

Code No: X0221

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD****B.Tech II Year I Semester Examinations, November/December - 2017****MATHEMATICS – III****(Common to EEE, ECE)****Time: 3 hours****Max. Marks: 80****Answer any five questions****All questions carry equal marks**

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- 1.a) Prove that  $\int_0^{\pi/2} \sqrt{\cos x} \, dx \cdot \int_0^{\pi/2} \frac{1}{\sqrt{\cos x}} \, dx = \pi$
- b) Prove that  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  [8+8]
- 2.a) Find the conjugate harmonic function of the harmonic function  $u = x^2 - y^2$
- b) Show that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin although Cauchy-Riemann equations are satisfied at the point. [8+8]
- 3.a) Find all roots of the equation i)  $\tanh z + 2 = 0$  ii)  $\sin z = i$
- b) If  $(x + iy)^{\frac{1}{3}} = a + ib$ , then prove that  $4(a^2 - b^2) = \frac{x}{a} + \frac{y}{b}$
- c) Determine all the principal values of  $\log(-9)$ . [8+4+4]
- 4.a) Evaluate  $\int_{(0,0)}^{(1,1)} [3x^2 + 5y + i(x^2 - y^2)] dz$  along  $y^2 = x$ .
- b) State and prove Cauchy's Integral theorem. [8+8]
- 5.a) Find the Laurent series expansion of the function  $\frac{z^2 - 1}{z^2 + 5z + 6}$  about  $z = 0$  in the region  $2 < |z| < 3$ .
- b) Find Taylor's expansion for the function  $f(z) = \frac{1}{(1+z)^2}$  with centre  $-i$  [8+8]
6. Show that  $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ . [16]
- 7.a) State and prove Rouché's theorem.
- b) Apply Rouché's theorem to find number of zeros for the function  $f(z) = z^4 - 5z + 1$  in the annulus region  $1 < |z| < 2$  [8+8]
- 8.a) Show that the transformation  $w = z + \frac{1}{z}$  converts the radial lines  $\theta = \text{constant}$  in the  $z$ -plane into a family of confocal hyperbolas in the  $w$ -plane.
- b) Under the transformation  $w = \frac{1}{z}$  find the image of the circle  $|z - 2i| = 2$ . [8+8]

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