Our brilliant masters project final report

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Abstract

In this report, we describe our final project ....

1 Current Progress

1.1 Local Vol models

The Local Vol model I am using is the parameterized SVI model provided by professor Gatheral. $\sigma_{SVI}(k,t)$ where k is the log strike and t is the current time. I modified it to take S_t and t to be used in the Monte-Carlo local vol pricer. $\sigma_{SVI}(k = log(\frac{S_t}{S_0}), t)$

1.2 Exotic Volga

Exotic Volga is computed in this way

$$P(\sigma_{KT} - \delta\sigma) - 2P(\sigma_{KT}) + P(\sigma_{KT} + x_{KT}^{Volga})$$

$P(\sigma_{KT} - \delta\sigma)$ means the price of an exotic option using a local vol surface with a constant shift.
$P(\sigma_{KT})$ means the price of an exotic under a local vol surface.
$P(\sigma_{KT} + x_{KT}^{Volga})$ means the price of an exotic under a local vol surface with each $(S_t = K, t = T)$ has a different shift x_{KT}^{Volga}

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Average Exotic Greeks across K,T	Exotic Volga	Exotic Vanna
Vanilla Call	-1.8113e-05	-0.002063
Down and Out Call	0.0107	0.0590

Table 1: exotic greeks

x_{KT}^{Volga} is obtained by solving the following equation for each strike price K and each time to maturity T.

$$0 = C(\sigma_{KT} - \delta\sigma) - 2C(\sigma_{KT}) + C(\sigma_{KT} + x_{KT}^{Volga})$$

$\Longrightarrow$

$$\sigma_{BS} + x_{KT}^{Volga} \approx C^{-1}(-C(\sigma_{KT} - \delta\sigma) + 2C(\sigma_{KT}))$$

where σ_{BS} is the Black-Scholes implied vol when strike is K and time to maturity is T for the vanilla call options , using the local vol surface we have.
C^{-1} is the Black-Scholes implied vol solver.
$C(\sigma_{KT} - \delta\sigma)$ is the price of a vanilla call with strike K and time to maturity T, under the local vol surface we have with a constand drift $-\delta\sigma$.
$C(\sigma_{KT})$ is the price of a vanilla call with strike K and time to maturity T, under the local vol surface we have.

1.3 Current result

With $S_0 = 1$, log strikes $k = log(K/S_0) \in (-0.6, 0.2)$, time to maturity $T \in (0, 1)$. The average exotic greeks are concluded in the table 1

References

[1] Gatheral, J., The Volatility Surface: A Practitioner's Guide, Wiley Finance (2006).

[2] Gatheral, J., Hsu, E.P., Laurence, P., Ouyang, C., and Wang, T.-H., Asymptotics of implied volatility in local volatility models, *Mathematical Finance* (2011) forthcoming.