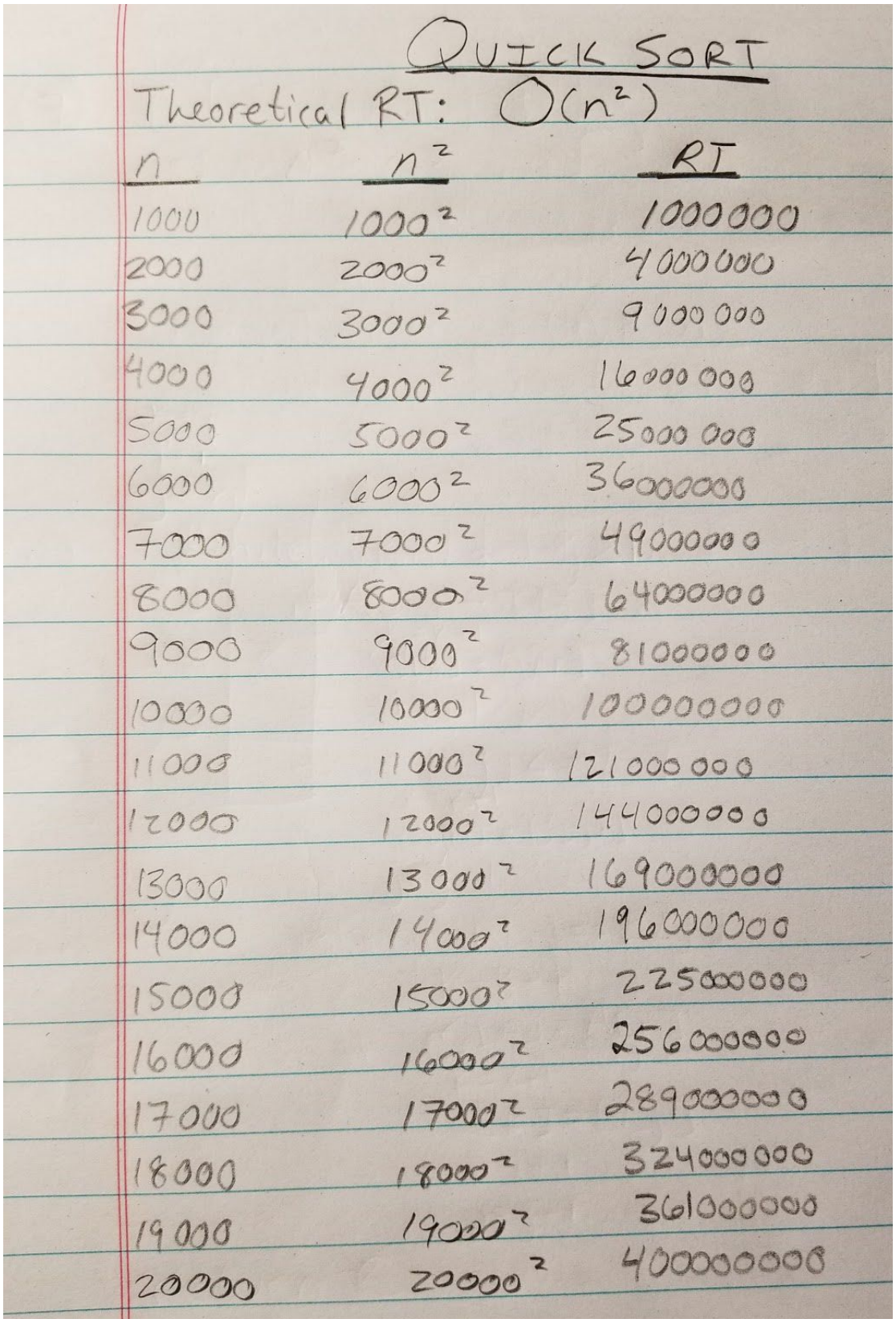
MAXIMUM C value: $C = 1.4394 \times 10^{-7}$

