R07

Code No: X0221

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICS – III
(Common to EEE, ECE)

Time: 3 hours **Max. Marks: 80**

Answer any five questions
All questions carry equal marks
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1.a) Using Beta and Gamma function, evaluate the integral $\int_{-1}^{1}\left(1-x^2\right)^n dx$ where n is a positive integer.

b) Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ where $m > 0$. [8+8]

2.a) Show that the function $f(z) = z$ is not analytic at $z = \infty$.

b) Find the orthogonal trajectories of the family of curves $r^2\cos 2\theta = c$ = constant. [8+8]

3. If $A + iB = \tan^{-1}(x + iy)$, prove that $B=\frac{1}{4}\log\frac{x^2+(1+y)^2}{x^2+(1-y)^2}$. [16]

4.a) State and prove Cauchy integral theorem.

b) Evaluate $\oint_C \frac{\cos z-\sin z}{(z+i)^3}dz$ where $c : |z|=2$. [8+8]

5.a) Find the Taylor's series expansion of $f(z) = \frac{1}{1+z^2}$ about the point $z = 0$. Determine the region of convergence.

b) Determine and classify the singularities of $\frac{1}{(2\sin z-1)^2}$. [8+8]

6. Evaluate $\int_{-\infty}^{\infty}\frac{z^2-z+2}{z^4+10z^2+9}dz$. [16]

7.a) State and prove Fundamental theorem of Algebra.

b) If $f(z) = z^5-3iz^2+2z-1+i$, evaluate $\oint_C \frac{f'(z)}{f(z)}dz$ where c encloses all the zeros of $f(z)$. [8+8]

8. Find the transformation which maps the points $z = 1, -i, -1$ to the points $w = i, 0, i$ respectively. Also show that this transformation maps the region outside the circle $|z| = 1$ into the half plane Real $(w) \geq 0$. [16]

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