Code No: 123AH **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, November/December - 2018
MATHEMATICS – III
(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the complementary function of $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$. [2]

b) Find the P.I of $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$. [3]

c) Prove that $P_n(-x)=(-1)^n P_n(x)$. [2]

d) Evaluate $\int x^{-1}J_4(x)dx$. [3]

e) Prove that $u=y^3-3x^2y$ is a harmonic function. [2]

f) Evaluate $\int_c \frac{z+2}{z}dz$ where C is the semi circle $z=2e^{i\theta}, 0\le\theta\le\pi$. [3]

g) Expand $\frac{1}{z^2}$ where $|z+1|<1$. [2]

h) Evaluate $\int_c \frac{2e^z}{z(z-3)}dz$ where $c:|z|=2$. [3]

i) Define cross ratio of four points. [2]

j) Prove that $f(z)=\bar{z}$ is not conformal. [3]

PART-B

(50 Marks)

2. Solve $\frac{d^2y}{dx^2}+xy=0$ by power series. [10]

OR

3. Solve in series the equation $x\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$. [10]

4.a) Show that $p_5^1(x) = 9p_4(x) + 5p_2(x) + p_0(x)$.

b) Show that $\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)z^n p_n(x)$. [5+5]

OR

5. If α and β are the roots of $J_n(x) = 0$ then prove that:

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = \begin{cases} 0, & \text{if } \alpha \neq \beta \\ \frac{1}{2} J^2{}_{n+1}(\alpha), & \text{if } \alpha = \beta \end{cases}$$ [10]

6. If $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is analytic function of z then find $f(z)$. [10]

OR

7.a) Prove that $\int_c \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz = \pi i$ if $c: |z| = 1$.

b) If $F(a) = \int_c \frac{3z^2 + 7z + 1}{z - a} dz$ where c is the circle $|z| = 2$ find the value of F(1), F(3), $F'(1-i)$ and $F''(1-i)$. [5+5]

8.a) Find the Taylor's and Laurent's series which represent the function $\frac{z^2 - 1}{(z+2)(z+3)}$ in $2 < |z| < 3$.

b) Evaluate $\int_c \frac{e^z}{(z^2 + \pi^2)^2} dz$ where $c: |z| = 4$ by residue theory. [5+5]

OR

9. Evaluate $\int_{-\infty}^{\infty} \frac{\cos mx}{(a^2 + x^2)} dx \ (a > 0, b > 0)$. [10]

10.a) Show that the transformation $w = \frac{2z+3}{z-4}$ changes the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$.

b) Determine the region of the w-plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$. [5+5]

OR

11.a) Show that the relation $w = \frac{5-4z}{4z-2}$ transform the circle $|z| = 1$ into a circle of radius unity in the w-plane.

b) Find the bilinear transformation that maps the points $1, i, -1$ into the points $2, i, -2$. [5+5]

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