## QUICK SORT RUN TIME TABLE

| Number of Elements to be Sorted (n) | Quick Sort RT(μs) | Theoretical RT *O(n^2)* | Constant C |
|---|---|---|---|
| 1000 | 107.2 | 1000000 | 1.07200E-10 |
| 2000 | 304.8 | 4000000 | 7.62000E-11 |
| 3000 | 575.7 | 9000000 | 6.39660E-11 |
| 4000 | 977.4 | 16000000 | 6.10875E-11 |
| 5000 | 1458.1 | 25000000 | 5.83200E-11 |
| 6000 | 2100.5 | 36000000 | 5.83470E-11 |
| 7000 | 2882.3 | 49000000 | 5.88224E-11 |
| 8000 | 3786 | 64000000 | 5.91562E-11 |
| 9000 | 4839.5 | 81000000 | 5.97469E-11 |
| 10000 | 5927.9 | 100000000 | 5.92700E-11 |
| 11000 | 7133.5 | 121000000 | 5.89545E-11 |
| 12000 | 8590.8 | 144000000 | 5.96583E-11 |
| 13000 | 10123.3 | 169000000 | 5.99012E-11 |
| 14000 | 11773.3 | 196000000 | 6.00679E-11 |
| 15000 | 13384.7 | 225000000 | 5.94876E-11 |
| 16000 | 15353 | 256000000 | 5.99727E-11 |
| 17000 | 17559.4 | 289000000 | 6.07592E-11 |
| 18000 | 19521.2 | 324000000 | 6.02506E-11 |
| 19000 | 21598.2 | 361000000 | 5.98299E-11 |
| 20000 | 24060.9 | 400000000 | 6.01523E-11 |

**MAXIMUM C VALUE : C =** $7.620 \times 10^{-11}$

**CALCULATIONS FOR QUICK SORT ANALYSIS**



QUICK SORT

Theoretical RT: $O(n^2)$

| n | $n^2$ | RT |
|---|---|---|
| 1000 | $1000^2$ | 1000000 |
| 2000 | $2000^2$ | 4000000 |
| 3000 | $3000^2$ | 9000000 |
| 4000 | $4000^2$ | 16000000 |
| 5000 | $5000^2$ | 25000000 |
| 6000 | $6000^2$ | 36000000 |
| 7000 | $7000^2$ | 49000000 |
| 8000 | $8000^2$ | 64000000 |
| 9000 | $9000^2$ | 81000000 |
| 10000 | $10000^2$ | 100000000 |
| 11000 | $11000^2$ | 121000000 |
| 12000 | $12000^2$ | 144000000 |
| 13000 | $13000^2$ | 169000000 |
| 14000 | $14000^2$ | 196000000 |
| 15000 | $15000^2$ | 225000000 |
| 16000 | $16000^2$ | 256000000 |
| 17000 | $17000^2$ | 289000000 |
| 18000 | $18000^2$ | 324000000 |
| 19000 | $19000^2$ | 361000000 |
| 20000 | $20000^2$ | 400000000 |

# QUICKSORT

Hidden Constant C: $0 \leq RT \leq C(n^2)$

$\boxed{C \geq RT/n^2}$

| n | |
|---|---|
| 1000 | $0 \leq 107.2\,ms \leq C(1000^2) \Rightarrow C \geq \frac{0.0001072}{1000000}$<br>$C \geq 1.072 \times 10^{-10}$ |
| 2000 | $0 \leq 304.8\,ms \leq C(2000^2) \Rightarrow C \geq \frac{0.0003048}{4000000}$<br>$C \geq 7.620 \times 10^{-11}$ ← Maximum C value |
| 3000 | $0 \leq 575.7\,ms \leq C(3000^2) \Rightarrow C \geq \frac{0.0005757}{9000000}$<br>$C \geq 6.3966 \times 10^{-11}$ |
| 4000 | $0 \leq 977.4\,ms \leq C(4000^2) \Rightarrow C \geq \frac{0.0009774}{16000000}$<br>$C \geq 6.1087 \times 10^{-11}$ |
| 5000 | $0 \leq 1458.1\,ms \leq C(5000^2) \Rightarrow C \geq \frac{0.0014581}{25000000}$<br>$C \geq 5.832 \times 10^{-11}$ |
| 6000 | $0 \leq 2100.5\,ms \leq C(6000^2) \Rightarrow C \geq \frac{0.0021005}{36000000}$<br>$C \geq 5.8347 \times 10^{-11}$ |
| 7000 | $0 \leq 2882.3\,ms \leq C(7000^2) \Rightarrow C \geq \frac{0.0028823}{49000000}$<br>$C \geq 5.8822 \times 10^{-11}$ |
| 8000 | $0 \leq 3786\,ms \leq C(8000^2) \Rightarrow C \geq \frac{0.003786}{64000000}$<br>$C \geq 5.91562 \times 10^{-11}$ |
| 9000 | $0 \leq 4839.5\,ms \leq C(9000^2) \Rightarrow C \geq \frac{0.0048395}{81000000}$<br>$C \geq 5.974691 \times 10^{-11}$ |

