Assignment 2

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1 QUESTION No. 2.106 - LINEAR FORMS

2.106 Find the values of k for which the line

$$\begin{pmatrix} k-3 & -(4-k^2) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \quad (1)$$

is
a) Parallel to the x-axis
b) Parallel to the y-axis
c) Passing through the origin

Solution Given equation of the line,

$$\begin{pmatrix} k-3 & -(4-k^2) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \quad (2)$$

of a general line equation $\mathbf{nx} = c$
here $\mathbf{n} = \begin{pmatrix} k-3 & -\{4-k^2\} \end{pmatrix}$
and $c = -k^2 + 7k - 6$

a) Parallel to x-axis
$\mathbf{n} = \begin{pmatrix} 0 & 1 \end{pmatrix}$ if the line is parallel to x-axis
Equation of x-axis is $\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 0$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} k-3 \\ -\{4-k^2\} \end{pmatrix} = 0$$
$$k - 3 = 0 \quad (3)$$
$$\implies k = 3$$

Substituting $k = 3$ in (2)
Equation of line is,

$$\begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} = 6 \quad (4)$$

b) Parallel to y-axis
$\mathbf{n} = \begin{pmatrix} 1 & 0 \end{pmatrix}$ if the line is parallel to y-axis
Equation of y-axis is $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = 0$

$$\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} k-3 \\ -(4-k^2) \end{pmatrix} = 0$$
$$4 - k^2 = 0 \quad (5)$$
$$\implies k = \pm 2$$

Substituting $k = 2$ in (2). Equation of line is,

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\mathbf{x} = 4 \quad (6)$$

Substituting $k = -2$ in (2). Equation of line is,

$$\begin{pmatrix} -5 & 0 \end{pmatrix}\mathbf{x} = -24 \quad (7)$$

c) Passing through origin
$c = 0$ if the line passes through origin
Equation of line when passing through origin is

$$\mathbf{n}^\top \mathbf{x} = 0 \quad (8)$$

Hence

$$-k^2 + 7k - 6 = 0$$
$$= -k^2 + k + 6k - 6$$
$$= (k-1)(k-6) \quad (9)$$
$$\implies k = 1, k = 6$$

Substituting $k = 1$ in (2). The equation of line is,

$$\begin{pmatrix} -2 & -3 \end{pmatrix}\mathbf{x} = 0 \quad (10)$$

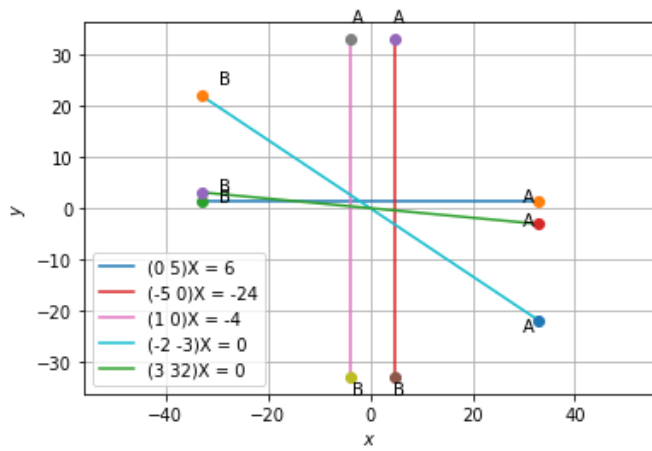Substituting $k = 6$ in (2). The equation of line is,

$$\begin{pmatrix} 3 & 32 \end{pmatrix}\mathbf{x} = 0 \quad (11)$$

Fig. 3: Plot of line equations