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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICS – II
(Common to CE, AE)

Time: 3 hours   **Max. Marks: 80**

Answer any five questions
All questions carry equal marks
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1.a) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{pmatrix}$ by reducing to echelon form.

b) Show that the system of equations $x + y + z = 7$, $x + 2y + 3z = 16$ and $x + 3y + 4z = 22$ is consistent and hence solve it. [8+8]

2.a) If λ is an eigen value of a non-singular matrix A, then show that $\dfrac{1}{\lambda}$ is an eigen value

of A^{-1} and $\dfrac{|A|}{\lambda}$ is an eigen value of $Adj\, A$.

b) If $A = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$, find the modal matrix P which diagonalizes A and the diagonal

matrix D. [8+8]

3.a) Show that the matrix $B = (I - A)\,(I + A)^{-1}$, where $A = \begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$, is a Unitary

matrix.

b) Reduce the quadratic form $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2 x_3$ to canonical form. [8+8]

4.a) Obtain the Fourier series for $f(x) = x^2$, $-\pi < x < \pi$ and hence deduce that:

i) $\sum_{n=1}^{\infty} \dfrac{1}{n^2} = \dfrac{\pi^2}{6}$ and ii) $\sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{1}{n^2} = \dfrac{\pi^2}{12}$.

b) Find the half-range sine and cosine series expansions of $f(x) = x$ in [0, 2]. [8+8]

5.a) Eliminate the arbitrary functions f and g to find a partial differential equation from
$z = y\, f(x) + g(y)$.

b) Solve $z^2 = 1 + p^2 + q^2$. [8+8]

6.a) Solve $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u,\ u(0, y) = e^{-5y}$ by the method of separation of variables.

b) Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ which satisfies

$u(0, y) = u(\pi, y) = u(x, 0) = 0$ and $u(x, a) = \sin nx$. [6+10]

7.a) Express the function $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ as Fourier integral and hence evaluate

$\displaystyle\int_0^\infty \dfrac{\sin \lambda \cos \lambda x}{\lambda}\, d\lambda$.

b) Find the Fourier sine and cosine transforms of $f(x) = e^{-2x}$. [8+8]

8.a) If $Z\{f_n\} = \dfrac{3z^2 - 4z + 7}{(z-1)^3}$, find f_0, f_1, f_2 and f_3.

b) Using convolution theorem, find $Z^{-1}\left\{\left(\dfrac{z}{z-1}\right)^3\right\}$. [8+8]

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