Code No: R101410 **Pre-Ph.D. Examinations, March-2018**

MATHEMATICALMETHODS

Time: 3hours **Max.Marks:100**

Answer any FIVE questions
All questions carry equal marks
- - -

1.a) Suppose $f(t) = t^2e^{-t}$. is continuous for $t \geq 0$ and $f'(t)$ is piecewise continuous of exponential order at infinity with $|f'(t)| \leq Me^{at}, t \geq C$ Then show that
(i) f(t) is of exponential order at infinity
(ii) $L[f'(t)] = sL[f(t)] - f(0) = sF(s) - f(0),\ s > \max\{a,0\}+1.$

(b) Draw the Graph of the function f(t) = h(t − 1) + h(t − 3) for t ≥ 0, where h(t) is the Heaviside step function, and find L[f(t)]. [10+10]

2.a) Consider the initial value problem
$ay'' + by' + cy = f(t),\ y(0) = y'(0) = 0,\ t > 0$
Suppose that the Laplace transform of the differential equation takes the form $Y(s) = \varphi(s)F(s)$ where the transfer function of this system is given by

$$\Phi(s) = \frac{1}{2s^2 + 5s + 2}.$$

(i) What are the constants a, b, and c?
(ii) If $f(t) = e^{-t}$, determine F(s), Y (s), and y (t).

b) Sketch the following functions and express those interms of unit step functions. Hence obtain the Laplace transform

(i) $f(t) = \begin{cases} t^2, 0 < t \leq 2 \\ 4t, t > 2 \end{cases}$ (ii) $f(t) = \begin{cases} \cos(wt+\phi), 0 < t \leq T \\ 0, t > T \end{cases}$ [12+8]

3.a) Prove that $\int_0^\infty\int_0^\infty e^{-(ax^2+by^2)}x^{2m-1}y^{2n-1}dxdy = \frac{\Gamma(m)\Gamma(m)}{4a^mb^n}$,where a,b,m,n are positive

b) Show that $\iint x^{m-1}y^{n-1}dxdy$ over the positive quadrant of the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is $\frac{a^mb^n}{2n}\beta\left(\frac{m}{2},\frac{n}{2}+1\right)$ [10+10]

4.a) Prove that the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$

b) Prove that $\left(1-2xz+z^2\right)^{1/2}$ is a solution of the equation

$$z\frac{\partial^2}{\partial z^2}(zv) + \frac{\partial}{\partial x}\left(\left(1-x^2\right)\frac{\partial v}{\partial x}\right) = 0$$

c) Derive the generating function for Bessel function of the first kind of order n and hence prove that $J_0(x) = \frac{1}{\pi}\int_0^\pi \cos(x\cos\phi)d\phi$ [5+5+10]

PTO..

5.a) Prove that all the eigenvalues of the Sturm–Liouville problem are real.

b) If φ_1 and φ_2 are two eigen functions of the Sturm–Liouville problem corresponding to eigenvalues λ_1 and λ_2, respectively, and if $\lambda_1 = \lambda_2$, then prove that $\int_0^1 r(x)\phi_1(x)\phi_2(x)\,dx = 0$

c) Determine the normalized eigenfunctions of the boundary value problem $y'' + \lambda y = 0$, $y(0)=0$, $y(1)=0$. [6+8+6]

6. Solve the given boundary value problems by determining the appropriate Green's function and expressing the solution as a definite integral

(a) $-y'' = f(x), y(0) = 0, y(1) + y'(1) = 0$

(b) $-(y'' + y) = f(x), y'(0) = 0, y(1) = 0$ [10+10]

7.a) Using the method of Fourier transform, determine the displacement y(x , t) of an infinite string given that the string is initially at rest and that the initial displacement is f(x), $-\infty < x < \infty$ .Also show that the solution can also be put in the form

$$y(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct) \right]$$

b) Find Fourier sine and cosine transform of $F(x) = x^n e^{-ax}; a > 0, n > -1$.

Hence find Fourier sine and cosine transform of

(i) x^{m-1} (ii) x^{-m} [10+10]

8.a) Apply appropriate Fourier transform to solve the partial differential equation

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}; x > 0, t > 0$$

Subject to the conditions

(i) $V_x(0,t) = 0$ (ii) $V(x,0) = \begin{cases} x, 0 \le x \le 1 \\ 0, x > 1 \end{cases}$ (iii) $V(x,t)$ is bounded

b) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$; *for* $x \ge 0, t \ge 0$ under the conditions

(i) $u(0,t) = u_0, t > 0$ (ii) $u(x,0) = 0, x \ge 0$ (iii) $u(x,t)$ is bounded [10+10]

--ooOoo--