# Quick Sort Continued

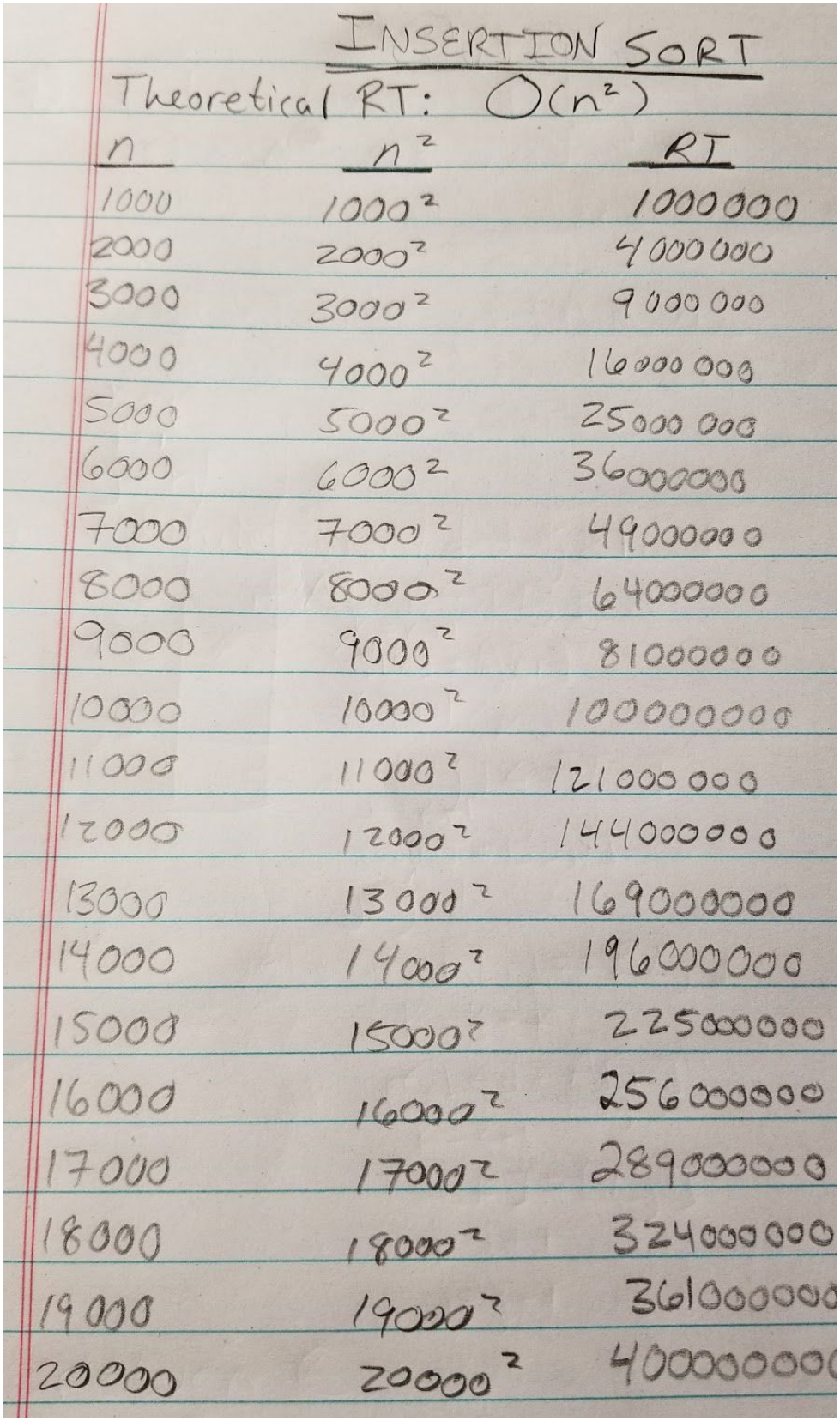
| n | |
|---|---|
| 10000 | $0 \leq 5927.9\,ms \leq C(10000^2) \Rightarrow C \geq \frac{0.0059279}{100000000}$<br>$C \geq 5.927 \times 10^{-11}$ |
| 11000 | $0 \leq 7133.5\,ms \leq C(11000^2) \Rightarrow C \geq \frac{0.0071335}{121000000}$<br>$C \geq 5.895454 \times 10^{-11}$ |
| 12000 | $0 \leq 8590.8\,ms \leq C(12000^2) \Rightarrow C \geq \frac{0.0085908}{144000000}$<br>$C \geq 5.96583 \times 10^{-11}$ |
| 13000 | $0 \leq 10123.3\,ms \leq C(13000^2) \Rightarrow C \geq \frac{0.0101233}{169000000}$<br>$C \geq 5.99018 \times 10^{-11}$ |
| 14000 | $0 \leq 11773.3\,ms \leq C(14000^2) \Rightarrow C \geq \frac{0.0117733}{196000000}$<br>$C \geq 6.0067 \times 10^{-11}$ |
| 15000 | $0 \leq 13384.7\,ms \leq C(15000^2) \Rightarrow C \geq \frac{0.0133847}{225000000}$<br>$C \geq 5.94875 \times 10^{-11}$ |
| 16000 | $0 \leq 15353\,ms \leq C(16000^2) \Rightarrow C > \frac{0.015353}{256000000}$<br>$C \geq 5.997265 \times 10^{-11}$ |
| 17000 | $0 \leq 17559.4\,ms \leq C(17000^2) \Rightarrow C \geq \frac{0.0175594}{289000000}$<br>$C \geq 6.075916 \times 10^{-11}$ |
| 18000 | $0 \leq 19521.2\,ms \leq C(18000^2) \Rightarrow C \geq \frac{0.0195212}{324000000} \Rightarrow C \geq 6.025061 \times 10^{-11}$ |
| 19000 | $0 \leq 21598.2\,ms \leq C(19000^2) \Rightarrow C \geq \frac{0.0215982}{361000000} \Rightarrow C \geq 5.98299 \times 10^{-11}$ |
| 20000 | $0 \leq 24060.9\,ms \leq C(20000^2) \Rightarrow C \geq \frac{0.0240609}{400000000} \Rightarrow C \geq 6.01523 \times 10^{-11}$ |

