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ARYABHATEEYA

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(ENGLISH TRANSLATION ONLY)

BY ARYABHATA - I (499 AD)

MEANING AS GIVEN BY PROF. K. S. SUKLA & PROF. K. V. SARMA

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Indian Institute of Scientific Heritage

Thiruvananthapuram - 695 018

Heritage Publication Series – 64

INTRODUCTION

Aryabhata I, who wrote the book Aryabhateeya at the age of 23, has contributed many novel information to the world of astronomy and mathematics. His contributions, have gone unnoticed by almost all the Indian and foreign scientists. In fact he lived more than one millennia before Sir Isaac Newton, Galileo, Copper Nicus, Keplier and many famous European scientists. Many ideas clearly put forth by Aryabhatta I are now attributed to these European scientists.

It is accepted that science and scientific approach are universal. Hence we should reevaluate the old thoughts that every knowledge came from Europe, and submit to the forum of the world scientific community that 1500 years ago, Indian scientists like Aryabhatta I, has given the data with numerical values, the spherical shape , diameter, rotation and, declination, of earth. The sine and cosine values and their tables, the method to find out square root and cube root, and also many novel methods, facts and the concepts.

We Indians should learn first, then submit those information to the world class scholars and prove that we deserve the credit of many discoveries which we are due !

Indian Institute of Scientific Heritage is spreading the message of our scientists and great ancient Indian scholars who have contributed in all the field of knowledge. The most authentic translation of Sri. K.S Sukla and K.V. Sarma is taken as the basis of this book and wherever possible simple commentary has been given.

The aim of publishing this book is to spread the hitherto unknown message, of our motherland’s heritage, to millions of Indians and also those who love and respect India !

Indian Institute of Scientific Heritage submits this book to you, on 2003, March 21, the day this great book (Aryabhateeya) is celebrating its 1504 th birth day. We request you , to Spread the message of the great Indians ! World over people are looking to India for knowledge ! Let us know scientifically our heritage and inform others with the true spirit and vision of modern science !

Dr. N. Gopalakrishnan
21 March 2003

Chapter I

1. Having paid obeisance to God Brahma – who is one and many the real God, the Supreme Brahman – Aryabhata sets forth the three, viz., mathematics (ganita), reckoning of time (kalakriya) and celestial sphere (gola).

In the first line , like all other astronomers and mathematicians of ancient India, the author of the book Aryabhateeya does pranams to the divine power and tells that he is going to write the book on the celestial spheres. This was the practice followed by all the ancient Indian scientists.

2. The varga letters (ka to ma) should be written in the varga places and the avarga letters (ya to ha) in the avarga places. (The varga letters take the numerical values 1, 2. 3 etc.) from ka onwards; (the numerical value of the initial avarga letters) ya (30) is equal to nga (5) plus ma (25) (i.e., 5+25). In the places of the two nines of zeros (which are written to denote the notational places), the nine vowels should be written (one vowel in each pair of the varga and avarga places). In the varga (and avarga) places beyond (the places denoted by) the nine vowels too (assumed vowels or other symbols should be written, if necessary).

During the period of Aryabhatta, (5^{th} century AD) there were three methods for writing the numbers. The most commonly used system was Sanskrit number system. The second method was writing through bhootha sankhya. This number system was used by Varahamihira, Bhaskaracharya I and II and many other ancient Indian mathematicians. However, Aryabhatta created his own number system and made a new contribution to the number systems. It can be seen that for the commentary of Aryabhateeya other mathematicians have used this number, but they have not used Aryabhateeya number systems for their original contribution in the subject

The meaning of the above stanza is that the Sanskrit letters ka to ma carry values from 1 to 25. From ya to ha , the values are in the order 30, 40, … 80. Whenever I (I-kaara) is used, the value for the number becomes the multiplication of 100, when u (u-kaara) is used the value becomes multiplication product of 10000, when ru is used the value becomes multiplication of 1000000. And the number will be the addition of all the number values used. Say cha = 6; chi 600; chu 60000 and chru 6000000 and so on. When kuchi is written it is ku + chi = 10000 + 600 = 10600. Thus very large numbers can be written using the Aryabhateeya number system.

3-4 In a yuga, the eastward revolutions of the Sun, are 43,20,000; of the Moon, 5,77,53,336 ; of the Earth, 1,58,22,37,500 ; of Saturn, 1,46,564; of Jupiter, 3,64,224 ; of Mars, 22,96,824; of Mercury and Venus, the same as those of the Sun ; of the Moon's apogee, 4,88,219 ; of (the sighrocca of) Mercury, 1,79,37,020; of (the sighrocca of Venus, 70,22,388 ; of (the sighrochas of) the other planets, the same as those of the Sun; of the moon's ascending node in the opposite direction (i.e., westward), 2,32,226. These revolutions commenced at the beginning of the sign Aries on Wednesday at sunrise at Lanka (when it was the commencement of the current yuga).

One yuga is 4320000 years . During that period the revolutions of the Sun and all other planets and the moon are given, in the above lines. Interestingly the numerical values given for Earth is not for its revolution but for the rotation. This is obvious from the value available when the above number is divided by 4320000 we get 366.25868, the number of sidereal days in a year. The rotations of apogee and perigee given for the planets are the number of rotations taking place for the imaginary point of apogee and perigee of those planets. The number of rotation given for the Sun is the number for the revolution of earth and not that of the Sun , actually.

5. A day of Brahma (or a Kalpa) is equal to (a period of) 14 Manus, and (the period of one) Manu is equal to 72 yugas. Since Thursday, the beginning of the current Kalpa, 6 Manus, 27 yugas and 3 quarter yugas had elapsed before the beginning of the current Kaliyuga (lit. before Bharata - before Kurukshetra war).

A kalpa is one day of Brahma which is equal to 14 manvantharas. Each manvanthara is composed of 72 Mahayugas. Each Mahayuga is composed of 4 yugas which are krutha- thretha- dvapara and Kaliyuga. The total period of these four yugas is equal to 4320000 years. Given here is the present period , which is in the 7th Manvanthara known as Vaivaswatha manvanthara and on the 28th Mahayuga . in this Mahayuga after krutha , thretha and dvaaparayuga, we are in the kaliyuga which was started during 3102 , Feb. 17th Thursday BC. In fact Aryabhatta wanted to inform that , when the Kurukshetra war took place, 6 Manvantharas and 27 Mahayugas and three parts of the 28th Mahayugas were elapsed, since the beginning of this Kalpa.

