

Code No: X0121

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2017

MATHEMATICS – II

(Common to CE, AE)

Time: 3 hours

Max. Marks: 80

Answer any five questions
All questions carry equal marks

- 1.a) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to canonical form. Hence find its rank.
- b) Test for consistency the following equations and solve them if consistent:
 $5x + 3y + 7z = 4$; $3x + 2y + 2z = 9$; $7x + 2y + 10z = 5$ [8+8]
- 2.a) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.
- b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and hence find its inverse. [8+8]
- 3.a) Prove that the Eigen values of Hermitian matrix are real.
- b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$ into canonical form and also write the nature of the quadratic form. [6+10]
- 4.a) Obtain the Fourier series $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$. Deduce that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}.$$
- b) Obtain the Fourier expansion of $f(x) = x \sin x$ as a cosine series in $(0, \pi)$. [10+6]
- 5.a) Form partial differential equation by eliminating the arbitrary functions from $z = f(x) + e^y g(x)$
- b) Find the general solution of the partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2zx$.
- c) Solve the partial differential equation $p(1 + q) = qz$. [5+5+6]

6.a) Solve by the method of separation of variables

$$4u_x + u_y = 3u \text{ and } u(0, y) = e^{-5y}.$$

b) A string of length L is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3\left(\frac{\pi x}{L}\right)$. Find displacement $y(x, t)$.

[8+8]

7.a) Express $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$ as a Fourier sine integral and hence

evaluate $\int_0^\infty \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda) d\lambda$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence evaluate

$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$. [8+8]

8.a) If $Z(u_n) = \bar{u}(z)$, prove that $Z(a^{-n} u_n) = \bar{u}(az)$. Hence find $Z(n^2 a^{-n})$.

b) Find the inverse Z-transform of $\frac{z^2 + z}{(z-1)(z^2+1)}$.

c) Solve the difference equation $u_{n+2} - 4u_n = 0$ given that $u_0 = 0, u_1 = 2$ by using Z-transform. [5+5+6]

---ooOoo---