Code No: R101410 **Pre-Ph.D. Examinations, September-2018**

MATHEMATICAL METHODS

Time: 3 hours **Max.Marks:100**

Answer any FIVE questions
All questions carry equal marks
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1.a) Find the Laplace Transforms of Bessel's function of order zero.

b) Find the inverse Laplace Transform of $cot^{-1}\left(\frac{2}{s+1}\right)$

c) Use convolution theorem to find the inverse Laplace Transform of $\frac{s+2}{(s^2+4s+5)^2}$ [7+7+6]

2.a) Solve the initial value problem $\frac{d^2y}{dt^2}+a^2y=f(t)$, y(0) = 1, y(0) = -2 by using Laplace Transform.

b) Solve the integral equation by using Laplace Transform
$y(t)= sint+2\int_0^t y(u)(t-u)^3du$ [10+10]

3.a) Find Cosine series for the function 'f' defined by $f(x)=\begin{cases} x \ for\ 0\le x\le \frac{1}{2} \\ L-x\ for\ \frac{1}{2}\le x\le L\end{cases}$

b) Using Parseval's identity, find the value of the integral $\int_0^\infty \frac{sin^2t}{t^2}\,dt$ [10+10]

4.a) Find the Fourier series for $f(x)=\frac{(\pi-x)^2}{4}$ in the interval (0, 2π) and hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.........=\frac{\pi}{6}$.

b) Find the function whose Cosine transform is $\frac{\sin aw}{w}$, a > 0. [10+10]

5.a) Evaluate $\int_a^b (x-a)^m(b-x)^n dx$, m and n being positive integers.

b) Prove that $J_n^1(x)=\frac{1}{4}\{J_{n-2}(x)-2J_n(x)+J_{n+2}(x)\}$

c) Express x^3+2x^2-x-3 in terms of Legendre polynomials. [7+7+6]

6.a) When 'n' is a positive integer show that $J_n(x)=\frac{1}{\pi}\int_0^\pi \cos(n\theta-xsin\theta)\,d\theta$

b) Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ [10+10]

7.a) Find the eigen values and eigen function of
$\{xy'(x)\}'+\left(\frac{\lambda}{x}\right)y(x)=0, y(1)=y'\left(e^{2\pi}\right)=0$

b) Find the Green's function for $y''=0\ in\ [0,1],\ y'(0)=y(1)=0.$ [10+10]

8.a) Solve the Boundary value problem $y''=3x+4y, y(0)=0, y(1)=1.$

b) Find Green's function for $y''+\frac{1}{4x^2}=0\ in\ [1,2]y(1)=y(2)=0.$ [10+10]

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