6. Reduce the Moon's revolutions (in a yuga) to signs, multiplying them by 12 (lit. using the fact that there are 12 signs in a circle or revolution). Those signs multiplied successively by 30, 60 and 10 yield degrees, minutes and yojanas, respectively. (These yojanas give the length of the circumference of the sky). The Earth rotates through (an angle of) one minute of arc in on respiration (= 4 sidereal seconds). The circumference of the sky divided by the revolutions of a planet in a yuga gives (the length of) the orbit on which the planet moves. The orbit of the asterisms divided by 60 gives the orbit of the Sun.

Given here is an explanation for the angular measurement of celestial sphere, in signs, angular values, which in the last line is equated with linear measurement. The rotation speed of earth is clearly and correctly given by Aryabhatta in this line.

7. 8000 nr make a yojana. The diameter of the Earth is 1050 yojanas; of the Sun and the Moon, 4410 and 315 yojanas, (respectively) ; of Meru, 1 yojana; of Venus, Jupiter, Mercury, Saturn and Mars (at the Moon's mean distance), one-fifth, one-tenth, one-fifteenth, one-twentieth, and one-twenty fifth, (respectively), of the Moon's diameter. The years (used in this work) are solar years.

One nara (nr) is also equal to the Vedic measurement of one purusha unit. It is equal to the average height of a man. 8000 times the height of the nara is equal to one yojana and using this linear measurement unit, he gave the diameters of the Sun, moon and other planets. However except for Earth, the values are not correct. For earth it is almost the actual values. Remember here that, the spherical shape of earth was discovered by Aryabhata and the credit is attributed to European scientists. But Aryabhata has given the diameter of the earth 1000 years before these European scientists !

8. The greatest declination of the Sun is 24^o . The greatest celestial latitude (lit. deviation from the ecliptic) of the Moon is 4 ½, of Saturn, Jupiter and Mars, 2^o, 1^o and 1 $½^o$ respectively; and of Mercury and Venus (each), 2^o . 96 angulas or 4 cubits make once nr.

The greatest declination according to modern calculation is 23 1/2 for the earth. Also given here is the declination for other planets. The linear measurements and also the angular equivalent to that are given in the above stanza. Except for Mercury the declination for other planets agree very well with the modern values.

9. The ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn having moved to 20^o, 60^o, 40^o, 80^o and 100^o respectively (from the beginning of the sign Aries) (occupy those positions; 3 and the apogees of the Sun and the same planes (viz., Mercury, Venus, Mars. Jupiter and Saturn) having moved to 78^o,

210^{o}, 90^{o}, 118^{o}, 180o and 236 respectively (from the beginning of the sign Aries) (occupy those positions).

The ascending nodes, of the planets described here by Aryabhatta, are for the year 499 AD, the year during when he has completed the book Aryabhateeya. The same is true for the apogee of the planets also. Aryabhatta's values for ascending nodes agree with modern calculations to a great extend except for Mercury. Significant variations for the apogee can be seen for Venus and Mercury , when compared with the modern measurements .

10. The manda epicycles of the Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn (in the first and third anomalistic quadrants) are respectively, 7, 3, 7, 4, 14, 7 and 9 (degrees) each multiplied by 4½ (i.e., 31.5, 13.5, 31.5, 18, 63, 31.5 and 40.5 degrees, respectively); the sighra epicycles of Saturn, Jupiter, Mars, Venus and Mercury (in the first and third anomalistic quadrants) are, respectively, 9, 16, 53, 59 and 31 (degrees) each multiplied by 4½ (i.e., 40.5, 72, 238.5, 265.5 and 139.5 degrees, respectively).

11. The manda epicycles of the retrograding planets (viz., Mercury, Venus, Mars, Jupiter and Saturn) in the second and fourth anomalistic quadrants are, respectively, 5, 2, 18, 8 and 13 (degree) each multiplied by 4½ (i.e., 22.5, 9, 81, 36 and 58.5 degrees, respectively; and the sighra epicycles of Saturn, Jupiter, Mars, Venus and Mercury (in the second and fourth anomalistic quadrants) are, respectively, 8, 15, 51, 57 and 29 (degree) each multiplied by 4½ (i.e., 36, 67.5. 229.5., 256.5 and 130.5 degrees, respectively). 3375 is the outermost circumference of the terrestrial wind.

12. 12. 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22 and 7 – these are the Rsine-differences (at intervals of 225 minutes of arc) in terms of minutes of arc.

The above given are the Rsine differences for the angles falling at interval of 225' (3^{o} 45'). The values given here is as 225', 450', 675'....in that order and respective Rsine differences. The modern values and the Aryabhatta's table agree with the accuracy level of first decimal place. Here for the first time Aryabhatta has given the Sine values for angular measurements falling at specific intervals. They are in full agreement with the modern calculated values.

13. Here Aryabhatta is describing the importance of knowing this Dasagitika-sutra, which are the basic principles and concepts giving the motion of the earth and the planets, on the Celestial sphere (Sphere of asterisms of Bhagola). According to him one attains the Supreme Brahma after piercing through the orbits of the planets and stars.

In fact Aryabhatta has mentioned here the very fundamental and the most important information on astronomy very systematically. They are all universally facts/truths. Every Indian scholar aims at attaining the ultimate point of realization of the Brahma chaitanya through their on pathway of attaining the knowledge. Hence Aryabhatta used the words “piercing through the orbits of celestial bodies ….”

Chapter II

1. Having bowed with reverence to Brahma, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn and the asterisms, Aryabhata sets forth here the knowledge honored at Kusumapura.

Every mathematician and astronomer of ancient India use to do pranams to the celestial bodies which are part of the prapancha purusha , the universal person concept of India. Here too as in the first chapter Aryabhata has written the invocation lines to the Indian trinity concept and also the celestial bodies. Kusumapura is said to be the ancient Pataleeputra (present Patna) or Kodungallur in the Trissur district of Keralam)

2. Eka (units place), dasa (tens place), sata (hundred place), sahasra (thousands place), ayuta (ten thousands place), niyuta (hundred thousands place), prayuta (millions place) koti (ten millions place), arbuda (hundred millions place), and nyarbuda (thousand millions place) are, respectively, from place to place, each ten times are preceding.

The first ten notational places are given here as mentioned in Yajurveda also. Many astronomers of ancient India have given this system of numbering. Aryabhatta has specifically mentioned the decimal places of numbers.

Here information connected with mathematical and geometrical knowledge are described. As the subjects are connected with astronomy they are inevitable component of the Jyothisha. In the fifth vedanga, (the jyothisha) both astronomy and ganitha/mathematics are included.

3. (a-b) An equilateral quadrilateral with equal diagonals and also the area thereof are called ‘square’. The product of two equal quantities is also ‘square’

Definitions of square and squaring are given here very specifically. Modern definition is in perfect agreement with this definition given 15 centuries ago.

3. (c-d) The continued product of three equals as also the (rectangular) solid having twelve (equal) edges are called a ‘cube’.

The definition of cube and cubing is also given as for squaring. Here too it is in agreement with the modern approach.