**INSERTION SORT RUN TIME TABLE**

| Number of Elements to be Sorted (n) | Insertion Sort RT(μs) | Theoretical RT *O(n^2)* | Constant *C* |
|---|---|---|---|
| 1000 | 1033.8 | 1000000 | 1.0338E-09 |
| 2000 | 4706.3 | 4000000 | 1.7660E-09 |
| 3000 | 11405.6 | 9000000 | 1.2673E-09 |
| 4000 | 24005.2 | 16000000 | 1.5003E-09 |
| 5000 | 42053.2 | 25000000 | 1.6820E-09 |
| 6000 | 69027.7 | 36000000 | 1.9174E-09 |
| 7000 | 105854 | 49000000 | 2.1603E-09 |
| 8000 | 152712 | 64000000 | 2.3861E-09 |
| 9000 | 214960 | 81000000 | 2.6538E-09 |
| 10000 | 291404 | 100000000 | 2.9140E-09 |
| 11000 | 384800 | 121000000 | 3.1802E-09 |
| 12000 | 509976 | 144000000 | 3.5415E-09 |
| 13000 | 644692 | 169000000 | 3.8147E-09 |
| 14000 | 797054 | 196000000 | 4.0666E-09 |
| 15000 | 971586 | 225000000 | 4.3182E-09 |
| 16000 | 1.17E+06 | 256000000 | 4.5588E-09 |
| 17000 | 1.39E+06 | 289000000 | 4.8226E-09 |
| 18000 | 1.64E+06 | 324000000 | 5.0652E-09 |
| 19000 | 1.92E+06 | 361000000 | 5.3053E-09 |
| 20000 | 2.22E+06 | 400000000 | 5.5489E-09 |

