Code No: 131AB **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year I Semester Examinations, December – 2019/January - 2020
MATHEMATICS-II
(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find $L\left[e^{2t}.t\right]$. [2]

b) Find inverse Laplace of $\frac{1}{s^2+2s+2}$. [3]

c) Evaluate $\int_0^{\pi/2} \sin^{3/4} x \cos^{3/4} x dx$ using Beta -Gamma functions. [2]

d) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, using the relation $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [3]

e) Evaluate $\int_0^1\int_0^1\int_0^1 dxdydz$ [2]

f) Evaluate $\int_0^2\int_0^{\sqrt{2x}} dydx.$ [3]

g) Find the directional derivative of $x^2+y^2+z^2=r^2$, at the point (0, r, 0) in the direction of $\bar{j}$ [2]

h) Find Curl (grad f) where f= x^2+y^2-z. [3]

i) State Green's theorem in a plane. [2]

j) Find the work done by $\bar{F}=2x\bar{i}+2y\bar{j}+3z\bar{k}$ in moving a particle from (-1, 2, 1) to (2, 3, 4) along the line joining them. [3]

PART-B

(50 Marks)

2.a) Find inverse Laplace transform $L^{-1}\left\{\log\left(\frac{s^2+1}{(s-1)^2}\right)\right\}$

b) Find $L\{\text{te}^{2t}\sin 3t\}$ [5+5]

OR

3. Using Laplace transform solve $(D^2+3D+2)y=3, \quad y(0)=y'(0)=1.$ [10]

4.a) Prove the relation between Beta and Gamma functions.

b) If $\frac{\beta(m,n)}{k}=\frac{\beta(m-1,n+1)}{p}$ find k+p in terms of m and n. [5+5]

OR

5.a) Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

b) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}}dx=\frac{\beta(m,n)}{a^n(1+a)^m}$. [5+5]

6.a) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates.

b) Evaluate $\iiint_V (xy+yz+zx)dxdydz$ where V is the region of space bounded by $x=0, x=1, y=0, y=2, z=0, z=3$. [5+5]

OR

7.a) Evaluate $\int_0^1\int_{x^2}^{2-x} xydydx$.

b) Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)dxdydz$. [5+5]

8.a) Prove that $\nabla(\bar{A}\cdot\bar{B})=(\bar{B}\cdot\nabla)\bar{A}+(\bar{A}\cdot\nabla)\bar{B}+\bar{B}\times(\nabla\times\bar{A})+\bar{A}\times(\nabla\times\bar{B})$.

b) Evaluate the angle between the normals of the surface $xy=z^2$ at the points (4,1,2) and (3,3,-3). [5+5]

OR

9.a) Find the unit normal vector to the surface $x^2y+2xz=4$ at the point (2,-2,3). Also find the directional derivative of the surface in the direction normal to the surface $x\log z-y^2=1$ at (-1, 2, 1).

b) For any vector field $\bar{V}$, prove that Div Curl $\bar{V}$ =0. [5+5]

10. Verify Divergence Theorem for $\bar{F}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}$ over the surface S of the solid cut of by the plane x+y+z=a in the 1^{st} octant. [10]

OR

11.a) Find the work done by the force $\bar{F}=(xy)\bar{i}-z\bar{j}+(x^2)\bar{k}$ along the curve $x=t^2; y=2t; z=t^3$ from t=0 to t=1.

b) Apply Stokes Theorem to evaluate $\int_c \bar{F}.d\bar{R}$, where C is the curve of intersection of the sphere $x^2+y^2+z^2=a^2$ and x + z=a and $\bar{F}=y\bar{i}+z\bar{j}+x\bar{k}$. [5+5]

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