4. (Having subtracted the greatest possible square from the last odd place and then having written down the square root of the number subtracted in the line of square root) always divide the even place (standing on the right) by twice the square root. Then, having subtracted the square (of the quotient) from the odd place (standing on the right), set down the quotient at the next place (i.e., on the right of the number already written in the line of the square root). This is the square root. (Repeat the process if there are still digits on the right)

According all the available information , Aryabhatta is the first mathematician who described the method for calculating the square root of a large number. Of course the same for small numbers like 2 and 3 are described in Sulbasutras. Aryabhatta's method is correct. It has been falsely described that Chinese have described the method for determining the square root, before Aryabhatta I. However with proof it can be submitted that till 11^{th} century Chinese have not arrived at any method for determining the square root and later also they used only Aryabhatta's method

5. (Having subtracted the greatest possible cube from the last cube place and then having written down the cube root of the number subtracted in the line of the cube root), divide the second non-cube place (standing on the right of the last cube place) by thrice the square of the cube root (already obtained); (then) subtract from the first non-cube place (standing on the right of the second non-cube place) the square of the quotient multiplied by thrice the previous (cube root); and (then subtract)the cube (of the quotient) from the cube place (standing on the right of the first non-cube place) (and write down the quotient on the right of the previous cube root in the line of the cube root, and treat this as the new cub root. Repeat the process if there are still digits on the right).

Aryabhatta has not only mentioned the method for square root, but also have described the method for cube root also. The method, even though appears to be complicated, by practice it can be seen that, this method is very easy.

6. The area of a triangle is the product of half the base and altitude. (In modern geometry, this equation is very well known)

6. (c-d) Half the product of that area (of the triangular base) and the height is the volume of as six-edged solid.

Volume of Right Pyramid is given here, which is not correct. However the correct formula is given by Brahma Gupta in his work Brahmasphuta siddhanta.

7.(a-b) Half of the circumference, multiplied by the semi-diameter certainly gives the area of a circle.

The area of a circle is the product of half of the circumference which is $2\pi r / 2$ which is equal to πr; this when multiplied with half of the diameter (= r) gives the correct value as πr^2

7.(c-d) That area multiplied by its own square root gives the exact volume of a sphere

However this explanation of finding out the volume of the sphere is not correct. The correct value for determining the volume of the sphere was given by Bhaskaracharya II in 1148 AD

8. Area of a Trapezium will be obtained by (Severally) multiplying the base and the face (of the trapezium) by the height, and divide (each product) by the sum of the base and the face : the results are the lengths of the perpendiculars on the base and the face (from the point of intersection of the diagonals). The results obtained by multiplying half the sum of the base and the face by the height is to be known as the area (of the trapezium).

Here the area of the trapezium given by Aryabhatta is correct. In fact it appears that the method for determining the area of a trapezium was first discovered by him.

9. (a-b) In the case of all the plane figures, one should determine the adjacent sides (of the rectangle into which that figure can be transformed) and find the area by taking their product.

Area of plane figures can be determined by making them into smaller figures (whose area can be easily determined) like triangle, square, rectangle etc. then the areas are added and finally the area of the figure will be obtained.

9. (c-d) The chord of one-sixth of the circumference (of a circle) is equal to the radius.

Infact in this line Aryabhatta is giving a theorem that “a chord of one-sixth circle has the length of the radius of the circle”.

10. 100 Plus 4, multiplied by 8, and added to 62,000 : this is the nearly approximate measure of the circumference of a circle whose diameter is 20,000.

Indirectly in this line the value of π is given through the dimension of the circumference of a circle having diameter 20,000 unit, as 62832 . The circumference-diameter ratio will directly give the approximate value for π as 62832/ 20,000 (Aryabhatta has specifically mentioned that the circumference will be approximate (using the Sanskrit word *Asanno*), hence the value of π obtained will also be approximate.

11. Divide a quadrant of the circumference of a circle (into as many parts as desired). Then from (right) triangles and quadrilaterals, one can find as many Rsines of equal arcs as one likes, for any given radius.

Aryabhatta is giving here the method for determining the Rsine value of angles from the table which he has given in the first chapter. Through geometrical methods by submitting the proof.

12. The first Rsine divided by itself and then diminished by the quotient gives the second Rsine-difference. The same first Rsine diminished by the quotients obtained by dividing each of the preceding Rsines by the first Rsine gives the remaining Rsine-differences.

Here too the method for the determination of Rsine values for angles falling between the tabular values (given earlier) is mentioned

13. A circle should be constructed by means of a pair of compasses; a triangle and a quadrilateral by means of the two hypotenuses (karma). The level of ground should be tested by means of water; and vertically by means of a plumb.

Method for constructing the circle and quadrilateral is precisely explained

14. Add the square of the height of the gnomon to the square of its shadow. The square root of that sum is the semi-diameter of the circle of shadow.

The shadow circle and their dimension connected with the light lamp is described here and also in the following lines.

15. Multiply the distance between the gnomon and the lamp-post (the latter being regarded as base) by the height of the gnomon and divide (the product) by the difference between (the heights of) the lamp-post (base) and the gnomon. The quotient (thus obtained) should be known as the length of the shadow measured from the foot of the gnomon.

16. (When there are two gnomons of equal height in the same direction from the lamp-post), multiply the distance between the tips of the shadows (of the two gnomons) by the (larger or shorter) shadow and divide by the larger shadow diminished by the shorter one : the result is the upright (i.e., the distance of the tip of the larger or shorter shadow from the foot of the lamp-post). The upright multiplied by the height of the gnomon and divided by the (larger or shorter) shadow gives the base (i.e., the height of the lamp-post).

17. In a right-angled triangle) the square of the base plus the square of the upright is the square of the hypotenuse. In a circle (when a chord divides it into two arcs), the product of the arrows of the two arcs is certainly equal to the square of half the chord.

This is the theorem on square of Hypotenuse and on Square of half-chord

18. (When one circle intersects another circle) multiply the diameters of the two circles each diminished by the erosion, by the erosion and divide (each result) by the sum of the diameters off the two circles after each has been diminished by the erosion: then are obtained the arrows of the arcs)of the two circles) intercepted in each other.

Arrows of intercepted arcs off intersecting circles are connected in the geometrical figure to get various other parameters of two intersecting circles. The relations between the arrow, the diameters and the length of the points of the intersection of the circles were all connected here in this description.

After giving few lines on geometry, Aryabhatta is giving the description on the arithmetical parameters of progression. Sum (or partial sum) of a series In A.P is given here.

19. Diminish the given number of terms by one, then divide by two, then increase by the number of the preceding terms (if any), then multiply by the common difference, and then increase by the first term of the (whole) series : the result is the arithmetic mean (of the given number of terms). This multiplied by the given number of terms is the sum of the given terms. Alternatively, multiply the sum of the first and last terms (of the series or partial series or partial series which is to be summed up by half the number of terms.

