Assignment 2

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Download all python codes from

https://github.com/TGURUBALAJI/INTERNSHIP–IITH/Assignment2/code

and latex-tikz codes from

https://github.com/TGURUBALAJI/INTERNSHIP–IITH/Assignment2/gbalaji.tex

1 Question No. 2.106 - LINEAR FORMS

2.106 Find the values of k for which the line

$$\begin{pmatrix} k-3 & -\left(4-k^2\right) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \tag{1}$$

is

a) Parallel to the x-axis
b) Parallel to the y-axis
c) Passing through the origin

Solution Given equation of the line,

$$\begin{pmatrix} k-3 & -(4-k^2) \end{pmatrix}\mathbf{x} + k^2 - 7k + 6 = 0 \tag{2}$$

of a general line equation

$$\mathbf{n}\mathbf{x} = c \tag{3}$$

here

$$\mathbf{n} = \begin{pmatrix} k-3 & -\{4-k^2\} \end{pmatrix} \tag{4}$$

and

$$c = -k^2 + 7k - 6 \tag{5}$$

a) Parallel to x-axis

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{6}$$

if the line is parallel to x-axis Equation of x-axis is

$$\begin{aligned} \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} &= 0 \\ \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} k-3 \\ -\{4-k^2\} \end{pmatrix} &= 0 \\ k-3 &= 0 \\ \implies k &= 3 \end{aligned} \tag{7}$$

Substituting $k = 3$ in (2) Equation of line is,

$$\begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} = 6 \tag{8}$$

b) Parallel to y-axis

$$\mathbf{n} = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{9}$$

if the line is parallel to y-axis Equation of y-axis is

$$\begin{aligned} \begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} &= 0 \\ \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} k-3 \\ -(4-k^2) \end{pmatrix} &= 0 \\ 4-k^2 &= 0 \\ \implies k &= \pm 2 \end{aligned} \tag{10}$$

Substituting $k = 2$ in (2). Equation of line is,

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\mathbf{x} = 4 \tag{11}$$

Substituting $k = -2$ in (2). Equation of line is,

$$\begin{pmatrix} -5 & 0 \end{pmatrix}\mathbf{x} = -24 \tag{12}$$

c) Passing through origin $c = 0$ if the line passes through origin Equation of line when passing through origin is

$$\mathbf{n}^{\top}\mathbf{x} = 0 \tag{13}$$

Hence

$$\begin{aligned} -k^2 + 7k - 6 &= 0 \\ &= -k^2 + k + 6k - 6 \\ &= (k-1)(k-6) \\ \implies k &= 1, k = 6 \end{aligned} \tag{14}$$

Substituting $k = 1$ in (2). The equation of line is,

$$\begin{pmatrix} -2 & -3 \end{pmatrix}\mathbf{x} = 0 \tag{15}$$

Substituting $k = 6$ in (2). The equation of line is,

$$\begin{pmatrix} 3 & 32 \end{pmatrix}\mathbf{x} = 0 \tag{16}$$

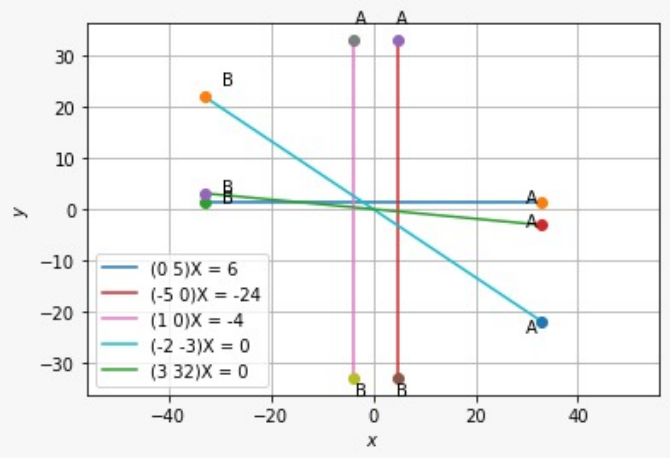

Fig. 3: Plot of line equations