

Code No: 53007

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**B.Tech II Year I Semester Examinations, November/December - 2017****MATHEMATICS-III****(Common to EEE, ECE, EIE, ETM)****Time: 3 hours****Max. Marks: 75****Answer any five questions****All questions carry equal marks**

- 1.a) If m, n are positive then prove that $\frac{\beta(m, n+1)}{n} = \frac{\beta(n+1, m)}{m} = \frac{\beta(m, n)}{m+n}$.
- b) If m_1, m_2 are roots of $J_n(x) = 0$, then prove that $\int_0^1 x J_n(m_1 x) J_n(m_2 x) dx = 0$. [8+7]
- 2.a) Prove that $\int_{-1}^1 (P_n')^2 dx = n(n+1)$.
- b) Show that $\frac{\sqrt{1-x^2}}{1-2xt+t^2} = \sum_{n=1}^{\infty} U_{n+1}(x) t^n$. [7+8]
- 3.a) If $f(z)$ is analytic at z_0 , prove that it must be continuous at z_0 . Give an example to show that the converse is not true.
- b) If $\tan \log(x + iy) = a + ib$, where $a^2 + b^2 \neq 1$, prove that $\tan \log(x^2 + y^2) = \frac{2a}{1-a^2-b^2}$. [8+7]
- 4.a) State and prove Cauchy integral formula.
- b) If $F(a) = \int_c \frac{3z^2 + 7z + 1}{z-a} dz$ where c is the circle $|z|=2$. Find the values of $F(1)$, $F(3)$, $F''(1-i)$. [7+8]
- 5.a) Evaluate $\oint_c \frac{dz}{z^2 + 6iz}$ where c is the circle $|z| = 1$.
- b) Evaluate $\int_c \frac{z \sec z}{(1-z^2)} dz$ where c is the ellipse $4x^2 + 9y^2 = 9$. [7+8]
6. Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} d\theta$; ($a > b > 0$). [15]
- 7.a) Show that every bilinear transformation maps the circles in the z -plane onto the circles in the w -plane.
- b) Determine the region of the w -plane into which the first quadrant of z -plane is mapped by the transformation $w = z^2$. [7+8]
- 8.a) Show that every complete graph is regular.
- b) If $G = (V, E)$ be an undirected graph with ' e ' edges. Then prove that the sum of the degrees of all the vertices of the graph is twice the number of edges. [7+8]

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