

Code No: 111AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, May - 2018

MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, BME, IT, MCT, ETM, MMT, AE, AME, MIE, PTM, CEE, AGE)

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A****(25 Marks)**

- 1.a) If A and B are square symmetric matrices of same order then prove that  $AB + BA$  is symmetric. [2]

- b) If one of Eigen vectors of  $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is  $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ , then find the corresponding Eigen value. [3]

- c) Find the value of c in Roll's theorem for  $f(x) = \sin x$  in  $(0, \pi)$ . [2]

- d) Find the stationary points of the following functions  $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ . [3]

- e) Evaluate  $\int_0^{\infty} x^2 e^{-x^4} dx$  [2]

- f) Evaluate  $\int_0^2 \int_0^{x^2} y dx dy$  [3]

- g) Solve the differential equation  $(D^2 - 4D + 13)y = 0$  [2]

- h) Evaluate  $\frac{1}{D^2 - 1}(x^2 + x)$ . [3]

- i) Find  $L[te^t]$  [2]

- j) Find f(t), if  $L[f(t)] = \frac{1}{(s-1)^2}$ . Hence find  $L^{-1}\left[\frac{1}{s(s-1)^2}\right]$  using any theorem of Laplace transforms. [3]

**PART-B****(50 Marks)**

- 2.a) Test for the consistency and hence solve the system.

$$x + y + z = 6, x - y + 2z = 5, 3x + y + z = 8, 2x - 2y + 3z = 7$$

- b) If  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$  the Eigen values of a non singular matrix A of order 'n' then prove that the Eigen values of  $A^{-1}$  are  $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$  [5+5]

**OR**

3. Reduce the quadratic form  $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$  by orthogonal reduction to the canonical form. [10]

4.a) Verify Lagrange's mean value theorem for  $f(x) = \log_e x$  in  $[1, e]$ .

b) Find the maximum and minimum values of  $xy + \frac{a^3}{x} + \frac{a^3}{y}$ . [5+5]

OR

5. If  $x + y = 2e^\theta \cos \phi$ ,  $x - y = 2ie^\theta \sin \phi$ , find  $\frac{\partial(x, y)}{\partial(\theta, \phi)}$  and verify that  $JJ^1 = 1$  [10]

6.a) Evaluate  $\int_0^a x^4 \sqrt{a^2 - x^2} dx$

b) Change the order of integration and evaluate  $\int_0^a \int_{x^2/a}^{2a-x} xy^2 dy dx$ . [5+5]

OR

7.a) Prove that  $\int_0^1 (1-x^n)^{1/n} dx = \frac{1}{n} \left( \frac{\Gamma(1/n)}{\Gamma(1/n)} \right)^2 = \frac{2}{2\Gamma(2/n)}$

b) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dx dy$  by changing into polar coordinates. [5+5]

8.a) Find the orthogonal Trajectory of the family of  $ay^2 = x^3$ .

b) Solve the differential equation  $(D^2 + 9)y = \cos 3x + \sin 2x$  [5+5]

OR

9.a) If a population is increasing exponentially at the rate of 2% per year. What will be the percentage increase over a period of 10 years?

b) Solve by the method of variation of Parameters  $\frac{d^2 y}{dx^2} + y = \sec x$  [5+5]

10.a) Evaluate  $\int_0^\infty \frac{\sin t}{t} dt$

b) Find the inverse Laplace transform of  $\log \left( \frac{s+1}{s-1} \right)$  [5+5]

OR

11. Solve the differential equation  $(D^2 + D)y = t^2 + 2t$ , using Laplace transform given that

$y(0) = 4, \frac{dy(0)}{dt} = 2$ . [10]