20. The number of terms (is obtained as follows) : Multiply (the sum of the series) by eight and by the common difference, increase that by the square of the difference between twice the first term and the common difference, and then take the square root; then subtract twice the first term, then divide by the common difference, then add one (to the quotient), and then divide by two.

Number of terms of a series in an Arithmetic Progression of the type a + (a + d) + (a + 2d) + (a +3 d) + ….to n terms is given here.

21. Of the series (upaciti) which has one for the first term and one for the common difference, take three terms in continuation, of which the first is equal to the given number of terms, and find their continued product. That (product), or the number of terms plus one subtracted from the cube of that, divided by 6 gives the citighana.

Sum of the Series 1 + (1+2) + (1+2+3) +… … to n terms is described above. Variety of the problems connected with this arithmetic progression has been given by Bhaskaracharya I and Sreedharacharya in their books. Aryabhatta I has given only the required method for finding out the sum, without giving examples.

22. The continued product of the three quantities, viz., the number of terms plus one, the same increased by the number of terms, and the number of terms, when divided by 6 gives the sum of the series of squares of natural numbers (vargacitighana). The square of the sum of the series of natural numbers (citi) gives the sum of the series of cubes of natural numbers (ghanacitighana).

Determination of the sum of the series of □ N^2 and □ N^3 is given in the above description.

23. From the square of the sum of the two factors subtract the sum of their squares. One-half of that (difference) should be known as the product of the two factors.

Product of factors from their sum and squares is given in the above explanation, as it is commonly followed in the modern method

24. Multiply the product by four, then add the square of the difference of the two (quantities), and then take the square root. (Set down this square root in

two places). (In one place) increase it by the difference (of the two quantities), and (in the other place) decrease it by the same. The results thus obtained, when divided by two, give the two factors (of the given product).

Method of determining the quantities from their difference and product has been described above. As it is known that (X - Y) is multiplied by itself (squaring) then the value will have a product of X and Y (X x Y) , this can be obtained directly from the other parameters obtained during the calculation

25. Multiply the interest on the principal plus the interest on that interest by the time and by the principal ; (then) add the square of half the principal; (then) take the square root ; (then) subtract half the principal; and (then) divide by the time : the result is the interest on the principal.

A unique method for the determination of interest of the principal amount is described in these lines. This method is different from the method followed for calculating the interest using the formula PNR / 100 = I.

26. In the rule of three, multiply the 'fruit' (phala) by the 'requisition' (iccha) and divide the resulting product by the 'argument' (pramana). Then is obtained the 'fruit' corresponding to the requisition' (icchaphala).

Determination of unknown factors using the rule of three was a common approach followed in ancient times. Here too Aryabhatta has given the phala, and pramana and from that iccha is to be calculated.

27. The numerators and denominators of the multipliers and divisors should be multiplied by one another. multiply the numerator as also the denominator of each fraction by denominator of the other fraction; then the (given) fractions are reduced to a common denominator.

Reduction of two fractions to a common denominator , as in the modern method the LCM is taken and then addition or subtraction of the numerator is followed . Here too, indirectly the same method is adopted

28 In the method of inversion multipliers become divisors and divisors become multipliers, additive becomes subtractive and subtractive becomes additive.

In this method the determination of the unknown number, from a series of processing is done by starting from the result, thus it can be called as the method of inversion.

29. The sums of all combinations of the (unknown) quantities except one (which are given) separately should be added together ; and the sum should be written down separately and divided by the number of (unknown) quantities less one : the quotient thus obtained is certainly the total of all the (unknown) quantities. (This total severally diminished by the given sums gives the various unknown quantities).

Determination of the unknown quantities from sums of all but one, is followed here by a series of steps.

30. Divide the difference between the rupakas with the two persons by the difference between their gulikas. The quotient is the value of one gulika, if the possessions of the two persons are of equal value.

Determination of the unknown quantities from equal sums can determined by this equation as we follow in the algebraic methods. Here the description is given as an example of comparing the value of the gulika and rupaka after equating them. A series of very interesting mathematical exercises are given by many mathematicians based on this explanations

Meeting of two moving bodies are very important problems connected with the travelers and moving celestial bodies. As it is well known that ancient astronomy particularly focussed on the graha sphuta which means calculating the position of every planet/celestial bodies connected with astrology. Here occulting and eclipses are clearly predicted based on the relative motions of the planets and stars. For this calculation the relative motion in the same direction or opposite direction are important. In the following stanza such an explanation is given

31. Divide the distance between the two bodies moving in the opposite directions by the sum of their speeds, and the distance between the two bodies moving in the same direction by the difference of their speeds ; the two quotients will give the time elapsed since the two bodies met or to elapse before they will meet.

32-33. Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder. (Discard the quotient). Divide the remainder obtained (and the divisor) by one another (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the difference of the remainders (corresponding to the greater and smaller divisors; then divide this sum by the last divisor of the mutual division. The optional number is to be so chosen that this division one below the other in a column ; below them write the optional number and underneath it the quotient just obtained. Then reduce the chain of numbers which have been written down one below the other, as follows): Multiply by the last but one number (in the bottom) the number just above it and then add the number just below it (and then d9iscard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number) by the divisor corresponding to the smaller remainder, then multiply the remainder, and then add the greater remainder : the result is the dvicchedagra (i.e., the number answering to the two divisors). (This is also the remainder corresponding to the divisor equal to the product of the two divisors).

The above method and explanations are mainly for coming to the required values from numerator, denominator and remainder obtained after a series of mathematical processing. Thus it can be called as the pulverizing technique for arriving at a result from other known factors.

32-33. Divide the greater number (denoting the divisor) by the smaller number (denoting the dividend) (and by the remainder obtained the smaller number and so on. Dividing the greater and the smaller numbers by the last non-zero remainder of the mutual division, reduce them to their lowest terms.) Divide the resulting numbers mutually (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the (given) additive (or subtract the subtractive). (Divide this sum or difference by the last divisor of the mutual division. The optional number is so chosen that this division is exact. Now place the quotient of the mutual division one below the other in a column ; below them write the optional number and underneath it the quotient just obtained. Then reduce this chain of numbers as follows). Multiply by the last but one number (in the bottom) the number just below it and then add the number just below it (and then discard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number by the abraded greater number and the lower number) by the abraded smaller number. (The remainders thus obtained are the required values of the unknown multiplier and quotient).

The translation is said to be very complicated hence additional information is given within the parenthesis. Here too when the divisor, dividend, subtractive etc are given, to arrive at the unknown figure, the above said explanations can be followed.

