Code No: 121AB **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year Examinations, May/June - 2019
MATHEMATICS-I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, AE, AME, MIE, PTM, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the matrix of the quadratic form $Q = 2x^2 + 3y^2 + z^2 - 2xy - 4yz + 6zx$. [2]

b) Determine the values of λ and μ such that the system of equations $x + 2y + 3z = 6,\ x + 3y + 5z = 9,\ 2x + 5y + \lambda z = \mu$ has unique solution. [3]

c) Show that the functions $u = xe^y \sin z,\ v = xe^y \cos z,\ w = x^2 e^{2y}$ are functionally dependent. [2]

d) Write the geometrical interpretation of Rolle's theorem. [3]

e) Find the value of the integral $\int_0^{\pi}\int_0^{x} x \sin y \, dy \, dx$. [2]

f) Evaluate $\Gamma\left(\frac{-5}{2}\right)$. [3]

g) Find the complementary function of $\frac{d^3y}{dx^3} + 8y = 0$. [2]

h) State Newton's law of cooling. [3]

i) Express $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & t > 2 \end{cases}$ in terms of unit step function. [2]

j) Find $L^{-1}\left\{\frac{1}{s^2 + 2s + 2}\right\}$. [3]

PART-B

(50 Marks)

2.a) Show that every square matrix A can be written as the sum of a Hermitian matrix and a Skew-Hermitian matrix.

b) Reduce the matrix $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ to normal form and hence find its rank. [5+5]

OR

3. Reduce the quadratic form $Q = x^2 + 2y^2 + 3z^2 + 2yz - 2zx + 2xy$ to its canonical form. [10]

4.a) Using Lagrange's mean value theorem, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, $0 < a < b < 1$.

b) Verify Cauchy's mean value theorem for $f(x) = x^3 - 3x^2 + 2x$ and $g(x) = x^3 - 5x^2 + 6x$ in $\left[0, \frac{1}{2}\right]$. [5+5]

OR

5.a) Verify $J\left(\frac{x,y,z}{u,v,w}\right).J'\left(\frac{u,v,w}{x,y,z}\right) = 1$, for $x = u$, $y = u\tan v$, $z = w$.

b) Find the maximum and minimum values of the function $f(x,y) = x^4 + y^4 - x^2 - y^2 + 1$. [5+5]

6.a) Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx$.

b) Evaluate $\int_0^\infty \frac{x^a}{a^x}\, dx$, *where* $a > -1$. [5+5]

OR

7.a) Change the order of integration in $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$ and hence evaluate the integral.

b) Find volume of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$. [5+5]

8.a) Solve $\left(D^2 - 5D + 6\right)y = e^{2x} + \sin 3x$.

b) A radioactive substance disintegrates at a rate proportional to its mass. When the mass is 10 mgm, the rate of disintegration is 0.051 per day. How long will it take for the mass to reduce from 10 to 5 mgm? [5+5]

OR

9.a) Show that the family of curves $y^2 = 4c(c+x)$ is self orthogonal.

b) Solve $y'' + y = x^2 e^x + 2x + \sin x$. [5+5]

10.a) Find the Laplace transform of $f(t) = t\, e^{-t} \sin t + (\sin t - \cos t)^2$.

b) Find $L\left\{\frac{\cos 4t - \cos 2t}{t}\right\}$. [5+5]

OR

11.a) Apply convolution theorem to find $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\}$.

b) Solve $y'' + 4y' + 3y = e^{-t}, y(0) = 1, y'(0) = 1$ using Laplace transforms. [5+5]

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