Code No: R101410 **Pre-Ph.D. Examinations, September - 2019**

MATHEMATICAL METHODS

Time: 3 hours **Max.Marks:100**

Answer any FIVE questions
All questions carry equal marks
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1.a) Find the Laplace transform of the function $f(t)=\begin{cases} t, & 0<t\le a \\ -t+2a, & a<t<2a \end{cases}$

b) Using Laplace transform, solve $y''-2y'-8y=0;\ y(0)=3,\ y'(0)=6.$ [10+10]

2.a) Find Laplace transform of the function $\frac{e^{-at}-e^{-bt}}{t}$.

b) Use Laplace transform, solve $y(t) = 1-e^{-t}+\int_0^t y(t-u)\,sinu\,du.$ [10+10]

3.a) Find Fourier sine transform of $f(x) = \frac{1}{x(x^2+a^2)}$ and hence deduce cosine transform of $\frac{1}{x^2+a^2}$.

b) Find the inverse Fourier sine transform of $F_s\{p\} = \frac{e^{-ap}}{p}$ and hence deduce $F_s^{-1}\{1/p\}$. [10+10]

4.a) Find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, using Beta, gamma functions.

b) Evaluate $\iint(x^2+y^2)dx\,dy$ over the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant by using the transformation $x=au$ and $y=bv$, using Beta, Gamma functions. [10+10]

5.a) Find the eigen values and eigen functions of the Sturm-Liouville problem $y''+\lambda y=0$ with boundary conditions $y(0)=0, y(\pi)=0$,

b) Using Parseval's identity, evaluate $\int_0^\infty \frac{1-\cos 2x}{x^2}dx$. [10+10]

6.a) If $n>-1$,prove that $\int_0^x x^{-n}J_{n+1}(x)\,dx = \frac{1}{2^n\Gamma(n+1)}-x^{-n}J_n(x)$.

b) Prove that $P_n' - P_{n-2}' = (2n-1)P_{n-1}$. [10+10]

7.a) Show that the given exponential series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$ converges absolutely for all x.

b) If $f(x)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \pi/2\le x<\pi \end{cases}$ and $f(x+2\pi)=f(x)$ then show that $f(x) = \frac{4}{\pi}[sinx-\frac{sin3x}{3^2}+\frac{sin5x}{5^2}-\cdots]$. [10+10]

8.a) Solve the initial value problem $(y+2)y' = \sin x, y(0)=0$, using Laplace Transform.

b) Find half range Fourier sine series for $f(x) = \pi - x$ in $[0, \pi]$. [10+10]

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