Chapter III

Aryabhateeya has been divided into four chapters , in which the kaalakirya paada is the third. As the title means, it gives the measurement of time and processing with time. In the beginning itself the author gives specific definition for the angular and linear dimensions. This is particularly important because many a times, the dimensions mentioned in puranas are taken as reference. To avoid confusion Aryabhata has made it a specific point to create a new number system of his own and clear definition for angular and linear values and also for the time parameters.

1. A year consists of 12 months. A month consists of 30 days. A day consists of 60 nadis. A nadi consists of 60 vinadikas (or vinadis).

This definition is well known which are commonly used through out India, for many centuries till the last three or four decades as nadika and vinadika were also in use.

2. A sidereal vinadika is equal to (the time taken by a man in normal condition in pronouncing) 60 long syllabus (with moderate flow of voice) or (in taking) 6 respirations (pranas). This is the division of time. The division of a circle (lit. the ecliptic) proceeds in a similar manner from the revolution.

Here the definition of the vinakida is given. One nadika is approximately equal to 24 minutes and one vinadika is approximately equal to 24 seconds. Interestingly as in the modern system, the angular measurements in lower units are known as minutes and seconds and same units are used for time measurements also. Similarly the angular measurements and time measurements are nadika and vinadika , this approach appears to be selected first by Aryabhatta I.

3. (a-b) The difference between the revolution-numbers of any two planets is the number of conjunctions of those planets in a yuga.

Given here is the conjunctions of two planets in a Yuga. That is the difference between the number of revolutions of the planets in 432000 years. The number of revolution of each planets (both grahas and thaara grahas are given by Aryabhatta I) in the first chapter.

3. (c-d) The (combined) revolutions of the Sun and the Moon added to themselves is the number of Vyatipatas (in a yuga).

In the above lines explanation for the Vyatipatas in a Yuga is given. It is mentioned that there two types of the phenomenon called Vyateepaata. That Laata vyateepaata and the Vaidhruta Vyateepaata. The Lata Vyateepaata occurs when the sum of the tropical longitude of the Sun and the moon amounts to 180 degrees and the Vaidhurta Vyateepaata occurs when the tropical longitude of the Sun and the moon amounts to 360 degree. It is said that in one combined revolution of the Sun and the moon there occur two Vyatipaataas

4. (a-b) The difference between the revolution-numbers of a planet and its ucca gives the revolutions of the planet's epicycle (in a yuga).

Given here is the number of anomalistic revolutions of any planet, which is the difference between the number of revolution of the planet in one Mahayuga and the number of revolution of the apogee (mandoccha) of that planet in one Mahayuga. If X is the number of revolutions of the planet in 4320000 years and Y is the number of revolutions of the apogee of that planet (which is a very small number) then the anomalistic revolution number is X - Y

According to Aryabhata the period for this anomalistic revolution is calculated as 27 days 13 hrs 18 min 36.6 second where as the modern value is 27 days 13 hrs 18 min and 33.1 seconds !

4. (c-d) The revolution-number of Jupiter multiplied by 12 gives the number of Jovian years beginning with Asvayuk (in a yuga).

The period taken by Jupiter for completing one revolution around the Sun is known as the Jovian year. It is named clearly from the Vedic period itself. In fact through out India the period 12 years is named connecting the Jupiter planet.

5. The revolutions of the Sun are solar years. The conjunctions of the Sun and the Moon are lunar months. The conjunctions of the Sun and Earth are (civil) days. The rotations of the Earth are sidereal days.

The definition for Solar Year and Lunar, Civil and Sidereal Days are given above. Here one can see that Aryabhatta I has clearly given the definition and explanation for the rotation of the earth. All these definitions are true.

6. The lunar months (in a yuga) which are in excess of the solar months (in a yuga) are (known as) the intercalary months in a yuga; and the lunar days (in a yuga) diminished by the civil days (in a yuga) are known as the omitted lunar days in a yuga.

Given here are the facts on Intercalary months and omitted lunar days in one Mahayuga.

7. A solar year is a year of men. Thirty times a year of men is a year of the Manes. Twelve times a year of the Manes is called a divine year (or a year of the gods).

8. 12000 divine years make a general planetary yuga. 1008 (general) planetary yugas make a day of Brahma.

9. The (first) half of a yuga is Utsarpini and the second half Apasarpini. Susama occurs in the middle and Dussama in the beginning and end. (The time elapsed or to elapse is to be reckoned) from the position of the Moon's apogee.

Names of different parts of the yuga are given above as Utsarpini, Apasarpini, Susama and Dussama

10. When sixty times sixty years and three quarter yugas (of the current yuga) had elapsed, twenty three years had then passed since my birth.

It is very important to note that ancient Indian mathematicians and astronomers have specifically given their date of their birth or the date on which the writing of their book was completed. This was given either in Saka era, or in Kali era. Sometimes they give , as a factor for correcting the astronomical parameters or position of the planets , in which the year will be given clearly. Here Aryabhatta has given his age on the day he completed the writing of the book based on the Kali era. Which has been fixed on back calculation as 3102 BC, Feb. 17th midnight on Thursday.

11. The yuga, the year, the month, and the day commenced simultaneously at the beginning of the light half of Caitra. This time, which is without beginning

and end, is measured with the help of the planets and the asterisms on the Celestial Sphere.

Mention is made on the beginning of measuring the time for fixing the starting and ending point. As we now do January 1 as the beginning and December 31st as ending.

12. The planets moving with equal linear velocity in their own orbits complete (a distance equal to) the circumference of the sphere of the asterisms in a period of 60 solar years, and (a distance equal to) the circumference of the sphere of the sky in a yuga.

This explanation appears to be not in agreement with the actual scientific facts.

13. The Moon completes its lowest and smallest orbit in the shortest time ; Saturn completes its highest and largest orbit in the longest time.

Here the comparison is made with the satellite moon and the planet Saturn, However the facts remain the same as the moon takes 27 days for revolving around the earth and the Saturn takes nearly 30 years for revolving around the Sun

14. (The linear measured of) the signs are to be known to be small in small orbits and large in large orbits ; so also (the linear measures of) the degrees, minutes, etc. The circular division is however, the same in the orbits of the various planets.

15 Beneath the asterisms lie (the planets) Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon (one below the other) ; and beneath them all lies the Earth like the hitching peg in the midst of space.

16 The (above-mentioned) seven planets beginning with Saturn, which are arranged in the order of increasing velocity, are the lords of the successive hours. The planets occurring fourth in the order of increasing velocity are the lords of the successive days, which are reckoned from sunrises (at Lanka).

In fact , based on the above explanations the days are named as Sun - day, Moon - day and so on because the lord of the hour of Sunrise is linked with the days name

17 (The mean planets move on their orbits and the true planets on their eccentric circles). All the planets, whether moving on their orbits (kaksya-mandala) or on the eccentric circles (prati-mandala), move with their own (mean) motion, anticlockwise from their apogees and clockwise from their sighroccas.

