**Code No: 133BD** **R16**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year I Semester Examinations, May/June - 2019**
**MATHEMATICS – IV**
**(Common to CE, EEE, ME, ECE, CSE, EIE, IT MCT, ETM MMT, AE, MIE, PTM, CEE, MSNT)**
**Time: 3 Hours** **Max. Marks: 75**

**Note:** This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A**

**(25 Marks)**

1.a) State the necessary and sufficient conditions for a function $f(z) = u + iv$ to be analytic. [2]

b) Show that $f(z) = |z|^2$ is not analytic at any point. [3]

c) State Cauchy's integral theorem. [2]

d) Find the poles and the residues at the poles of the function $f(z) = \frac{e^z}{\cos \pi z}$. [3]

e) Define bilinear transformation and cross ratio. [2]

f) Find the image of the circle $|z| = 2$, under the transformation $w = z + 3 + 2i$. [3]

g) State Fourier integral theorem. [2]

h) Expand $f(x) = \pi x - x^2$ in a half range sine series in $(0, \pi)$. [3]

i) Classify the partial differential equation $u_{xx} + 6u_{xy} + 2u_{yy} + 2u_x - 2u_y + u = x^2 y$. [2]

j) Write the three possible solutions of the heat equation.
$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ [3]

**PART-B**

**(50 Marks)**

2.a) If $f(z)$ is a regular function of $z$, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2.$$

b) Let $f(z) = u(r,\theta) + iv(r,\theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function $f(z)$ in terms of $z$. [5+5]

**OR**

3.a) Show that the function $f(z)$ defined by
$f(z) = \frac{x^2 y^3 (x+iy)}{x^6 + y^{10}} \quad for\ z \neq 0$, is not analytic at the origin, even though it satisfies the
$f(0) = 0$
Cauchy-Riemann equations at the origin.

b) Determine the analytic function whose real part is $\log\sqrt{x^2 + y^2}$. [5+5]

4. Represent the function $\frac{1}{z^2-4z+3}$ in the domain
(a) $1< |z| < 3$ (b) $|z| < 1$. [10]

**OR**

5.a) Expand the function $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$, and name the series thus obtained.
b) Evaluate $\oint_C \frac{e^z}{(z+3)(z+2)} dz$, where $C$ is the circle $|z-1| = \frac{1}{2}$. [5+5]

6. Evaluate the integral using contour integration $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. [10]

**OR**

7. Show that the transformation $w = i\,\frac{1-z}{1+z}$ transforms the circle $|z| = 1$ into the real axis of $w$ plane and the interior of the circle $|z| < 1$ into the upper half of the $w$ plane. [10]

8. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$. Hence evaluate
$\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}\,dx$. [10]

**OR**

9.a) Obtain the half range cosine series for
$$f(x) = \begin{cases} kx, & for\ 0 \le x < L/2 \\ k(L-x), & for\ L/2 \le x \le L \end{cases}.$$
b) Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence show that $\int_0^\infty \frac{x\sin mx}{x^2+1}dx = \frac{\pi}{2}e^{-m}$.
[5+5]

10. A string is stretched and fastened to two points $L$ apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{L}$ from which it is released at time $t = 0$. Find the displacement of any point at a distance $x$ from one end at time $t$. [10]

**OR**

11. Write down the one dimensional heat equation. Find the temperature $u(x,t)$ in a slab whose ends $x = 0$ and $x = L$ are kept at zero temperature and whose initial temperature $f(x)$ is given by
$$f(x) = \begin{cases} k, & when\ 0 < x < \frac{1}{2}L \\ 0, & when\ \frac{1}{2}L < x < L \end{cases}.$$ [10]

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