**MAXIMUM C VALUE : C =** $5.5489 \times 10^{-9}$

**CALCULATIONS FOR INSERTION SORT ANALYSIS**

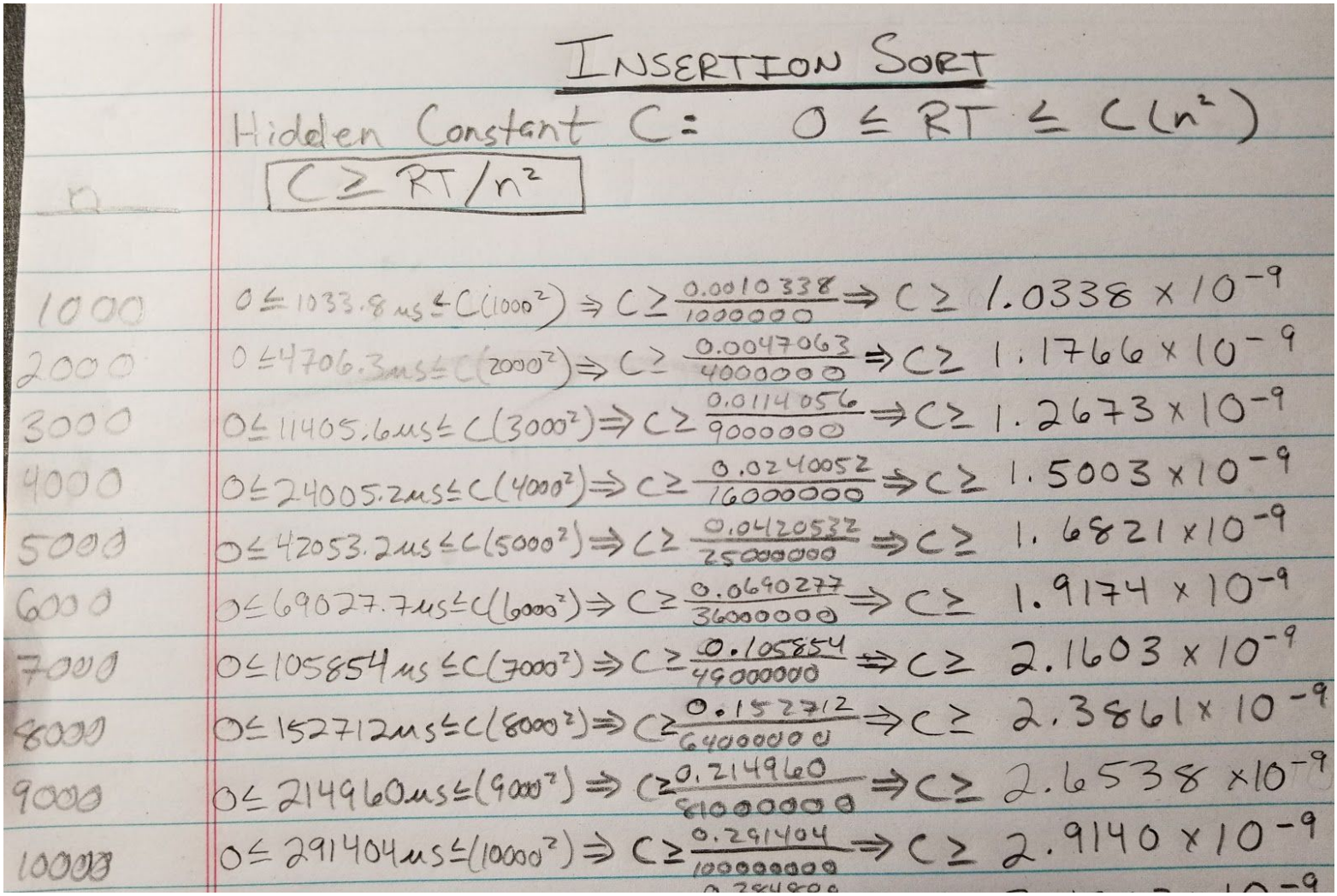


INSERTION SORT

Theoretical RT: $O(n^2)$

| n | $n^2$ | RT |
|---|---|---|
| 1000 | $1000^2$ | 1000000 |
| 2000 | $2000^2$ | 4000000 |
| 3000 | $3000^2$ | 9000000 |
| 4000 | $4000^2$ | 16000000 |
| 5000 | $5000^2$ | 25000000 |
| 6000 | $6000^2$ | 36000000 |
| 7000 | $7000^2$ | 49000000 |
| 8000 | $8000^2$ | 64000000 |
| 9000 | $9000^2$ | 81000000 |
| 10000 | $10000^2$ | 100000000 |
| 11000 | $11000^2$ | 121000000 |
| 12000 | $12000^2$ | 144000000 |
| 13000 | $13000^2$ | 169000000 |
| 14000 | $14000^2$ | 196000000 |
| 15000 | $15000^2$ | 225000000 |
| 16000 | $16000^2$ | 256000000 |
| 17000 | $17000^2$ | 289000000 |
| 18000 | $18000^2$ | 324000000 |
| 19000 | $19000^2$ | 361000000 |
| 20000 | $20000^2$ | 400000000 |