Here motion of the Planets are explained through eccentric circles

18 The eccentric circle of each of these planets is equal to its own orbit, but the center of the eccentric circle lies at the distance from the center of the solid Earth.

19 The distance between the center of the Earth and the center of the eccentric circle is (equal to) the semi-diameter of the epicycle (of the planet). (c-

d) All the planets undoubtedly move with mean motion on the circumference of the epicycles.

20 A planet when faster than its ucca moves clockwise on the circumference of its epicycle and when slower than its ucca moves anticlockwise on its epicycle.

21. The epicycles move anticlockwise from the apogees and clockwise from the sighroccas. The mean planet lies at the center of its epicycle, which is situated on the (planet's) orbit.

22. (a-b) The corrections from the apogee (for the four anomalistic quadrants) are respectively minus, plus, plus, and minus. Those from the sighrocca are just the reverse.

A Special Pre-correction is given below for the superior planets. This also shows that the knowledge on the position of the planets from the Sun, was clearly understood 15 centuries ago.

22.(c-d) In the case of (the superior planets) Saturn, Jupiter and Mars, first apply the mandaphala negatively or positively (as the case may be).

Procedure of Mandaphala and Sighraphala corrections for Superior Planets, can be taken as a proof on the knowledge for ancient Indians on giving correction for various astronomical parameters. Here superior planets are those which falls outside the circle of the earth during the revolution of the planets.

23. Apply half the mandaphala and half the sighraphala to the planet and to the planet's apogee negatively or positively (as the case may be). The mean planet (then) corrected for the mandaphala (calculated afresh from the new mandakendra) is called the true-mean planet and that (true-mean planet) corrected for the sighraphala (calculated afresh) is known as the true planet.

Mandaphala and Sighraphala Corrections are also given for Inferior Planets. Where the inferior planets are those which revolves within the orbit of the earth's revolution around the Sun. As it is well known that Mercury and Venus are the inferier planets corrections are given for those .

24. (In the case of Mercury and Venus) apply half the sighraphala negatively or positively to the longitude of the planet's apogee (according as the sighrakendra is less than or greater than 180^{o}). From the corrected longitude of the planet's apogee (calculate the mandaphala afresh and apply it to the mean longitude of the planet ; then) are obtained the true-mean longitude of Mercury and Venus. The sighraphala, calculated afresh, being applied to them), they become true (longitudes).

25. The product of the mandakarna and the sighrakarna when divided by the radius gives the distance between the Earth and the planet.

The velocity of the (true) planet moving on the (sighra) epicycle is the same as the velocity of the (true-mean) planet moving in its orbit (of radius equal to the mandakarna).

Chapter IV

CELESTIAL SPHERES (GOLA)

Detailed explanations on the celestial bodies are given in this chapter. In every astronomical and mathematical writings of ancient India, major part of the subject matter will be on the Gola. Many scientific astronomical parameters are explained under this title.

1. One half of the ecliptic, running from the beginning of the sign Aries to the end of the sign Virgo, lies obliquely inclined (to the equator) northwards. The remaining half (of the ecliptic) running from the beginning of the sign Libra to the end of the sign Pisces, lies (equally inclined to the equator) southwards.

2. The nodes of the star-planets (Mars, Mercury, Jupiter, Venus and Saturn) and of the Moon incessantly move on the ecliptic. So also does the sun. From the Sun, at a distance of half a circle, moves thereon the Shadow of the Earth.

Star planets are those which are really planets, whereas the moon and the Sun are not included in the above list. This is the difference between the ancient astrology and astronomy. In fact the word graha might have used in astrology with the meaning « that is influencing». The specific knowledge on the real planets and their difference between the moon and the Sun, etc were at the finger tips of the ancient Indian astronomers.

2. The Moon moves to the north and to the south of the ecliptic (respectively) from its (ascending and descending) nodes. So also do the planets Mars, Jupiter and Saturn. Similar is also the motion of the sighroccas of Mercury and Venus.

3. When the Moon has no latitude it is visible when situated at a distance of 12 degrees (of time) from the Sun. Venus is visible when 9 degrees (of time) distant from the Sun. The other planets taken in the order of decreasing sizes viz., Jupiter, Mercury, Saturn, and Mars) are visible when they are 9 degrees (of time) increased by two-s (i.e., when they are 11, 13, 15 and 17 degrees of time) distant from the Sun.

Here one degree can be taken as equivalent to 4 minutes of time, as it is the time taken by the earth for one degree rotation in its own axis. Here the degree separation for each planets has been correlated indirectly with the brightness of the planets also, for the visibility when compared with the brightness of the Sun.

4. Halves of the globes of the Earth, the planets and the stars are dark due to their own shadows; the other halves facing the sun are bright in proportion to their sizes.

6. The globe of the Earth stands (supportless) in space at the center of the circular frame of the asterisms (i.e., at the center of the Bhagola) surrounded by the orbits (of the planets) ; it is made up of water, earth, fire and air and is spherical (lit. circular on all sides).

With perfect clarity Aryabhatta gives the shape of the earth again and also on the orbits of rotation of the planets. Its composition is also given in the definition itself and also the shape. (note specifically the word Bhoogola)

7. Just as the bulb of a Kadamba flower is covered all around by blossoms, just so is the globe of the Earth surrounded by all creatures, terrestrial as well as aquatic.

8. During a day of Brahma, the size of the Earth increases externally by one yojana; and during a night of Brahma, which is as long as a day, this growth of the earth is destroyed.

9. Just as a man in a boat moving forward sees the stationary objects (on either side of the river) as moving backward, just so are the stationary stars seen by people at Lanka (on the equator), as moving exactly towards the west.

Here with specific example Aryabhata has given the Apparent Motion of the Stars due to the Earth's rotation. The example stands unique in its scientific narration methodology.

10. (It so appears as if) the entire structure of the asterisms together with the planets were moving exactly towards the west of Lanka, being constantly driven by the pro-vector wind, to cause their rising and setting.

11. The Meru (mountain) is exactly one yojana (in height). It is light-producing, surrounded by the Himavat mountain, situated in the middle of the Nandana forest, made of jewels, and cylindrical in shape.

Here the Meru may be the arctic ocean or circle where throughout the year shining sun rays reflect. The people, are the Eskimos.

12. The heaven and the Meru mountain are at the center of the land (i.e., at the north pole); the hell and the Badavamukha are at the center of the water (i.e., at the south pole). The gods (residing at the Meru mountain) and the demons (residing at the Badavamukha) consider themselves positively and permanently below each other.

