

Code No: 123AH

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD****B. Tech II Year I Semester Examinations, November/December - 2017****MATHEMATICS – III****(Common to EEE, ECE, EIE, ETM)****Time: 3 Hours****Max. Marks: 75****Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A****(25 Marks)**

- 1.a) Solve  $(D^4 + 13D^2 + 36)y = 0$ . [2]
- b) Find the P.I of  $(D^2 + 4)y = \cos 2x$ . [3]
- c) Prove that  $P_n^1(1) = \frac{1}{2}n(n+1)$ . [2]
- d) Prove that  $\int x J_0^2(x) dx = \frac{1}{2}x^2 [J_0^2(x) - J_1^2(x)]$ . [3]
- e) If  $u = e^x(x \cos y - y \sin y)$  then find analytic function of  $f(z)$ . [2]
- f) Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the path  $y = x^2$ . [3]
- g) Expand  $f(z) = \frac{z+3}{z(z^2 - z - 2)}$  for  $|z| > 2$ . [2]
- h) Find the residue of  $f(z) = \frac{e^z}{(z-1)^2}$  at the singular point. [3]
- i) Find the fixed points of  $w = \frac{3z-2}{z+1}$ . [2]
- j) Prove that  $w = \frac{1}{z}$  is circle preserving. [3]

**PART-B****(50 Marks)**

- 2.a) Solve  $(D+2)(D-1)^2 = e^{-2x} + 2 \sinh x$ .
- b) Solve  $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$ . [5+5]

**OR**

3. Obtain the series solution of the equation  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 4)y = 0$  [10]

4.a) Prove that  $\int_{-1}^1 P_m(x)P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1}, & \text{if } m = n \end{cases}$

b) Show that  $\frac{2}{5}P_3(x) + \frac{3}{5}P_1(x) = x^3$ . [5+5]

**OR**

5.a) Prove that  $\frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)] = 2 \left[ \frac{n}{x} J_n^2(x) - \frac{(n+1)}{x} J_{n+1}^2(x) \right]$ .

b) Show that  $\left[ J_{\frac{1}{2}}(x) \right]^2 + \left[ J_{-\frac{1}{2}}(x) \right]^2 = \frac{2}{\pi x}$ . [5+5]

6.a) Prove that the function of  $f(z)$  defined by

$$f(z) = \frac{x^3(1+x) - y^3(1-i)}{x^2 + y^2}, z \neq 0$$

$$= 0, \quad z = 0$$

is continuous and C – R equations at the origin, yet  $f'(0)$  does not exist.

b) If  $f(z)$  is an analytic function of  $z$ , prove that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$ . [5+5]

**OR**

7.a) Evaluate  $\int_c \frac{z+4}{z^2+2z+5} dz$  where  $c$  is the circle.

i)  $|z+1-i|=2$

ii)  $|z+1+i|=2$

b) State and prove Cauchy's inequalities. [5+5]

8.a) State and prove residue theorem.

b) Evaluate  $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta} (a > 0)$ . [5+5]

**OR**

9.a) Evaluate  $\int_0^\infty \frac{dx}{x^4 + a^4} (a > 0)$ .

b) Prove that  $\int_0^\infty \frac{\sin mx}{x} dx = \frac{\pi}{2}$ . [5+5]

10.a) Plot the image of  $1 < |z| < 2$  under the transformation  $w = 2iz + 1$ .

b) Find the graph of the region  $-\frac{\pi}{2} < x < \frac{\pi}{2}, 1 < y < 2$  under the mapping  $w = \sin z$ . [5+5]

**OR**

11.a) Find the image of the region in the  $z$ -plane between the lines  $y=0$  and  $y=\frac{\pi}{2}$  under the transformation  $w = e^z$ .

b) Find the bilinear transformation which maps the points  $\infty, i, 0$  in the  $z$ -plane into  $-1, -i, 1$  in the  $w$ -plane. [5+5]