Assignment 2

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Download all python codes from

https://github.com/TGURUBALAJI/INTERNSHIP−IITH/Assignment2/code

and latex-tikz codes from

https://github.com/TGURUBALAJI/INTERNSHIP−IITH/Assignment2/gbalaji.tex

1 Linear forms Q:2.106

Find the values of k for which the line

$$\begin{pmatrix} k-3 & -\left(4-k^2\right) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \tag{1}$$

is
a) Parallel to the x-axis
b) Parallel to the y-axis
c) Passing through the origin

Solution Given equation of the line,

$$\begin{pmatrix} k-3 & -\left(4-k^2\right) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \tag{2}$$

of a general line equation $\mathbf{nx} = c$
here $\mathbf{n} = \begin{pmatrix} k-3 & -\{4-k^2\} \end{pmatrix}$
and $c = -k^2 + 7k - 6$

a) Parallel to x-axis
$\mathbf{n} = \begin{pmatrix} 0 & 1 \end{pmatrix}$ if the line is parallel to x-axis
Equation of x-axis is $\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 0$

$$\begin{aligned} \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} k-3 \\ -\{4-k^2\} \end{pmatrix} &= 0 \\ k-3 &= 0 \\ \implies k &= 3 \end{aligned} \tag{3}$$

Substituting $k = 3$ in (2)
Equation of line is,

$$\begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} = 6 \tag{4}$$

b) Parallel to y-axis
$\mathbf{n} = \begin{pmatrix} 1 & 0 \end{pmatrix}$ if the line is parallel to y-axis
Equation of y-axis is $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = 0$

$$\begin{aligned} \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} k-3 \\ -(4-k^2) \end{pmatrix} &= 0 \\ 4-k^2 &= 0 \\ \implies k &= \pm 2 \end{aligned} \tag{5}$$

Substituting $k = 2$ in (2). Equation of line is,

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\mathbf{x} = 4 \tag{6}$$

Substituting $k = -2$ in (2). Equation of line is,

$$\begin{pmatrix} -5 & 0 \end{pmatrix}\mathbf{x} = -24 \tag{7}$$

c) Passing through origin
$c = 0$ if the line passes through origin
Equation of line when passing through origin is

$$\mathbf{n}^\top\mathbf{x} = 0 \tag{8}$$

Hence

$$\begin{aligned} -k^2 + 7k - 6 &= 0 \\ &= -k^2 + k + 6k - 6 \\ &= (k-1)(k-6) \\ \implies k &= 1, k = 6 \end{aligned} \tag{9}$$

Substituting $k = 1$ in (2). The equation of line is,

$$\begin{pmatrix} -2 & -3 \end{pmatrix}\mathbf{x} = 0 \tag{10}$$

Substituting $k = 6$ in (2). The equation of line is,

$$\begin{pmatrix} 3 & 32 \end{pmatrix}\mathbf{x} = 0 \tag{11}$$

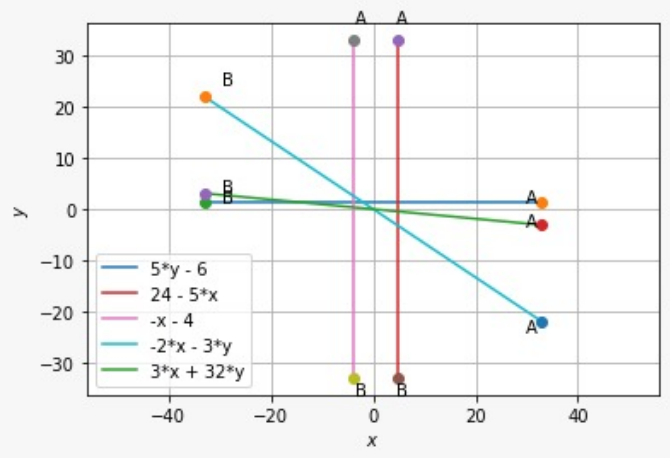

Fig. 3: Plot of line equations