The Meru is taken as the north pole and the Bhadavamukha as the south pole. The facts agree with explanation given by Araybhatta. I personally feel that the penguins in Southpole Antarctica may be termed as demons as they are looking like that. In Hindu Puranas the Gods are dwarf, it is so in the case of the Eskimos. Hence, even though nothing can be told conclusively, the fact remains that the explanations agree to a great extend.

13. When it is sunrise at Lanka, it is sunset at Siddhapura, midday at Yavakoti, and midnight at Romaka.

The explanation on the four cardinal cities on the four quadrants of earth gives another set of perfect proof for the spherical shape of earth. It appears that Siddhapura is the Gautimaala, Yavakoti as Korea Romaka desa as Rome and Lanka either equator or Lanka itself.

14. From the centers of the land and the water, at a distance of one quarter of the Earth's circumference, lies Lanka; and from Lanka, at a distance of one-fourth thereof, exactly northwards, lies Ujjaini.

The latitude given by Aryabhatta for Ujjaini is in full agreement with the modern knowledge which is 23.5 degree.

15. One half of the Bhagola as diminished by the Earth's semi -diameter is visible from a level place (free from any obstructions). The other one-half as increased by the Earth's semi-diameter remains hidden by the Earth.

16. The gods living in the north at the Meru mountain (i.e., at the north pole) see one half of the Bhagola as revolving from left to right (or clockwise); the demons living in the south at the Badavamukha (i.e., at the south pole), on the other land, see the other half as revolving from right to left (or anti-clockwise).

The geographical explanation given is in full agreement with the scientific knowledge on Arctic and Antarctic ocean.

17. The gods see the Sun, after it has risen, for half a solar year; so is done by the demons too. The manes living on (the other side of) the Moon see the Sun for half a lunar month; the men here see it for half a civil day.

Here Gods are those who are in North pole, manes are those who reside in Antarctica and men are people like us live neither of the poles of the earth

Given below are a series of definitions for the geographical and astronomical parameters, based on which all the mathematical calculations were arrived at.

18. The vertical circle which passes through the east and west points is the prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above circles (like a girdle) and on which the stars rise and set is horizon.

19. The circle which passes through the east and west points and meets (the meridian above the north point and below the south point) at distances equal to the latitude (of the place) from the horizon is the equatorial horizon (or six o' clock circle) on which the decrease and increase of the day and night are measured.

20. The east-west line, the nadir-zenith line, and the north-south line intersect where the observer is.

21. The great circle which is vertical in relation to the observer and passes through the planet is the drnmandala (i.e., the vertical circle through the planet). The vertical circle which passes through that point of the ecliptic which is three signs behind the rising point of the ecliptic is the drkksepavrtta.

22. The sphere (Gola-yantra) which is made of wood, perfectly spherical, uniformly dense all round but light (in weight) should be made to rotate keeping pace with time with the help of mercury, oil and water by the application of one's own intellect.

Here again the spherical shape declination, rotation, etc. of earth are clarified by making a small model of the earth as we do now in making the globe.

Below, Aryabhatta I is discussing a series of parameters by focussing on the mathematical method of calculating those facts. Very complicated methodologies are adopted, including the use of Sine and cosine values of angles connected with various astronomical parameters.

23. Divide half of the Bhagola lying in the visible half of the Khagola by means of Rsines (so as to form latitude-triangles). The Rsine of the latitude is the base of a latitude-triangle. The Rsine of the latitude is the upright of the same (triangle).

24. Subtract the square of the given declination from the square of the radius, and take the square root of the difference. The result is the radius of the day circle, whether the heavenly body is towards the north or towards the south of equator.

25. Multiply the day radius corresponding to the greatest declination (on the ecliptic) by the desired Rsine (of one, two or three signs) and divide by the corresponding day radius: the result is the Rsine of the right ascension (of one, tow or three signs), measured from the first point of Aries along the equator.

26. The Rsine of latitude multiplied by the Rsine of the given declination and divided by the Rsine of latitude gives the earthsine, lying in the plane of the day circle. This is also equal to the Rsine of the half the excess of defect of the days or night (in the plane of the day circle).

27. The First as well as the last quadrant of the ecliptic rises (above the local horizon) in one quarter of a sidereal day diminished by (the ghatis of) the ascensional difference. The other two (viz. the second and third quadrants) rise in one quarter of a sidereal day as increased by the same (i.e. the ghatis of the ascensional difference). The times of rising of the individual signs (Aries, Taurus and Gemini) in the first quadrant are obtained by subtracting their ascensional differences from their right ascensions in the serial order; in the second quadrant by adding the ascensional differences of the same signs to the corresponding right ascensions in the reverse order. The times of risings of the six signs in the first and second quadrants (Aries, etc) taken in the reverse order give the risings of the six signs in the third and fourth quadrants (Libra, etc.).

28. Find the Rsine of the are of the day circle form the horizon (up to the point occupied by the heavenly body) at the given time; multiply that by the Rsine of the co-latitude and divide by the radius: the result is the Rsine of the altitude (of the heavenly body) at the given time clasped since sunrise in the forenoon or to elapse before sunset in the afternoon.

29. Multiply the Rsine of the Sun's altitude for the given time by the Rsine of latitude and divide by the Rsine of co-latitude: the result is the Sun's sankvagra, which is always to the south of the Sun's rising-setting line.

30. Multiply the Rsine of the (Sun's tropical) longitude for the given time by the Rsine of the Sun's greatest declination and then divide by the Rsine of colatitude : the resulting Rsine is the Sun's agra on the eastern or western horizon.

31. When that (agra) is less than the Rsine of the latitude and the Sun is in the northern hemisphere, multiply that (Sun's agra) by the Rsine of colatitude and divide by the Rsine of latitude: the result is the Rsine of the Sun's altitude when the Sun is on the prime vertical.

32. The Rsine of the degrees of the (Sun's) altitude above the horizon (at midday when the Sun is on the Meridian) is the greatest gnomon (on that day). The Rsine of the (Sun's) zenith distance (at that time) is the shadow of the same gnomon.

33. Divide the product of the Madhyajya and the udayajya by the radius. The square root of the difference between the squares of that (result) and the madhyajya is the (Sun's or Moon's) own drkksepa.

34. (i) The square root of the difference between the squares of (i) the Rsine of the zenith distance (of the Sun or Moon) and (ii) the drkkshepajya, is the (Sun's or Moon's) own drggatijya.

34. (ii) On account of (the sphericity of) the Earth, parallax increases from zero at the zenith to the maximum value equal to the Earth's semi-diameter (as measured in the spheres of the Sun and the Moon) at the horizon.