**HIDDEN CONSTANT CALCULATIONS**



## INSERTION SORT

Hidden Constant C: $0 \leq RT \leq C(n^2)$

$C \geq RT/n^2$

| n | |
|---|---|
| 1000 | $0 \leq 1033.8\,ms \leq C(1000^2) \Rightarrow C \geq \frac{0.0010338}{1000000} \Rightarrow C \geq 1.0338 \times 10^{-9}$ |
| 2000 | $0 \leq 4706.3\,ms \leq C(2000^2) \Rightarrow C \geq \frac{0.0047063}{4000000} \Rightarrow C \geq 1.1766 \times 10^{-9}$ |
| 3000 | $0 \leq 11405.6\,ms \leq C(3000^2) \Rightarrow C \geq \frac{0.0114056}{9000000} \Rightarrow C \geq 1.2673 \times 10^{-9}$ |
| 4000 | $0 \leq 24005.2\,ms \leq C(4000^2) \Rightarrow C \geq \frac{0.024005 2}{16000000} \Rightarrow C \geq 1.5003 \times 10^{-9}$ |
| 5000 | $0 \leq 42053.2\,ms \leq C(5000^2) \Rightarrow C \geq \frac{0.0420532}{25000000} \Rightarrow C \geq 1.6821 \times 10^{-9}$ |
| 6000 | $0 \leq 69027.7\,ms \leq C(6000^2) \Rightarrow C \geq \frac{0.0690277}{36000000} \Rightarrow C \geq 1.9174 \times 10^{-9}$ |
| 7000 | $0 \leq 105854\,ms \leq C(7000^2) \Rightarrow C \geq \frac{0.105854}{49000000} \Rightarrow C \geq 2.1603 \times 10^{-9}$ |
| 8000 | $0 \leq 152712\,ms \leq C(8000^2) \Rightarrow C \geq \frac{0.152712}{64000000} \Rightarrow C \geq 2.3861 \times 10^{-9}$ |
| 9000 | $0 \leq 214960\,ms \leq (9000^2) \Rightarrow C \geq \frac{0.214960}{81000000} \Rightarrow C \geq 2.6538 \times 10^{-9}$ |
| 10000 | $0 \leq 291404\,ms \leq (10000^2) \Rightarrow C \geq \frac{0.291404}{100000000} \Rightarrow C \geq 2.9140 \times 10^{-9}$ |

11000 $0 \leq 384800\mu s \leq (11000^2) \Rightarrow C \geq \frac{0.384800}{121000000} \Rightarrow C \geq 3.1802 \times 10^{-9}$

12000 $0 \leq 509976\mu s \leq (12000^2) \Rightarrow C \geq \frac{0.509976}{144000000} \Rightarrow C \geq 3.5415 \times 10^{-9}$

13000 $0 \leq 644692\mu s \leq (13000^2) \Rightarrow C \geq \frac{0.644692}{169000000} \Rightarrow C \geq 3.8147 \times 10^{-9}$

14000 $0 \leq 797054\mu s \leq (14000^2) \Rightarrow C \geq \frac{0.797054}{196000000} \Rightarrow C \geq 4.0666 \times 10^{-9}$

15000 $0 \leq 971586\mu s \leq (15000^2) \Rightarrow C \geq \frac{0.971586}{225000000} \Rightarrow C \geq 4.3182 \times 10^{-9}$

16000 $0 \leq 1.167055 \leq (16000^2) \Rightarrow C \geq \frac{1.16705}{256000000} \Rightarrow C \geq 4.5588 \times 10^{-9}$

17000 $0 \leq 1.393725 \leq (17000^2) \Rightarrow C \geq \frac{1.39372}{289000000} \Rightarrow C \geq 4.8226 \times 10^{-9}$

18000 $0 \leq 1.641145 \leq (18000^2) \Rightarrow C \geq \frac{1.64114}{324000000} \Rightarrow C \geq 5.0652 \times 10^{-9}$

19000 $0 \leq 1.915230s \leq (19000^2) \Rightarrow C \geq \frac{1.915230}{361000000} \Rightarrow C \geq 5.3053 \times 10^{-9}$

20000 $0 \leq 2.21959s \leq (20000^2) \Rightarrow C \geq \frac{2.21959}{400000000} \Rightarrow C \geq 5.5489 \times 10^{-9}$

MAXIMUM C VALUE: $C = 5.5489 \times 10^{-9}$