R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year I Semester Examinations, December - 2017
MATHEMATICS-II
(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART- A

(25 Marks)

1.a) Find the Laplace transform of $\cosh^3 2t$. [2]

b) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$. [3]

c) Evaluate the improper integral $\int_0^\infty \sqrt{x}e^{-x^2}dx$ using Gamma function. [2]

d) Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ using Beta and Gamma functions. [3]

e) Find the area bounded by the curves $x^2 = y^3, x = y$ using double integration. [2]

f) Change the order of the integration $\int_{y=0}^{1}\int_{x=0}^{y+4} \frac{2y+1}{x+1}dxdy$ and evaluate the integral. [3]

g) Find $\nabla\phi$, when $\phi = 3x^2y - y^3z^2$ at the point (1, -2, -1). [2]

h) Find the directional derivative of the function $f(x, y, z) = 2xy + z^2$ at the point (1, -1, 3) in the direction of the vector $i + 2j + 2k$. [3]

i) If $R = t\bar{i} - t^2\bar{j} + (t-1)\bar{k}$ and $S = 2t^2\bar{i} + 6t\bar{k}$, evaluate $\int_0^2 R.\,S\;dt$. [2]

j) Evaluate the line integral $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region $y = \sqrt{x}, y = x.$ [3]

PART-B

(50 Marks)

2.a) Find the Laplace transform of $\sin\sqrt{t}$. Hence find $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$.

b) Prove that $\int_{t=0}^{\infty}\int_{u=0}^{t} e^{-t}\left(\frac{\sin u}{u}\right)dudt = \frac{\pi}{4}$. [5+5]

OR

3.a) Find the inverse Laplace transform of $\ln\left(\frac{s+1}{s-1}\right)$.

b) Find the inverse Laplace transform of $\frac{1}{s^3(s^2+a^2)}$ using the convolution theorem.

[5+5]

4.a) Prove that $\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n}\operatorname{cosec}\left(\frac{\pi}{n}\right)$.

b) Evaluate $\int_0^{\pi} x\sin^7 x\cos^4 x\,dx$ using Beta and Gamma functions. [5+5]

OR

5. Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}} = \frac{\left[\Gamma(1/4)\right]^2}{4\sqrt{\pi}}$. [10]

6.a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to spherical polar coordinates.

b) Evaluate the integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dzdydx$. [5+5]

OR

7. Find by triple integration, the volume of the paraboloid of revolution $x^2+y^2=4z$ cut off by the plane $z=4$. [10]

8. Prove the following vector identities.

a) $\nabla(\phi_1\phi_2) = \phi_1\nabla(\phi_2) + \phi_2\nabla(\phi_1)$ b) $\nabla\left(\frac{\phi_1}{\phi_2}\right) = \frac{\phi_2\nabla\phi_1 - \phi_1\nabla\phi_2}{\phi_2^2}, \phi_2 \neq 0$. [5+5]

OR

9. If $R = xi + yj + zk$, show that: a) $\nabla r = \frac{R}{r}$ b) $\nabla\left(\frac{1}{r}\right) = -\frac{R}{r^3}$ c) $\nabla r^n = nr^{n-2}R$

d) $\nabla(a.R) = a$, where a is a constant vector and $r = |R|$. [10]

10. State the Stokes' theorem. Verify it for the vector field $F = (2x-y)i - yz^2 j - y^2 zk$ over the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on the xy - plane. [10]

OR

11. State the Green's theorem in a plane. Verify it for $\oint_C e^{-x}(\sin y\,dx + \cos y\,dy)$ where C is the rectangle with the vertices (0,0), $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$. [10]

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