R09

Code No: 53007

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICS-III
(Common to EEE, ECE, EIE, ETM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks
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1.a) Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$.

b) Evaluate $\int_0^1 \left(\ln\frac{1}{x}\right)^{n-1} dx,\ n>0$.

c) Express $J_3'(x)$ in terms of $J_0(x)$ and $J_1(x)$. [7+4+4]

2.a) State and prove orthogonal property of Legendre polynomials $P_n(x)$.

b) Prove that i) $T_n(1)=1$ and ii) $T_n(-x)=(-1)^n T_n(x)$. [10+5]

3.a) Find the analytic function $f(z)=u+iv$ if $u+v=2x(1-y)$.

b) Show that $u(x,y)=y^3-3x^2y$ is harmonic and find its conjugate harmonic function $v(x,y)$.

c) Find the principal value of $\log(1-\sqrt{3}\,i)$. [7+5+3]

4.a) Evaluate $\int_0^{1+i} (x^2-iy)\,dz$ along i) the straight line joining (0,0) and (1,1) and ii) along the curve $y^2=x$.

b) Evaluate $\oint_c \frac{z\,e^z}{(z-1)(z-2)(z-3)}\,dz$, where $C:|z|=4$, using Cauchy's integral formula. [8+7]

5.a) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ valid in the regions i) $|z|<1$ ii) $|z|>3$ iii) $1<|z|<3$ and iv) $0<|z+1|<2$.

b) Locate and classify the singularities of i) $\frac{2z+1}{(z-1)(z-2)^2}$ and ii) $z^2\sin(\frac{1}{z})$. [9+6]

6. Evaluate:

a) $\int_{-\infty}^{\infty} \frac{x^2}{\left(x^2+1\right)^3} dx$ and b) $\int_{0}^{\infty} \frac{\cos mx}{x^2+1} dx$. [7+8]

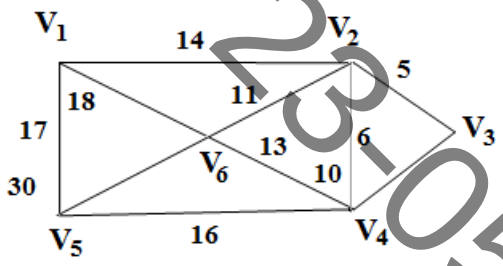
7.a) Find the image and draw a rough sketch of the region $1 \le x \le 2$ and $2 \le y \le 3$ under the transformation $w = e^z$.

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ to the points $w = 2, i, -2$ respectively. Also find the fixed points of the transformation. [7+8]

8.a) Define adjacency matrix. Construct the graph whose adjacency matrix is given by

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

b) Apply Prim's algorithm to find the shortest spanning tree of the following graph. [5+10]



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