35. Multiply the Rsine of the latitude of the local place by the Moon's latitude and divide (the resulting product) by the Rsine of the colatitude : (the result is the aksadrkkarma) for the Moon). When the Moon is to the north (of the ecliptic), it should be subtracted from the Moon's longitude in the case of the rising of the Moon and added to the Moon's longitude in the case of the setting of the Moon; when the Moon is to the south (of the ecliptic), it should be added to the Moon's longitude (in the case of the rising of the Moon) and subtracted from the Moon's longitude (in the case of the setting of the Moon).

36. Multiply the Reversed sine of the Moon's (tropical) longitude (as increased by three signs) by the Moon's latitude and also by the (Rsine of the Sun's) greatest declination and divide (the resulting product) by the square of the radius. When the Moon's latitude is north, it should be subtracted from or added to the Moon's longitude, according as the Moon's ayana is north or south (i.e., according as the Moon is in the six signs beginning with the tropical

sign Capricorn or in those beginning with the tropical sign Cancer) ; when the Moon's latitude is south, it should be added or subtracted, (respectively).

37. The Moon is water, the Sun is fire, the Earth is earth and what is called Shadow is darkness (caused by the Earth's Shadow). The Moon eclipses the Sun and the great Shadow of the Earth eclipses the Moon.

Here a clear picture on the shadow as the area where light is dim or absent is given. It is nothing but the darkness. Hence an excellent definition on the eclipse is given in this line itself.

38. When at the end of a lunar month, the Moon, lying near a node (of the Moon), enters the Sun, or at the end of a lunar fortnight, enters the Earth's Shadow, it is more or less the middle of an eclipse, (solar eclipse in the former case and lunar eclipse in the latter case.

The best explanation for the solar and lunar eclipse and the period during when this can occur are also given scientifically. What is mentioned in Puranic stories are generally taken and Indians are labeled as superstitious. However none of the thousands of astronomical books written in ancient India gives the explanation of the so called serpent Rahu as the cause of eclipse.. All the astronomical books follow Aryabhatta's type of explanation only.

Further in the next line, other parameters connected with the shadow, diameter of the Sun, moon and earth, are given correctly.

39. Multiply the distance of the Sun from the Earth by the diameter of the Earth and divide (the product) by the difference between the diameters of the Sun and the Earth: the result is the length of the Shadow of the Earth (i.e., the distance of the vertex of the Earth's shadow) from the diameter of the Earth (i.e., from the center of the Earth).

This method also agrees systematically based on the simple calculations adopted for the eclipses. That is the diameters of shadow, moon and the Sun. Given below is the Earth's Shadow at the Moon's Distance

40. Multiply the difference between the length of the Earth's shadow and the distance of the Moon by the Earth's diameter and divide (the product) by the length of the Earth's shadow : the result is the diameter of the Tamas (i.e., the diameter of the Earth's shadow at the Moon's distance).

This can be proved correct by simple geometrical method. Further calculation on half Duration of a Lunar Eclipse is also given below.

41. From the square of half the sum of the diameters of that (Tamas) and the Moon, subtract the square of the Moon's latitude, and (then) take the square root of the difference : the result is known as half the duration of the eclipse (in terms of minutes of arc). The corresponding time (in ghatis etc.) is obtained with the help of the daily motions of the Sun and the Moon.

While giving various parameters connected with the lunar and solar eclipses, even minute mathematical aspects are derived systematically. One can see the method for the calculation of the half-duration of totality of a lunar eclipse

42. Subtract the semi-diameter of the Moon from the semi-diameter of that Tamas and find the square of that different. Diminish that by the square of the (Moon's) latitude and then take the square root of that : the square root (thus obtained) is half the duration of totality of the eclipse as follows.

While the above line gives the eclipsed part of Sun or Moon and the next stanza is devoted for the part which is not affected by the shadow/ eclipse. (The part of the moon not eclipsed.) Followed by this explanation, the time parameters related with the eclipse are given. The contact time and releasing time of eclipse are also discussed with the support of calculations.

43. Subtract the Moon's semi-diameter from the semi-diameter of the Tamas; then subtract whatever is obtained from the Moon's latitude : the result is the part of the Moon not eclipsed (by the Tamas).

44. For getting the measure of the Eclipse at the given time Subtract the ista from the semi-duration of the eclipse; to (the square of) that (difference) add the square of the Moon's latitude (at the given time) ; and take the square root of this sum. Subtract that (square root) from the sum of the semi-diameters of the Tamas and the Moon: the remainder (thus obtained) is the measure of the eclipse at the given time.

45. (a-b) Multiply the Reversed sine of the hour angle (east or west) by the (the Risen of) the latitude, and divide by the radius : the result is the aksavalana. Its direction (towards the east of the body in the afternoon and towards the west of the body in the forenoon) is south. (In the contrary case, it is north).

46. (c-d) Marking of the semi-duration of the eclipse, calculate the longitude of the Sun or Moon (whichever is eclipsed for the time of the first contact. Increase that longitude by three sign and (multiplying the Rversed sine thereof by the Rsine of the Sun's greatest declination and dividing by the radius) calculate the Rsine of the corresponding declination : this is the ayanavalana (or krantivalana) for the time of the first contact.

(Its direction in the eastern side of the eclipsed body is the same as that of ayana of the eclipsed body ; in the western side it is contrary to that).

47. Color of the Moon During Eclipse: at the beginning and end of its eclipse, the Moon (i.e., the obscured part of the Moon) is smoky; when half obscured, it is black; when (just) totally obscured, (i.e., at immersion or emersion), it is tawny; when far inside the Shadow, it is copper-colored with blackish tinge.

48. When the discs of the Sun and the Moon come into contact , a solar eclipse should not be predicted when it amounts to one-eighth of the Sun's diameter (or less) (as it may not be visible to the naked eye) on account of the brilliancy of the Sun and the transparency of the Moon.

Due to the brightness of the Sun, when a very small part of the moon gets eclipsed, it cannot be seen, because of the former phenomenon.

However the minimum level of shadowing for which the prediction of the eclipse can be made clearly is described in the above line.

In the final line, Aryabhatta bows his head before the creator for giving him the knowledge and experience to inform the world on the truth of the universe, and those related with the celestial bodies hence the acknowledgement to Brahma is included in the end of the book..

49. By the grace of Brahma, the precious jewel of excellent knowledge (of astronomy) has been brought out by me by means of the boat of my intellect from the sea of true and false knowledge by diving deep into it.

In the conclusion Aryabhata says that the knowledge existed in the world. However, he has only reproduced it. It is also specifically instructed that none should imitate it. This gives an addition push to conduct the research and include more data , so that refinement is possible. It is the Indian spiritual way of protecting the copy right of the book

50. This work, Aryabhatiya by name, is the same as the ancient svayambhuva (which was revealed by Svayambhu) and as such it is true for all times. One who imitates it or finds fault with it shall lose his good deeds and longevity.

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